Article

Optimize the Banker's Multi-Stage Decision-Making and the Mechanism of Pay Contract Influencing on Bank Default Risk in the Long-Term Model

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Abstract: In recent years, researchers have been devoted to illustrating the correlation between bankers' pay contracts and a bank's risk-taking behavior where corporate governance is concerned, especially throughout the past four decades and by using empirical analysis. Despite being a widespread concern, the causality of this relationship is not thoroughly understood. We initiate this research by modeling bankers' multi-stage decisions of option investment and bond investment from the perspective of theoretical analysis, and by analyzing the function image results using data from Wells Fargo & Co. from the ExecuComp, BvD Orbis, and CRSP-COMPUSTAT databases. We aim to deeply explore the mechanism of how compensation influencing on risk. We are the first to find that it has a "risk cap", which is the optimal risk level to maximize the return of decision-making. We are also the first to discover the optimal decision coefficient level to maximize the decision return, during which the internal causes and mechanisms of the impact of bankers' compensation on a bank's default risk are revealed. We also illustrate the influence of the number of periods. We expect our findings to provide advice for establishing policies when designing pay contracts.

Keywords: decision-making; multi-stage; pay contract; bank default risk; option investment; bond investment; long-term model; BS model

1. Introduction

1.1. Research Background and Framework Diagram

In 1976, Jensen and Meckling [1] first published their paper regarding executive compensation, which belongs to the field of corporate governance. Since then, this field has become an object of increasing concern for related scholars. Furthermore, after the financial depression of 2007–2009, researchers realized that banks have unique features different from other non-financial institutions, which could lead to the systematic risk of the whole economic system, even cross-nationally, when the risk level is high or bankruptcy occurs. However, the way in which the pay contracts of bankers influence a bank's risk-taking behavior or, in other words, the mechanism behind the influence of pay contracts on a bank's default risk in the long term is still not thoroughly understood.

Figure 1 outlines a schematic diagram of the effect of the decision-making of bankers on the relationship between compensation and the default risk of banks. As is shown in Figure 1, we divide the total compensation into two parts, that is, inside equity and inside debt. Inside equity refers to equity-based compensation, for instance, restricted stock, stock options, and other instruments, of which the value lies in the equity returns in the future. Meanwhile, inside debt is defined as the sum of benefit pensions and deferred compensation, as per Jensen and Meckling [1]. The multi-stage

decision-making of a CEO contains the decisions of option investment and bond investment for some times (or in multiple stages) in this paper. The objective of a banker is to maximize the decision return when making decisions. According to the different proportions of option investment and bond investment in the total decision-making process, and the variance of default risk level, the return of executives varies. This, in turn, affects the compensation levels and the structure of pay contracts. The aim of this paper is to determine the optimal level of the default risk and decision weight level (invest more option or bond) in order to find out the deeper mechanism regarding how compensation influences the risk levels of banks—namely, through the decisions of bankers.

1.2. Related Studies

Many researchers have studied structure design from the following perspectives. From the empirical works of their predecessors, Wang and Lin (2009) [2] revealed that risk management can be taken into consideration to select a strategy for the Bank of Kaohsiung. Riachi et al. (2013) [3] examined the effect of corporate asset-backed securitization on managerial compensation. Li et al. (2011) [4] investigated the relationship between stock mispricing, the investment of the firm, and the compensation of bankers from an empirical perspective. Latham and Braun (2010) [5] indicated that there is a positive relationship between corporate debt level and the decision of an IPO (Initial Public Offering). Kokkinis (2019) [6] explored the impacts of the rule of a bonus cap on managers' decision incentives, which is linked to different degrees of risk. Kim (2015) [7] found that, as the compensation for executives increases, the greater both the investment in R&D (research and development) and the profits become. Kang et al. (2006) [8] implied that executive incentive compensation is endogenously determined, as are the corporate investment policies. Huang et al. (2012) [9] discussed, in detail, the results of the specific interactions between investment and financing decisions. Ho et al. (2016) [10] offered CEO overconfidence as an explanation for the cross-sectional heterogeneity in a bank's risk-taking behavior. Gulati and Yasin (1994) [11] presented the application of an expert system to time the market right.

On a theoretical level, Grundy and Li (2010) [12] developed a model that is able to predict the increase of corporate investment levels. Eisdorfer et al. (2013) [13] examined the factors that affect the quality of the investment decisions of the corporation. Eastburn and Boland (2015) [14] revealed that bankers narrow the focus of their concentration due to the complexity of their decisions. Bliss and Rosen (2001) [15] examined the relationship between mergers and CEO compensation during 1986 to 1995.

From the perspective of the relationship of bank risk and compensation, Belkhir and Boubaker (2013) [16] showed that CEO inside debt holdings yield a positive influence on hedging. Li et al. [17] provided an advanced inside debt metric superior to the previously used ones. Li and Zhang [18] highlighted the positive relationship between the proportion of female directors and short-maturity debt. Li et al. [19] found that the greater the amount of inside debt that a CEO holds, the lower the possibility to issue convertibles rather than straight debt by firms. Merton (1974) [20] used a framework to design the compensation of bankers based on the concept of the European call option of Black and Scholes.

In addition to the abovementioned research, our study provides significant insight to better understand the stability of banks or other financial institutions. Just as Abad-Segura et al. [21] wrote, companies should be obliged to work toward social responsibility by using a sustainability approach, including banks.

Overall, there are limited studies regarding the mechanism behind the influence of compensation on risk, especially on a theoretical level. Thus, attempting to uncover this mechanism on a theoretical level is also a particular aim of our research.

Figure 1. Schematic diagram of the effect of the decision-making of bankers on the relationship between compensation and the default risks of banks. (Figure 1 was created by the author of this article according to the framework of the decision-making model presented in this paper).

1.3. Research Progress

In this article, we initiate this research by setting up a multi-stage decision-making model, containing both option investment and bond investment, to deeply explore the mechanism of how compensation influencing on risk, based on our previous work (Ma et al., 2020 [22,23]) on a banker's long-term compensation and the design of pay contracts. We demonstrate the relationship between the decision return of multi-stages and both the risk and the decision weight of the bank by developing a particular formula. After setting up this model, we analyze the function image result using data from Wells Fargo & Co. (New York, NY, USA). We use CEO remuneration data, U.S. bank financial data, and stock price data from the ExecuComp, BvD Orbis, and CRSP-COMPUSTAT databases.

The objective of our study is to illustrate the mechanism responsible for how a banker's compensation influences the bank's risk level. Firstly, we find an optimal situation regarding the default risk of the bank to maximize the banker's decision return in multi-stages of option investment and bond investment. Secondly, we are the first to find the presence of a "risk cap", where the maximum point in this cap is the optimal risk level to maximize the return of decision-making. At the same time, we are also the first to determine the optimal decision structure (to invest option or bond more) to maximize the decision return, during which the internal causes and mechanisms of the impact of a banker's compensation on the bank's default risk are revealed. Thirdly, we also illustrate the sensitivity of decision return on the number of periods. We expect our findings to provide advice for establishing policies when designing pay contracts.

The remainder of this paper is organized as follows. In Section 2, we set up a model of multi-stage decision-making, containing both option investment and bond investment, based on long-term total compensation. Then, we analyze the Vega and sensitivity, as well as the nature of other functions, and find the optimal level of the default risk and the decision weight to maximize the decision return in Section 3 (also shown in Figure 1). Then, we conduct curve fitting and analyze the function images in Section 4. The last section concludes the paper.

2. Materials and Methods

In this section, we made a multi-stage decision-making model of the banker to illustrate the mechanism responsible for how a banker's compensation influences the bank's risk level. We divided the decision into two parts, namely option investment and bond investment, and structured their relationship with respect to the decision return when the risk or decision weight varied. Firstly, Table 1 summarizes the notations and definitions used in our model.

Table 1. Notations and definitions in theoretical analysis.

Classification	Notations	Definition	Notations	Definition	Notations	Definition	Notations	Definition	Notations	Definition
Bank	A	the bank asset	E	the bank equity	D	the bank debt	ROE	return on equity	σ_θ	volatility of ROE
Compensation	TC	total long-term compensation	IE	inside equity	ID	inside debt	ω_1	the fraction of equity	ω_2	the fraction of debt
Decision	D_T	the banker's decision-making	D_1	option investment	D_2	bond investment	α	the fraction of D_1	R	return on D_T
Option investment	R_1	return on D_1	Δ	the time interval	$\kappa/kappa$	the executive's tenure	$E_q(i\Delta)$	the bank equity	λ	number of decision stages
	$C(\Delta, K_C)$	Black and Scholes call option price	K_C	strike price of the call option	$N_x(x)$	standard cumulative normal distribution	$n_x(x)$	standard normal density	r	the risk-free rate
Bond investment	R_2	return on D_2	C_i	cash flow for period i	SR_r	the stock return ratio	ε_1	slope in Equation (23)	ε_2	intercept in Equation (23)
	ζ_1	slope in Equation (21)	ζ_2	intercept in Equation (21)	η_1	slope in Equation (22)	η_2	intercept in Equation (22)	σ_S	volatility of stock return ratio
Optimization	$Vega_R$	Vega of decision return	SEN_R	sensitivity of decision return	DS_α	decision structure incentives	$\sigma_{\theta 1}$	minimum point of σ_θ	$\sigma_{\theta 2}$	maximum point of σ_θ
	$f_1(\sigma_\theta)$	option investment	$f_2(\sigma_\theta)$	bond investment	$g_1(\lambda)$	for the monotonicity of DS_α	$g_2(\lambda)$	For the monotonicity of DS_α	ν	part of expression of $\sigma_{\theta(1, 2)}$
	σ_θ^*	optimal default risk	α^*	optimal decision structure	SSE	sum of squares due to error	$R - Square$	coefficient of determination	$RMSE$	root mean square error

Note: The notation E in classification bank and $E_q(i\Delta)$ in option investment are both the bank equity; D_1 in classification option investment and $f_1(\sigma_\theta)$ in optimization are both the decision of option investment; D_2 in classification bond investment and $f_2(\sigma_\theta)$ in optimization are both the decision of bond investment.

2.1. Model Assumptions

To develop the banker's multi-stage decision-making model, we made the following assumptions:

- Meet All Assumptions 1–7 of the BS (Black and Scholes) Model

See Assumptions 1–7 of Black and Scholes (1973) [24].

That is,

“(a) The short-term interest rate is known and is constant through time. (b) The variance rate of the return on the stock is constant. (c) The stock pays no dividends or other distributions. (d) The option can only be excised at maturity. (e) There are no transaction costs in buying or selling the stock or the option. (f) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate. (g) There are no penalties to short selling.”

- Assumption 8

We divided total compensation TC in the long term into two parts: inside equity (or equity-based compensation) IE and inside debt (or debt-based compensation) ID :

$$TC = IE + ID \quad (1)$$

- Assumption 9

We defined the decision-making during the executive's tenure D_T as the sum of the following two parts: decision of option investment D_1 and decision of bond investment D_2 :

$$D_T = \alpha D_1 + (1 - \alpha) D_2 \quad (2)$$

where α is the decision structure coefficient, which stands for the fraction of D_1 in D_T . As shown in Equation (2), we assumed bankers to make investment decisions (bond and option investment) and to choose which type of decision was more appropriate (bond or option investment).

- Assumptions 10–12

See Assumptions 10–12 of Ma et al., 2020 [22].

- Assumption 13

We assumed that the decision number and the frequency of option investment was the same as that of bond investment.

2.2. Model Setup

2.2.1. Compensation and Investment Decision

We used a similar framework to Merton [20] to build our model.

$$IE = \omega_1 E \quad (3)$$

where IE is the inside equity of the banker, E is the bank equity, and ω_1 is the proportion of inside equity paid from bank equity. Similarly:

$$ID = \omega_2 D \quad (4)$$

where ID is the inside debt of the banker, D is the bank debt, and ω_2 is the fraction of inside debt paid from bank debt. Besides, bank asset A equals bank debt D plus bank equity E .

Basing on Equation (1), (3), and (4), we have the following

$$TC = \omega_1 E + \omega_2 D \quad (5)$$

Consider a bank with zero-coupon debt with face value $E_q((i-1)\Delta)$, maturity $E_q(i\Delta)$, and equity for all $i \in \{1, 2, \dots, \lambda\}$. If the value $E_q(i\Delta)$ of the bank's net assets on date κ exceeds $E_q((i-1)\Delta)$, the debt is paid off and the balance is paid to the bank's equity holders. As a consequence, the decision return to the banker's option investment for once on date κ is as follows:

$$R_1 = \Delta E = \max[E_q(i) - E_q(i-1), 0] \quad (6)$$

Assume the decision stage λ and the frequency of option investment are the same as that of bond investment (Assumption 10). The decision return for λ stages of option investment is:

$$R_1 = \max[E_q(i\Delta) - E_q((i-1)\Delta), 0] \quad (7)$$

where:

$$\Delta = \kappa / \lambda \quad (8)$$

Similarly, according to the traditional bond price model, the Dividend Discount Model (DDM) [25], the decision return to the banker's bond investment for once is as follows:

$$R_2 = \Delta D = SR_r \frac{C_i}{(1+r_i)^i} \quad (9)$$

where C_i is the cash flow for period i , SR_r is the stock return ratio, and r is the risk-free rate.

Furthermore, the decision return for λ stages of bond investment is:

$$R_2 = SR_r \sum_{i=1}^{\lambda} \frac{C_i}{(1+r_i)^i} \quad (10)$$

Pay attention to the fact that, in Equation (9), debts are the opposite of investment signs and their absolute values are equal. Besides, for ease of calculation, we used C_1 to replace C_i .

Thus, on the one hand, given the default risk value, the banker's objective is to maximize the decision return value:

$$\max R \quad (11)$$

subject to:

$$R = \alpha R_1 + (1 - \alpha) R_2 \quad (12)$$

where α is the decision structure coefficient, which stands for the fraction of R_1 in R .

On the other hand, on the basis of Equations (5), (6), and (9), we have:

$$\Delta A = \Delta E + \Delta D \quad (13)$$

where ΔA , ΔE , and ΔD are the increments of bank assets, bank equity, and bank debt value, respectively. We also have:

$$\Delta IE = \omega_1 R_1 \sim \sigma_\theta \quad (14)$$

$$\Delta ID = \omega_2 R_2 \sim \sigma_\theta \quad (15)$$

where ΔIE is the increment of inside equity, with respect to σ_θ , and ΔID is the increment of inside debt, with respect to σ_θ , too. From Equations (6), (9), (14), and (15), we can see how pay contracts affect bank risk, that is, through the decisions of the banker. Different decisions yield different returns, which in turn change risk. The risk further affects the compensation, iterates, and repeats, as shown in Figure 1.

2.2.2. Decision of Option Investment

The value of the decision return of the banker's option investment can now be determined using the European call option pricing theory of Black and Scholes [24]. The first step is to define $C(\Delta, K_C)$, the current value of a call option on the bank with strike price K_C , and then we can obtain the following:

$$C(\Delta, K_C) = E[\exp(-r\Delta) \max\left(\frac{E_q(\Delta)}{E_q(0)} - K_C, 0\right)] = N_x(d_1(\Delta)) - K_C \exp(-r\Delta) N_x(d_2(\Delta)) \quad (16)$$

where $N_x(x) = \int_{-\infty}^x n_x(t) dt$ is the standard cumulative normal distribution and $n_x(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$ is the standard normal density:

$$d_1(\Delta) = \frac{1}{\sigma_\theta \sqrt{\Delta}} \left[\ln\left(\frac{1}{K_C}\right) + \left(\frac{1}{2}\sigma_\theta^2 + r\right)\Delta \right] \quad (17)$$

$$d_2(\Delta) = d_1(\Delta) - \sigma_\theta \sqrt{\Delta} \quad (18)$$

Based on Assumption 12, the decision return for λ stages of option investment can be written as follows, using the risk-neutral pricing in Equation (19), which is similar to Jokivuolle et al. [26]:

$$R_1 = \sum_{i=1}^{\lambda} E[\exp(-ri\Delta) \max[E_q(i\Delta) - E_q((i-1)\Delta), 0]] \quad (19)$$

2.2.3. Decision of Bond Investment

Stock returns are affected by the company's profitability. Therefore, the bank's stock return is related to changes in the bank's net assets (see Vuolteenaho and Tuomo (2002) [27]). That brings:

$$\sigma_S = \zeta_1 \sigma_\theta + \zeta_2 \quad (20)$$

where σ_S is the volatility of the stock return ratio SR_r and σ_θ is the volatility of the return on equity ROE. Further:

$$SR_r = \eta_1 \sigma_S + \eta_2 \quad (21)$$

Based on Equations (20) and (21), we have:

$$SR_r = \varepsilon_1 \sigma_\theta + \varepsilon_2 \quad (22)$$

where:

$$\varepsilon_1 = \eta_1 \zeta_1 \quad (23)$$

$$\varepsilon_2 = \eta_1 \zeta_2 + \eta_2 \quad (24)$$

2.2.4. The Banker's Decision Return

Proposition 1. The banker's decision return R satisfies

$$R = \alpha \lambda E_q(0) C(\Delta, 1) + (1 - \alpha) C_1(\varepsilon_1 \sigma_\theta + \varepsilon_2) \frac{1}{r} [1 - (1 + r)^{-\lambda}] \quad (25)$$

Proof. On the basis of Equations (10), (19), and (22) and the iterated expectation theory, we have:

$$R = \alpha \sum_{i=1}^{\lambda} E(\exp(-ri\Delta) E_q((i-1)\Delta) \max[\frac{E_q(i\Delta)}{E_q((i-1)\Delta)} - 1, 0]) + (1 - \alpha) SR_r \sum_{i=1}^{\lambda} \frac{C_i}{(1 + r_i)^i} \quad (26)$$

$$R = \alpha \sum_{i=1}^{\lambda} E[E(\exp(-ri\Delta)E_q((i-1)\Delta)\max[\frac{E_q(i\Delta)}{E_q((i-1)\Delta)} - 1, 0]|F_{(i-1)\Delta})] \\ + (1-\alpha)C_1(\varepsilon_1\sigma_{\theta} + \varepsilon_2)\left(\frac{1}{1+r} + \dots + \frac{1}{(1+r)^{\lambda}}\right) \quad (27)$$

$$R = \alpha \sum_{i=1}^{\lambda} E[\exp(-r(i-1)\Delta)E_q((i-1)\Delta)E(\exp(-r\Delta)\max[\frac{E_q(i\Delta)}{E_q((i-1)\Delta)} - 1, 0]|F_{(i-1)\Delta})] \\ + (1-\alpha)C_1(\varepsilon_1\sigma_{\theta} + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}] \quad (28)$$

$$R = \alpha \sum_{i=1}^{\lambda} \exp(-r(i-1)\Delta)C(\Delta, 1)E[E_q(i-1)\Delta] + (1-\alpha)C_1(\varepsilon_1\sigma_{\theta} + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}] \quad (29)$$

Since $E[E_q(i-1)\Delta] = E_q(0)\exp(r(i-1)\Delta)$, and from using Equation (16), Proposition 1 is established. $\square$

According to Proposition 1 and Equations (7) and (10), we have the formula of D_1 for λ stages:

$$R_1 = \Delta E = \lambda E_q(0)C(\Delta, 1) = f_1(\sigma_{\theta}) \quad (30)$$

where $\kappa = S - K + A = \lambda\Delta$, $C(\Delta, 1)$ is the call option price from Equation (16), and $E_q(0)$ is the initial net asset value. Furthermore, there is a theoretically positive relationship between the decision of option investment and the default risk, that is:

$$\frac{\partial f_1}{\partial \sigma_{\theta}} > 0 \quad (31)$$

and the formula of D_2 for λ stages:

$$R_2 = \Delta D = C_1(\varepsilon_1\sigma_{\theta} + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}] = f_2(\sigma_{\theta}) \quad (32)$$

where C_1 is the stock market value that the banker holds. Additionally, there is a theoretically negative relationship between the decision of bond investment and the default risk, that is:

$$\frac{\partial f_2}{\partial \sigma_{\theta}} < 0 \quad (33)$$

3. Results

3.1. Decision Return Vega

Corollary 1. *Decision Return Vega satisfies*

$$Vega_R = \alpha \lambda E_q(0) \frac{\sqrt{\Delta}}{\sqrt{2\pi}} \exp\left[-\frac{\Delta}{8\sigma_{\theta}^2}(\sigma_{\theta}^2 + 2r)^2\right] + (1-\alpha)C_1\varepsilon_1\frac{1}{r}[1 - (1+r)^{-\lambda}] \quad (34)$$

Further, $Vega_R$ is increased by σ_{θ} in $(\sigma_{\theta 1}, \sigma_{\theta 2})$ and decreased in $(0, \sigma_{\theta 1})$ and $(\sigma_{\theta 2}, \infty)$ where:

$$\sigma_{\theta(1,2)} = \frac{\sqrt{-\frac{8v}{\Delta}} \pm \sqrt{-\frac{8v}{\Delta} - 8r}}{2} \quad (35)$$

and:

$$v = \ln\left(-\frac{\sqrt{2\pi}(1-\alpha)C_1\varepsilon_1[1 - (1+r)^{-\lambda}]}{r\sqrt{\Delta}\alpha\lambda E_q(0)}\right) \quad (36)$$

Proof. According to Proposition 1:

$$Vega_R = \frac{\partial R}{\partial \sigma_\theta} = \alpha \lambda E_q(0) \sqrt{\Delta} \exp(-r\Delta) N_x(d_2(\Delta)) + (1-\alpha) C_1 \varepsilon_1 \frac{1}{r} [1 - (1+r)^{-\lambda}] \quad (37)$$

When:

$$Vega_R = \frac{\partial R}{\partial \sigma_\theta} = 0 \quad (38)$$

we have:

$$\exp\left[-\frac{\Delta}{8\sigma_\theta^2}(\sigma_\theta^2 + 2r)^2\right] = -\frac{\sqrt{2\pi}(1-\alpha)C_1\varepsilon_1\frac{1}{r}[1-(1+r)^{-\lambda}]}{\sqrt{\Delta}\alpha\lambda E_q(0)} \quad (39)$$

$$-\frac{\Delta}{8\sigma_\theta^2}(\sigma_\theta^2 + 2r)^2 = \ln\left(-\frac{\sqrt{2\pi}(1-\alpha)C_1\varepsilon_1[1-(1+r)^{-\lambda}]}{r\sqrt{\Delta}\alpha\lambda E_q(0)}\right) \quad (40)$$

Set:

$$v = \ln\left(-\frac{\sqrt{2\pi}(1-\alpha)C_1\varepsilon_1[1-(1+r)^{-\lambda}]}{r\sqrt{\Delta}\alpha\lambda E_q(0)}\right) \quad (41)$$

Then we have:

$$-\frac{\Delta}{8\sigma_\theta^2}(\sigma_\theta^2 + 2r)^2 = v \quad (42)$$

$$\sigma_\theta^2 - \sqrt{-\frac{8v}{\Delta}}\sigma_\theta + 2r = 0 \quad (43)$$

Thus, on the basis of the nature of the quadratic function, the relationship between the derivative of the function and the monotonicity was proved by Corollary 1.

On the basis of Corollary 1 and Equation (11), the optimal default risk level to maximize the decision return is:

$$\sigma_\theta^* = \sigma_{\theta 2} = \frac{\sqrt{-\frac{8v}{\Delta}} + \sqrt{-\frac{8v}{\Delta} - 8r}}{2} \quad (44)$$

where σ_θ^* is the optimal default risk level to maximize the decision return when α is fixed. $\square$

This finding is consistent with IMF (International Monetary Fund, 2014), who concluded that the inside debt may reduce the bank's risk taking, and vice versa with inside equity [28].

3.2. The Sensitivity of Decision Return

Corollary 2. The sensitivity of the decision return value (SEN_R) with respect to the bank's default risk σ_θ increases in stage number λ :

$$\frac{\partial(R_{\lambda+1})}{\partial \sigma_\theta} \geq \frac{\partial(R_\lambda)}{\partial \sigma_\theta} \quad (45)$$

Proof. Based on Black and Scholes (1973) [24], $\frac{\partial C(\Delta, 1)}{\partial \sigma_\theta} = \sqrt{\Delta} N_x(d_1(\Delta)) > 0$. As a consequence, we obtained:

$$SEN_R = \frac{\partial^2(R)}{\partial \sigma_\theta \partial \lambda} = \frac{\partial}{\partial \lambda} \left(\alpha \lambda E_q(0) \frac{\partial C(\Delta, 1)}{\partial \sigma_\theta} + (1-\alpha) C_1 \varepsilon_1 \frac{1}{r} [1 - (1+r)^{-\lambda}] \right) \quad (46)$$

$$SEN_R = \alpha E_q(0) \frac{\partial C(\Delta, 1)}{\partial \sigma_\theta} + \frac{\partial}{\partial \lambda} \left((1-\alpha) C_1 \varepsilon_1 \frac{1}{r} [1 - (1+r)^{-\lambda}] \right) \quad (47)$$

$$SEN_R = \alpha E_q(0) \frac{\partial C(\Delta, 1)}{\partial \sigma_\theta} + (1-\alpha) C_1 \varepsilon_1 \frac{1}{r} (1+r)^{-\lambda} \ln(1+r) > 0 \quad (48)$$

As a result, we obtained the following:

$$\frac{\partial(R_{\lambda+1})}{\partial\sigma_\theta} - \frac{\partial(R_\lambda)}{\partial\sigma_\theta} = \int_\lambda^{\lambda+1} \frac{\partial(R_\varphi)}{\partial\sigma_\theta\partial\varphi}|_{\varphi=i} di > 0 \quad (49)$$

So, we drew the following conclusion. $\square$

On the basis of Corollary 2, the stage number (λ) was positively related to the decision return value (R). This finding is consistent with Gopalan et al. (see prediction 2), who concluded that the shorter the pay duration of a firm, the more volatile the cash flows [29].

3.3. Decision Return and the Decision Structure Coefficient

Corollary 3. Decision structure incentives DS_α satisfy

$$DS_\alpha = \frac{\partial R}{\partial\alpha} = \lambda E_q(0)C(\Delta, 1) - C_1(\varepsilon_1\sigma_\theta + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}] \quad (50)$$

Additionally, D_α is increased when $g_1(\lambda) > g_2(\lambda)$, and decreased when $g_1(\lambda) < g_2(\lambda)$ where:

$$g_1(\lambda) = \frac{1}{(1+r)^\lambda} \quad (51)$$

and:

$$g_2(\lambda) = 1 - \frac{rE_q(0)C(\Delta, 1)}{C_1(\varepsilon_1\sigma_\theta + \varepsilon_2)}\lambda \quad (52)$$

Proof. According to Proposition 1:

$$\frac{\partial R}{\partial\alpha} = \lambda\omega E_q(0)C(\Delta, 1) - C_1(\varepsilon_1\sigma_\theta + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}] \quad (53)$$

When:

$$\frac{\partial R}{\partial\alpha} > 0 \quad (54)$$

we have:

$$\lambda > \frac{C_1(\varepsilon_1\sigma_\theta + \varepsilon_2)\frac{1}{r}[1 - (1+r)^{-\lambda}]}{E_q(0)C(\Delta, 1)} \quad (55)$$

$$1 - \frac{rE_q(0)C(\Delta, 1)}{C_1(\varepsilon_1\sigma_\theta + \varepsilon_2)}\lambda < \frac{1}{(1+r)^\lambda} \quad (56)$$

Let the left part of Equation (57) be $g_2(\lambda)$ and the right part be $g_1(\lambda)$. On the basis of the nature of the quadratic function, the relationship between the derivative of the function and monotonicity was proved by Corollary 3.

On the basis of Corollary 1 and Equation (11), the optimal decision structure level to maximize the decision return is:

$$\alpha^* = f(\lambda_a)|_{g_1(\lambda_a)=g_2(\lambda_a)} \quad (57)$$

where α^* is the optimal decision structure level to maximize the decision when σ_θ is set to be fixed. $\square$

This finding is consistent with Grundy and Li (2010) [12], who developed a model in which the firm investment level increases with the investors' optimism.

3.4. Decision Return and the Number of Stages

Corollary 4. The Decision Return R increases in the stage number λ :

$$R_{\lambda+1} \geq R_{\lambda} \quad (58)$$

Proof. According to Proposition 1, we have the following:

$$\frac{\partial(R)}{\partial\lambda} = \alpha E_q(0) C\left(\frac{\kappa}{i}, 1\right) + (1 - \alpha) C_1(\varepsilon_1 \sigma_{\theta} + \varepsilon_2) \frac{1}{r} (1 + r)^{-\lambda} \ln(1 + r) > 0 \quad (59)$$

In conclusion, Corollary 4 was established. $\square$

On the basis of Corollary 4, the number of decision stages (λ) was positively correlated with the decision return of the banker (R). This finding is consistent with Sundaram and Yermack (2007), who concluded that the tenure of executives is an influencing factor on bank default risk [30].

4. Discussion

4.1. A Case Study: John G. Stumpf of Wells Fargo & Co.

Based on the results we obtained in Section 3, we used the case of Wells Fargo & Co. to further illustrate the mechanism responsible for how compensation influences the risk, the optimal risk, and the decision structure level to maximize the decision return.

4.1.1. Data

For calibrating the parameters of the decision functions and the related equations introduced in Proposition 1 and Corollaries 1–4, we used CEO remuneration data, U.S. bank financial data, and stock price data from the ExecuComp, BvD Orbis, and CRSP-COMPUSTAT databases. Table 2 lists the annual stock price data and other financial data for John G. Stumpf of Wells Fargo & Co. All values are reported in dollars as of December 31 of each year.

The following are the resources of each column. (a) The collection methods of the variables of the first two columns and the sixth and eighth columns in Table 2 are similar to Ma et al., 2020 [22,23]. (b) The fifth and seventh columns are calculated as the variance of SR_r and ROE , respectively. (c) The third and fourth columns are the variables related to the stock price, used to calculate the values of ε_1 and ε_2 in Equation (22). C_i is the market value of the total fiscal year and SR_r is the earnings per share, including extraordinary items from the CRSP-COMPUSTAT database.

Table 2. John G. Stumpf's decision-related variables as CEO of Wells Fargo & Co. (\$).

Year	Age	Stock Market Value Per Year (C_i)	Stock Return Ratio (SR_r)	Volatility of SR_r (σ_S)	Return on Equity (ROE)	Volatility of ROE (σ_{θ})	Total Equity Value ($E_q(t)$)
2007	53	99,539.5094	2.41	-	0.2443	-	47,432,000,000
2008	54	124,660.0419	0.7	1.46205	0.0324	0.022451	70,949,000,000
2009	55	139,771.0888	1.76	0.5618	0.1574	0.007813	105,846,000,000
2010	56	163,078.1502	2.23	0.11045	0.1486	0.0000387	119,155,000,000
2011	57	145,037.5867	2.85	0.1922	0.167	0.000169	130,189,000,000
2012	58	180,002.6125	3.4	0.15125	0.1792	0.0000744	145,953,000,000
2013	59	238,675.2002	3.95	0.15125	0.1908	0.0000673	154,646,000,000
2014	60	283,438.5322	4.17	0.0242	0.1829	0.0000312	166,075,000,000
2015	61	276,808.1324	4.18	0.00005	0.1735	0.0737	171,567,000,000
2016	62	276,437.767	4.03	0.01125	0.1602	0.0000884	175,820,000,000

4.1.2. Curving Fitting Functions

The statistics ε_1 and ε_2 based on Equations (20)–(22) calculated by MATLAB are as follows. As a consequence, Equations (60)–(62) are good fits for equations (20)–(22), as shown in Table 3.

$$\sigma_S = 61.24\sigma_\theta + 0.08665 \quad (60)$$

$$SR_r = -2.278\sigma_S + 3.704 \quad (61)$$

$$SR_r = -134.9\sigma_\theta + 3.491 \quad (62)$$

Table 3. The determination of parameters in Equations (20)–(22).

Curving Fitting		a	b	c
Corresponding Equation		(20)	(21)	(22)
Fit slope		$\zeta_1 = 61.24$	$\eta_1 = -2.278$	$\varepsilon_1 = -134.9$
Fit intercept		$\zeta_2 = 0.08665$	$\eta_2 = 3.704$	$\varepsilon_2 = 3.491$
SSE		0.03544	3.188	3.953
R-square		0.9799	0.7413	0.6791
Goodness of fit	Adjusted R-square	0.977	0.7043	0.6333
	RMSE	0.07116	0.6748	0.7515

Note: Goodness of fit contains: (a) SSE: the sum of squares due to error; (b) R-Square: coefficient of determination; (c) Adjusted R-Square: degree-of-freedom adjusted coefficient of determination; (d) RMSE: root mean square error.

On the basis of Equations (60), (62), (30), and (32), we drew the following figure.

Figure 2a illustrates the volatility of stock return ratio (σ_S) and the corresponding stock return ratio (SR_r) with respect to the bank's default risk (σ_θ) based on Equations (60) and (62) for Wells Fargo & Co. The image of the stock return ratio (σ_S) was obtained from Equation (62) (Figure 2a, blue dotted line), and the image of the stock return ratio (SR_r) was directly drawn by Equation (62) (Figure 2a, orange-red solid line).

Figure 2. (a) Function image of the volatility of stock return ratio (σ_S) and the corresponding stock return ratio (SR_r) with respect to the default risk (σ_θ) based on Equations (60) and (62) (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, $\zeta_1 = 61.24$ and $\zeta_2 = 0.08665$); (b) Function image of the decision return of the option investment (R_1) and the corresponding bond investment (R_2) with respect to the default risk (σ_θ) based on Equations (30) and (32) (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.).

Figure 2b illustrates the decision return of the option investment (R_1) and the corresponding bond investment (R_2) with respect to the default risk (σ_θ) based on Equations (30) and (32) for the

example bank. Similar to Figure 2a, the image of the decision return of the option investment (R_1) was obtained from Equation (30) (Figure 2b, blue dotted line). The image of the decision return of the bond investment (R_2) was directly drawn using Equation (32) (Figure 2b, orange-red solid line).

As shown in Figure 2a, the volatility of the return on equity has a positive influence on the volatility of the stock return ratio. This finding is consistent with Vuolteenaho and Tuomo [28]. Furthermore, the volatility of the return on equity is negatively related with the stock return ratio. Figure 2b shows that the decision of option investment exerts positive impacts on the default, and vice versa for the decision of the bond investment. This empirical finding is consistent with Equations (31) and (33) of our theoretical model.

4.2. Simulation Analysis

Figure 3a illustrates the decision return value (R) of the bankers with respect to the default risk (σ_θ) when the decision structure coefficient (α) varies based on Proposition 1 for Wells Fargo & Co. The image of the inside debt was obtained by fitting the default risk (σ_θ) to the corresponding decision return value (R). Figure 3b illustrates the decision return incentives ($Vega_R$) with respect to the default risk (σ_θ) when the decision structure coefficient (α) varies based on Equation (34) of Corollary 1 for the example bank. Similar to Figure 3a, the image of the decision return incentives ($Vega_R$) was obtained by fitting the σ_θ value to the corresponding $Vega_R$ value.

Figure 3. (a) Function image of the decision return value (R) of the bankers with respect to the default risk (σ_θ) when the decision structure coefficient (α) varies based on Proposition 1 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.); (b) Function image of the decision return incentives ($Vega_R$) with respect to the default risk (σ_θ) when the decision structure coefficient (α) varies based on Equation (34) of Corollary 1 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.).

As shown in Figure 3a, the volatility of the return on equity has a positive-to-negative influence on the decision return. As a consequence, it has a “risk cap”, where the maximum point in this cap is the optimal risk level to maximize the return of decision-making. Figure 3b shows that the impact of the default risk on the Vega of the decision return becomes stronger and stronger as the decision weight grows.

Figure 4a illustrates the decision return value (R) of the bankers with respect to the decision structure coefficient (α) when the default risk (σ_θ) varies based on Proposition 1 for Wells Fargo & Co. The image of the inside debt was obtained by fitting the decision structure coefficient (α) to the corresponding decision return value (R). Figure 4b illustrates the decision return incentives

($Vega_R$) with respect to the decision structure coefficient (α) when the default risk (σ_θ) varies based on Equation (34) of Corollary 1 for the example bank. Similar to Figure 4a, the image of the decision return incentives ($Vega_R$) was obtained by fitting the decision structure coefficient (α) to the corresponding $Vega_R$ value.

Figure 4. (a) Function image of the decision return value (R) of the bankers with respect to the decision structure coefficient (α) when the default risk (σ_θ) varies based on Proposition 1 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.); (b) Function image of the decision return incentives ($Vega_R$) with respect to the decision structure coefficient (α) when the default risk (σ_θ) varies based on Equation (34) of Corollary 1 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.).

As shown in Figure 4a, the decision weight has a positive influence on the decision return. Furthermore, this effect becomes weaker and weaker as the default risk grows. Figure 4b shows that the decision weight has different directions of influence on the Vega of the decision return as the risk value varies.

Figure 5a illustrates the minimum point ($\sigma_{\theta 1}$) and the maximum point ($\sigma_{\theta 2}$) of the default risk (σ_θ) with respect to the decision structure coefficient (α) based on Equations (35) and (36) of Corollary 1 for Wells Fargo & Co. The image of the minimum point ($\sigma_{\theta 1}$) was obtained from Equation (35) (Figure 5a, blue dotted line), and the image of the maximum point ($\sigma_{\theta 2}$) was directly drawn by Equation (36) (Figure 5a, orange-red solid line).

Figure 5. (a) Function image of the minimum point ($\sigma_{\theta 1}$) and the maximum point ($\sigma_{\theta 2}$) of the default risk (σ_{θ}) with respect to the decision structure coefficient (α) based on Equations (35) and (36) of Corollary 1 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276,808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.); (b) Function image of $G_1(\lambda)$ and $G_2(\lambda)$ with respect to the number of periods (λ) when $\sigma_{\theta} = 0.001$ based on Equations (51) and (52) of Corollary 3 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276,808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, $\varepsilon_2 = 3.491$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.).

Figure 5b illustrates $G_1(\lambda)$ and $G_2(\lambda)$ with respect to the number of periods (λ) when $\sigma_{\theta} = 0.001$ based on Equations (51) and (52) of Corollary 3 for the example bank. Similar to Figure 5a, the image of $G_1(\lambda)$ was obtained from Equation (51) (Figure 5b, blue dotted line), and the image of $G_2(\lambda)$ was directly drawn using Equation (52) (Figure 5b, orange-red solid line).

As shown in Figure 5a, the decision weight value yields a negative impact on the optimal risk value $\sigma_{\theta 2}$. Based on Figure 5b, Equation (57), and Corollary 3, when $\lambda = 27$, the decision weight reaches its optimal value to maximize the decision return value.

Figure 6a illustrates the sensitivity of the decision revenue (SEN_R) of the bankers with respect to the default risk (σ_{θ}) when the decision structure coefficient (α) varies based on Equation (48) and Corollary 2 for Wells Fargo & Co. The image of the sensitivity of the decision revenue (SEN_R) was obtained by fitting the default risk (σ_{θ}) to the corresponding SEN_R value. Figure 6b illustrates the sensitivity of the decision revenue (SEN_R) of the bankers with respect to the decision structure coefficient (α) when the default risk (σ_{θ}) varies based on Equation (48) and Corollary 2 for the example bank. Similar to Figure 6a, the image of the sensitivity of the decision revenue (SEN_R) was obtained by fitting the decision structure coefficient (α) to the corresponding SEN_R value.

Figure 6. (a) Function image of the sensitivity of the decision revenue (SEN_R) of the bankers with respect to the default risk (σ_{θ}) when the decision structure coefficient (α) varies based on Equation (48) and Corollary 2 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276,808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.); (b) Function image of the sensitivity of the decision revenue (SEN_R) of the bankers with respect to the decision structure coefficient (α) when the default risk (σ_{θ}) varies based on Equation (48) and Corollary 2 (Note: Parameter values (example bank: Wells Fargo & Co., year: 2015): $E_q(0) = 171,567,000,000$, $C_1 = 276,808.1324$, $r = 0.07$, $\kappa = 9.326$, $\varepsilon_1 = -134.9$, and $\lambda = 9$. The risk-free rate r is the mean of the one-month interest rate of national debt during November 2015 in the U.S.).

As shown in Figure 6a, the impact of the default risk on the sensitivity of the decision return becomes stronger and stronger as the decision weight grows. Figure 6b shows that the decision weight has different directions of influence on the sensitivity of the decision return as the risk value varies.

4.3. Robustness

We used the data of Richard M. Adams, Sr. of United Bankshares to perform the robust test. He has been the chairman and CEO of United Bankshares since 1 January 1984. The results showed that our conclusion is robust and effective. The trends and relationships of the decision return, the decision structure, and the default risk are still established. For instance, Equations (63)–(65) are good fits for equations (20)–(22) where United Bankshares is concerned.

$$\sigma_S = 44.31\sigma_{\theta} + 0.0004784 \quad (63)$$

$$SR_r = -2.345\sigma_S + 1.845 \quad (64)$$

$$SR_r = -113\sigma_{\theta} + 1.848 \quad (65)$$

5. Conclusions

In this research, we built a multi-stage decision-making model, containing option investment and bond investment, to explore the mechanism of how compensation influencing on risk, based on our previous work (Ma et al., 2020 [22,23]) on a banker's long-term compensation and the design of pay contracts. As is shown in Table A1 of Appendix A, we illustrated the relationship between the decision return of multi-stages and both the bank's default risk and decision weight by formulating a particular formula. After setting up the model, we analyzed the function image results using data from Wells Fargo & Co. Firstly, we found an optimal situation regarding the default risk of the bank to maximize the banker's decision return in multi-stages of option investment and bond investment.

Secondly, we were the first to find the presence of a “risk cap”, where the maximum point in this cap is the optimal risk level to maximize the return of decision-making. At the same time, we were the first to determine the optimal decision structure (to invest option or bond more) to maximize the decision return, during which the internal causes and mechanisms of the impact of the banker’s compensation on the bank’s default risk were revealed. Thirdly, this decision-making model sheds light on further theoretical analysis, because it provides a route to bridge the gap of the relationship between the compensation of the banker and the bank’s risk-taking. Fourthly, based on this model, we were able to explain how the banker’s decision influences the decision return and further influences the risk and future compensation, as shown in Figure 1. The limitation of our study is that the banker’s decision of investment contained option investment and bond investment, but did not include other kinds of investment that were difficult to express using the particular formula. We will focus on the particular influence of the decision-making of the banker on the relationship between compensation and the default risk of banks, and we will improve the long-term compensation models by adding an equity incentive.

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Appendix A

Table A1. Main conclusions of this paper.

Decision	Definition	Long-Term Model	Relationship with Risk/Decision Structure/Number of Periods			
			From Predecessors or Intuition	From Theoretical Analysis	From Simulation or Empirical Results	Corresponding Figure
Return on decision (R)	Equation (12): $R = \alpha R_1 + (1 - \alpha) R_2$	Proposition 1: $R = \alpha \lambda E_q(0) C(\Delta, 1) + (1 - \alpha) C_1 (\varepsilon_1 \sigma_\theta + \varepsilon_2) \frac{1}{r} [1 - (1 + r)^{-\lambda}]$	(?)	Corollary 1: $Vega_R$ is increased by σ_θ in $(\sigma_{\theta 1}, \sigma_{\theta 2})$ and decreased in $(0, \sigma_{\theta 1})$ and $(\sigma_{\theta 2}, \infty)$ where $\sigma_{\theta(1,2)} = \frac{\sqrt{-\frac{8v}{\Delta}} \pm \sqrt{-\frac{8v}{\Delta} - 8r}}{2}$ $v = \ln \left(-\frac{\sqrt{2\pi}(1-\alpha)C_1\varepsilon_1[1-(1+r)^{-\lambda}]}{r\sqrt{\Delta}\alpha\lambda E_q(0)} \right)$ Equation (44) $\sigma_\theta^* = \sigma_{\theta 2} = \frac{\sqrt{-\frac{8v}{\Delta}} + \sqrt{-\frac{8v}{\Delta} - 8r}}{2}$	(+) and (−)	Figures 3a and 5a
Return on decision of option investment (R_1)	Equation (6): $R_1 = \Delta E = \max[E_q(i) - E_q(i - 1), 0]$	Equation (30): $R_1 = \lambda E_q(0) C(\Delta, 1) = f_1(\sigma_\theta)$	(+)	Equation (31): $\frac{\partial f_1}{\partial \sigma_\theta} > 0$	(+)	Figure 2b
Return on decision of bond investment (R_2)	Equation (9): $R_2 = \Delta D = SR_r \sum_{i=1}^{\lambda} \frac{C_i}{(1+r_i)^i}$	Equation (32): $R_2 = C_1 (\varepsilon_1 \sigma_\theta + \varepsilon_2) \frac{1}{r} [1 - (1 + r)^{-\lambda}] = f_2(\sigma_\theta)$	(−)	Equation (33): $\frac{\partial f_2}{\partial \sigma_\theta} < 0$	(−)	Figure 2b
Vega of return on decision ($Vega_R$)	$Vega_R = \frac{\partial R}{\partial \sigma_\theta}$	Equation (34): $Vega_R = \alpha \lambda E_q(0) \frac{\sqrt{\Delta}}{\sqrt{2\pi}} \exp \left[-\frac{\Delta}{8\sigma_\theta^2} (\sigma_\theta^2 + 2r)^2 \right] + (1 - \alpha) C_1 \varepsilon_1 \frac{1}{r} [1 - (1 + r)^{-\lambda}]$	(?)	Corollary 4: $\frac{\partial(R)}{\partial \lambda} = \alpha E_q(0) C\left(\frac{\lambda}{r}, 1\right) + (1 - \alpha) C_1 (\varepsilon_1 \sigma_\theta + \varepsilon_2) \frac{1}{r} (1 + r)^{-\lambda} \ln(1 + r) > 0$ $R_{\lambda+1} \geq R_\lambda$	(+) and (−)	Figures 3b and 5b

Table A1. Cont.

Decision	Definition	Long-Term Model	Relationship with Risk/Decision Structure/Number of Periods			
			From Predecessors or Intuition	From Theoretical Analysis	From Simulation or Empirical Results	Corresponding Figure
Sensitivity of return on decision (SEN_R)	$SEN_R = \frac{\partial^2(R)}{\partial \sigma_\theta \partial \lambda}$	Equation (48): $SEN_R = \alpha E_q(0) \frac{\partial(C(\Delta, 1))}{\partial \sigma_\theta} + (1 - \alpha) C_1 \varepsilon_1 \frac{1}{r} (1 + r)^{-\lambda} \ln(1 + r) > 0$	(?)	Corollary 2: $\frac{\partial(R_{\lambda+1})}{\partial \sigma_\theta} \geq \frac{\partial(R_\lambda)}{\partial \sigma_\theta}$	(+)	Figure 6a,b
Decision structure incentives (DS_α)	$D_\alpha = \frac{\partial R}{\partial \alpha}$	Equation (50): $DS_\alpha = \lambda E_q(0) C(\Delta, 1) - C_1(\varepsilon_1 \sigma_\theta + \varepsilon_2) \frac{1}{r} [1 - (1 + r)^{-\lambda}]$	(?)	Corollary 3: DS_α is increased when $g_1(\lambda) > g_2(\lambda)$ and decreased when $g_1(\lambda) < g_2(\lambda)$ where $g_1(\lambda) = \frac{1}{(1+r)^\lambda}$ $g_2(\lambda) = 1 - \frac{r E_q(0) C(\Delta, 1)}{C_1(\varepsilon_1 \sigma_\theta + \varepsilon_2)} \lambda$ Equation (57) $\alpha^* = f(\lambda_a) _{g_1(\lambda_a)=g_2(\lambda_a)}$	(+) and (−)	Figure 4a,b

Note: (+) stands for a positive relationship between the two variables, (−) stands for a negative one, and (?) stands for an unknown one; σ_θ^* is the optimal default risk level to maximize the decision return when α is fixed, and α^* is the optimal decision structure level to maximize the decision when σ_θ is fixed.

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