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Optimal Pricing Strategy and Government Consumption Subsidy Policy in Closed-Loop Supply Chain with Third-Party Remanufacturer

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Abstract: Due to the increasing awareness of sustainable manufacturing, remanufacturing has been widely accepted by enterprises in many countries. In the process of Closed-Loop Supply Chain (CLSC) development, to stimulate the demand for remanufactured products, the Chinese government's interventions such as the "Trade old for Remanufactured" program cannot be ignored. However, prior research has not answered the questions of whether governments should offer consumption subsidies and how to determine the optimal subsidy value. This paper investigates the optimal government consumption subsidy policy and its impact on the operation of Closed-Loop Supply Chain (CLSC) where an Original Equipment Manufacturer (OEM) produces new products, while a Third-Party Remanufacturer (TPR) remanufactures the used products collected from consumers. A game model with a leader (government) and two followers (OEM and TPR) is then introduced. The government determines the consumption subsidy to maximize the social welfare, while the TPR and OEM attempt to maximize their own profit functions. Game theoretic models are proposed to explore and compare the scenarios, i.e., CLSC with a consumption subsidy policy and without a consumption subsidy policy. The equilibrium characteristics with respect to the government's consumption subsidy decisions and the price decisions for chain members are derived. Based on the theoretical and numerical analysis, the results show that: (1) governments should not always offer a consumption subsidy; (2) the consumption subsidy cannibalizes demand for new products while boosting the demand for remanufactured products; (3) the consumption subsidy should be shared between the TPR and consumers when the TPR raises the sales price of remanufactured product; (4) the members of the CLSC do not always benefit from the consumption subsidy policy.

Keywords: closed-loop supply chain; remanufacturing; third-party remanufacturer; remanufacturing; consumption subsidy; pricing

1. Introduction

A supply chain is a complex system that needs careful coordination and well-thought design from the parties involved [1,2]. A Closed-Loop Supply Chain (CLSC) adds another level of complexities due to its remanufacturing process. Remanufacturing is "the process whereby some components of used products are disassembled, cleaned, reprocessed, inspected, and then reassembled to be used again" [3]. Because of environmental, financial and marketing benefits, both the industry and academia have paid more and more attention to remanufacturing activities [4–7].

Since remanufacturing is profitable and environmentally efficient [8], more and more firms, such as HP, BMW, DEC, Apple, Kodak, IBM, Xerox, AT&T, Caterpillar and Cannon have participated in voluntary product remanufacturing. Meanwhile, many countries have enacted legislation such as the Waste Electrical and Electronic Equipment (WEEE) in Europe, or implemented "trade old for remanufactured" programs to incentivize the remanufacturing practice.

In the process of CLSC's development, the role of the Chinese government cannot be ignored [9]. The Chinese government has announced several programs such as a "Trade old for remanufactured" program or establishing a "model remanufacturing factories" program to attract more consumer attention to remanufactured products and to boost the demand for remanufactured products. However, due to low consumers' acceptance for remanufactured products [5,10], the marketing share of remanufactured product is less than 5% [11], and the growth of the remanufacturing industry in China still lags behind some developed countries [12]. Hence a key issue that should be addressed first is how to effectively attract more consumers to remanufactured products [13]. Establishing reasonable pricing schemes [12] or providing government consumption subsidies [9] are two important solutions to this problem.

However, very limited research exists on the impact of consumption subsidy policies on remanufacturing [5,9,13–15], and especially on government policy related to consumption subsidies involving third-party remanufacturers. In order to have a better understanding of whether the government should implement a consumption subsidy policy and its related impact on the CLSC's members' optimal strategy, the following important managerial questions must be answered:

1. Under what conditions should the government offer a consumption subsidy of remanufactured products?
2. How does the consumption subsidy policy affect the firm's decisions?
3. Will consumption subsidies of remanufactured products incentivize consumers to purchase the remanufactured products? Will the consumption subsidy policy cannibalize new product sales?
4. What benefits or costs can consumption subsidy policies bring to firms, consumers and society?

Our paper aims at looking for the answers of the above research questions. The main features of this paper are summarized in four aspects: first, a mathematical model is proposed, where the government's decision is incorporated; second, the analytical expressions of the optimal consumption subsidy, chain members' optimal decisions and maximal profits are derived; third, the above results are compared with the case without consumption subsidies; finally, the impacts of consumption subsidies on the product demand and chain members' profits are analyzed.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 gives the model description and the key assumptions and notations of the modeling framework. Section 4 proposes the models with and without consumption subsidies. In Section 5, we compare the analytical results obtained from Section 4. Numerical stimulations are conducted in Section 6 to validate the theoretical results and then some managerial insights are provided. Finally, the paper concludes in Section 7 and future research suggestions are provided.

2. Literature Review

Two aspects of research are directly related to our research: (1) pricing strategy for CLSC; and (2) government policy for CLSC. Each aspect will be reviewed separately and our research will be positioned accordingly.

2.1. Pricing Strategy for Closed-Loop Supply Chains

Pricing decisions of CLSCs directly affect the demand for remanufactured products and the operational efficiency of the CLSC; hence some research regarding the problem of pricing strategies in CLSCs has been done. For instance, Ferrer and Swaminathan [16,17] studied the joint pricing strategy of new and remanufactured products in monopoly and duopoly scenarios. Atasu et al. [18]

found that monopolistic competition can improve the profitability of remanufacturing. Based on the different consumer preferences for new and remanufactured products, Abbey et al. [19] confirmed that introducing a remanufactured product is beneficial for Original Equipment Manufacturers (OEMs). Wu [20] and Wu and Wu [21] focused on the impacts of OEM-remanufactured products on the independent remanufacturers' operational decisions, and the results revealed that consumer loyalty to OEMs is not beneficial to the independent remanufacturer. Zhao et al. [22] studied the problem of pricing, collecting channel choices and collecting effort decisions under the scenario where the remanufacturer adopts dual collecting channels to collect the used product.

Wu [3] extended the previous research by considering price and service competition among new products and remanufactured products. Hong et al. [23,24] focused on the impact of advertising investment on the decisions of the CLSC's members. Differing from the other studies, Wei et al. [25] extended the previous research by considering symmetric and asymmetric information environments. Zhang and Ren [26] proposed a pricing and coordination model for CLSC systems under patent licensing. Xie et al. [27] looked for the optimal price, wholesale price and advertising investment in a dual-channel CLSC. Gan et al. [28] developed a model for a short life-cycle product in a CLSC consisting of a monopolist OEM, a retailer and a collector, and then investigated the impact of consumer acceptance of remanufactured products and their direct channel preference for buying remanufactured products on the pricing strategy and on CLSC chain members' profits. Based on the assumption that the presence of remanufactured products and the identity of remanufacturers influence the consumers' perceived value of new products, Li et al. [29] considered the product life cycle in the context of OEMs' decision whether to remanufacture themselves, and the corresponding pricing strategy. Liu et al. [30] proposed a model to analyze the optimal pricing strategies for a monopolistic manufacturer who engages in remanufacturing. Kleber et al. [31] studied the robustness of the assumption that consumers' willingness to pay for remanufactured products is a fraction of that for new products, and then analyzed the pricing strategy and quality choice of new products for three different remanufacturing scenarios simultaneously. Zhao et al. [32] developed decision models for the pricing, service and recycling of CLSCs, considering the technology authorization and different remanufacturing roles, and revealed that the degree of consumer willingness to purchase the remanufactured products has an important impact on the remanufacturing role choice (OEMs remanufacturing or the retailers remanufacturing under the technology authorization from OEMs) and the retail price of new or remanufactured products. Wu and Zhou [33] compared a buyer-specific pricing policy with a uniform pricing policy in CLSCs with a third-party remanufacturer. Wang et al. [34] studied the optimal pricing strategy for new products, remanufactured products made by OEMs and remanufactured products made by third-party remanufacturers (TPRs) simultaneously. Zhang and He [35] studied a centralized CLSC with consumers having different preferences regarding new and recycled products; they found that given that the consumers' demand can be satisfied, there will be more "green profits" despite remanufactured products cannibalizing the new product market.

The above works have obtained many valuable findings on the pricing strategies for OEMs and TPRs. However, most of the previous work only focused on the game between the chain members of the CLSC. Our study differs from the above works by incorporating the government as a game player.

2.2. Government Policy of Closed-Loop Supply Chains

Government plays an important role in the development of CLSCs [36]. Therefore, more and more researchers pay attention to the government's intervention in CLSCs [36]. Many scholars have studied the impact of government subsidies to enterprises on the operation of CLSCs. For example, Webster and Mitra [37] analyzed the impact of different take-back laws on the operation of the remanufacturing industry. Mitra and Webster [38] considered the effect of government subsidies to chain members on the CLSC, and confirmed the validity of subsidies to promote the remanufacturing industry. Wang et al. [39] stated that subsidies to remanufacturers could incentivize remanufacturing activity, and a moderate subsidy results in cooperation between OEMs and TPRs. Wang et al. [40] analyzed the impacts of

different subsidy policies (initial subsidy, recycling subsidy, R&D subsidy and production subsidy) on the development of the remanufacturing industry. Hong et al. [41] analyzed programs including the advanced recycling fees that are paid by manufacturers and subsidy fees that are paid to recyclers. Heydari et al. [42] studied the efficiency of exemptions or subsidies, and found that governments should offer exemptions or subsidies to manufacturers rather than retailers. Zhu et al. [11] analyzed the policy options for product remanufacturing, that is, remanufactured product resale subsidies or donation subsidies, and confirmed that resale subsidies could aggravate the market cannibalization of new products by remanufactured products. Zhao et al. [43] confirmed that remanufacturers' profits could be improved by subsidy sharing between remanufacturers and consumers. Wan and Hong [7] found that government subsidies will incentivize market demand and benefit CLSC members. Guo et al. [44] pointed out that governments should adopt an appropriate subsidy policy to encourage the recovery of returned items. Cao et al. [45] compared Remanufacturing Subsidy Policy (RSP) with Carbon Tax Policy (CTP) to examine which policy is better for society, and found that if the environmental damage coefficient is relatively small, the government should implement the RSP to gain greater social welfare.

Comprehensive study has been centered on the issue of government subsidies to members of the CLSC (such as remanufacturers or recyclers), while very little study has focused on government subsidies to consumers of remanufactured product. Some studies contribute to the literature on CLSCs with 'trade old for new' (trade-in) subsidies or 'trade old for remanufactured' subsidies, in which government will offer subsidies to replacement consumers who return their used products and then buy new or remanufactured ones. For example, Ma et al. [9] confirmed that when the government subsidizes the replacement consumers, the profit of the CLSC's members and consumers improve. Zhu et al. [8] revealed that governments should offer a proper trade-in subsidy in order to attract more consumers to return their used products. Miao et al. [46] confirmed that an increase in trade-in subsidies could increase the system players' profits; the adoption of trade-ins could improve environmental performance under certain conditions. Shu et al. [15] demonstrated that trade-in subsidies could encourage consumers to replace their existing products with new products and remanufactured products. Cao et al. [47] focused on the competition between the trade-in service of a manufacturer and the recycling of a third party. Zhang et al. [48] confirmed that simple government subsidies or taxes on products upgrading with trade-in remanufacturing could induce a social optimum and optimal remanufacturing efficiency. Zhao et al. [49] analyzed three different government subsidy scenarios, and their results showed that government subsidies for consumers had a significantly impact on the market competitiveness of remanufactured products. Xiao et al. [50] examined the conditions for manufacturers and retailers to implement a trade-in policy voluntarily and then identified the optimal channel choice for implementing trade-in policies.

To our knowledge, Ma et al. [9], Ma et al. [13], Shu et al. [15,51], Han et al. [5], Miao et al. [46,52], He et al. [14] and Zhu et al. [53] are the only theoretical papers that have considered the influence of consumption subsidies to the consumers of remanufactured products on the remanufacturing industry. Han et al. [5] focused on whether consumption subsidy policies could encourage firms to offer "trade old for remanufactured" programs. Ma et al. [13] revealed that the coexistence of "trade-in" and "trade old for remanufactured" programs is not always conductive to firms. Shu et al. [15] compared the efficiency of two different subsidy mechanisms (remanufacturing subsidy or tax rebate) on the profits of a monopolist manufacturer who engages in product remanufacturing and on the achievement of a low-carbon economy. Zhu et al. [53] investigated the conditions under which a trade-in program for remanufactured products should be adopted and then derived an optimal pricing strategy. Shu et al. [51] analyzed the "trade old for remanufactured" model with carbon tax and government subsidies, and confirmed that the government subsidy policy could achieve a "win–win " between profits and carbon missions. Miao et al. [52] focused on the impact of carbon regulations on the coexistence of "trade-in" and "trade old for remanufactured" programs, and highlighted that the

introduction of carbon regulations can promote sales of remanufactured products and the willingness of governments to propose subsidy policies.

The major difference between prior research and our work is that consumption subsidies are an exogenous variable in prior research. However, as government has had a key role in the development of CLSCs [9], it is necessary to involve it as a game player in the game model, and then analyze the impact of government consumption policies on the CLSC's operation.

He et al. [14] and Nielsen et al. [54] are the closest studies to ours. He et al. [14] considered the optimal consumption subsidy level by incorporating the government as a decision-making party in a CLSC system, where the OEM as a monopolist offers both new products and remanufactured products simultaneously through different channel structures without considering the existence of third-party remanufacturers. Nielsen et al. [54] compare the outcomes of three government policies (i.e., consumption subsidies to consumers, used product collection subsidies to manufacturers and subsidies to manufacturers to improve product quality) in a CLSC comprising of an OEM and a retailer with the assumption that there is no difference between the new and the remanufactured products. They considered that the OEM is responsible for used product collection and remanufacturing. This paper focuses on the outcome comparison between three different policies, but does not answer the question: is government intervention necessary?

In fact, not all OEMs engage in remanufacturing due to some barriers, such as lack of remanufacturing strategy [55] and cannibalization of new product sales [33,55]. Therefore, third-party remanufacturers, such as Caterpillar, have engaged in collection and remanufacturing operations [56,57]. Especially in China, the majority of the remanufacturers approved by China's national Development and Reform Commission (CNDRC) are third-party remanufacturers [57]. Many scholars have focused on the impact of the TPRs' entrance on CLSC operations [33]. However, there has been little research considering the design of government consumption subsidy policy in the CLSC with OEM and TPR.

To fill this important research gap, unlike He et al. [14] and Nielsen et al. [54], we consider a CLSC system composed of one OEM for producing new products and one TPR for providing remanufactured products, and focus on how the government should design its consumption subsidy policy to regulate the market and enhance social welfare.

At the same time, previous works such as [5,51] make a common assumption that the government would offer consumption subsidies for replacement consumers holding a used product if they would choose to buy a remanufactured product. Under this assumption, the market of remanufactured products is limited to replacement consumers. We propose a model with the assumption that both first-time (new) consumers and replacement consumers will choose to buy the remanufactured product, which is consistent with Ma et al. [13] and Shu et al. [15].

This research contributes to the literature in two ways. First, we propose a two-stage game model by incorporating the government as a game player. Most of the existing literature considered the government policy as an exogenous variable, and only focused on the channel members' response to the government policy. The government is not involved in the decision-making process. Very little has been done to explore how to make the optimal government policy decision to balance the members' economic profits and the welfare of the entire society. Second, in the two-stage game model, we focus on the consumption subsidy policy. Most of the existing literature focuses on the subsidy policy to CLSC's members, while fewer researches focus on the consumption subsidy policy, that is, the government providing subsidies to consumers who choose to buy the remanufactured product.

3. Model Development

In this study, we consider a CLSC with an OEM and a TPR. The OEM is responsible for manufacturing new products from raw materials. The TPR collects the end-of-life products from which the useful material is extracted and remanufactured; the remanufactured product then is sold to consumers directly [3,58]. Consumer demand can be satisfied by purchasing new products manufactured by the OEM, or items remanufactured by the TPR. Both types of products are assumed

to have the same function and quality with a different price [26]. The government implements the consumption subsidy policy aiming to encourage consumers to buy remanufactured products: consumers will receive a consumption subsidy if they buy remanufactured products.

For the sake of clarity, the relevant notations are provided in Table 1.

Table 1. Notations.

Parameter	Definition
c_n	Unit manufacturing cost
c_r	Unit remanufacturing cost
v	Unit recycling price of used product
χ	Basic collected quantity of zero reward money
δ	Recycling price elasticity coefficient of used product
p_n	Unit sales price of new product
p_r	Unit sales price of remanufactured product
s	Unit consumption subsidy for consumers who purchase the remanufactured product
α_n	Potential market size of new product
α_r	Potential market size of remanufactured product
β	Price elasticity coefficient
γ	Substitution coefficient
m, r, SC	Original equipment manufacturer, third-party remanufacturer, the supply chain
D_n , D_r	Demand volume of new product, demand volume of remanufactured product
Q_c	Quantity of used product recycled
CS_n , CS_r , CS_u	Consumer surplus for new product, consumer surplus for remanufactured product, consumer surplus for recycled product

The following modeling assumptions were used in constructing the model (note that most of these assumptions have been used in previous literature):

1. All the members of the CLSC are risk-neutral and profit-maximizing and have access to the same information [3,23,58,59].
2. All of the recycled used products can be used for remanufacturing, and c_r is the same for all of the used products.
3. $c_n > c_r + v$. Let $\Delta = c_n - c_r$, which denotes unit cost savings for remanufacturing. This assumption guarantees that remanufacturing is profitable [60].
4. When consumers chooses to purchase the remanufactured product from the TPR, they can obtain a consumption subsidy s $(0 < s < p_r)$ from government.
5. There is a substitution relationship between the new products and the remanufactured product [26]. D_n is the demand function, which is defined by the sales price of the new product, the cross price competition and the consumption subsidy: $D_n = \alpha_n - \beta p_n + \gamma(p_r - s)$ Similarly, D_r can be defined as follows: $D_r = \alpha_r - \beta(p_r - s) + \gamma p_n$
6. In the above definition, we assume that $\beta > \gamma > 0$, which means that the market demand is more sensitive to its own sales price than to the sales price of its substitutable product. In addition, we assume $\alpha_n > \alpha_r$, which indicates that a consumer's acceptance of new products is relatively higher than that of remanufactured products [26].
7. The consumer surplus for the new product and the remanufactured product [61] can be computed and expressed as: $CS_n = \frac{(D_n)^2}{2\beta} = \frac{[\alpha_n - \beta p_n + \gamma(p_r - s)]^2}{2\beta}$, $CS_r = \frac{(D_r)^2}{2\beta} = \frac{[\alpha_r - \beta(p_r - s) + \gamma p_n]^2}{2\beta}$, respectively.
8. Consumer surplus for the recycled product [61], i.e., the difference between the recycling price and the fee level that consumers are willing to pay to bring their used products to remanufacturers can be expressed as $CS_u = vQ_c - \frac{\delta v^2}{2}$.
9. The TPR is a rational decision maker that wants to maximize its profit and will set the price p_r so that D_r would be lower than the supply Q_c, i.e., $D_r \leq Q_c$.
10. A product can be recycled unlimited times.

11. Q_c is a function of the recycling price and can be expressed as: $Q_c = \chi + \delta v$ [26,41,61].
12. Both the OEM and TPR have ample capacity to meet demand for new products and to remanufacture all of the recycled product, respectively. Therefore, all demand can be satisfied.
13. OEM and TPR supply chains are disruption-free and the cost of recovery from disruption is negligible [62].

4. Basic Model without Government Consumption Subsidy (Model I)

Before exploring the impact of the consumption subsidy on CLSC operation, a base model where there is no government consumption subsidy was developed.

The OEM, as the market leader, has sufficient channel power over the TPR to act as a Stackelberg's leader [3,63] who sets the sale price of new products, while the TPR follows, trying to maximize profit after observing the OEM's pricing policy.

In the base model, the market demand D_n^0 of the new product and the market demand D_r^0 of the remanufactured product are as follows, respectively:

$$D_n^0 = \alpha_n - \beta p_n + \gamma p_r \tag{1}$$

$$D_r^0 = \alpha_r - \beta p_r + \gamma p_n. \tag{2}$$

Given market demand, we can formulate the members' profit function.

The profit functions of the OEM and TPR are, respectively:

$$\Pi_m^0(p_n, p_r, v) = (p_n - c_n)D_n^0 \tag{3}$$

$$\Pi_r^0(p_r, v, p_n) = (p_r - c_r)D_r^0 - vQ_c \; s.t. \chi \le D_r^0 \le Q_c \tag{4}$$

By backward induction, we can obtain the results outlined in Proposition 1.

Proposition 1. *In the base model, the TPR's optimal pricing strategy of the remanufactured product and the used products and the OEM's optimal pricing strategy of the new product are given in the following equations, respectively:*

$$p_n^{0^*} = \frac{2\beta(\beta+\delta)(\beta c_n + \alpha_n) + \gamma[(2\beta+\delta)(\alpha_r - \gamma c_n) + \beta\delta c_r - \beta\chi]}{4\beta^2(\beta+\delta) - 2\gamma^2(2\beta+\delta)} \tag{5}$$

$$\begin{aligned} p_r^{0^*} = & \frac{\gamma(2\beta+\delta)\left[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]c_n + 2\beta\gamma(\beta+\delta)(2\beta+\delta)\alpha_n}{4\beta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \\ & + \frac{[(2\beta+\delta)\alpha_r - \beta\chi + \beta\delta c_r]\left[4\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]}{4\beta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \end{aligned} \tag{6}$$

$$\begin{aligned} v^{0^*} = & \frac{\gamma\delta\left[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]c_n + 2\beta\gamma\delta(\beta+\delta)\alpha_n + \delta\left[4\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]\alpha_r}{4\delta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \\ & + \frac{\left(-4\beta^4+4\beta^2\gamma^2-12\beta^3\delta+9\beta\gamma^2\delta-8\beta^2\delta^2+4\gamma^2\delta^2\right)\chi - \beta\delta\left[4\beta^2(\beta+\delta)-\gamma^2(4\beta+3\delta)\right]c_r}{4\delta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \end{aligned} \tag{7}$$

All proofs are provided in the Appendix A.

The corresponding optimal profits of the TPR and OEM and the total profit of the CLSC can be obtained respectively as follows:

$$\Pi_r^{0^*} = \frac{\left\{\begin{array}{c} \beta\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right](\chi-\delta c_r) + \gamma\delta\left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n \\ +2\beta\gamma\delta(\beta+\delta)\alpha_n + \delta\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r \end{array}\right\}^2}{16\beta\delta(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]^2} \tag{8}$$

$$\Pi_m^{0^*} = \frac{\left\{-\beta\gamma\chi - \left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n + \beta\gamma\delta c_r + 2\beta(\beta+\delta)\alpha_n + \gamma(2\beta+\delta)\alpha_r\right\}^2}{8\beta(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]} \tag{9}$$

$$\Pi_{SC}^{0^*} = \Pi_m^{0^*} + \Pi_r^{0^*} \tag{10}$$

The corresponding optimal consumer surplus, optimal social welfare and the optimal demand for the new product and the remanufactured product without a consumption subsidy can be expressed respectively as: $\Pi_{CS}^{0^*} = CS_n^{0^*} + CS_r^{0^*} + CS_u^0, \Pi_{gov}^{0^*} = \Pi_m^{0^*} + \Pi_r^{0^*} + \Pi_{CS}^{0^*}$, $D_n^{0^*}, D_r^{0^*}$.

We investigated the CLCS without a government consumption subsidy (Model I) as the benchmark.

5. Model with Government Consumption Subsidy (Model II)

In this section, we study the model where the government implements a consumption subsidy, which has impacts on various decision makers in the CLSC. We consider a CLSC comprising of an OEM, a TPR and a government. The OEM produces a new product and the TPR remanufactures used products that are collected from consumers. The government acts as a Stackelberg leader to determine the consumption subsidy policy for the remanufactured product, while the OEM and the TPR are followers seeking to maximize their own profit after observing the government's subsidy policy. At the same time, in the game between the OEM and the TPR, the OEM is a Stackelberg's leader who sets the optimal sales price of the new product while the TPR is a follower who determines the optimal recycling price of the used product and the optimal sales price of the remanufactured product.

Given the above description, we provide the sequence of events in Model II is as follows: first, the government announces its consumption subsidy policy for the remanufactured product; second, after observing the subsidy ratio, the OEM decides the optimal sales price of the new product; finally, the TPR decides the optimal sales price of remanufactured products and the optimal colleting price of used products.

The profit functions of the OEM and TPR are, respectively:

$$\Pi_m(p_n, p_r, v, s) = (p_n - c_n)D_n \tag{11}$$

$$\Pi_r(p_r, v, p_n, s) = (p_r - c_r)D_r - vQ_C \; s.t. \chi \le D_r \le Q_c \tag{12}$$

The government's objective function is social welfare, which is the sum of the OEM's profit, the TPR's profit and consumer surplus, minus the government's subsidy expenditure, sD_r.

The sum of consumer surplus in the CLSC can be expressed as:

$$\Pi_{CS} = CS_n + CS_r + CS_u = \frac{[\alpha_n - \beta p_n + \gamma(p_r - s)]^2}{2\beta} + \frac{[\alpha_r - \beta(p_r - s) + \gamma p_n]^2}{2\beta} + vQ_c - \frac{\delta v^2}{2} \tag{13}$$

Hence, the government objective function can be described as follows:

$$\Pi_{gov} = \Pi_m + \Pi_r + \Pi_{CS} - sD_r \tag{14}$$

The total profit of the CLSC can be written as:

$$\Pi_{SC} = \Pi_m + \Pi_r \tag{15}$$

Using backward induction, we first solve the TPR's sale price of remanufacturing p_r and the recycling price v.

Proposition 2. *The TPR's optimal pricing strategies, given the decision made by the OEM isp_n, are:*

$$p_r^*(p_n, s) = \frac{(2\beta+\delta)(\alpha_r + \gamma p_n + \beta s) - \beta\chi + \beta\delta c_r}{2\beta(\beta+\delta)} \tag{16}$$

$$v^*(p_n, s) = \frac{1}{2}\left(\frac{\alpha_r + \beta s + \gamma p_n - \chi - \beta c_r}{\beta + \delta} - \frac{\chi}{\delta}\right) \tag{17}$$

Corollary 1.

1. p_r^* and v^* increase in p_n.
2. p_r^* and v^* increase in s.

The proof for Proposition 2 and Corollary 1 can be found in the Appendix A.

Corollary 1 implies that both the OEM's pricing decision and the government's consumption subsidy have a positive relationship with the TPR's pricing decision. Therefore, the TPR and consumers with the used product are more willing to participate in the CLSC if theOEM increases its price or the government increases the consumption subsidy.

Using the TPR's decisions, the OEM's optimal sale price for the new product can be derived. Considering the TPR's decisions, the OEM chooses the optimal sale price of the new product to maximize its individual profit $\Pi_m(p_n, p_r^*(p_n, s), v^*(p_n, s), s)$, in which $(p_r^*(p_n, s), v^*(p_n, s))$ are defined as in Proposition 2.

Proposition 3. *The OEM's optimal decision (denoted as p_n^*) can be derived as follows:*

$$p_n^*(s) = \frac{2\beta(\beta + \delta)(\beta c_n + \alpha_n) + \gamma[(2\beta + \delta)(\alpha_r - \gamma c_n) + \beta\delta(c_r - s) - \beta\chi]}{4\beta^2(\beta + \delta) - 2\gamma^2(2\beta + \delta)} \tag{18}$$

Corollary 2. *p_n^* decreases as s* increases.

The proof for Proposition 3 and Corollary 2 can be found in the Appendix A.

Corollary 2 states that the government's consumption subsidy has a reverse relation to the new product's price. The government's consumption subsidy and the remanufactured product's cost advantage make the remanufactured product more competitive. In other words, if consumers of the new product will see a lower price if the government increases the consumption subsidy, it would thus lead to the loss of the remanufactured product's price advantage, and then induce the demand expansion of the new product.

Using the manufacturer's decision, the government's optimal consumption subsidy can be derived. The government chooses the optimal consumption subsidy s to maximize social welfare Π_{gov}.

Proposition 4. *The government's optimal consumption subsidy of a remanufactured product (denoted as s^*) is given by*

$$s^* = \max\left\{\frac{X_1\chi - \delta(X_2 c_n + X_1 c_r + X_3\alpha_n + X_4\alpha_r)}{\delta\beta(\beta^2 - \gamma^2)X_5 + 9\beta^4\delta^3\gamma^4}, 0\right\} \tag{19}$$

where $X_1 = \beta\Big[3\beta\delta^2\gamma^6 + 4\delta^3\gamma^6 + 16\beta^5\left(\beta^2 - \gamma^2\right)^2 + 4\beta^2\left(\beta^2 - \gamma^2\right)\left(12\beta^4 - 9\beta^2\gamma^2 - \gamma^4\right) + \beta\delta^2\left(\beta^2 - \gamma^2\right)\left(48\beta^4 - 24\beta^2\gamma^2 - \gamma^4\right) + \delta^3\left(\beta^2 - \gamma^2\right)\left(16\beta^4 - 4\beta^2\gamma^2 + 3\gamma^4\right)\Big]$, $X_2 = -\gamma\Big[\left(\beta^2 - \gamma^2\right)(2\beta + \delta) + \beta^2\delta\Big]\Big[4\beta^2\left(\beta^2 - \gamma^2\right)\left(4\beta^2 - \gamma^2\right) + \beta\delta\left(\beta^2 - \gamma^2\right)\left(32\beta^2 - \gamma^2\right) + \delta^2\left(16\beta^4 - 13\beta^2\gamma^2 + \gamma^4\right) + 3\beta\delta\gamma^4\Big]$, $X_3 = 2\beta\gamma(\beta + \delta)\Big[\left(4\beta^2 + \delta^2\right)\left(2\beta^2 - \gamma^2\right)\left(\beta^2 - \gamma^2\right) + 2\beta\delta\left(4\beta^2 - \gamma^2\right)\left(\beta^2 - \gamma^2\right) + \beta\delta\left(4\beta^4 + \gamma^4\right) + 6\delta^2\beta^4\Big]$, $X_4 = 8\beta^3\left(-2\beta^6 + 6\beta^4\gamma^2 - 5\beta^2\gamma^4 + \gamma^6\right) + 2\beta^2\left(-24\beta^6 + 66\beta^4\gamma^2 - 43\beta^2\gamma^4 + 6\gamma^6\right) + \beta\delta^2\left(-48\beta^6 + 120\beta^4\gamma^2 - 59\beta^2\gamma^4 + 6\gamma^6\right) + \delta^3\left(-16\beta^6 + 36\beta^4\gamma^2 - 13\beta^2\gamma^4 + \gamma^6\right)$, $X_5 = 16\beta^5\left(\beta^2 - \gamma^2\right) + 4\beta^2\delta\left(12\beta^4 - 9\beta^2\gamma^2 + \gamma^4\right) + \delta^3\left(16\beta^4 - 4\beta^2\gamma^2 + \gamma^4\right) + \beta\delta^2\left(48\beta^4 - 24\beta^2\gamma^2 + 4\gamma^4\right)$.

Using Equations (19)–(22), the optimal profit of the TPR and OEM and the total profit of the CLSC are as follows, respectively:

$$\Pi_r^* = \frac{\left\{\begin{array}{c}\beta\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right](\chi+\delta s^*-\delta c_r)+\gamma\delta\left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n \\ +2\beta\gamma\delta(\beta+\delta)\alpha_n+\delta\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r\end{array}\right\}^2}{16\beta\delta(\beta+\delta)\left[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)\right]^2} \tag{20}$$

$$\Pi_m^* = \frac{\left\{-\beta\gamma(\chi+\delta s^*)-\left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n+\beta\gamma\delta c_r+2\beta(\beta+\delta)\alpha_n+\gamma(2\beta+\delta)\alpha_r\right\}^2}{8\beta(\beta+\delta)\left[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)\right]} \tag{21}$$

$$\Pi_{SC}^* = \Pi_m^* + \Pi_r^* \tag{22}$$

The corresponding consumer surplus, the optimal social welfare, and the demand for the new product and the remanufactured product with a consumption subsidy can be expressed, respectively, as $\Pi_{CS}^* = CS_n^* + CS_r^* + CS_u^*$, $\Pi_{gov}^* = \Pi_m^* + \Pi_r^* + \Pi_{CS}^* - sD_r^*, D_n^*, D_r^*$.

Corollary 3. *The government's consumption subsidy of remanufactured products increases in χ or c_n, but decreases as c_r increases.*

The proof for Proposition 4 and Corollary 3 can be found in the Appendix A.

Corollary 3 states that government intervention relates to factors such as the manufacturing cost, the remanufacturing cost and the level of consumer environmental awareness. The government is willing to offer a consumption subsidy if the manufacturing cost, the environmental awareness of consumers or the cost saving of remanufacturing is relatively larger.

6. Comparisons with Managerial Implications

In this section, the optimal results derived in Section 4 will be compared, and some preliminary corollaries will be obtained.

From Proposition 4, it is easy to see that the government will only offer the consumption subsidy policy provided that the condition $X_1\chi - \delta(X_2c_n + X_1c_r + X_3\alpha_n + X_4\alpha_r) > 0$ is satisfied. By analyzing this condition, we find that a high manufacturing cost, a high basic collected quantity of zero reward money, and a low remanufacturing cost can motivate the government to offer the consumption subsidy policy. Details are given in Corollary 4.

Corollary 4. *The region in which the government should implement the consumption subsidy policy:*

(i) *Increases in c_n;*
(ii) *Increases in χ;*
(iii) *Decreases asα_n increases;*
(iv) *Decreases as c_r increases.*

The proof for Corollary 4 can be found in the Appendix A.

Corollaries 4(i) and 4(iv) imply that the higher the manufacturing cost, or the lower the remanufacturing cost, the more likely is the government willing to offer the consumption subsidy. In others words, cost saving in remanufacturing positively influences government policy on the remanufacturing industry. Corollary 4(ii) shows that the government will be more likely to offer the consumption subsidy policy when facing environmentally-friendly consumers. Corollary 4(iii) indicates that the market status of the new product also positively impacts government policy.

From the comparisons of the optimal decisions of ModelIand ModelII, we derive Corollary 5 and then summarize the influence of the consumption subsidy policy on the CLSC operation.

Corollary 5. *When the government offers the consumption subsidy policy (i.e.,* $X_1\chi - \delta(X_2c_n + X_1c_r + X_3\alpha_n + X_4\alpha_r) > 0$*),*

(i) *The consumption subsidy reduces the consumer's net payment for the remanufactured product;*
(ii) *The consumption subsidy will be shared between the consumers of the remanufactured product and the TPR;*
(iii) *For remanufactured products,* $p_r^* > p_r^{0^*}$*;*
(iv) *For the demand for remanufactured products:*$D_r^* > D_r^{0^*}$

The proof for Corollary 5 can be found in the Appendix A.

Corollary 5 implies that as the government offers the consumption subsidy, (1) the consumer's net payment for remanufactured products decreases, which in turn boosts the consumer demand for remanufactured products (demand expansion) [64]; (2) with the pursuit of profit maximization, the TPR could adjust its sales price of the remanufactured product to get more shares from the government policy, thus leading to a demand reduction for remanufactured products; (3) for the remanufactured product , the demand expansion due to the consumption subsidy is higher than the demand reduction due to the higher price, and thus the demand for remanufactured items increases due to the consumption subsidy. That is, the consumption subsidy policy is beneficial to the marketing of the remanufactured product.

Corollary 6. *When the government offers the consumption subsidy policy, we have:*

i. *For new products,* $p_n^* < p_n^{0^*}$*;*
ii. *For the demand for new products,* $D_n^* < D_n^{0^*}$*;*
iii. *For used product s,* $v^* > v^{0^*}$, $Q_c^* > Q_c^{0^*}$*;*
iv. *The total demand increases due to the consumption subsidy, that is,* $D_n^* + D_r^* > D_n^{0^*} + D_r^{0^*}$.

The proof for Corollary 6 can be found in the Appendix A.

Corollary 6 demonstrates that as the government subsidizes the remanufactured items, (1) since the consumer's net payment for the remanufactured product decreases, the OEM has to reduce the price of the new product to compete with the remanufactured product. However, the demand expansion of the new product due to the price reduction is smaller than the demand reduction due to the lower net payment for the remanufactured product. Therefore, the demand for the new product is decreased due to the consumption subsidy policy, which shows the remanufactured product cannibalizes the sales of the new product. However, the above subsidy enhances the overall product demand; (2) the higher collecting price of the used product under the consumption subsidy leads to a higher recycling quantity of the used product, which means the policy is effective at promoting the collection of the used product.

Corollary 7. *When the government offers the consumption subsidy, we have:*

(1) For the TPR's profit, if $Y_1 > 0$, where $Y_1 = \beta\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right](\chi-\delta c_r) + \gamma\delta\left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n + 2\beta\gamma\delta(\beta+\delta)\alpha_n + \delta\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r$, $\Pi_r^* > \Pi_r^{0^*}$; otherwise, $\Pi_r^* < \Pi_r^{0^*}$;
(2) For the OEM's profit, if $Y_2 > 0$, where $Y_2 = \left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]c_n + \beta\gamma(\chi-\delta c_r) - 2\beta(\beta+\delta)\alpha_n - \gamma(2\beta+\delta)\alpha_r$, $\Pi_m^* > \Pi_m^{0^*}$; otherwise, $\Pi_m^* < \Pi_m^{0^*}$;
(3) For the CLSC's profit, if $\beta A_1(\delta s^* + 2\chi) + 2\gamma\delta A_2c_n - 2\beta\delta A_1c_r + 2\delta A_3\alpha_r - 4\beta\gamma^3\delta^2(\beta+\delta)\alpha_n > 0$, where $A_1 = 16\left(\beta^3-\beta\gamma^2\right)^2 + 4\beta\delta\left(8\beta^4-13\beta^2\gamma^2+5\gamma^4\right) + \delta^2\left(16\beta^4-20\beta^2\gamma^2+7\gamma^4\right)$, $A_2 = \left[2\beta\left(\beta^2-\gamma^2\right)+\delta\left(2\beta^2-\gamma^2\right)\right]\left[8\beta\left(\beta^2-\gamma^2\right)+\delta\left(8\beta^2-5\gamma^2\right)\right]$, $A_3 =$

$\beta\left[16\left(\beta^3-\beta\gamma^2\right)^2+2\beta\delta\left(16\beta^4-26\beta^2\gamma^2+9\gamma^4\right)+\delta^2\left(16\beta^4-20\beta^2\gamma^2+5\gamma^4\right)\right]$, $\Pi^*_{SC} > \Pi^{0^*}_{SC}$; otherwise, $\Pi^*_{SC} < \Pi^{0^*}_{SC}$.

Corollary 7 indicates that the members of the CLSC do not always benefit from the consumption subsidy.

7. Numerical Examples

In this section, a numerical study is carried out to illustrate the above models. We compare the results obtained in the above sections and discuss the influence of the government's consumption subsidy on the supply chain member's profit, consumer surplus and social welfare. Based on these results, some managerial insights are generated.

7.1. Parameter Design

The values of parameters and coefficients used in the numerical examples are assumed. The basic relationships between different parameters are considered when we give the specific values. In Section 3, $c_n > c_r + v, \beta > \gamma > 0$ and $\alpha_n > \alpha_r$ should be guaranteed. In fact, the specific value may not coincide with reality, but the changes of the parameters have no impact on the conclusions and their analysis in our paper.

The value of parameters in each models are set as follows: $c_n = 100, f(v) = 50 + 8v, \alpha_n = 1000$, $\alpha_r = 800$, $\beta = 5$ and $\chi = 50$. Then, we can discuss the profit, price, consumer surplus and social welfare differences between the CLSC with the consumption subsidy and the CLSC with no consumption subsidy.

7.2. The Conditions of the Government's Consumption Subsidy Policy

The government should implement a consumption subsidy only when a certain condition is satisfied. Figure 1 displays the optimal strategy regions in which the government should implement a consumption subsidy policy.

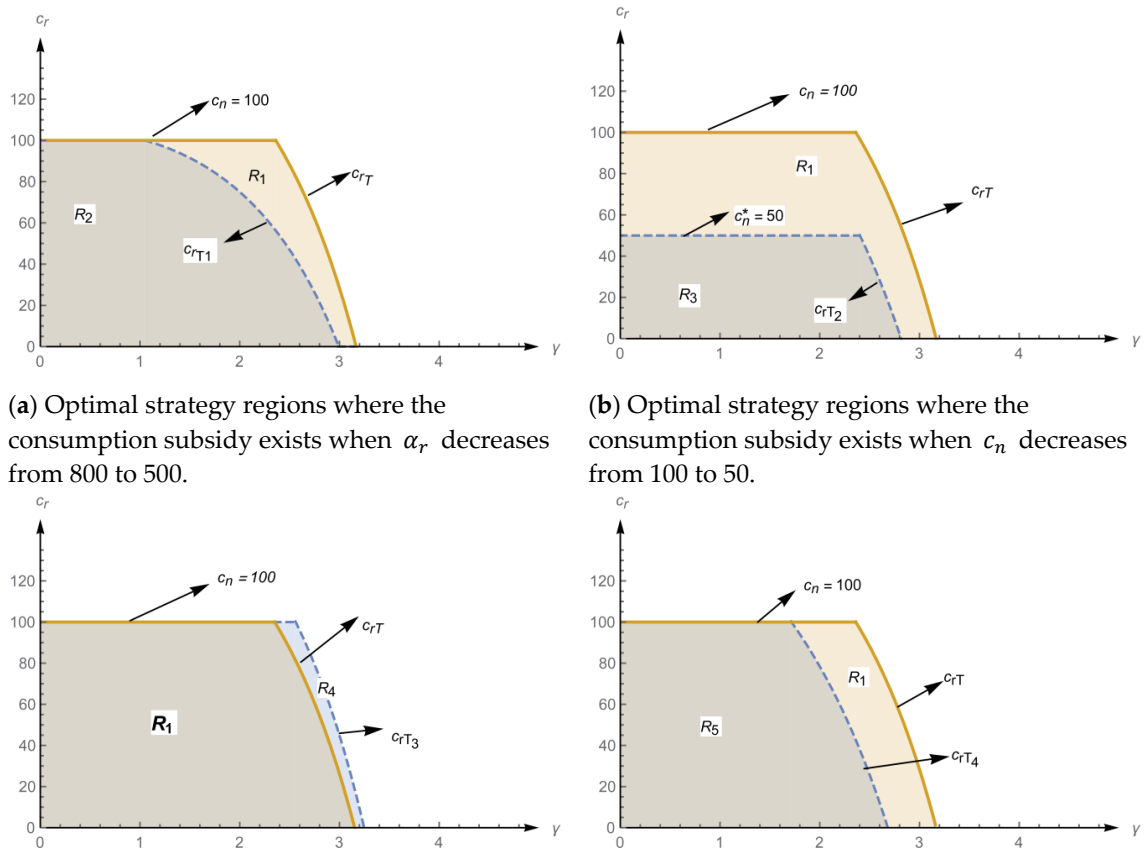
Note that in Figure 1a, $c_{r_T} = -\frac{1415}{4} - 45\gamma + \frac{1625\{10985000+3\gamma[316875+\gamma(-270400+9\gamma(-3250+\gamma(520+59\gamma)))]\}}{34328125+\gamma^2(6\gamma^2-325)(6500+27\gamma^2)}$, $c_{r_{T1}} = -\frac{875}{4} - 45\gamma + \frac{1625\{6865625+3\gamma[316875+\gamma(-169000+9\gamma(-3250+\gamma(329+59\gamma)))]\}}{34328125+\gamma^2(6\gamma^2-325)(6500+27\gamma^2)}$; in Figure 1b, $c_{r_{T2}} = \frac{5}{4}\left\{-283-18\gamma+\frac{650\{21970000+\gamma[105625+3\gamma(-540800+\gamma(-20150+9\gamma(1040+47\gamma)))]\}}{34328125+\gamma^2(6\gamma^2-325)(6500+27\gamma^2)}\right\}$; in Figure 1c, $c_{r_{T3}} = \frac{6350703125-5\gamma^2\{122102500+3\gamma[3168750+\gamma(-845975+18\gamma(-4550+\gamma(201+27\gamma)))]\}}{34328125+\gamma^2(6\gamma^2-325)(6500+27\gamma^2)}$; and in Figure 1d, $c_{r_{T4}} = -\frac{1415}{4} - 45\gamma + \frac{1625\{10985000+\gamma[528125+3\gamma(-270400+9\gamma(-24700+\gamma(520+53\gamma)))]\}}{34328125+\gamma^2(6\gamma^2-325)(6500+27\gamma^2)}$.

When the condition is satisfied ($X_1\chi - \delta(X_2c_n + X_1c_r + X_3\alpha_n + X_4\alpha_r) > 0$ in Proposition 4), corresponding to $c_r < c_{r_T}$ in Figure 1a, and the Assumption 3 is satisfied, corresponding to $0 < c_r < c_n$, there is only one region r_1 when $\alpha_r = 800$. The government will offer the consumption subsidy policy in this region R_1. Similarly, when $c_r < c_{r_{T1}}$ and $0 < c_r < c_n$ in Figure 1a, the government will offer the consumption subsidy policy in the region R_2 when $\alpha_r = 500$.

From Figure 1a we find that the optimal strategy regions increase with α_r; that is, when the basic market capacity of remanufactured product α_r decreases from 800 to 500, the region that government should implement the consumption subsidy policy is significantly reduced from the region R_1 to the region R_2.

When $c_r < c_{r_T}$ and $0 < c_r < c_n$, government will offer the consumption subsidy policy in this region R_1 when $c_n = 100$ in Figure 1b. Similarly, when $c_r < c_{r_{T2}}$ and $0 < c_r < c_n$ in Figure 1b, government will offer the consumption subsidy policy in the region r_3 when $c_n = 50$.

From Figure 1b, we find that the optimal strategy regions increase with c_n; that is, when the manufacturing cost c_n decreases from 100 to 50, the region in which the government should implement the consumption subsidy policy is significantly reduced, which validates Corollary 4(i).



(**a**) Optimal strategy regions where the consumption subsidy exists when α_r decreases from 800 to 500.

(**b**) Optimal strategy regions where the consumption subsidy exists when c_n decreases from 100 to 50.

(**c**) Optimal strategy regions where the consumption subsidy exists when χ increases from 50 to 200.

(**d**) Optimal strategy regions where the consumption subsidy exists when α_n increases from 1000 to 1500.

Figure 1. The changes of optimal strategy regions.

When $c_r < c_{r_T}$ and $0 < c_r < c_n$, the government will offer the consumption subsidy policy in region R_1 when $\chi = 50$ in Figure 1c. Similarly, when $c_r < c_{r_{T3}}$ and $0 < c_r < c_n$ in Figure 1c, the government will offer the consumption subsidy policy in the region R_4 when $\chi = 200$.

From Figure 1c, we find that the optimal strategy regions increase with χ; that is, when the basic collected quantity of zero reward money χ improves from 50 to 100, the region that government should implement the consumption subsidy policy is significantly enlarged, which validates Corollary 4(ii).

When $c_r < c_{r_T}$ and $0 < c_r < c_n$, the government will offer the consumption subsidy policy in region R_1 when $\alpha_n = 1000$ in Figure 1d. Similarly, when $c_r < c_{r_{T4}}$ and $0 < c_r < c_n$ in Figure 1d, the government will offer the consumption subsidy policy in the region R_5 when $\alpha_n = 1500$.

From Figure 1d, we find that the optimal strategy regions decrease with α_n; that is, when the basic market capacity of the new product α_n improves from 1000 to 1500, the region in which the government should implement the consumption subsidy policy is significantly narrowed. This result validates Corollary 4(iii).

7.3. Impact of Remanufacturing Cost (c_r) on CLSC

Higher remanufacturability is associated with lower remanufacturing cost. Figure 2 displays the impact of remanufacturability on the government's subsidy decision, the CLSC member's decision and so on. The proof of the impact of c_r on the government's subsidy decision, the member's decisions and so on can be found in the Appendix A.

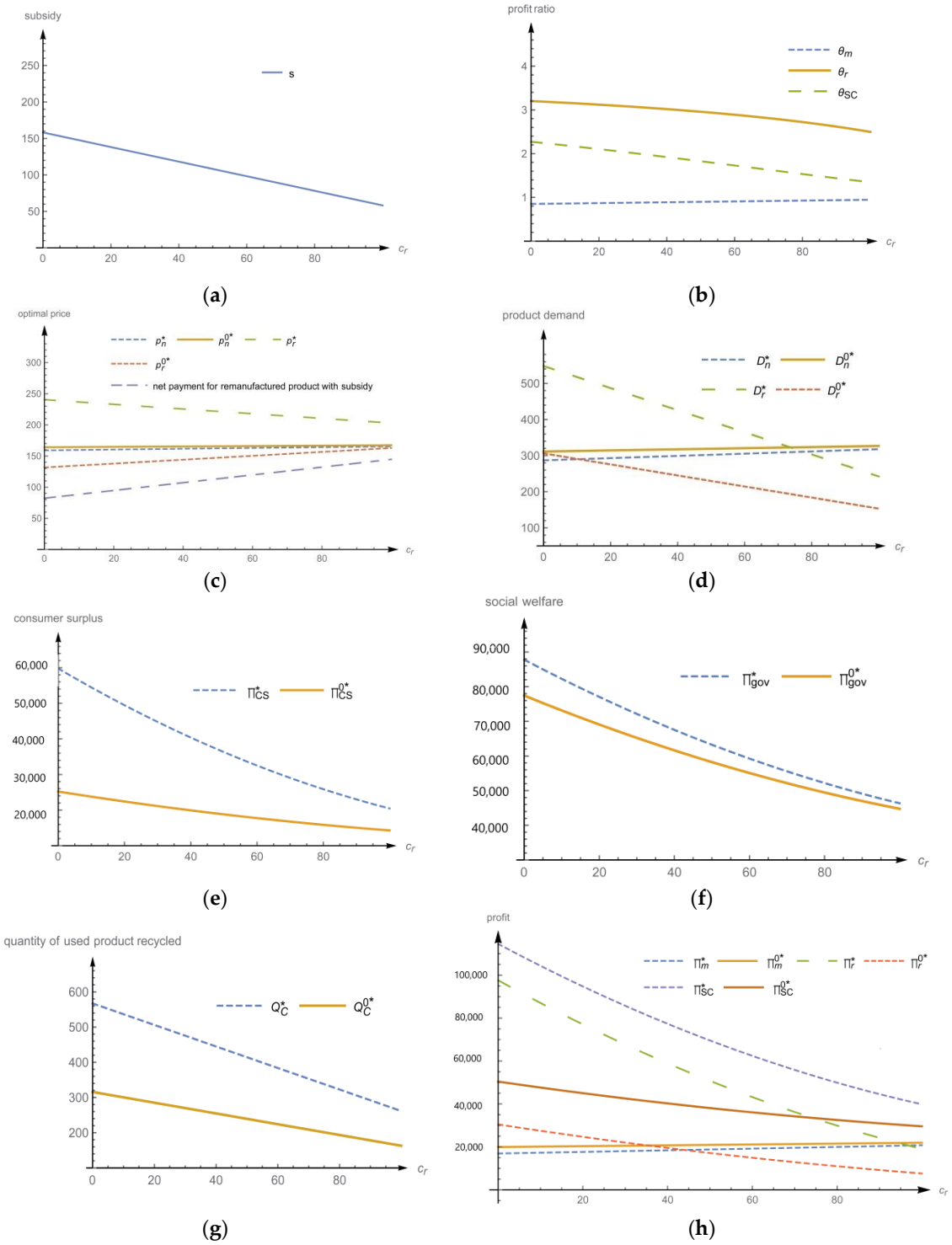


Figure 2. The impact of remanufacturing cost (c_r) on. (**a**) government consumption-subsidy; (**b**) profit ratio; (**c**) pricing strategies; (**d**) demand; (**e**) consumer surplus; (**f**) social welfare; (**g**) quantity of used product recycled; (**h**) chain member's profit.

From Figure 2, we can derive the following results:

1. When the remanufacturing cost c_r rises, the optimal government consumption subsidy declines.
2. θ_m increases with c_r, while θ_r, θ_{SC} and the recycled quantity of the used product, with or without a consumption subsidy, decrease with c_r, and $\theta_r > 1$ (corresponding to $Y_1 = 10064400 - 50240c_r > 0$ in Corollary 7), $\theta_m < 1$ (corresponding to $Y_2 = -80950 - 40c_r < 0$ in Corollary 7) and $\theta_{SC} > 1$ (corresponding to $Y_3 = 424426154620255 - 2399957878716\ c_r > 0$ in Corollary 7), suggesting that the consumption subsidy policy increases the TPR's profit and the total supply chain profit, and decreases the OEM's profit. High product remanufacturability and a government consumption subsidy policy are strong incentives for the TPR to recycle used products and remanufacture.
3. The optimal price of the new product and the net payment for the remanufactured product increase with c_r, with or without a consumption subsidy. If the government offers the consumption subsidy, the optimal price of the new product and the net payment for the remanufactured product will be reduced, while the optimal price of the remanufactured product will be raised. These results match those in Corollaries 1, 2, 5(iii) and 6(i), suggesting that the effort to lower the remanufacturing cost could not only positively influence the net payment for the remanufactured product, but also the new product's price.
4. With rising c_r, the demand for remanufactured products declines gradually, with or without a consumption subsidy. Additionally, the demand for remanufactured products increases when the government offers the consumption subsidy. These results suggest that the demand for remanufactured product can be improved by the TPR decreasing the remanufactured cost or the government providing the consumption subsidy, which validates Corollary 5(iv).
5. The demand for new products increases with c_r, with or without a consumption subsidy. The demand for new products with a consumption subsidy is lower than that without a consumption subsidy, suggesting that the consumption subsidy cannibalizes the new product's demand, which validates Corollary 6(ii).
6. The consumer surplus follows the order of $\Pi^*_{CS} > \Pi^{0^*}_{CS}$, and the social welfare follows the order of $\Pi^*_{gov} > \Pi^{0^*}_{gov}$. These results demonstrate that the consumption subsidy is beneficial to both consumers and society.
7. With or without a consumption subsidy, the quantity of used products recycled decreases with c_r, meaning that high remanufacturability induces the TPR to recycle more used products. The quantity of used products recycled with a consumption subsidy is higher than that without a consumption subsidy, suggesting that the consumption subsidy could enhance the environmental performance.

7.4. Impact of γ on CLSC

A higher substitution coefficient (γ) is associated with higher consumer receptivity of remanufactured products. In this section, we analyze the impact of γ on the government's subsidy decision, the CLSC members' decisions and so on. Due to the complexity of the equilibriums of the CLSC with the government consumption subsidy, from the perspective of γ we have to analyze how the substitution coefficient impacts the members' decisions, profits, demand etc. with numerical simulation in the following part. The impact of γ on the operation of CLSC without a consumption subsidy can be found in the Appendix A.

According to Proposition 4, let $X_1\chi - \delta(X_2c_n + X_1c_r + X_3\alpha_n + X_4\alpha_r) = 0$, we can obtain $\gamma_{max} = 2.77$. If $\gamma > \gamma_{max}$, the government should not offer the consumption subsidy.

From Figure 3, we can derive the following results:

1. The government consumption subsidy decreases with γ when $\gamma < \gamma_{max}$;
2. θ_m, θ_r and θ_{SC} decrease with γ (corresponding to $\gamma < \gamma_{max}$), while $\theta_r > 1$ (corresponding to $Y_1 = 27625 + \gamma[7800 - \gamma(103 + 72\gamma)] > 0$ in Corollary 7), $\theta_m < 1$ (corresponding to $Y_2 = -50[13000 + \gamma(331 + 36\gamma)] < 0$ in Corollary 7) and $\theta_{SC} > 1$ (corresponding to $Y_3 > 0$ in Corollary

7). These results show that the consumption subsidy policy increases the TPR's profit and the total supply chain profit, while it decreases the OEM's profit; when the consumer acceptance of remanufactured products increases, the growth in the TPR's profit and in the total supply chain's profit decreases, suggesting that the efficiency of the consumption subsidy policy declines when the consumer's acceptance of the remanufactured product improves.

3. The optimal price of the new product and the net payment for the remanufactured product increase with γ, with or without a consumption subsidy. If the government offers the consumption subsidy, the optimal price of the new product and the net payment for the remanufactured product will be reduced. These results mean that the consumer acceptance of the remanufactured product could positively influence the price of the new product and that of the remanufactured product.
4. With rising γ (corresponding to $\gamma < \gamma_{max}$), demand for remanufactured products without a consumption subsidy declines gradually, while that with a consumption subsidy first increases and then declines. Demand for remanufactured products increases when the government offers the consumption subsidy. These results suggest that demand for remanufactured products can be improved by consumers improving the acceptance of remanufactured products or the government providing the consumption subsidy policy.
5. Demand for new products increases with γ, with or without a consumption subsidy, suggesting that the competition from the remanufactured product is beneficial for the sales of the new product. The demand for new products with a consumption subsidy is lower than that without a consumption subsidy, verifying the consumption subsidy's effect of cannibalization of the new product's demand.
6. The consumer surplus follows the order of $\Pi^*_{CS} > \Pi^{0^*}_{CS}$, and the social welfare follows the order of $\Pi^*_{gov} > \Pi^{0^*}_{gov}$. These results demonstrate that the consumption subsidy is beneficial to both consumers and society.
7. The quantity of used product recycled, with or without a consumption subsidy, increases with γ, suggesting that high consumer acceptance of remanufactured products encourages the TPR to recycle more used products, which leads to an improvement of environmental performance. The quantity of used product recycled with a consumption subsidy is higher than that without a consumption subsidy, suggesting that the consumption subsidy could enhance the environmental performance.

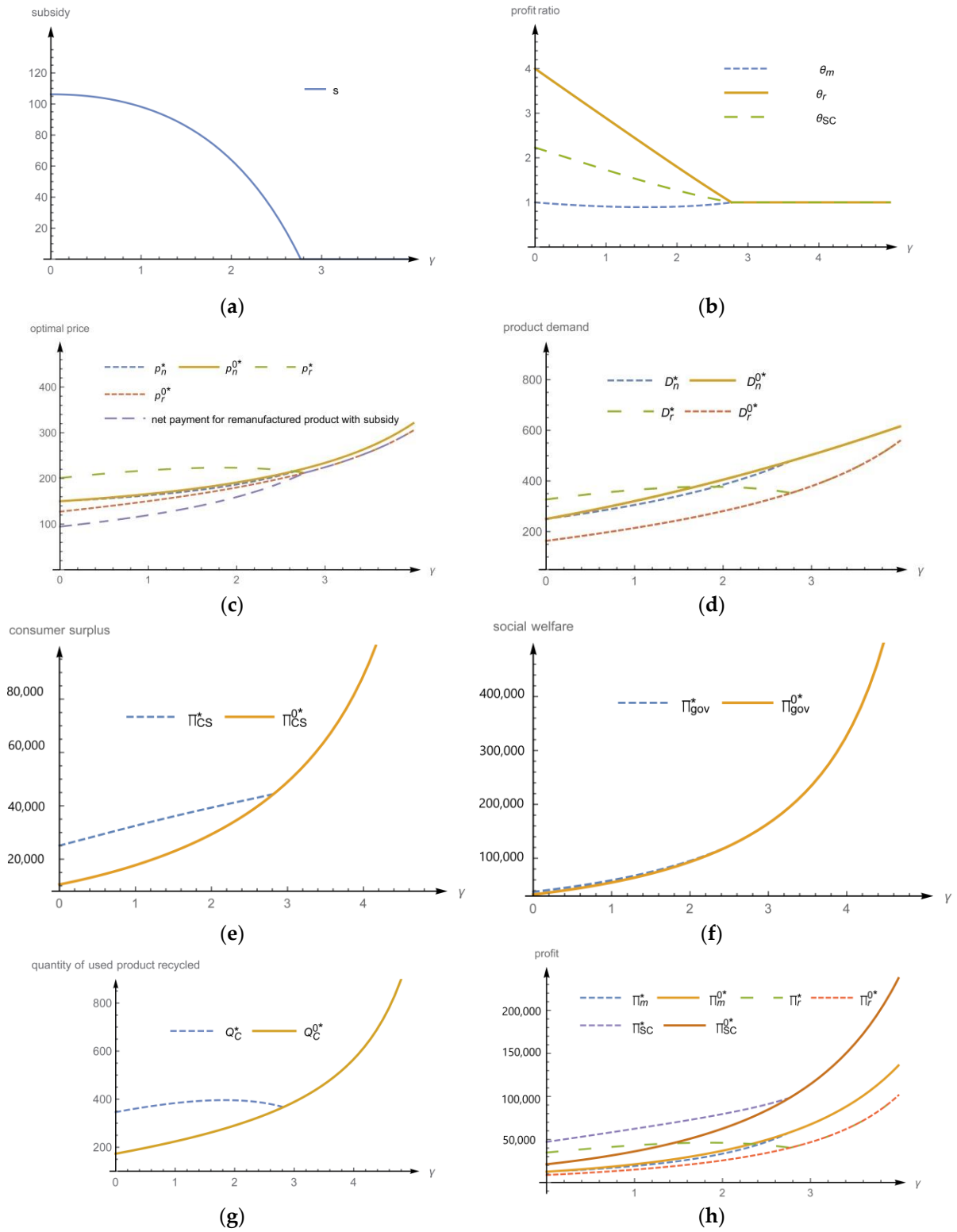


Figure 3. The impact of the substitution coefficient (γ) on. (**a**) government consumption-subsidy; (**b**) profit ratio; (**c**) pricing strategy; (**d**) demand; (**e**) customer surplus; (**f**) social welfare; (**g**) quantity of used product recycled; (**h**) chain member's profit.

8. Conclusions

In this paper, we investigated the optimal consumption subsidy policy of the government and the optimal pricing decisions of two profit-maximizing firms (OEM and TPR). Both firms determine

the optimal sales price of their product, while the government determines the optimal consumption subsidy policy for the remanufacturing industry. Several interesting conclusions are summarized as follows:

1. The government should implement the consumption subsidy policy only when certain conditions are satisfied.
2. We derived the optimal consumption subsidy policy for the government and the optimal pricing strategy for the OEM and TPR. These results are valuable as they provide practical guidance and insights for governments and enterprises to maximize their own objectives.
3. Both the remanufacturing cost and the substitution degree between new products and remanufactured products can significantly impact the government's consumption subsidy policy providing decision. A low remanufacturing cost and low consumers' acceptance of remanufactured product are crucial for incentivizing the government to offer the consumption subsidy policy.
4. The consumption subsidy policy could motivate the consumers to buy more remanufactured products while cannibalizing the market of new products.
5. Although the consumption subsidy policy cannibalizes the new product sales, it is not always harmful to the OEM. The OEM could gain more profit under the consumption subsidy when certain conditions are satisfied.
6. The consumption subsidy prompts the OEM to decrease the sales price of the new product. The consumption subsidy will be shared between the TPR and consumers while the TPR raises the sales price of the remanufactured product.
7. The consumption subsidy does not always improve the firm's profit, consumer surplus and social welfare.

The results provide insights for governments to determine whether they should offer the consumption subsidy or not and for the chain members to make optimal pricing decisions under the government decision. This study contributes to the literature in two aspects. On the one hand, the study is one of the first works, to our knowledge, that considers the consumption subsidy value as an endogenous variable, and obtains the conditions under which the government should implement the consumption subsidy policy. Therefore, both the optimal pricing decisions of firms and the optimal government subsidy policy can be obtained simultaneously. On the other hand, through the theoretical analysis and numerical theory, it is confirmed that the consumption subsidy is beneficial to promotions of the remanufactured product market, but not all the supply chain members could benefit from the consumption subsidy.

Furthermore, this paper is based on some assumptions. Thus, our work can be extended in ways such as:

1. Our models consider a duopolistic setting in which the OEM only manufactures new products and the TPR only remanufactures remanufactured products. It is worth studying the duopolistic setting in which the OEM also takes back used products to restrict the capacities of the TPR.
2. Our models only consider the price competition between new products and remanufactured products; insights would be gained by expanding our models to incorporate quality competition or warranty competition.

For future study, we plan to collect empirical data to empirically estimate the parameters of our model to provide more practical and realistic managerial insights. In addition, we also plan to integrate supply a chain network design for the TPR and OEM as suggested by [65]. At the meanwhile, it will be interesting to incorporate the environmental impact of production of new product and that of remanufactured product supply chain into the government objective [66,67].

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Appendix A

Appendix A.1. Proof of Proposition 1

Taking the derivatives of $\prod_r^0(p_r, v, p_n)$ with respect to p_r and v, we have:

$$\frac{\partial \Pi_r^0(p_r, v, p_n)}{\partial p_r} = \beta c_r + \gamma p_n - 2\beta p_r + \alpha_r \tag{A1}$$

$$\frac{\partial \Pi_r^0(p_r, v, p_n)}{\partial v} = -2\delta v - \chi \tag{A2}$$

The Hessian matrix of $\Pi_r^0(p_r, v, p_n)$ is as follows:

$$H_1 = \begin{bmatrix} \frac{\partial^2 \Pi_r^0(p_r,v,p_n)}{\partial {p_r}^2} & \frac{\partial^2 \Pi_r^0(p_r,v,p_n)}{\partial p_r \partial v} \\ \frac{\partial^2 \Pi_r^0(p_r,v,p_n)}{\partial v \partial p_r} & \frac{\partial^2 \Pi_r^0(p_r,v,p_n)}{\partial v^2} \end{bmatrix} = \begin{bmatrix} -2\beta & 0 \\ 0 & -2\delta \end{bmatrix} \tag{A3}$$

We can get $\frac{\partial^2 \Pi_r^0(p_r,v,p_n)}{\partial {p_r}^2} = -2\beta < 0$ and the determinate of the Hessian $|H_1| = 4\beta\delta > 0$, hence, $\prod_r^0(p_r, v, p_n)$ is jointly concave in p_r and v.

The Lagrangian and KT optimality conditions for the TPR's optimization problem are as follows:

$$L^0(p_r, v, \lambda_1, \lambda_2) = \Pi_r^0(p_r, v, p_n) + \lambda_1 g_1(p_r, v) + \lambda_2 g_2(p_r, v), \text{where } g_1(p_r, v) = Q_c - D_r^0,\ g_2(p_r, v) = D_r^0 - \chi \tag{A4}$$

$$\begin{cases} \frac{\partial L^0(p_r,v,\lambda_1,\lambda_2)}{\partial p_r} = \frac{\partial \Pi_r^0(p_r,v,p_n)}{\partial p_r} + \lambda_1 \frac{\partial g_1(p_r,v)}{\partial p_r} + \lambda_2 \frac{\partial g_2(p_r,v)}{\partial p_r} = 0 \\ \frac{\partial L^0(p_r,v,\lambda_1,\lambda_2)}{\partial v} = \frac{\partial \Pi_r^0(p_r,v,p_n)}{\partial v} + \lambda_1 \frac{\partial g_1(p_r,v)}{\partial v} + \lambda_2 \frac{\partial g_2(p_r,v)}{\partial v} = 0 \\ \lambda_1 g_1(p_r, v) = 0 \\ \lambda_2 g_2(p_r, v) = 0 \\ \lambda_1, \lambda_2 \geq 0 \end{cases} \tag{A5}$$

Solving the KT optimality conditions, we obtain the best response functions:

$$p_r^{0^*}(p_n) = \frac{(2\beta + \delta)(\alpha_r + \gamma p_n) - \beta\chi + \beta\delta c_r}{2\beta(\beta + \delta)} \tag{A6}$$

$$v^{0^*}(p_n) = \frac{1}{2}\left(\frac{\alpha_r + \gamma p_n - \chi - \beta c_r}{\beta + \delta} - \frac{\chi}{\delta}\right) \tag{A7}$$

Then, the OEM makes the decision about the optimal p_n according to the TPR's best response function.

By substituting $p_r^{0^*}(p_n)$ and $v^{0^*}(p_n)$ to the OEM's profit function in Equation (6), the profit OEM can be expressed as:

$$\Pi_m^0\left(p_n, p_r^{0^*}(p_n), v^{0^*}(p_n)\right) = (p_n - c_n)\left\{\alpha_n - \beta p_n + \frac{\gamma[(\gamma p_n + \alpha_r)(2\beta + \delta) - \beta\chi + \beta\delta c_r]}{2\beta(\beta + \delta)}\right\} \tag{A8}$$

The second order derivative to check for the optimality is as follows: $\frac{\partial^2 \Pi_m^0\left(p_n, p_r^{0^*}, v^{0^*}\right)}{\partial {p_n}^2} = \frac{2\beta\left(\gamma^2-\beta^2\right)+\delta\left(\gamma^2-2\beta^2\right)}{\beta(\beta+\delta)}$. Hence $\Pi_m^0\left(p_n, p_r^{0^*}(p_n), v^{0^*}(p_n)\right)$ is concave in p_n. We have:

$$p_n^{0^*} = \frac{2\beta(\beta+\delta)(\beta c_n + \alpha_n) + \gamma[(2\beta+\delta)(\alpha_r - \gamma c_n) + \beta\delta c_r - \beta\chi]}{4\beta^2(\beta+\delta) - 2\gamma^2(2\beta+\delta)} \tag{A9}$$

Further, we obtain the optimal sales price of remanufactured product and the optimal recycling price of used product as follows:

$$\begin{aligned} p_r^{0^*} = & \frac{\gamma(2\beta+\delta)\left[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]c_n+2\beta\gamma(\beta+\delta)(2\beta+\delta)\alpha_n}{4\beta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \\ & + \frac{[(2\beta+\delta)\alpha_r-\beta\chi+\beta\delta c_r]\left[4\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]}{4\beta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \end{aligned} \tag{A10}$$

$$\begin{aligned} v^{0^*} = & \frac{\gamma\delta\left[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]c_n+2\beta\gamma\delta(\beta+\delta)\alpha_n+\delta\left[4\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)\right]\alpha_r}{4\delta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \\ & + \frac{\left(-4\beta^4+4\beta^2\gamma^2-12\beta^3\delta+9\beta\gamma^2\delta-8\beta^2\delta^2+4\gamma^2\delta^2\right)\chi-\beta\delta\left[4\beta^2(\beta+\delta)-\gamma^2(4\beta+3\delta)\right]c_r}{4\delta(\beta+\delta)[2\beta^2(\beta+\delta)-\gamma^2(2\beta+\delta)]} \end{aligned} \tag{A11}$$

Hence, Proposition 1 is proven.

Appendix A.2. Proof of Proposition 2

Proof: The first order derivatives of $\Pi_r(p_r, v, p_n, s)$ to p_r and v can be shown as:

$$\frac{\partial \Pi_r(p_r, v, p_n, s)}{\partial p_r} = \beta(s + c_r) + \gamma p_n - 2\beta p_r + \alpha_r \tag{A12}$$

$$\frac{\partial \Pi_r(p_r, v, p_n, s)}{\partial v} = -2\delta v - \chi \tag{A13}$$

The second order derivatives to check for the optimality are as follows:

$$\frac{\partial^2 \Pi_r(p_r, v, p_n, s)}{\partial {p_r}^2} = -2\beta < 0, \ \frac{\partial^2 \Pi_r(p_r, v, p_n, s)}{\partial v^2} = -2\delta, \ \frac{\partial^2 \Pi_r(p_r, v, p_n, s)}{\partial p_r \partial v} = \frac{\partial^2 \Pi_r(p_r, v, p_n, s)}{\partial v \partial p_r} = 0$$

.

So, the determinant of the Hessian matrix can be described as follows:

$$\begin{vmatrix} \frac{\partial^2 \Pi_r(p_r,v,p_n,s)}{\partial {p_r}^2} & \frac{\partial^2 \Pi_r(p_r,v,p_n,s)}{\partial p_r \partial v} \\ \frac{\partial^2 \Pi_r(p_r,v,p_n,s)}{\partial v \partial p_r} & \frac{\partial^2 \Pi_r(p_r,v,p_n,s)}{\partial v^2} \end{vmatrix} = \begin{vmatrix} -2\beta & 0 \\ 0 & -2\delta \end{vmatrix} = 4\beta\delta > 0$$

So, $\Pi_r(p_r, v, p_n, s)$ is jointly concave in p_r and v.

The TPR's objective function can be described as:

$$\Pi_r(p_r, v, p_n, s) \ s.t. \chi \le D_r \le Q_c \tag{A14}$$

The Lagrangian and Karush-Kuhn-Tucker(KKT) optimality conditions for the TPR's optimization problem are as follows:

$$L(p_r, v, \lambda_1, \lambda_2) = \Pi_r(p_r, v, p_n, s) + \lambda_1 g_1(p_r, v) + \lambda_2 g_2(p_r, v), \text{where} g_1(p_r, v) = Q_c - D_r, g_2(p_r, v) = D_r - \chi \tag{A15}$$

$$\begin{cases} \frac{\partial L(p_r,v,\lambda_1,\lambda_2)}{\partial p_r} = \frac{\partial \Pi_r(p_r,v,p_n)}{\partial p_r} + \lambda_1 \frac{\partial g_1(p_r,v)}{\partial p_r} + \lambda_2 \frac{\partial g_2(p_r,v)}{\partial p_r} = 0 \\ \frac{\partial L(p_r,v,\lambda_1,\lambda_2)}{\partial v} = \frac{\partial \Pi_r(p_r,v,p_n)}{\partial v} + \lambda_1 \frac{\partial g_1(p_r,v)}{\partial v} + \lambda_2 \frac{\partial g_2(p_r,v)}{\partial v} = 0 \\ \lambda_1 g_1(p_r, v) = 0 \\ \lambda_2 g_2(p_r, v) = 0 \\ \lambda_1, \lambda_2 \ge 0 \end{cases}$$

Solving the TPR's optimization problem, we have:

$$p_r^*(p_n, s) = \frac{(2\beta + \delta)(\alpha_r + \gamma p_n + \beta s) - \beta\chi + \beta\delta c_r}{2\beta(\beta + \delta)} \tag{A16}$$

$$v^*(p_n, s) = \frac{1}{2}\left(\frac{\alpha_r + \beta s + \gamma p_n - \chi - \beta c_r}{\beta + \delta} - \frac{\chi}{\delta}\right) \tag{A17}$$

$$\lambda_1^* = \frac{\alpha_r + \beta s + \gamma p_n - \chi - \beta c_r}{\beta + \delta} \tag{A18}$$

$$\lambda_2^* = 0 \tag{A19}$$

Hence, Proposition 2 is proven.

Appendix A.3. Proof of Corollary 1

The first-order partial derivatives of p_r^* to p_n and s can be shown as follows: $\frac{\partial p_r^*}{\partial p_n} = \frac{\gamma(2\beta+\delta)}{2\beta(\beta+\delta)}$, $\frac{\partial p_r^*}{\partial s} = \frac{2\beta+\delta}{2(\beta+\delta)}$. The first order partial derivatives of v^* to p_n and s can be shown as follows: $\frac{\partial v^*}{\partial p_n} = \frac{\gamma}{2(\beta+\delta)}$, $\frac{\partial v^*}{\partial s} = \frac{\beta}{2(\beta+\delta)}$. Obviously, $\frac{\partial p_r^*}{\partial p_n} > 0$, $\frac{\partial p_r^*}{\partial s} > 0$, $\frac{\partial v^*}{\partial p_n} > 0$, $\frac{\partial v^*}{\partial s} > 0$. Thus, the Corollary 1 is proven.

Appendix A.4. Proof of Proposition 3

Proof: To find the optimal sale price of new product p_n, the first order derivative of $\Pi_m(p_n, p_r^*(p_n, s), v^*(p_n, s), s)$ to p_n can be shown as:

$$\frac{\partial \Pi_m(p_n, p_r^*(p_n,s), v^*(p_n,s), s)}{\partial p_n} = \beta c_n - 2\beta p_n + \alpha_n + \frac{\gamma[(2\beta+\delta)(2\gamma p_n + \alpha_r - \gamma c_n) + \beta\delta(c_r - s) - \beta\chi]}{2\beta(\beta+\delta)} \tag{A20}$$

The second order derivative to check for the optimality is as follows:

$$\frac{\partial^2 \Pi_m(p_n, p_r^*(p_n, s), v^*(p_n, s), s)}{\partial {p_n}^2} = \frac{2\beta\left(\gamma^2 - \beta^2\right) + \left(\gamma^2 - 2\beta^2\right)}{\beta(\beta + \delta)} \tag{A21}$$

Based on the assumption 5, it can be obtained that $\frac{\partial^2 \Pi_m(p_n, p_r^*(p_n,s), v^*(p_n,s), s)}{\partial {p_n}^2} < 0$. Thus, $\Pi_m(p_n, p_r^*(p_n, s), v^*(p_n, s), s)$ is concave in p_n. Furthermore, by setting $\frac{\partial^2 \Pi_m(p_n, p_r^*(p_n,s), v^*(p_n,s), s)}{\partial {p_n}^2} = 0$, the optimal sale price of the new product can be derived as follows:

$$p_n^*(s) = \frac{2\beta(\beta + \delta)(\beta c_n + \alpha_n) + \gamma[(2\beta + \delta)(\alpha_r - \gamma c_n) + \beta\delta(c_r - s) - \beta\chi]}{4\beta^2(\beta + \delta) - 2\gamma^2(2\beta + \delta)} \tag{A22}$$

Thus, Proposition 3 is proven.

Appendix A.5. Proof of Proposition 4

The second order derivative to check for the optimality is as follows:

$$\frac{\partial^2 \Pi_{gov}(s, p_n^*, p_r^*, v^*)}{\partial s^2} = \frac{-\delta\left(\beta^2 - \gamma^2\right)\left[\begin{array}{c} 16\beta^5\left(\beta^2 - \gamma^2\right) + 4\beta^2\left(12\beta^4 - 9\beta^2\gamma^2 + \gamma^4\right) \\ +\delta^3\left(16\beta^4 - 4\beta^2\gamma^2 + \gamma^4\right) + \beta\delta^2\left(48\beta^4 - 24\beta^2\gamma^2 + 5\gamma^4\right) \end{array}\right]^{-\beta\delta^2\gamma^6}}{16\beta(\beta+\delta)^2(-2\beta^3 + 2\beta\gamma^2 - 2\beta^2\delta + \gamma^2\delta)^2} \tag{A23}$$

Based on Assumption 5, i.e., $\beta > \gamma > 0$, it can be proven that $12\beta^4 - 9\beta^2\gamma^2 + \gamma^4 > 0$, $\left(16\beta^4 - 4\beta^2\gamma^2 + \gamma^4\right) > 0$ and $48\beta^4 - 24\beta^2\gamma^2 + 5\gamma^4 > 0$. Thus $\frac{\partial^2 \Pi_{gov}(s, p_n^*, p_r^*, v^*)}{\partial s^2} < 0$. Hence, $\Pi_{gov}(s, p_n^*, p_r^*, v^*)$ is concave in s.

Thus, the optimal government's consumption subsidy of the remanufactured product can be derived by setting $\frac{\partial \Pi_{gov}(s,p_n^*,p_r^*,v^*)}{\partial s}$ to zero as follows: $s|_{\frac{\partial \Pi_{gov}(s,p_n^*,p_r^*,v^*)}{\partial s}=0} = \frac{X_1\chi-\delta(X_2c_n+X_1c_r+X_3\alpha_n+X_4\alpha_r)}{\delta\beta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4}$, where $X_1 = \beta\Big[3\beta\delta^2\gamma^6+4\delta^3\gamma^6+16\beta^5\left(\beta^2-\gamma^2\right)^2+4\beta^2\left(\beta^2-\gamma^2\right)\left(12\beta^4-9\beta^2\gamma^2-\gamma^4\right)+\beta\delta^2\left(\beta^2-\gamma^2\right)\left(48\beta^4-24\beta^2\gamma^2-\gamma^4\right)+\delta^3\left(\beta^2-\gamma^2\right)\left(16\beta^4-4\beta^2\gamma^2+3\gamma^4\right)\Big]$, $X_2 = -\gamma\Big[\left(\beta^2-\gamma^2\right)(2\beta+\delta)+\beta^2\delta\Big]\Big[4\beta^2\left(\beta^2-\gamma^2\right)\left(4\beta^2-\gamma^2\right)+\beta\delta\left(\beta^2-\gamma^2\right)\left(32\beta^2-\gamma^2\right)+\delta^2\left(16\beta^4-13\beta^2\gamma^2+\gamma^4\right)+3\beta\delta\gamma^4\Big]$, $X_3 = 2\beta\gamma(\beta+\delta)\Big[\left(4\beta^2+\delta^2\right)\left(2\beta^2-\gamma^2\right)\left(\beta^2-\gamma^2\right)+2\beta\delta\left(4\beta^2-\gamma^2\right)\left(\beta^2-\gamma^2\right)+\beta\delta\left(4\beta^4+\gamma^4\right)+6\delta^2\beta^4\Big]$, $X_4 = 8\beta^3\left(-2\beta^6+6\beta^4\gamma^2-5\beta^2\gamma^4+\gamma^6\right)+2\beta^2\left(-24\beta^6+66\beta^4\gamma^2-43\beta^2\gamma^4+6\gamma^6\right)+\beta\delta^2\left(-48\beta^6+120\beta^4\gamma^2-59\beta^2\gamma^4+6\gamma^6\right)+\delta^3\left(-16\beta^6+36\beta^4\gamma^2-13\beta^2\gamma^4+\gamma^6\right)$, $X_5 = 16\beta^5\left(\beta^2-\gamma^2\right)+4\beta^2\delta\left(12\beta^4-9\beta^2\gamma^2+\gamma^4\right)+\delta^3\left(16\beta^4-4\beta^2\gamma^2+\gamma^4\right)+\beta\delta^2\left(48\beta^4-24\beta^2\gamma^2+4\gamma^4\right)$.

Since $\beta > \gamma$, we can easily verify that $X_1 > 0$, $X_2\langle 0$, $X_3\rangle 0$ and $X_5 > 0$ hold.

We obtain the optimal consumption subsidy as follows:

$$s^* = max\left\{\frac{X_1\chi-\delta(X_2c_n+X_1c_r+X_3\alpha_n+X_4\alpha_r)}{\delta\beta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4},0\right\} \tag{A24}$$

Hence, Proposition 4 is proven.

Appendix A.6. Proof of Corollary 2

The first order partial derivatives of p_n^* to s can be shown as follows: $\frac{\partial p_n^*}{\partial s} = \frac{-\gamma\beta\delta}{4\beta(\beta^2-\gamma^2)+2\gamma(2\beta^2-\gamma^2)}$. Obviously, it is easy to prove that $\frac{\partial p_n^*}{\partial s} < 0$. Thus, Corollary 2 is proven.

Appendix A.7. Proof of Corollary 3

The first order derivative of s^* with respect to χ can be shown as follows: $\frac{\partial s^*}{\partial \chi} = \frac{X_1}{\beta\delta(\beta^2-\gamma^2)X_5+\beta^2\delta^3\gamma^6}$. Since $\beta > \gamma$, $X_1 > 0$ and $X_5 > 0$, we can easily verify that $\frac{\partial s^*}{\partial \chi} > 0$ holds. Similarly, $\frac{\partial s^*}{\partial c_n} > 0$, $\frac{\partial s^*}{\partial c_r} < 0$. Thus, Corollary 3 is proven.

Appendix A.8. Proof of Corollary 5

Under the consumption subsidy policy, the consumers' net payment for the remanufactured product (denoted as ap) can be written as $ap = p_r^* - s^*$. From Propositions 1–4, we have $p_r^{0^*} - ap = \frac{\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\delta s^*}{4(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]}$. From Assumption 5, it can be easily verified that $p_r^{0^*} > ap$.

From Proposition 1–4 and Corollary 6, the consumer net payment for the remanufactured product is reduced (denoted as $p_r^{0^*} - ap$), while consumers can get a higher recycling price of end-of-use products (denoted as $v^* - v^{0^*}$) due to the consumption subsidy. Therefore, the consumer gains from the consumption subsidy policy (denoted as C_g) can be expressed as:

$$C_g = \left(p_r^{0^*} - ap\right) + \left(v^* - v^{0^*}\right) = \frac{\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]s^*}{8\beta(\beta^2-\gamma^2)+4\delta(2\beta^2-\gamma^2)} \tag{A25}$$

Similarly, the TPR can get a higher sale price of remanufactured products (denoted as $p_r^* - p_r^{0^*}$) and meanwhile pay a higher price to obtain the end-of-use product from consumers (denoted as $v^* - v^{0^*}$). Therefore, the TPR's gains from the consumption subsidy policy (denoted as R_g) can be expressed as:

$$R_g = \left(p_r^* - p_r^{0^*}\right) - \left(v^* - v^{0^*}\right) = \frac{\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right]s^*}{8\beta(\beta^2-\gamma^2)+4\delta(2\beta^2-\gamma^2)} \tag{A26}$$

Based on Assumption 5, it can be derived that $C_g > R_g > 0$ and $C_g + R_g = s^*$.

$$p_r^* - p_r^{0^*} = \frac{(2\beta+\delta)\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right]s^*}{4(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]} \tag{A27}$$

$$D_r^* - D_r^{0^*} = \frac{\beta\delta s^*\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right]}{4(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]} \tag{A28}$$

Based on Assumption 5, we obtain $p_r^* - p_r^{0^*} > 0$, $D_r^* - D_r^{0^*} > 0$

Hence, Corollary 5 is proven.

Appendix A.9. Proof of Corollary 6

From Propositions 1–4, we have:

$$p_n^* - p_n^{0^*} = \frac{-\beta\gamma\delta s^*}{4\beta(\beta^2-\gamma^2)+2\delta(2\beta^2-\gamma^2)} \tag{A29}$$

$$D_n^* - D_n^{0^*} = -\frac{\gamma\delta s^*}{4(\beta+\gamma)} \tag{A30}$$

$$v^* - v^{0^*} = \frac{\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right]\beta s^*}{4(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]} \tag{A31}$$

$$(D_n^* + D_r^*) - \left(D_n^{0^*} + D_r^{0^*}\right) = \frac{(\beta-\gamma)\delta s^*\left[2\beta\left(2\beta^2-\gamma^2\right)+\gamma\delta(2\beta-\gamma)+2\beta^2(\gamma+2\delta)\right]}{4(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]} \tag{A32}$$

Based on Assumption 5, we obtain $p_n^* - p_n^{0^*} < 0$, $D_n^* - D_n^{0^*} < 0$, $v^* - v^{0^*} > 0$ and $(D_n^* + D_r^*) - \left(D_n^{0^*} + D_r^{0^*}\right) > 0$.

Thus, Corollary 6 is proven.

Appendix A.10. Proof of Corollary 7

From Propositions 1–5, we have $\theta_r = \frac{\Pi_r^*}{\Pi_r^{0^*}} = \left\{\frac{\beta\delta\left[4\beta\left(\beta^2-\gamma^2\right)+\delta\left(4\beta^2-3\gamma^2\right)\right]s^*+Y_1}{Y_1}\right\}^2$.

We can easily verify that if $Y_1 > 0$, $\Pi_r^* > \Pi_r^{0^*}$, otherwise, $\Pi_r^* < \Pi_r^{0^*}$.

Similarly, we have $\theta_m = \frac{\Pi_m^*}{\Pi_m^{0^*}} = \left(1 + \frac{\beta\gamma\delta s^*}{Y_2}\right)^2$

It can easily verified that if $Y_2 > 0$, $\Pi_m^* > \Pi_m^{0^*}$, otherwise, $\Pi_m^* < \Pi_m^{0^*}$.

Similarly, we have

$$\Pi_{SC}^* - \Pi_{SC}^{0^*} = \frac{\left[\beta A_1(\delta s^*+2\chi)+2\gamma\delta A_2 c_n - 2\beta\delta A_1 c_r + 2\delta A_3\alpha_r - 4\beta\gamma^3\delta^2(\beta+\delta)\alpha_n\right]s^*}{16(\beta+\delta)[2\beta(\beta^2-\gamma^2)+\delta(2\beta^2-\gamma^2)]^2} \tag{A33}$$

We can easily verify that if $\beta A_1(\delta s^*+2\chi)+2\gamma\delta A_2 c_n - 2\beta\delta A_1 c_r + 2\delta A_3\alpha_r - 4\beta\gamma^3\delta^2(\beta+\delta)\alpha_n > 0$, $\Pi_{SC}^* > \Pi_{SC}^{0^*}$, otherwise, $\Pi_{SC}^* < \Pi_{SC}^{0^*}$. Thus, Corollary 7 is proven.

Appendix A.11. Proof of the Impact of c_r on Optimal Decisions of CLSC Members, Government Decision and So On

Differentiating the equilibriums and the CLSC's performance without a government consumption subsidy, with respect to c_r, the following relationships are obtained:

$$\frac{\partial\, p_n^{0^*}}{\partial c_r} = \frac{\gamma\beta\delta}{4\beta^2(\beta+\delta)-2\gamma^2(2\beta+\delta)} > 0 \tag{A34}$$

$$\frac{\partial\, p_r^{0^*}}{\partial c_r} = \frac{\delta\left[4\beta^2(\beta+\delta) - \gamma^2(2\beta+\delta)\right]}{4(\beta+\delta)[2\beta^2(\beta+\delta) - \gamma^2(2\beta+\delta)]} > 0 \tag{A35}$$

$$\frac{\partial v^{0^*}}{\partial c_r} = \frac{-\beta\left[4\beta^2(\beta+\delta) - \gamma^2(4\beta+3\delta)\right]}{4(\beta+\delta)[2\beta^2(\beta+\delta) - \gamma^2(2\beta+\delta)]} < 0 \tag{A36}$$

$$\frac{\partial \Pi_r^{0^*}}{\partial c_r} = -\frac{B_1}{8(\beta+\delta){B_2}^2}\left\{\beta B_1(\chi-\delta c_r) + \gamma\delta B_2 c_n + \delta\left[2\beta\left(2\beta^2-\gamma^2\right) + \delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r + 2\beta\gamma\delta(\beta+\delta)\alpha_n\right\} < 0 \tag{A37}$$

$$\frac{\partial \Pi_m^{0^*}}{\partial c_r} = \frac{\gamma\delta\{-\beta\gamma\chi - B_2 c_n + \beta\gamma\delta c_r + 2\beta(\beta+\delta)\alpha_n + \gamma(2\beta+\delta)\alpha_r\}}{4(\beta+\delta)B_2} > 0 \tag{A38}$$

$$\frac{\partial D_n^{0^*}}{\partial c_r} = \frac{\gamma\delta}{4(\beta+\delta)} > 0 \tag{A39}$$

$$\frac{\partial D_r^{0^*}}{\partial c_r} = \frac{-\beta\delta B_1}{4(\beta+\delta)B_2} < 0 \tag{A40}$$

$$\begin{aligned}\frac{\partial \Pi_{gov}^{0^*}}{\partial c_r} = \frac{1}{8(\beta+\delta){B_2}^2}\Big\{&2\beta\gamma^3\delta(\beta+\delta)\alpha_n \\ &-\delta\left\{16\beta^2\left(\beta^2-\gamma^2\right)^2 + 2\beta\delta\left(8\beta^2-9\gamma^2\right)\left(2\beta^2-\gamma^2\right) + \delta^2 B_3\right\}\alpha_r \\ &-\beta\left\{16\beta^2\left(\beta^2-\gamma^2\right)^2 + 4\beta\delta\left(\beta^2-\gamma^2\right)\left(8\beta^2-5\gamma^2\right) + \delta^2\left[B_3+2\gamma^4\right]\right\}(\chi-\delta c_r) \\ &-\gamma\delta B_2\left[8\beta\left(\beta^2-\gamma^2\right) + \delta\left(8\beta^2-5\gamma^2\right)\right]c_n\Big\}\end{aligned} \tag{A41}$$

$$\begin{aligned}\frac{\partial \Pi_{CS}^{0^*}}{\partial c_r} &= \frac{\gamma\delta\{2\beta(\beta+\delta)\alpha_n + \gamma(2\beta+\delta)\alpha_r - \beta\gamma(\chi-\delta c_r) - B_2 c_n\}}{16\beta^2(\beta+\delta)^2} \\ &+ \frac{B_1\left\{-2\beta\gamma\delta(\beta+\delta)\alpha_n - \delta\left[2\beta\left(2\beta^2-\gamma^2\right) + \delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r - \beta B_1(\chi-\delta c_r) - \gamma\delta B_2 c_n\right\}}{16(\beta+\delta){B_2}^2}\end{aligned} \tag{A42}$$

where $B_1 = 4\beta\left(\beta^2-\gamma^2\right) + \delta\left(4\beta^2-3\gamma^2\right)$,$B_2 = \delta\left(2\beta^2-\gamma^2\right) + 2\beta\left(\beta^2-\gamma^2\right)$,$B_3 = 4\left(\beta^2-\gamma^2\right)\left(4\beta^2-\gamma^2\right) + \gamma^4$.

When $2\beta\gamma^3\delta(\beta+\delta)\alpha_n - \delta\left\{16\beta^2\left(\beta^2-\gamma^2\right)^2 + 2\beta\delta\left(8\beta^2-9\gamma^2\right)\left(2\beta^2-\gamma^2\right) + \delta^2 B_3\right\}\alpha_r - \beta\left\{16\beta^2\left(\beta^2-\gamma^2\right)^2 + 4\beta\delta\left(\beta^2-\gamma^2\right)\left(8\beta^2-5\gamma^2\right) + \delta^2\left[B_3+2\gamma^4\right]\right\}(\chi-\delta c_r) - \gamma\delta B_2\left[8\beta\left(\beta^2-\gamma^2\right) + \delta\left(8\beta^2-5\gamma^2\right)\right]c_n > 0$, $\frac{\partial \Pi_{gov}^{0^*}}{\partial c_r} > 0$; otherwise, $\frac{\partial \Pi_{gov}^{0^*}}{\partial c_r} \le 0$.

When $\gamma\delta\{2\beta(\beta+\delta)\alpha_n + \gamma(2\beta+\delta)\alpha_r - \beta\gamma(\chi-\delta c_r) - B_2 c_n\}{B_2}^2 + \beta^2(\beta+\delta)B_1\left\{-2\beta\gamma\delta(\beta+\delta)\alpha_n - \delta\left[2\beta\left(2\beta^2-\gamma^2\right) + \delta\left(4\beta\right.\right.\right.$ 0, $\frac{\partial \Pi_{CS}^{0^*}}{\partial c_r} > 0$; otherwise, $\frac{\partial \Pi_{gov}^{0^*}}{\partial c_r} \le 0$.

Differentiating the equilibriums and the CLSC's performance with a government consumption subsidy, with respect to c_r, the following relationships are obtained:

$$\frac{\partial s^*}{\partial c_r} = \frac{-\delta X_1}{\beta\delta(\beta^2-\gamma^2)X_5 + 9\beta^4\delta^3\gamma^4} < 0 \tag{A43}$$

$$\frac{\partial p_n^*}{\partial c_r} = \frac{2\beta^4\delta^2\gamma(\beta+\delta)B_1}{\beta\delta(\beta^2-\gamma^2)X_5 + 9\beta^4\delta^3\gamma^4} > 0 \tag{A44}$$

$$\begin{aligned}\frac{\partial p_r^*}{\partial c_r} = -\frac{1}{\beta\delta(\beta^2-\gamma^2)X_5 + 9\beta^4\delta^3\gamma^4}\Big\{&16\beta^5\left(\beta^2-\gamma^2\right)^2 + 4\delta\left(8\beta^2+\gamma^2\right)\left(\beta^2-\gamma^2\right)^2 \\ &+\beta\delta^2\left[4\left(\beta^2-\gamma^2\right)\left(2\beta^2-\gamma^2\right)^2 + 8\gamma^6 + \beta^2\gamma^4\right]16\beta^5\left(\beta^2-\gamma^2\right)^2 \\ &+4\delta\left(8\beta^2+\gamma^2\right)\left(\beta^2-\gamma^2\right)^2 + \beta\delta^2\left[4\left(\beta^2-\gamma^2\right)\left(2\beta^2-\gamma^2\right)^2 + 8\gamma^6 + \beta^2\gamma^4\right]\Big\} < 0\end{aligned} \tag{A45}$$

$$\frac{\partial v^*}{\partial c_r} = -\frac{\beta^4\delta{B_1}^2}{\beta\delta(\beta^2-\gamma^2)X_5 + 9\beta^4\delta^3\gamma^4} < 0 \tag{A46}$$

$$\begin{aligned}\frac{\partial \Pi_m^*}{\partial c_r} &= -\frac{4\beta^5\gamma\delta^2(\beta+\delta)B_1B_2}{[\beta\delta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4]^2}\\ &\Big\{\Big[4\beta^2\big(\beta^2-\gamma^2\big)^2+\beta\delta\big(\beta^2-\gamma^2\big)\big(8\beta^2-\gamma^2\big)+\big(4\beta^4-\beta^2\gamma^2-2\gamma^4\big)\delta^2\Big]c_n\\ &+\beta\gamma B_1(\chi-\delta c_r)-\beta(\beta+\delta)\big(B_1+2\gamma^2\delta\big)\alpha_n-\gamma\Big[4\beta^2\big(\beta^2-\gamma^2\big)+\beta\delta\big(4\beta^2-\gamma^2\big)+2\gamma^2\delta\Big]\alpha_r\Big\}>0\end{aligned} \tag{A47}$$

$$\begin{aligned}\frac{\partial \Pi_r^*}{\partial c_r} &= \frac{2\beta^3(\beta+\delta)\delta{B_1}^2}{[\beta\delta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4]^2}\{\beta^3{B_1}^2(-\chi+\delta c_r)\\ &-\gamma\delta B_2\Big[2\beta\big(\beta^2-\gamma^2\big)\big(5\beta^2-\gamma^2\big)+\delta\big(10\beta^4-10\beta^2\gamma^2+\gamma^4\big)\Big]c_n\\ &+\beta\gamma\delta\Big[4\beta^2\big(\beta^2-\gamma^2\big)^2+2\beta\delta\big(\beta^2-\gamma^2\big)\big(4\beta^2-\gamma^2\big)+2\beta\delta\gamma^4+\delta^2\big(4\beta^4-2\beta^2\gamma^2+\gamma^4\big)\Big]\alpha_n\\ &-\Big[4\beta^2\big(\beta^2-\gamma^2\big)^2\big(4\beta^2-\gamma^2\big)+2\beta\delta\big(16\beta^6-32\beta^4\gamma^2+17\beta^2\gamma^4-2\gamma^6\big)\\ &+\delta^2\big(16\beta^6-28\beta^4\gamma^2+11\beta^2\gamma^4-\gamma^6\big)\Big]\alpha_r\}\end{aligned} \tag{A48}$$

$$\frac{\partial D_n^*}{\partial c_r} = \frac{\beta^2\delta^2\gamma B_1B_2}{\beta\delta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4}>0 \tag{A49}$$

$$\frac{\partial D_r^*}{\partial c_r} = -\frac{\beta^4\delta^2{B_1}^2}{\beta\delta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4}<0 \tag{A50}$$

$$\begin{aligned}\frac{\partial \Pi_{gov}^*}{\partial c_r} &= \frac{\beta\delta}{[\beta\delta(\beta^2-\gamma^2)X_5+9\beta^4\delta^3\gamma^4]}\{\beta^3{B_1}^2(-\chi+\delta c_r)\\ &-\gamma\delta B_2\Big[2\beta\big(\beta^2-\gamma^2\big)\big(5\beta^2-\gamma^2\big)+\delta\big(10\beta^4-10\beta^2\gamma^2+\gamma^4\big)\Big]c_n\\ &+\beta\gamma\delta\Big[4\beta^2\big(\beta^2-\gamma^2\big)^2+2\beta\delta\big(\beta^2-\gamma^2\big)\big(4\beta^2-\gamma^2\big)+2\beta\delta\gamma^4\\ &+\delta^2\big(4\beta^4-2\beta^2\gamma^2+\gamma^4\big)\Big]\alpha_n\\ &-\Big[4\beta^2\big(\beta^2-\gamma^2\big)^2\big(4\beta^2-\gamma^2\big)+2\beta\delta\big(16\beta^6-32\beta^4\gamma^2+17\beta^2\gamma^4-2\gamma^6\big)\\ &+\delta^2\big(16\beta^6-28\beta^4\gamma^2+11\beta^2\gamma^4-\gamma^6\big)\Big]\alpha_r\}\end{aligned} \tag{A51}$$

When $\beta^3{B_1}^2(-\chi+\delta c_r)-\gamma\delta B_2\Big[2\beta\big(\beta^2-\gamma^2\big)\big(5\beta^2-\gamma^2\big)+\delta\big(10\beta^4-10\beta^2\gamma^2+\gamma^4\big)\Big]c_n$, $\frac{\partial \Pi_{gov}^*}{\partial c_r}>0, \frac{\partial \Pi_r^*}{\partial c_r}>0$; otherwise, $\frac{\partial \Pi_{gov}^*}{\partial c_r}\le 0$, $\frac{\partial \Pi_r^*}{\partial c_r}\le 0$.

Appendix A.12. Proof of the Impact of γ on the Optimal Decisions of CLSC Members, Government Decisions and So On

Differentiating the equilibriums and the CLSC's performance without a government consumption subsidy, with respect to γ, the following relationships are obtained:

$$\frac{\partial p_n^{0^*}}{\partial \gamma} = \frac{\beta B_4(-\chi+\delta c_r)+4\beta\gamma(\beta+\delta)(2\beta+\delta)\alpha_n+(2\beta+\delta)B_4\alpha_r}{2{B_2}^2}>0 \tag{A52}$$

$$\frac{\partial p_r^{0^*}}{\partial \gamma} = \frac{(2\beta+\delta)\Big\{{B_2}^2c_n+2\beta(\beta+\delta)\Big[2\beta^2\gamma(-\chi+\delta c_r)+B_4\alpha_n+2\beta\gamma(2\beta+\delta)\alpha_r\Big]\Big\}}{4\beta(\beta+\delta){B_2}^2}>0 \tag{A53}$$

$$\frac{\partial v^{0^*}}{\partial \gamma} = \frac{c_n}{4(\beta+\delta)}+\frac{\beta\Big\{\Big[2\beta^2\gamma(-\chi+\delta c_r)+B_4\alpha_n+2\beta\gamma(2\beta+\delta)\alpha_r\Big]\Big\}}{2(\beta+\delta){B_2}^2}>0 \tag{A54}$$

$$\begin{aligned}\frac{\partial \Pi_m^{0^*}}{\partial \gamma} &= \frac{[-B_2c_n+\beta\gamma(-\chi+\delta c_r)+2\beta(\beta+\delta)\alpha_n+\gamma(2\beta+\delta)\alpha_r]}{4\beta(\beta+\delta){B_2}^2}\{\gamma(2\beta+\delta)B_2c_n\\ &+2\beta(\beta+\delta)\Big[\beta^2(-\chi+\delta c_r)+(2\beta+\delta)(\gamma\alpha_n+\beta\alpha_r)\Big]\}>0\end{aligned} \tag{A55}$$

$$\begin{aligned}\frac{\partial \Pi_r^{0^*}}{\partial \gamma} &= \frac{1}{8\beta(\beta+\delta){B_2}^3}\{\gamma\delta B_2 c_n + \beta B_1(\chi - \delta c_r) + 2\beta\gamma\delta(\beta+\delta)\alpha_n \\ &+\delta\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r\}\{{B_2}^2 c_n \\ &+2\beta(\beta+\delta)\left[2\beta^2\gamma(-\chi+\delta c_r)+B_4\alpha_n+2\beta\gamma(2\beta+\delta)\alpha_r\right]\} > 0\end{aligned} \tag{A56}$$

$$\frac{\partial \mathrm{D}_n^{0^*}}{\partial \gamma} = \frac{2\gamma(2\beta+\delta)c_n + \beta(-\chi+\delta c_r) + (2\beta+\delta)\alpha_r}{4\beta(\beta+\delta)} > 0 \tag{A57}$$

$$\frac{\partial \mathrm{D}_r^{0^*}}{\partial \gamma} = \frac{\delta c_n}{4(\beta+\delta)} + \frac{\beta\delta\left[B_4\alpha_n + 2\beta\gamma(2\beta+\delta)\alpha_r + 2\beta^2\gamma(-\chi+\delta c_r)\right]}{2{B_2}^2} > 0 \tag{A58}$$

$$\begin{aligned}\frac{\partial \Pi_{CS}^{0^*}}{\partial \gamma} &= \frac{(2\beta+\delta)\alpha_r + 2\gamma(2\beta+\delta)c_n + \beta(-\chi+\delta c_r)}{16(\beta+\delta)^2\beta^3}[-B_1 c_n + \beta\gamma(-\chi+\delta c_r) + 2\beta(\beta+\delta)\alpha_n \\ &+\gamma(2\beta+\delta)\alpha_r] + \frac{{B_2}^2 c_n + 2\beta(\beta+\delta)\left[B_4\alpha_n + 2\beta\gamma(2\beta+\delta)\alpha_r + 2\beta^2\gamma(-\chi+\delta c_r)\right]}{16(\beta+\delta)\beta{B_2}^3} \\ &\left\{2\beta\gamma\delta(\beta+\delta)\alpha_n + \delta\left[2\beta\left(2\beta^2-\gamma^2\right)+\delta\left(4\beta^2-\gamma^2\right)\right]\alpha_r + \gamma\delta B_2 c_n + \beta B_1(\chi-\delta c_r)\right\} > 0\end{aligned} \tag{A59}$$

$$\frac{\partial \Pi_{gov}^{0^*}}{\partial \gamma} = \frac{\partial \Pi_m^{0^*}}{\partial \gamma} + \frac{\partial \Pi_r^{0^*}}{\partial \gamma} + \frac{\partial \Pi_{CS}^{0^*}}{\partial \gamma} > 0 \tag{A60}$$

where $B_4 = 2\beta\left(\beta^2+\gamma^2\right) + \delta\left(2\beta^2+\gamma^2\right)$.

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