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Creating Realistic Mathematics Tasks Involving Authenticity, Cognitive Domains, and Openness Characteristics: A Study with Pre-Service Teachers

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Abstract: Creating mathematics tasks provide opportunities for students to develop their thinking, reasoning, communication, and creativity. This paper presents a study on teaching pre-service teachers to create realistic mathematics tasks in real contexts and amending them through an iterative process of analysis and refinement. The study was undertaken with pre-service teachers from two university training courses in Spain, undergraduate students from a primary teacher training course, and graduate students from an educational Master's course. The students worked in groups to collaborate in the creation of the requested tasks and improvement of them based on critical thinking and creativity. The tasks were not only evaluated concerning their level of realism, but also regarding their level of authenticity, the cognitive domains involved, and their openness characteristic. These are the key characteristics related to environmental and sustainability aspects. The outcomes confirmed that the creation of realistic mathematics tasks was a challenge for future primary teachers; however, they were able to create tasks with high levels of cognitive domain, authenticity, and openness. This evidences, on the one hand, the difficulty that future teachers have in understanding the realism of a mathematics task, and, on the other, the possibilities offered by the task's creation and the revision activity, which has educational implications and opens paths for future research.

Keywords: mathematics education; the creation of realistic tasks; creativity; initial teacher training

1. Introduction

Creating tasks should provide opportunities to develop thinking, creativity, and reasoning [1–8]. Creating or adapting mathematics tasks has a high cognitive demand [9,10], helps mathematics learning, and represents what is understood by mathematics [3,11–13]. Mathematics tasks can be created or adapted in different ways depending on the teaching objective. As a result of the aforementioned factors, mathematics task creation has become a crucial research line in mathematics education [14–17].

Realistic tasks are an especially relevant type of mathematics tasks. In these tasks, the solver needs to take into account relevant aspects of the everyday life context in order to attain the correct solution. Consequently, realistic mathematics tasks require the interpretation and understanding of a real-life situation to solve a problem. This process promotes reasoning and constructive learning [18]. Creating realistic tasks through reflection and communication will stimulate sustainable aspects in education, such as creativity and critical thinking (Sustainable Development Goals [19]).

There are also other relevant characteristics of mathematical tasks, apart from the realistic one. The levels of task authenticity, the cognitive domains involved, and the openness dimension of a task have been highly valued by several authors. For promoting constructive learning, authentic tasks are often considered to be better than believable or fictitious tasks. Similarly, tasks involving a reasoning domain are preferred over ones that just include applying or knowing domains. Regarding the openness dimension, open-ended tasks provide a better context for reasoning and creativity than closed-ended tasks, as the former generally imply working out several correct answers.

Considering the above, this paper provides and discusses a way of teaching creating realistic mathematics tasks to pre-service teachers. In this sense, we present an exploratory analysis with 62 pre-service primary teachers who were requested to create realistic tasks in order to evaluate their abilities in this matter. The subjects were introduced to the concepts of realistic tasks, authenticity levels, cognitive domains, and the openness characteristic. They worked in groups under an iterative process of creation and refinement based on critical thinking and creativity. The initially created tasks were amended according to the group discussions.

2. Theoretical Framework

2.1. Mathematics Tasks

The expression “mathematics tasks” adopts different meanings. Mathematics tasks are often seen as any activity carried out in order to learn mathematics regardless of its nature and application. Mathematics tasks can be, for example, used for introducing topics, relating knowledge, verifying theorems, or reinforcing concepts [20–22]. In the present study, “mathematics tasks” relate to specific activities proposed by teachers to mobilize student mathematical knowledge on a particular theme. This brings into play concepts, procedures, and skills contributing to students’ mathematics learning and application [23].

All types of tasks are important for developing mathematical competence, and each task should be used according to the learning purpose. For instance, to stimulate thinking and creativity we can, for example, use open-ended [13,24–26], inquiry-based [27], and real-life tasks [28]. Three main aspects could be considered to classify mathematics tasks: (1) the context in which the task is placed [29], (2) the variety of responses to the task [29], and (3) the level of cognitive domain activated when solving the task [30].

1. Real-life contexts (i.e., the non-mathematics aspects present in the task formulation) can be used as a didactic tool to support mathematics learning [31]. [32] establish three types of tasks based on their connection to the real world: intra-mathematics tasks, disguised tasks, and real-life application tasks. Intra-mathematics tasks have no connection to reality and require only mathematical procedures to be solved. Disguised tasks are those whose real context is given—i.e., all needed data to find the solution are provided and the outcome is achieved just by verifying the mathematics part. Real-life application tasks are often named authentic tasks; these are real-life situations where the solvers really feel the need to use mathematics [33]. Although National Council of Teachers of Mathematics (NCTM) claims that teachers should use such real-life tasks to facilitate the construction of student knowledge [20], these are rarely present in school lessons [34]. The authenticity of mathematics tasks has also been analyzed in both textbooks and assessments. For example, [35] analyzed the authenticity of mathematics tasks in Finnish and Swedish national evaluations. Their analysis included five dimensions: the chance to find the event of the task in everyday situations, the adequacy of the question (emerging from the task) for the event, the adequacy of the data offered in an everyday situation, the explicit presence of the task purpose, and the specificity of the information about the event. The results showed that the event was simulated in more than 90% of the tasks, while the others were simulated in a range from 25% (the presence of the purpose) to 60% (task adequacy to the event). Based on the aforementioned dimensions, [36] analyzed, over a seven-month period, the mathematics tasks of two primary

school teachers of 6th-grade pupils in Flanders. Their results showed that the context of the tasks as well as the data and specificity of the information were well simulated. [37] established levels of authenticity for 8373 mathematics tasks presented in the textbooks of a Spanish publisher across the six primary education courses. The results revealed that only 2% were authentic mathematics tasks, although about 26% may be easily convertible into authentic tasks.

2. Concerning the variety of responses to a task, we talk of open-ended or closed-ended tasks. The former has only one correct answer, such as the procedural activities appearing in textbooks for practicing a specific skill [38,39]. The latter allows more than one correct answer; that is, they are flexible enough to take into account the solver thinking, reasoning, and creativity [29]. Open-ended tasks are also divided into well- and ill-defined answers: the former occurs when the answer is clearly defined to be either correct or incorrect, and the latter takes place when the answer is subjective in the sense that there is no right or wrong answer. For example, an exploratory task such as “Make up a story where the answer is the result of 2.4×5.3 ” relates to a well-defined open answer [40], whereas a real-life task (e.g., “Design a playground for the school”) relates to an open ill-defined answer [13]. Referring to the realism of the tasks, realistic tasks could involve real-life situations. To solve them, mathematics calculations are not enough but also require the interpretation of when and how mathematics and non-mathematics knowledge should be applied [18,41,42]. An example of this would be: “John runs the 100 m in 17 s. How long will it take to run 1 km?” [43]. To solve this task, the use of proportionality is unsuitable because it is unusual for a person to run 1 km at the same constant speed, as when running 100 m. It is noteworthy that existing studies show the scarcity, in textbooks, of open-ended tasks [44,45] and the absence of realistic tasks [46].
3. Cognitive domains are understood as student thinking skills, which include aspects based on general processes such as working memory, attention, or language [47]. According to cognitive domains, mathematics tasks can be classified on three levels [48]: knowing, applying, and reasoning. Knowing tasks imply the evocation and repetition of knowledge that has already been taught. Applying tasks requires the integration and relationship of diverse mathematics knowledge, based on knowing and framed in non-routine situations related to familiar settings. Finally, reasoning tasks entail complex situations that, based on applying, demand reasoning and reflection to achieve the solution. Several authors have analyzed the cognitive domain involved in the mathematics tasks of secondary school textbooks, with a special focus on plane geometry [44] and linear equation tasks [45]. Such analyses have revealed that textbooks encompass mostly mathematics tasks involving routine activities related to the knowing or applying domain. Likewise, [46] showed that most tasks that appear in primary mathematics textbooks only aim to evoke knowledge. Similarly, in an analysis of tasks proposed by primary teachers, [49] found that about 81% and 19% of the mathematics tasks related to the knowing and applying domains, respectively.

Working with authentic, open-ended, realistic, or high-cognitive tasks becomes, however, important when trying to connect mathematics and reality. In real life, we often face environmental and sustainability issues where ill-defined answers arise and for which mathematics become a fundamental tool to be used [50]. The lack of such tasks in school textbooks forces mathematics teachers to create or design them.

2.2. Creating Mathematics Tasks by Pre-Service Teachers

Since solving mathematics tasks is the basis of many experiences proposed in the classroom [51], teachers must not only be good task solvers but also good task producers, capable of creating, modifying, and assessing tasks to suit the learning objectives. Teaching is an intentional and reflective practice where planning plays a fundamental role. Thus, teachers should be able to adequately select and adapt mathematics tasks to the learning context [16,52–55]. Research has shown that the selection and adaptation of such tasks often relate to our teaching and learning perceptions [56].

Several authors have demonstrated that professional knowledge is developed through the creation and modification of mathematics tasks [57]; this facilitates reasoning, communication, and creativity at different educational levels [58–60]. In the same way that mathematics knowledge is better acquired from tasks related to the solvers' contexts, professional knowledge increases when pre-service teachers are trained through tasks related to their future context [61]. The creation process of mathematics tasks requires high cognitive demands; this process can be approached from different ways, including task modification from textbooks or reflection on and revision of the work done when creating an initial task [61–63].

According to the above, initial teacher training should enable future teachers to create mathematics tasks and examine them didactically [64,65]. In fact, pre-service teachers often ask for such opportunities in the classroom [66], and research has revealed that they can certainly propose interesting and adequate mathematics tasks when possibilities to do so are given [67]. For example, in an empirical study about creating mathematics tasks from different contexts, [68] obtained that, although most pre-service teachers reached tasks involving routine applications, 47% of them successfully created authentic tasks, 18% open-ended tasks, and 8% realistic tasks. [69] found that—in a study where pre-service primary teachers had to create real-context mathematics tasks and to refine them through reflection on their own work and discussions with their classmates—the percent of the created “knowing tasks” (27%) remained unaffected from the initial to the final proposal, while the percent of “reasoning tasks” was reduced from 28% to 11%; in contrast, the percent of “open tasks” increased from 5% to 18%. In the same vein, [63] identified that half of their pre-service primary teachers were able to design mathematics tasks and transform them into authentic tasks. Although the tasks attained an applying cognitive domain, these achieved high levels of realism.

After a thorough review of the literature, we identify the need to teach pre-service teachers how to create realistic mathematics tasks and examine how these future teachers can amend their own tasks through the processes of critical thinking and creativity. Therefore, our research question is how to teach creating realistic math tasks to pre-service teachers. For this, a training proposal is presented and the realistic mathematics tasks created by pre-service teachers are analyzed. We assessed the specific characteristics of these created tasks, which are not often considered in previous studies, such as the realism, the levels of authenticity, the cognitive domains involved, and the openness task dimension.

3. Materials and Methods

3.1. Context and Participants

The practice was undertaken in the Faculty of Education at the University of Salamanca (Spain) during the academic year 2019–2020. In the study, 62 pre-service teachers from two training courses participated: 42 from a degree in primary teacher education and 19 from a Master's in advanced studies in learning difficulties. The study was undertaken through a convenience sample strategy; all the students enrolled in the above courses agreed to participate in the study.

The degree in primary teacher education encompasses four years of study, including two months of teaching practice in primary schools over the third and fourth years. The Master's degree combines one year of study with external internships in the learning difficulties unit of the university, the educational centers of Salamanca, or the educational and psychopedagogical guidance teams. The people enrolled in the Master's degree had normally graduated from the degree in primary teacher education and wanted to focus on learning difficulties. In both cases, the degrees intended to develop specific teacher competencies for instructing mathematics in primary education.

Both the undergraduate and graduate students who participated in the experience were trained to become mathematics teachers; it was thus assumed that they had already acquired the basic mathematical knowledge. However, they had no prior training in creating mathematics tasks, apart from the two formative sessions (of 2 h each) they received during the experience. In these two sessions, they learned the distinction between tasks and problems. They also were introduced to

realistic mathematics tasks and characteristics related to cognitive domains, authenticity, and openness. According to the objectives of this study, the pre-service teachers were requested to create realistic tasks using different real-contexts by means of an iterative process of task refinement. The experience was conducted by the usual faculty lecturer, who was also a member of the research team. The students were organized into groups: 9 groups were enrolled in the Master's degree and 16 in the primary teacher education degree. To ensure data confidentiality, each group was coded from G1 to G25. After this process, we analyzed not only the realism dimension of the created tasks but also the levels of cognitive domain involved, as well as their authenticity and openness characteristics.

3.2. Procedure and Data

To undertake the practice, each group of pre-service teachers was asked to follow seven consecutive steps:

1. Selecting three contexts appropriated for elementary school education. Step carried out in groups for 5 min.
2. Choosing a context and creating three realistic mathematics tasks for an elementary school lesson. Step carried out in groups for 15 min. Delivery 1: initial Created Tasks 0 (CT0).
3. Orally presenting the CT0s to the whole class. Proposing changes and improvements to other groups. Step carried out with the whole class for 15 min.
4. Modifying the CT0s according to the suggestions received in step 3 for attaining realistic mathematics tasks. Step carried out in groups for 15 min. Delivery 2: refined Created Tasks 1 (CT1).
5. Solving six CT1s: the three proposed tasks and another three from a different group. Step carried out in groups for 15 min.
6. Modifying the CT1s according to the outcomes of step 5 to better achieve realistic mathematics tasks. Step carried out in groups for 30 min. Delivery 3: final Created Tasks 2 (CT2).

Each group created 3 CT0s that were modified into 3 CT1s and finally into 3 CT2s, according to the process described above. Thus, 75 CT0 and their corresponding 75 CT2 were analyzed (Table 1). It should be noted that in some cases, given the difficulty of obtaining a realistic task on the first try, various CT1s and CT2s were completely different tasks from the ones initially created.

Table 1. Summary of tasks analyzed according to educational level.

Students	N	Group	CT0	CT2
Master	19	9	27	27
Degree	43	16	48	48
TOTAL	62	25	75	75

All the mathematics tasks created by the pre-service teachers used real-life contexts. From the total of CT2, 85% involved content related to numbers and 15% involved content related to measurement inspired by the two formative sessions and ideas taken from the Trends in International Mathematics and Science Study (TIMSS) 2019 project [48].

3.3. Categorization System for Analysis

An exploratory study was carried out. The initial and final mathematics tasks created by the pre-service teachers (CT0s and CT2s) were analyzed according to the four categories described above: Realism, Cognitive domains, Authenticity, and Openness. A qualitative approach was employed to gain in-depth information about how the pre-service teachers created and refined the tasks to attain the required characteristics. Two members of the research team worked together and agreed upon the information units to code. An independent third party (a researcher in mathematics education)

subsequently revised this work. Discussion among the research team and the independent third party helped endorse agreement [70,71] and validity for the undertaken analyses. A quantitative study was also carried out to find differences between the tasks created by the pre-service teachers at the beginning and end of this experience. The data were organized in tables, both in absolute values and percentages. McNemar's test for paired data (non-parametric test for dichotomous variables) and Wilcoxon's signed-rank test for related samples (non-parametric test for scale variables) were computed to compare the characteristics of the CT0s and CT2 in each group. Cohen's g and $r = \frac{Z}{\sqrt{n}}$ were used as measures of effect size in each case [72,73]. For the analysis of the data, the IBM SPSS Statistic 26 software was used. Below, we provide the categorization system utilized as a guide to analyze the task characteristics: Realism, Cognitive domains, Authenticity, and Openness.

3.3.1. Realism

The tasks that, in addition to mathematical calculations, required the interpretation of a situational context to be solved were categorized as realistic; otherwise, they were considered non-realistic [18,42].

3.3.2. Cognitive Domains

We considered three levels of Cognitive domain: *Knowing*, *Applying*, and *Reasoning*. In Table 2, we provide the indicators that allow us to categorize the CTs into the aforementioned levels [48].

Table 2. Indicators for the levels of Cognitive domain.

Levels	Indicators
<i>Knowing</i> : facts, concepts, and procedures students need to know	Recalling: definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
	Recognizing: numbers, expressions, quantities, shapes, and equivalent entities (e.g., equivalent familiar fractions, different orientations of simple geometric figures, etc.).
	Classifying/sorting: numbers, expressions, quantities, and shapes.
	Computing: algorithms $+$, $-$, \times , \div , or a combination of these with whole numbers, fractions, decimals, and integers. Undertaken straightforward algebraic procedures.
	Retrieving: information from graphs, tables, texts, or other sources.
<i>Applying</i> : knowledge and conceptual understanding to solve problems or answer questions	Measuring: using instruments; choosing appropriate units of measurement.
	Determining: efficient/appropriate operations, strategies, and tools for solving problems for which common methods exist.
	Representing/modeling: data in tables or graphs; equations, inequalities, geometric figures, or diagrams.
<i>Reasoning</i> : about unfamiliar situations, complex contexts, and multi-step problems	Implementing: strategies and operations to solve problems involving familiar mathematical concepts and procedures.
	Analyzing: relationships among numbers, expressions, quantities, and shapes.
	Integrating/synthesizing: different components of knowledge, representations, and procedures to solve problems.
	Evaluating: alternative problem-solving strategies and solutions.
	Drawing conclusions: valid inferences on the basis of information and evidence.
Generalizing: statements representing relationships in more general and more widely applicable terms.	
Justifying: mathematical arguments to support a strategy or solution.	

3.3.3. Authenticity

The CT Authenticity was analyzed by considering the proximity of the task Event to a real-life situation; the adequacy of the task Question to the proposed event; the concordance of the task Data with the question posed; the explicit Purpose given in the context for which an answer must be given; and the Specificity of information of the proposed situation [33,63]. In Table 3, we provide the indicators that allow us to categorize the CTs according to the aforementioned components.

Table 3. Components present in authentic tasks.

Components	Values	Indicators
Event	1	The proposed situation is feasible in real life.
	0	The proposed situation is imaginary, although it could be related to a real situation (e.g., odometers measuring in different units according to the time of day). It could happen in real life but in an unusual way (e.g., farmers with large greenhouses watered with domestic cans) or purely mathematics (e.g., children who draw the reflection of musical notes in a mirror).
Question	1	The question is formulated on a regular basis for the described event, and its answer has a practical value or is of interest to others who are not attracted by mathematics.
	0	The question would not be formulated in the real world, or it does not correspond to the event described.
Data	1	The data correspond with the real ones.
	0	The data does not correspond with the real ones or this information is only accessible through competencies that are required in a simulated situation (e.g., means or standard deviations).
Purpose	1	The purpose is explicitly mentioned and it is in accordance with that of the actual situation.
	0	The purpose is not clear enough or the task is described without referring to any specific situation. Thus, the task could be adjusted to many situations and purposes.
Specificity of information	1	The characters of the problem have proper names, the objects are defined or familiar, and the places are specific. The problem is formulated in the 1st or 2nd person or the origin of the graphics is mentioned. If the situation is not specific, the elements undergoing mathematical treatment provide, at least, their specific role.
	0	The situation is general without specifying objects and subjects, or the names of the characters are provided but not their role, which means that other aspects such as the realism of the data cannot be assessed. For example, it is not the same to say that “Andrew picked up 100 kg of potatoes” when he is farmer compared to when he is not.

Once the values of the above components for a particular CT were identified, we used them to obtain the specific task level of *Authenticity*—*Authentic*, *Believable*, or *Fictitious* [63]—as shown in Table 4.

Table 4. Task levels of *Authenticity*.

Authenticity	Values of the Matrix (Event, Question, Data, Purpose, Specificity of Information)
<i>Authentic</i>	(1,1,1,1,1)
<i>Believable</i>	(1,1,1,0,1) (1,1,1,1,0) (1,1,1,0,0)
<i>Fictitious</i>	(1,1,0,-,-) (1,0,1,-,-) (0,1,1,-,-) (1,0,0,-,-) (0,1,0,-,-) (0,0,1,-,-) (0,0,0,-,-)

3.3.4. Openness

Considering the response Openness, the CTs were categorized into *Open-* and *Closed-ended* tasks. The former were those implying several correct answers, and the latter were those with a single correct answer [13,29].

In the following section, we present the main results obtained from both the qualitative and quantitative analyses undertaken.

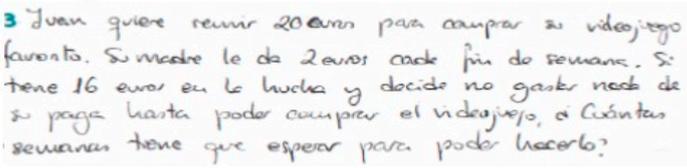
4. Results

The results are organized according to the four categories under analysis: Realism, Cognitive domain, *Authenticity*, and Openness.

4.1. Realism

Of the total CT0s developed by the undergraduate and the Master's students, 9% (7) were considered to be *Realistic* tasks. After the tasks refinement process, these percentages increased in the CT2 to 13% (10). Specifically, the values increased from 13% (6) to 17% (8) for the undergraduates and from 4% (1) to 7% (2) for the Master's students. It is noteworthy that the quantitative analysis revealed that the differences between the proportion of *Realistic* CT0s and *Realistic* CT2s were not significant globally (McNemar's test, $p > 0.999$, Cohen $g = 0.07$) and neither for any of the two groups (McNemar's test, $p = 0.5$, Cohen $g = 0.5$ undergraduate, $p > 0.999$, Cohen $g = 0.17$ Master's). However, a large effect size was obtained for undergraduates and a medium one for Master's students, which indicates that this slight improvement in creating *Realistic* tasks could be attributed to the training. This shows that changing from a *Non-Realistic* to a *Realistic* task was a difficult endeavor for the subjects participating in the study. Figure 1 provides an example of a *Non-Realistic* CT0 that turned out to be *Realistic* after the refinement process CT2.

CT0 original text:

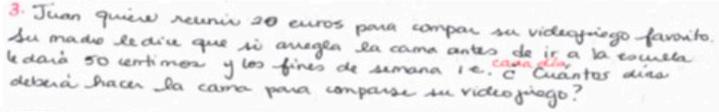


CT0 transcription:

Juan needs 20 euros to buy his favorite video game. His mother gives him €2 every weekend. If Juan has 16 euros in the piggybank and he decides not to spend any of his pay until he can buy the video game, how many weeks does he has to wait to be able to buy the game?



CT2 original text:



CT2 transcription:

Juan wants to collect €20 to buy his favorite video game. If Juan makes his bed before going to school, he will receive from his mother 50 cents per day and if he makes his bed during weekends, he will get €1 per day. How many days does he need to make his bed to be able to buy the video game?

Figure 1. A *Non-Realistic* CT0 that turned out to be a *Realistic* CT2 (created by Group G9).

In CT0, it is enough to carry out the operation $20 - 16 = 4$ to know that you still need €4 to buy the video game. In terms of time, you need to wait two weeks ($4:2 = 2$) to buy the game. CT2 is considered, however, a *Realistic* task because the solution requires the use of situational information. In this case, the solver must know, for instance, that weekends are two days per week. If Juan makes the bed for the entire week (7 days), he will get a total of €4.5 (2.5 euros from the week plus €2 from weekends: Saturday and Sunday). If we divide 20 by 4.5, we get 4.4. This result is not a whole number of weeks, so the solver will have to go back to the situational information of the task. In four complete weeks, we would have 18 euros (from 20 working days and 8 weekend days), and we will still need 2 more euros. These two euros can be obtained by making the bed one weekend (2 days more days), 1 weekend day and 2 working days (3 days more) or 4 working days (4 days more). In this case, the answer will depend on the day of the week that the bed-making routine begins: the answer would be 30 days if it starts on a Saturday, 31 days if it starts on a Thursday or Sunday, and 32 days otherwise. This shows that CT2 is a *Realistic* task because it not requires only calculations to be solved but also an interpretation of when and how to apply mathematics and non-mathematics knowledge.

4.2. Cognitive Domains

As Figure 2 shows, in most of the CT0s were identified aspects related to the *Applying* and *Reasoning* Cognitive domains. Of the total CT0s developed by the undergraduate and Master's students, 43% (32) were considered to involve the *Reasoning* domain. After the task refinement process, these percentages increased in the CT2 to 65% (49). For both groups (undergraduate and Master's students), the percentages of *Applying* and *Reasoning* domains decreased and increased, respectively, in the CT2s. The Cognitive domains levels in the CT0s increased significantly, and with a moderate effect size when transformed into CT2s (Wilcoxon test, $Z = 3.900$, $p < 0.001$, $r = 0.45$). For the undergraduate students, the Cognitive domains levels in the CT0s increased significantly, and with a large effect size, when transformed into CT2s (Wilcoxon test, $Z = 3.464$, $p = 0.001$, $r = 0.5$). Although the differences in the Cognitive domains levels were not significant for Master's students, a moderate effect size was obtained (Wilcoxon test, $Z = 1.890$, $p = 0.059$, $r = 0.36$). This shows that the training received influenced the students to consider *Reasoning* Cognitive domains as a characteristic of *Realistic* tasks.

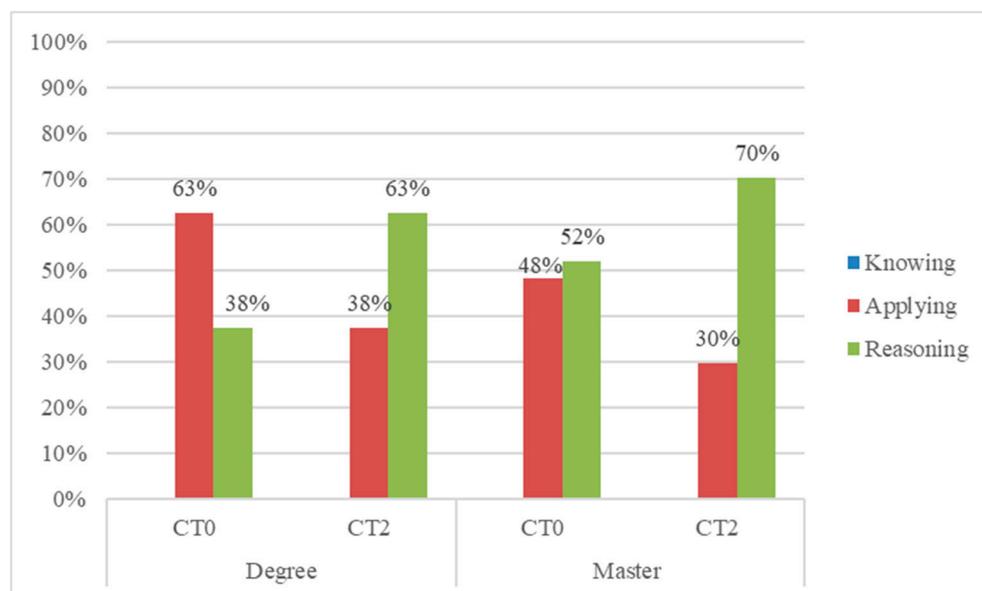


Figure 2. Percentages of the Cognitive domain in CT0 and CT2 for undergraduate and Master's students.

In Figure 3, we provide an example of how a CT0, involving the *Applying* Cognitive domain, was transformed into a CT2, which develops *Reasoning*.

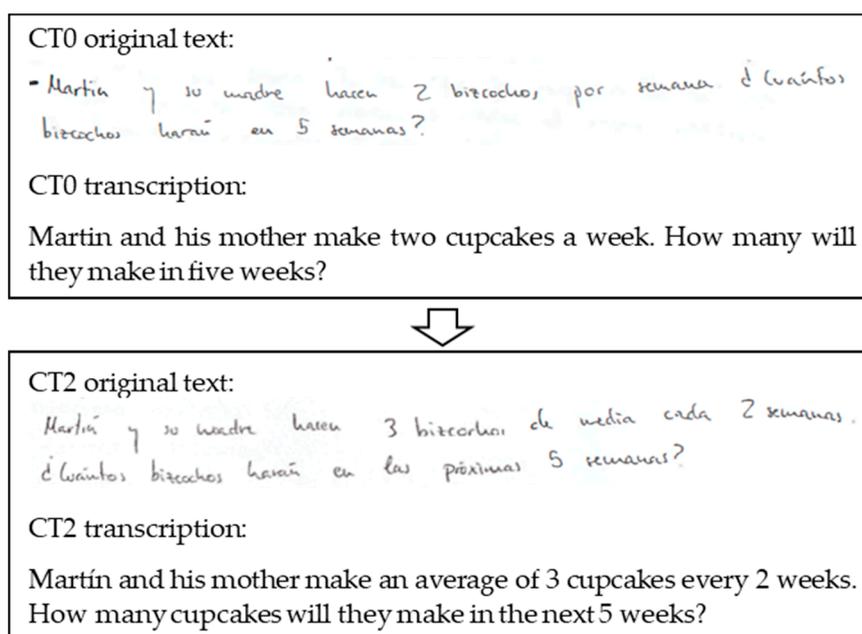


Figure 3. An *Applying* CT0 that turned out to be *Reasoning* CT2 (created by Group G14).

CT0 was found to be an *Applying* task because it only requires pupils to recognize the correct operation to be employed. That is a typical example when working multiplications and should not imply a big challenge for primary pupils. CT2 was considered a *Reasoning* task because it not only requires applying a multiplication to be solved but also connecting different elements of knowledge, representations, and procedures. Solving this task necessitates, for instance, evaluating the ways in which we can obtain an average of three cupcakes every 2 weeks and not confusing this with obtaining three cupcakes every week. Pupils must understand the concept of arithmetic mean and know how to calculate it. One way to solve this situation could be the following. We denote S_i as the number of cupcakes made per week. Considering that a mean of 3 cupcakes is made every two weeks $S_1 + S_2 = 3$. Then, the possible values are: $S_1 = 0$ and $S_2 = 3$; $S_1 = 1$ and $S_2 = 2$; $S_1 = 2$ and $S_2 = 1$; and $S_1 = 3$ and $S_2 = 0$. That is, the number of cupcakes per week varies between 0 and 3. To calculate how many cupcakes we will make in 5 weeks, we must add to these values the possible number of cupcakes made during the fifth week: 0, 1, or 3. This task requires reasoning and has different solutions. We could say that in the next 5 weeks, we can make between 6 and 9 cupcakes.

4.3. Authenticity

Authenticity was the characteristic of the CT0 better developed by the students. Of the total CT0s developed by the undergraduate and Master's students, 12% (9) were considered to be *Authentic* tasks. After the tasks refinement process, these percentages increased significantly in the CT2 to 43% (32). The Authenticity levels in CT0s increased significantly, with a large effect size, when transformed into CT2s (Wilcoxon test, $Z = 4.737$, $p < 0.001$, $r = 0.54$). As Figure 4 shows, the percentage of CT0s with a *Believable* dimension did not vary when modified into CT2s for any group. Just for the Master's students, the Authenticity levels in CT0s increased significantly, and with a large effect size, when transformed into CT2s (Wilcoxon test, $Z = 4.472$, $p < 0.001$, $r = 0.86$). The differences in Authenticity levels were not significant for degree's students and a small effect size was obtained (Wilcoxon test, $Z = 1.842$, $p = 0.066$, $r = 0.27$). This shows that the training received influenced the students to consider Authenticity as a characteristic of *Realistic* tasks.

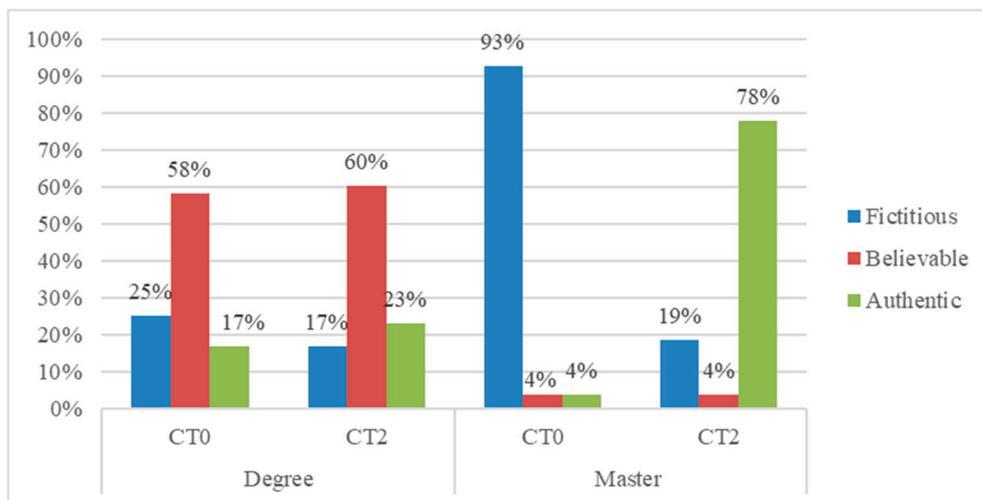


Figure 4. Percentages of Authenticity in CT0 and CT2 for undergraduate and Master's students.

Below, in Figure 5, we provide an example of a *Fictitious* CT0 that turned out to be an *Authentic* CT2 after the refinement process (created by Group G1)

CT0 original text:

Raúl y Ana salen de la tienda de chuches con un total de 20 regalices. De camino a casa Raúl se come 3 regalices y Ana se come 4. ¿Cuántos regalices les quedan al llegar a casa?

CT0 transcription:

Raúl and Ana leave the candy store with a total of 20 licorices. On the way home Raul eats 3 licorices and Ana eats 4; how many licorices did they have left when they got home?

CT2 original text:

Raúl y Ana van a la tienda de chuches que está al lado de su casa. Raúl compra 10 gominolas de sandía y 5 de plátano. Ana compra 7 gominolas de sandía. Al salir del quiosco a Ana se le caen sus gominolas y Raúl decide compartir las suyas con ella, pero como sólo le gustan los de sandía tiene que repartir sus gominolas de sandía entre los dos. ¿Cuántas gominolas de sandía les toca a cada uno para tener las mismas?

CT2 transcription:

Raul and Ana go to the candy store next to their house. He bought 10 watermelon jellybeans and 5 banana jellybeans, and she bought seven watermelon jellybeans. When they left the store, Ana drops her jellybeans and Raul decides to share his candies with her, but she only likes the watermelon jellybeans. So, they shared Raul's watermelon jellybeans. To share the same number of watermelon jellybeans, how many of them should Raul and Ana have?

Figure 5. A *Fictitious* CT0 that turned out to be an *Authentic* CT2 (created by Group G1).

To establish the level of Authenticity, we studied the values of the matrix: (event, question, data, purpose, specificity of the information). Concerning the event, the proposed situation was the same for CT0 and CT2; two children bought jellybeans at a candy store. This is a feasible real-life situation, and therefore the event was valued 1 in CT0 and CT2. Regarding the question, CT0 asks, “How many licorices did they have left when they got home?” It does not respond to a problematic situation, it seems more like an excuse to perform calculations. CT2 asks: “To share the same number of watermelon jellybeans, how many of them should Raul and Ana have?” This question is appropriate to the problem situation that arises in the context of the task. That is why it was valued 1. In relation to the data, the amount of candies that the children buy in CT0 and CT2 relates to a common real-life situation; consequently, both CT0 and CT2 were valued 1. Concerning the purpose: CT0 does not describe the intention for which we want to know the number of candies we will have when getting home. In CT2, it is clear that the aim, elucidated in the problem, is to distribute the watermelon jellybeans equally between the two children. For these reasons; CT0 and CT2 were valued 0 and 1 respectively. When referring to the specificity of the information, in CT0 and CT2 the protagonists have names and it is understood that they are children. Therefore, in both cases, CT0 and CT2 were valued 1. Taking in account the above analysis, the values of the matrix are (1,0,1,0,1) and (1,1,1,1,1) for CT0 and CT2, respectively. CT0 is thus considered a *Fictitious* task, while CT2 is understood as an *Authentic* task.

4.4. Openness

Of the total CT0s developed by the undergraduate and the Master’s students, 23% (17) were considered to be *Open* tasks. After the tasks refinement process, these percentages increased significantly in the CT2 to 43% (32). Specifically, the percentage increased from 17% (8) to 38% (18) in the case of the undergraduates and from 33% (9) to 52% (14) in the case of the Master’s students. The quantitative analysis revealed that the differences between the proportion of the total of *Open* CT0s and *Open* CT2s were significant, with a large effect size (McNemar’s test, $p = 0.012$, Cohen $g = 0.3$). The differences were significant, with a large effect size, for undergraduates (McNemar’s test, $p = 0.002$, Cohen $g = 0.5$), and were not significant, with a small effect size, for Master’s students (McNemar’s test, $p = 0.754$, Cohen $g = 0.1$). This shows that the training received influenced the undergraduate students to consider Openness as a characteristic of *Realistic* tasks, but the same did not happen with the Master’s students.

The video game task, described above in the Realism section, is an example of the *Open* tasks developed by the students. This particular task allows different approaches and solutions, depending on the variable taken. For example, you can consider the variable number of days; getting the money to buy the video game making the bed in the minimum number of days. Another possibility is, for example, the variable time—i.e., getting the money as early as possible.

5. Discussion

This work studies how to teach creating *Realistic* mathematics tasks to undergraduate and Master’s students who aim to become teachers and analyzes the tasks that the pre-service teachers created. We expected students to create *Realistic* tasks after two formative sessions. Initial tasks (CT0s) were designed from real contexts and, when necessary, transformed into CT2, developing their level of Realism as well as the Cognitive domain, Authenticity, and Openness characteristics learned across the training session. Innovation must be approached from a pedagogical point of view, not only as a means, but as a resource to achieve a more efficient way to create mathematics tasks [74].

Only 9% of the initially created tasks (CT0) turned out to be *Realistic*. This percentage increased to 13% (CT2) after the training sessions for reflective, creative, and cooperative work. Our results are similar to those of [68] that obtained 8% *Realistic* tasks, despite not having prepared their sample for this purpose. The training developed had a moderate effect in this aspect. That reveals the difficulty of pre-service teachers in understanding Realism and posing *Realistic* tasks. It is a crucial point to consider in mathematics education, because today mathematics learning is very much associated with solving real-life situations. The difficulty that pre-service teachers have in creating *Realistic* tasks may

relate to two facts: (1) not becoming familiar with *Realistic* tasks at the school level and (2) not receiving training in university programs.

The refinement process helped to improve other task characteristics, including Cognitive domains and Openness, with a large effect for undergraduates, and Authenticity, with a large effect for Master's student. *Reasoning* (the most reflective dimension of the Cognitive domain) was present in 65% of the CT2s. These results were far superior to the studies of [68] and [69], attaining a percentage not superior to 17%. Concerning Authenticity, 43% of the CT2s turned out to be *Authentic*; such findings concur with those of [68,69], as well as with the study of [63], specifically focused on creating and evaluating *Authentic* tasks with future teachers. Concerning the Openness characteristic, 43% of the CT2s were found to be *Open*; again, these figures were higher than those obtained by [68,69]. Considering the three characteristics aforementioned, we could stress that the tasks created in this study are more complete than those tasks usually proposed in textbooks, such as [37,43,45,46].

To some extent, we could also say that our pre-service teachers used their creativity and critical thinking to develop specific mathematics tasks. However, we detected that they were often unconscious of the possibilities offered by their tasks. The effort put into the iterative refinement process of the tasks seemed to be connected with the high level of engagement and motivation the future teachers exerted during the experience. Undertaking practical activities focused on their future students and working in real contexts, such as the one proposed here, turned out to be a source of motivation and inspiration for our future teachers, an aspect that has already been pointed out by other researchers (see, for example, [75]).

6. Conclusions

This study provides a form of training for pre-service teachers (undergraduate and Master's students) to create tasks in real contexts and amend them through an iterative process of analysis and refinement. This also suggests an approach to analyze mathematics tasks in relation to the four characteristics, Realism, Cognitive domains, Authenticity, and Openness. This approach can be helpful in future studies for evaluating the abilities to develop real-life mathematics tasks both in the context of teacher training and lifelong professional education. During this task development process, the future teachers worked in a collaborative environment, developing their creativity and critical thinking. The outcomes confirmed that the creation of *Realistic* mathematics tasks is a challenge for future primary teachers; consequently, training programs focused on creative learning and critical thinking are required to foster teachers' development of *Realistic* mathematics tasks. These pre-service teachers were, however, able to create tasks with high levels of Cognitive domain, Authenticity, and Openness. Although both the undergraduate and Master's students worked under similar conditions (receiving the same formative sessions and being supervised by the same lecturer), the undergraduate students were able to integrate higher Cognitive domains in their tasks and higher percentages of Openness tasks than the Master's students. The Master's students work, however, better on the Authenticity characteristic. Further research with a larger and more representative sample would be necessary to explain the reasons for such differences. In fact, one limitation of our study is that the sample was selected by a convenient strategy, as we only invited those future teachers to whom we had access to participate in the experience.

The results obtained suggested that this form of training, where pre-service teachers have to create specific tasks through an iterative process of design and refinement, is useful not only for instructing future teachers in task design but also for general professional development. It could thus have significant educational implications—for example, for designing adequate teacher training courses that may help primary students to become capable mathematics tasks solvers. As for future research, it would be interesting to develop and analyze tasks not only focused on numbers and measurement but also other mathematics domains such as geometry, algebra, data organization, and probability, as presented in the TIMSS 2019 framework [48]. It would be relevant to assess how primary students

solve mathematics tasks like the ones created in the present work. Further studies could also consider validating the approach presented in this work to analyze real-life tasks.

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