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The Hierarchical VIKOR Method with Incomplete Information: Supplier Selection Problem

Jong Hyen Kim and Byeong Seok Ahn *

College of Business and Economics, Chung-Ang University, 221 Heukseok, Dongjak, Seoul 156-756, Korea; kimjh0123@cau.ac.kr

* Correspondence: bsahn@cau.ac.kr; Tel.: +82-2820-5582

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Abstract: To solve a multi-criteria decision-making problem, many attempts have been made to alleviate difficulties of obtaining precise preference information attributed to time pressure, lack of data and domain knowledge, limited attention and information processing capabilities, etc. Structuring any decision problem hierarchically is known to be an efficient way of dealing with complexity and identifying the major components of the problem. In this paper, we propose the hierarchical VIKOR method that uses incomplete alternatives' values as well as incomplete criteria weights, extending previous works that consider mostly intervals or fuzzy under a flat structure of criteria. It ranks alternatives using the aggregated scores of group utility and individual regret scores which are computed from the linear programs. To show how to use our proposed method, we exemplified an international supplier selection problem that affects the organization's sustainable growth.

Keywords: VIKOR method; hierarchical VIKOR method; incomplete information

1. Introduction

Multi-criteria decision-making (MCDM) methods provide an effective means of assisting decision-makers to select the best alternative under various criteria. Especially, incomplete information has been widely-used in MCDM problems because of the diverse difficulties the decision might need to be made under lack of time, the decision-maker might not want to provide precise data, or the decision-maker might have a limited scope of information [1,2]. The incomplete information in the literature on MCDM encompasses incomplete weights and values. Many studies about incomplete weights are found, more frequently relative to incomplete values, in MCDM literature since they often occur in practice [3–11]. The linear types of incomplete values may include [10,12]: (a) Strict preference, (b) weak preference, (c) weak differences of preference, (d) ratio preferences.

The VIKOR method was developed for multi-criteria optimization of complex systems [13], which focuses on ranking and selecting from a set of available alternatives in the presence of conflicting criteria [14,15]. The VIKOR method is suitable for those situations where the goal is to maximize profit while the risk of the decisions is deemed to be less important. The major advantage of the VIKOR method is that it can trade off the maximum group utility of the “majority” and the minimum individual regret of the “opponent” [14,16,17].

A number of applications have been developed in the fields of studies (e.g., material selection, water resource planning, land-use restrictions, satisfaction measuring, investment, etc.), particularly in view of organization's sustainability under MCDM environment [18–28].

Fei et al. [29] proposed the DS-VIKOR method for supplier selection problem, which extends the VIKOR method by Dempster–Shafer evidence theory. Yang and Wu [30] presented the R-VIKOR method using the historical maximum and minimum data to resolve the rank reversal. Dev et al. [31] used the Entropy-VIKOR method to piston material selection problem.

In terms of information type, various forms of incomplete information (mainly fuzzy data) were dealt with in the context of the VIKOR method. Sayadi et al. [32] introduced the extended VIKOR method using interval criteria values. Opricovic [21] considered triangular fuzzy data in an application to water resources planning. Chatterjee and Chakraborty [33] analyzed the ranking performance of the original VIKOR method and its five variants, such as the comprehensive, the fuzzy, the regret theory-based, the modified, and the interval VIKOR methods. Kim and Ahn [34] proposed the VIKOR method using incomplete criteria weights under the flat structure of criteria weights. Moreover, they provided new insights that shed light on the relationship between the VIKOR and the decision-making under uncertainty (DMUU) method.

In this paper, we propose a new VIKOR method that makes use of incomplete alternatives' values, as well as incomplete criteria weights in the hierarchically structured problem. Structuring any decision problem hierarchically is an efficient way of dealing with complexity and identifying the major components of the problem. The hierarchical structure allows management in constructing a hierarchy to fit their idiosyncratic needs [35]. A number of studies [27,36–40], entitled the VIKOR with fuzzy AHP, are found in the literature on the VIKOR, but they use the hierarchical structure for determining the criteria weights and then apply the classical VIKOR method to aggregate the scores. Our hierarchical VIKOR method aggregates scores in bottom-up fashion until reaching top-most node under hierarchically structured criteria. Further, we propose the hierarchical VIKOR method featuring both (a) incomplete criteria weights instead of previous entropy (objective weights), equal weights, AHP, or fuzzy (subjective weights) and (b) incomplete alternatives' values instead of interval or fuzzy values.

The remaining of the paper is organized as follows. In Section 2, we review the VIKOR method, and then present types of incomplete alternatives' values and their extreme points. Further, we present an information processing procedure under our hierarchical VIKOR method incorporated by incomplete alternatives' values as well as incomplete criteria weights. A numerical example and the discussions of its results are presented in Sections 3 and 4 respectively. Finally, the conclusion is in Section 5.

2. The Hierarchical VIKOR Method with Incomplete Information

2.1. The VIKOR Method

In MCDM problems, one usually considers a finite discrete set of alternatives $A_i, i = 1, \dots, m$, each of which is evaluated by n multiple criteria $c_j, j = 1, \dots, n$. Table 1 shows a decision matrix composed of the alternatives, the criteria, and the consequence f_{ij} of alternative A_i with respect to the criterion, c_j . Further, we denote a set of unknown criteria weights as $W = \{w : \sum_{j=1}^n w_j = 1, w_j \geq 0, j = 1, \dots, n\}$.

Table 1. Decision matrix.

	c_1	c_2	$\dots$	c_n
A_1	f_{11}	f_{12}	$\dots$	f_{1n}
A_2	f_{21}	f_{22}	$\dots$	f_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	f_{m1}	f_{m2}	$\dots$	f_{mn}

Considering the L_p -metric below, the VIKOR method uses both a maximum group utility based on L_1 and a minimum individual regret of the "opponent" based on L_∞ (see Equations (3) and (4)).

$$L_{pi} = \left(\sum_{j=1}^n \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right)^p \right)^{\frac{1}{p}}, 1 \leq p < \infty, i = 1, 2, \dots, m. \quad (1)$$

We briefly describe the procedure of the VIKOR method in the following four steps (For convenience, we assume “more is better” criteria):

- (a) Determine the best f_j^* and the worst f_j^- for each criterion function.

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij}, \quad j = 1, \dots, n. \quad (2)$$

- (b) Compute S_i (group utility) and R_i (individual regret) for each alternative.

$$S_i = \sum_{j=1}^n w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-), \quad i = 1, \dots, m, \quad (3)$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-), \quad i = 1, \dots, m. \quad (4)$$

- (c) Compute the value Q_i for each alternative.

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*}, \quad i = 1, \dots, m \quad (5)$$

$$S^* = \min_i S_i, \quad S^- = \max_i S_i, \quad (6)$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i. \quad (7)$$

The constant v is the weight introduced to support the strategy of maximum group utility while $(1 - v)$ is used to weigh the individual regret, usually, $v = 0.5$.

- (d) Rank the alternatives by sorting the values S_i , R_i , and Q_i in descending order. The results are three ranking lists that can be used to propose and validate a compromise solution [14].

2.2. The Incomplete Information

The incomplete information has been widely-used in MCDM problems because of various difficulties associated with information acquisition the decision might need to be made under lack of time, the decision-maker might not want to provide precise data, or the decision-maker might have limited scope of information [1,2]. We find in the literature on the VIKOR that the incomplete information has been used mostly in the form of interval or fuzzy until Kim and Ahn [34] proposed a more general form of incomplete criteria weights. Ahn [12] presented the formulas for determining the extreme points of four types of incomplete criteria weights, such as weak inequalities, strict inequalities, ratio bounds, and preference differences (see [3–12,41–43] for more references). The incomplete information about values could occur in practice as well. For qualitative criteria, a decision-maker may say an alternative is best (100%), and another is in the level between 80% and 90% relative to the level of the first one [44]. This can be expressed in the form of ratio bounds. Similarly to the incomplete criteria weights, the formulas for determining the extreme points of incomplete values are summarized in Table 2.

Table 2. Extreme points of incomplete values.

Forms	Extreme Points
Strict preference $V_j = \{1 \geq v_j(x_i) - v_j(x_{i+1}) \geq \epsilon_i \geq 0,$ $i = 1, \dots, m-1, v_j(x_m) \geq 0\}$	$v_j^1 = (1, t_2, t_3, \dots, t_{m-1}, 0)^T,$ $v_j^2 = (1, 1 + t_2 - t_1, t_3, \dots, t_{m-1}, 0)^T,$ $v_j^3 = (1, 1 + t_2 - t_1, 1 + t_3 - t_1, t_4, \dots, t_{m-1}, 0)^T,$ $\dots$ $v_j^{m-1} = (1, 1 + t_2 - t_1, \dots, 1 + t_{m-1} - t_1, 0)^T,$ $(t_j = \sum_{k=j}^m \epsilon_k, j = 1, \dots, m-1)$
Weak preference $V_j = \{1 \geq v_j(x_1) \geq v_j(x_2) \geq \dots \geq v_j(x_m) \geq 0\}$	$v_j^1 = (1, 0, 0, \dots, 0)^T,$ $v_j^2 = (1, 1, 0, \dots, 0)^T,$ $\dots$ $v_j^{m-1} = (1, 1, 1, \dots, 1, 0)^T$
Weak differences in preference $V_j =$ $\{1 \geq v_j(x_1) - v_j(x_2) \geq \dots \geq v_j(x_{m-1}) - v_j(x_m) \geq v_j(x_m) \geq 0\}$	$v_j^1 = (1, 0, 0, \dots, 0)^T,$ $v_j^2 = (1, \frac{1}{2}, 0, \dots, 0)^T,$ $\dots$ $v_j^{m-2} = (1, \frac{m-3}{m-2}, \dots, \frac{1}{m-2}, 0, 0)^T,$ $v_j^{m-1} = (1, \frac{m-2}{m-1}, \dots, \frac{1}{m-1}, 0, 0)^T$
Ratio preference $V_j = \{1 \geq v_j(x_i) \geq \alpha_i v_j(x_{i+1}) \geq 0,$ $i = 1, \dots, m-1, v_j(x_m) \geq 0\}$	$v_j^1 = (1, 0, 0, \dots, 0)^T,$ $v_j^2 = (1, \frac{1}{\alpha_1}, 0, \dots, 0)^T,$ $v_j^3 = (1, \frac{1}{\alpha_1}, \frac{1}{\alpha_1 \alpha_2}, 0, \dots, 0, 0)^T,$ $\dots$ $v_j^{m-1} = (1, \frac{1}{\alpha_1}, \frac{1}{\alpha_1 \alpha_2}, \dots, \frac{1}{\alpha_1 \alpha_2 \dots \alpha_{m-2}}, 0)^T$

2.3. The Hierarchical VIKOR with Incomplete Information

2.3.1. The Hierarchical Structure

Given a hierarchical problem, the decision-maker will provide his or her preference information on the values of $\{w_j\}$ and $\{v_j(\cdot)\}$. It is assumed that both criteria weights and alternatives' values are not known precisely and hence information is to satisfy linear constraints. To model our hierarchical VIKOR with incomplete information, we adopt the notations that Ahn et al. [44] used in the mathematical programming technique for establishing dominance between alternatives under hierarchically structured criteria:

- $D(j) \subset J$ the set of criteria index which are structured immediately under criterion c_j . c_0 is the topmost goal.
- T the set of twig level criteria, $T = \{j \in J | D(j) = \emptyset\}$.
- $W_{D(j)}$ the set of constraints on the weights w_k of criteria $k \in D(j)$.
- V_j the set of constraints on the values $v_j(\cdot)$ of alternatives for each criterion $j \in T$.

Examples of $W_{D(j)}$ and V_j can be

$$W_{D(j)} = \{(w_k, w_l) | w_k \geq 2w_l, w_k, w_l \geq 0\}, k, l \in D(j),$$

$$V_k = \{(v_k(x), v_k(y)) | v_k(x) \geq 3v_k(y) \geq 0, 0.6 \leq v_k(x) \leq 0.8\}.$$

They proposed two approaches (a) weight-additive approach and (b) weight-product approach of which we adopt the former and denote the set of weights as follows:

$$W = \begin{cases} \sum_{j \in D(0)} w_j = 1 \\ \sum_{k \in D(j)} w_k = w_j & \text{for } j \notin T \\ W_{D(j)} & \text{for } j \notin T \end{cases} \quad (8)$$

When the criteria weights and values are incompletely known, our hierarchical VIKOR method aggregates information from bottom to top and finally rank alternatives via one of two approaches: (a) the extreme point approach and (b) the LP approach.

2.3.2. The Hierarchical VIKOR Method

In the following, we shall describe our hierarchical VIKOR method with incomplete information in five steps, as shown in Figure 1.

Figure 1. Flowchart of the hierarchical VIKOR method with incomplete information.

Step 0. Structurize the problem at hand to the hierarchical one when it is deemed appropriate to do so.

If it seems to be appropriate to decompose the criteria featuring the problem into multi-leveled criteria, we construct a multi-layered hierarchical structure of criteria in a spirit of divide and conquer. Required information is gathered from decision-maker in bottom-up according to the hierarchical structure.

Step 1. Compute the extreme points of incomplete alternatives' values and determine the lower and upper consequences of the i th alternative.

$$(d_1^L(x_i), \dots, d_n^L(x_i)) = \left(\left((1 - v_1^U(x_i), \dots, 1 - v_n^U(x_i)) \mid j \in \Psi \right), \left((v_1^L(x_i), \dots, v_n^L(x_i)) \mid j \in O \right) \right) \quad (9)$$

$$(d_1^U(x_i), \dots, d_n^U(x_i)) = \left(\left((1 - v_1^L(x_i), \dots, 1 - v_n^L(x_i)) \mid j \in \Psi \right), \left((v_1^U(x_i), \dots, v_n^U(x_i)) \mid j \in O \right) \right) \quad (10)$$

where Ψ and O are the sets of indices associated with benefit and cost criteria, respectively. Moreover, note that $v_j^L(x_i) = \min(\mathbf{v}_j(x_i))$ and $v_j^U(x_i) = \max(\mathbf{v}_j(x_i))$ where $\mathbf{v}_j(x_i)$ is a collection of elements associated with alternative x_i in the set of extreme points of V_j , i.e., $\mathbf{v}_j(x_i) = (v_j^1(x_i), v_j^2(x_i), \dots, v_j^{m-1}(x_i))$, $i = 1, \dots, m$, $j = 1, \dots, n$.

Step 2. Compute S and R intervals. In general, it is an efficient way to obtain S and R intervals via LPs for the hierarchical problem since the multi-leveled criteria require enormous calculations to obtain the extreme points of the criteria weights. Below are LPs for obtaining the intervals $[S_i^L, S_i^U]$ and $[R_i^L, R_i^U]$:

$$\begin{aligned} S_i^L &= \min \sum_{j \in J} d_j^L(x_i) w_j \\ \text{s.t. } &\sum_{k \in D(j)} w_k = w_j \text{ for } j \notin T, \\ &W_{D(j)} \text{ for } j \notin T. \end{aligned} \quad (11)$$

$$\begin{aligned} S_i^U &= \max \sum_{j \in J} d_j^U(x_i) w_j \\ \text{s.t. } &\sum_{k \in D(j)} w_k = w_j \text{ for } j \notin T, \\ &W_{D(j)} \text{ for } j \notin T. \end{aligned} \quad (12)$$

$$R_i^L = \min_j (R_j^L(x_i)), \quad R_i^U = \max_j (R_j^U(x_i)), \quad (13)$$

$$\begin{aligned} R_j^L(x_i) &= \max d_j^L(x_i) w_j \\ \text{s.t. } &\sum_{k \in D(j)} w_k = w_j \text{ for } j \notin T, \\ &W_{D(j)} \text{ for } j \notin T. \end{aligned} \quad (14)$$

$$\begin{aligned} R_j^U(x_i) &= \max d_j^U(x_i) w_j \\ \text{s.t. } &\sum_{k \in D(j)} w_k = w_j \text{ for } j \notin T, \\ &W_{D(j)} \text{ for } j \notin T. \end{aligned} \quad (15)$$

Calculating $R_i \in [R_i^L, R_i^U]$ is troublesome work requiring repeated calculations and thus we modify Equations (14)–(16) by Foroughi and Aouni [45].

$$\begin{aligned}
 R_i^L = \min \quad & \beta \\
 \text{s.t.} \quad & \sum_{k \in D(j)} w_k = w_j \text{ for } j \notin T, \\
 & W_{D(j)} \text{ for } j \notin T, \\
 & d_j^L(x_i)w_j - \beta \leq 0, \quad j = 1, \dots, n, \\
 & \beta \geq 0.
 \end{aligned} \tag{16}$$

The third constraint in Equation (16) means that $\beta \geq \max(d_1^L(x_i)w_1, d_2^L(x_i)w_2, \dots, d_n^L(x_i)w_n)$ and thus results in the mini-max of $d_j^L(x_i)w_j$ for all j in view of the objective function.

Step 3. Compute Q intervals. Compute the value Q_i for each alternative using the relation.

$$\begin{aligned}
 Q_i^L &= v \frac{S_i^L - S^*}{S^- - S^*} + (1 - v) \frac{R_i^L - R^*}{R^- - R^*}, \quad i = 1, \dots, m \\
 Q_i^U &= v \frac{S_i^U - S^*}{S^+ - S^*} + (1 - v) \frac{R_i^U - R^*}{R^+ - R^*}, \quad i = 1, \dots, m \\
 S^* &= \min_i S_i, S^- = \max_i S_i, \\
 R^* &= \min_i R_i, R^- = \max_i R_i.
 \end{aligned} \tag{17}$$

The constant v is the weight introduced to support the strategy of maximum group utility while $(1 - v)$ is used to weigh the individual regret, usually, $v = 0.5$.

Step 4. Determine the final ranking. Further computations are needed to obtain the final ranking of alternatives in the face of the intervals $[Q_i^L, Q_i^U]$ in Step 3. The methods for ranking intervals can be classified into three categories. One of the most widely-used methods is to consider each location of intervals (i.e., the gaps, overlapping, etc.) and the distributions of intervals [17,41]. Xu and Da [46] presented formulas for comparing intervals based on the degree of possibility. Ahn [47] presented a method to prioritize intervals by taking into account the strength of preference based on a probabilistic measure. The similarity between two intervals can be gauged by two measures that characterize the intervals: The ratio of the overlapping portion of two intervals and the level of closeness of the midpoints between two intervals [48].

3. Numerical Example

In this section, we present a numerical example to show how the proposed method can be used to solve an MCDM problem. We adopt an international supplier selection problem, which consists of a hierarchy of three levels [49], as shown in Figure 2. In an era of global sourcing, the multinational firm's sustainable growth often hinges on the most appropriate selection of its foreign suppliers. International supplier selection, however, is very complicated and risky owing to a variety of uncontrollable and unpredictable factors affecting the decision. These factors may include political situations, tariff barriers, cultural and communication barriers, trade regulations and agreements, currency exchange rates, cultural differences, ethical standards, quality standards, etc. Nevertheless, a vast majority of the purchasing literature still focuses on the domestic aspects and neglects international supplier selection research [25].

Figure 2. An example of a hierarchy of criteria.

The top level of the hierarchy represents the goal of the problem. The second level of the hierarchy contains the seven criteria (financial terms, quality assurance, perceived risks, service performance, buyer-supplier partnership, cultural and communication barriers, and trade restrictions), which are considered important in selecting the foreign supplier. Finally, the bottom level of the hierarchy is represented by sub-criteria, which are decomposed from a higher hierarchy. We consider five countries (A_i , $i = 1, \dots, 5$) as trading counterparts and the incomplete information pertinent to this supplier selection problem is given in Appendix A.

Step 1. In addition to the four types of incomplete alternatives' values in Table 2, we include verbal expression for the criteria c_{14} , c_{16} , c_{23} , and c_{25} , and use the Bipolar scale to convert it into the interval scale [50].

We illustrate how to obtain $d_{18}^L(x_1)$ and $d_{18}^U(x_1)$ when incomplete alternatives' values for "Delayed delivery" criterion are given as V_{18} ,

$$V_{18} = \{1 \geq v_{18}(A_2) \geq 1.5v_{18}(A_3) = 1.5v_{18}(A_4) \geq 3v_{18}(A_1) \geq 4.5v_{18}(A_5) \geq 0\}.$$

The ratio information in V_{18} can be interpreted as "the numbers of delayed deliveries occurred by A_2 is at least one and half times more than by A_3 and A_4 , the numbers by A_1 is half less than A_3 and A_4 , and finally the numbers by A_1 is at least one and half times more than by A_5 during the last decade".

The extreme points of the set V_{18} are determined by

$$v_{18}^1 = (0, 1, 0, 0, 0)^T, v_{18}^2 = (0, 1, 0.6667, 0.6667, 0)^T, v_{18}^3 = (0.3333, 1, 0.6667, 0.6667, 0)^T \text{ and } d_{18}^L(x_1) = 0, d_{18}^U(x_1) = 0.3333.$$

All the values of $d_j^L(x_i)$ and $d_j^U(x_i)$ are denoted in Table 3.

Step 2. Compute the intervals of $[S_i^L, S_i^U]$ and $[R_i^L, R_i^U]$, using Equations (11)–(16). We show how to compute the intervals $[S_1^L, S_1^U]$ and $[R_1^L, R_1^U]$ for A_1 for illustrative purposes:

$$\begin{aligned} S_1^L = & \min 0.4286w_{12} + w_{13} + 0.4494w_{14} + 0.2857w_{15} + 0.6742w_{16} + w_{17} + 0.3333w_{18} \\ & + w_{19} + w_{20} + 0.2532w_{23} + w_{24} + w_{26} \\ \text{s.t. } & w_j = \sum_{k \in D(j)} w_k, \quad j = 1, \dots, 7 \\ & W_{D(j)}, \quad j = 0, \dots, 7 \end{aligned} \quad (18)$$

$$\begin{aligned} S_1^U = & \max w_{11} + 0.7143w_{12} + w_{13} + 0.6629w_{14} + w_{15} + 0.8876w_{16} + w_{17} + w_{18} \\ & + w_{19} + w_{20} + w_{22} + 0.4937w_{23} + w_{24} + 0.5714w_{25} + w_{26} \\ \text{s.t. } & w_j = \sum_{k \in D(j)} w_k, \quad j = 1, \dots, 7 \\ & W_{D(j)}, \quad j = 0, \dots, 7 \end{aligned} \quad (19)$$

$$\begin{aligned} R_1^L = & \min \beta \\ \text{s.t. } & w_j = \sum_{k \in D(j)} w_k, \quad j = 1, \dots, 7 \\ & W_{D(j)}, \quad j = 0, \dots, 7 \\ & w_i \leq \beta, \quad i = 8, \dots, 26, \\ & \beta \geq 0. \end{aligned} \quad (20)$$

The optimal solutions to the Equations (18)–(20) are $[S_1^L, S_1^U] = [0, 0.8571]$ and $R_1^L = 0$.

We can also obtain R_1^U as follows:

$$\begin{aligned} R_1^U &= \max_j (R_j^U(x_1)) \\ &= \max(0, 0, 0, 0.5, 0.18, 0.33, 0.09, 0.08, 0.15, 0.07, 0.25, 0.13, 0.2, 0, 0.06, 0.04, 0.17, 0.08, 0.07) \\ &= 0.5 \end{aligned}$$

For example,

$$\begin{aligned} R_{12}^U(x_1) &= \max 0.1743w_{12} \\ \text{s.t. } & \sum_{k \in D(j)} w_k = w_j, \quad j = 1, \dots, 7 \\ & W_{D(j)}, \quad j = 0, \dots, 7. \end{aligned} \quad (21)$$

Step 3. Compute the intervals of $[Q_i^L, Q_i^U]$, $i = 1, \dots, 5$. We illustrate $[Q_1^L, Q_1^U]$ for alternative A_1 with $\alpha = 0.5$.

$$\begin{aligned} S^* &= 0, \quad S^- = 1, \\ R^* &= 0, \quad R^- = 1, \\ Q_1^L &= \frac{0.5(0-0)}{1-0} + \frac{0.5(0-0)}{1-0} = 0, \\ Q_1^U &= \frac{0.5(0.8571-0)}{1-0} + \frac{0.5(0.5-0)}{1-0} = 0.6786. \end{aligned}$$

The intervals $[S_i^L, S_i^U]$, $[R_i^L, R_i^U]$, $[Q_i^L, Q_i^U]$, and the midpoint of $[Q_i^L, Q_i^U]$ for all alternatives are listed in Table 4.

Step 4. According to the midpoints (Q_i^M) in the last column in Table 4, the final ranking of alternatives is determined by $A_1 > A_3 > A_5 > A_4 > A_2$. Other methods to rank intervals are discussed in Section 4.

Table 3. The values of $d_j^L(x_i)$ and $d_j^U(x_i)$ for each alternative with respect to each criterion.

Level 1		c_1			c_2
Sub-criteria	c_8	c_9	c_{10}	c_{11}	c_{12}
A_1	0	[0, 0]	[0, 0]	[0, 1]	[0.4286, 0.7143]
A_2	1	[1, 1]	[0, 0.9]	[0, 1]	[0.1429, 0.7143]
A_3	0.3913	[0, 0]	[0, 0.9]	[0, 1]	[0.8571, 1]
A_4	0.5652	[1, 1]	[0, 1]	[0, 1]	[0.1429, 0.4296]
A_5	0.2391	[1, 1]	[0, 1]	[1, 1]	[0, 0.5714]
Level 1		c_3			
Sub-criteria	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}
A_1	[1, 1]	[0.4494, 0.6629]	[0.2857, 1]	[0.6742, 0.8876]	[1, 1]
A_2	[0, 1]	[0.6742, 0.8876]	[0, 0.7619]	[0, 0.2135]	[0, 1]
A_3	[0, 1]	[0.8989, 1]	[0.5714, 1]	[0.8989, 1]	[0, 1]
A_4	[0, 1]	[0, 0.2135]	[0.8571, 0.9048]	[0.6742, 0.8876]	[0, 1]
A_5	[0, 0]	[0.2247, 0.4494]	[0.5714, 0.9524]	[0.2247, 0.4494]	[0, 1]
Level 1		c_4		c_5	
Sub-criteria	c_{18}	c_{19}	c_{20}	c_{21}	
A_1	[0.3333, 1]	[1, 1]	[1, 1]	[0, 0]	
A_2	[1, 1]	[0.6667, 1]	[0, 1]	[0, 1]	
A_3	[0, 0.6667]	[0, 0]	[0, 1]	[0, 1]	
A_4	[0, 0.6667]	[0.5, 1]	[0, 1]	[0, 1]	
A_5	[0, 0]	[0.75, 1]	[0, 0]	[1, 1]	
Level 1		c_6		c_7	
Sub-criteria	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}
A_1	[0, 1]	[0.2532, 0.4937]	[1, 1]	[0, 0.2405]	[1, 1]
A_2	[0, 1]	[0, 0.2405]	[0, 1]	[0, 0.2405]	[0, 1]
A_3	[0, 1]	[0.7595, 1]	[0, 0]	[0.7595, 1]	[0, 0]
A_4	[1, 1]	[0.2532, 0.4937]	[0, 1]	[0, 0.2405]	[0, 0]
A_5	[0, 0]	[0.2532, 0.4937]	[0, 1]	[0.5063, 0.7468]	[0, 1]

Table 4. S, R, and Q intervals with $v = 0.5$.

	S Interval		R Interval		Q Interval		
	S_i^L	S_i^U	R_i^L	R_i^U	Q_i^L	Q_i^U	Q_i^M
A_1	0	0.8571	0	0.5	0	0.6786	0.3393
A_2	0.2143	1	0.0588	1	0.1366	1	0.5683
A_3	0.0186	0.8261	0.0186	0.5	0.0186	0.6631	0.3408
A_4	0.0745	0.9517	0.0379	0.5652	0.0562	0.7585	0.4073
A_5	0.1056	0.8732	0.055	0.5	0.0803	0.6866	0.3834

4. Discussion

In this section, we look into how sensitive the ranking of alternatives is on the values of v , $0 \leq v \leq 1$ in (17) representing relative importance of group utilities to individual regrets. This is because different values of v , in due course, result in different Q intervals, which consequently can yield other rankings of alternatives. In fact, the midpoint approach dealt with in Step 4 of Section 3 fails to take into account useful information, such as, for example, distributions of intervals [17,41], degree of possibility [46,47], and the ratio of the overlapping portion and the level of closeness of the midpoints of the intervals [48].

As before, the values $Q_i \in [Q_i^L, Q_i^U]$ and $Q_j \in [Q_j^L, Q_j^U]$ are the aggregated intervals of group utilities and individual regrets for alternatives A_i and A_j for a given value of v . Then the degree of possibility of Q_i over Q_j (i.e., A_i over A_j) is defined as

$$p_{ij} = \max \left\{ 1 - \max \left\{ \frac{Q_j^U - Q_i^L}{(Q_i^U - Q_i^L) + (Q_j^U - Q_j^L)}, 0 \right\}, 0 \right\} \text{ where } p_{ij} \geq 0, p_{ij} + p_{ji} = 1, p_{ii} = 0.5.$$

The degree of possibility of A_i over the other alternatives is computed by

$$\psi_i = \sum_{j=1}^n p_{ij}, i = 1, \dots, n.$$

Applying the measure ψ_i to Q_i intervals in Table 4 leads to $\psi_1 = 2.2666$, $\psi_2 = 3.0279$, $\psi_3 = 2.2669$, $\psi_4 = 2.5141$, $\psi_5 = 2.4245$ to obtain a ranking of alternatives $A_1 \geq A_3 \geq A_5 \geq A_4 \geq A_2$. Note that this ranking is obtained based on the ascending order of ψ and $v = 0.5$.

Table 5 below lists the degree of possibility of each alternative over the other alternatives, depending on various v and its resulting ranking of alternatives.

Table 5. The degree of possibility of each alternative over others depending on the values of v .

	v										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ψ_1	2.2173	2.2279	2.2381	2.248	2.2574	2.2666	2.2756	2.2843	2.2927	2.3009	2.3087
ψ_2	3.2207	3.1801	3.1406	3.1021	3.0645	3.0279	2.9921	2.9571	2.923	2.8898	2.8571
ψ_3	2.3846	2.3598	2.3358	2.3124	2.2895	2.2669	2.2446	2.2225	2.2007	2.1789	2.1573
ψ_4	2.5992	2.5829	2.5661	2.549	2.5317	2.5141	2.4963	2.4783	2.4599	2.4414	2.4229
ψ_5	2.3423	2.3606	2.3779	2.3941	2.4097	2.4245	2.4386	2.4522	2.4652	2.4777	2.4899
Ranking	*	**	**	**	**	**	***	***	****	****	****

* $A_1 \geq A_5 \geq A_3 \geq A_4 \geq A_2$; ** $A_1 \geq A_3 \geq A_5 \geq A_4 \geq A_2$; *** $A_3 \geq A_1 \geq A_5 \geq A_4 \geq A_2$; **** $A_3 \geq A_1 \geq A_4 \geq A_5 \geq A_2$

Four different rankings can be derived from Figure 3. First, the ranking of $A_1 \geq A_5 \geq A_3 \geq A_4 \geq A_2$ is obtained when the decision-maker does not emphasize the group utility at all ($v = 0$). We have the ranking of $A_1 \geq A_3 \geq A_5 \geq A_4 \geq A_2$ in case of $0.1 \leq v \leq 0.5$. If the decision-maker stresses the group utility slightly more ($0.6 \leq v \leq 0.7$) or extremely ($0.8 \leq v \leq 1$) over the individual regret, we find that only the rank order of alternatives A_4 and A_5 changes, thus resulting in $A_3 \geq A_1 \geq A_5 \geq A_4 \geq A_2$ and $A_3 \geq A_1 \geq A_4 \geq A_5 \geq A_2$, respectively.

Figure 3. The rankings of alternatives based on the values of v .

5. Conclusions

Expressing a set of criteria hierarchically is a natural way in the process of decomposing composite criteria into their subordinate sub-criteria (if necessary, sub-sub criteria). Further, information regarding the criteria weights can be more easily obtained by comparing criteria versus criteria of the same layer in the hierarchical structure. A number of studies were presented in the literature, mostly entitled the VIKOR method with analytic hierarchy process (AHP) to benefit from such a hierarchical representation. Unfortunately, however, we find that the VIKOR method and hierarchical structure of criteria are used separately (to say rigorously, independently) in a sense that the hierarchical structure of criteria is used in determining the criteria weights to be fed into the conventional VIKOR. In this paper, we attempted to embody a true hierarchical VIKOR method by solving the MCDM problems in two steps: (1) Represent the MCDM problems hierarchically in top-down fashion and (2) aggregate information in bottom-up fashion.

The increasing complexity of the socio-economic environments makes it less and less possible for a single decision-maker to consider all relevant aspects of a decision problem. Therefore, many organizations employ a number of experts who exhibit their own domain knowledge. Our hierarchical VIKOR method in the paper is originally designed to solve a single decision-maker's problem and thus it is needed to cover a group decision-making problem frequently faced in the real world. With regard to this, we have to consider some strategies about how to aggregate individual preferences. In the first option, many efforts are made to communicate and negotiate among a group of members to narrow down their diverse preferences to a single aggregated preference. If this is successful, we directly apply our hierarchical VIKOR method. Otherwise, we solve individual decision-making problems (using our hierarchical VIKOR method) as many times as the number of decision-makers involved and finally attempt to aggregate individual decision outcomes expressed by the intervals of group utilities and individual regrets. Further, detailed development is left for future study.

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Appendix A

Table A1. Incomplete information about criteria weights.

Hierarchy	Incomplete Criteria Weights
Goal (c_0)	$W_{D(0)} = \{w_1 \geq w_2 \geq w_3 \geq w_4 \geq w_5 \geq w_6 \geq w_7 \geq 0\}$
Sub-criteria of c_1	$W_{D(1)} = \{w_8 \geq w_9 \geq w_{10} \geq 0\}$
Sub-criteria of c_2	$W_{D(2)} = \{w_{11} \geq w_{12} \geq 0\}$
Sub-criteria of c_3	$W_{D(3)} = \{w_{13} \geq w_{16} \geq w_{14} \geq w_{17} \geq w_{15} \geq 0\}$
Sub-criteria of c_4	$W_{D(4)} = \{w_{18} \geq w_{19} \geq 0\}$
Sub-criteria of c_5	$W_{D(5)} = \{w_{20} \geq w_{21} \geq 0\}$
Sub-criteria of c_6	$W_{D(6)} = \{w_{24} \geq w_{23} \geq w_{22} \geq 0\}$
Sub-criteria of c_7	$W_{D(7)} = \{w_{25} \geq w_{26} \geq 0\}$

Table A2. Incomplete information about alternatives' values.

Type	Incomplete Alternatives' Values
Precise values	$V_8 = \{v_8(A_1) = 136, v_8(A_2) = 182, v_8(A_3) = 154, v_8(A_4) = 162, v_8(A_5) = 147\}$
Weak preference	$V_9 = \{1 \geq v_9(A_1) = v_9(A_3) \geq v_9(A_2) = v_9(A_4) = v_9(A_5) \geq 0\}$
Strict preference	$V_{10} = \{1 \geq v_{10}(A_5) = v_{10}(A_4) \geq v_{10}(A_3) = v_{10}(A_2) \geq v_{10}(A_1) \geq 0\}$
Weak preference	$V_{11} = \{1 \geq v_{11}(A_5) \geq v_{11}(A_4) \geq v_{11}(A_3) \geq v_{11}(A_2) \geq v_{11}(A_1) \geq 0\}$
Interval values	$V_{12} = \{3 \leq v_{12}(A_1) \leq 5, 3 \leq v_{12}(A_2) \leq 7, 1 \leq v_{12}(A_3) \leq 2, 5 \leq v_{12}(A_4) \leq 7, 4 \leq v_{12}(A_5) \leq 8\}$
Weak preference	$V_{13} = \{1 \geq v_{13}(A_5) \geq v_{13}(A_4) \geq v_{13}(A_2) \geq v_{13}(A_3) \geq v_{13}(A_1) \geq 0\}$
Verbal	$V_{14} = \{v_{14}(A_1) = \text{Average}, v_{14}(A_2) = \text{High}, v_{14}(A_3) = \text{Very high}, v_{14}(A_4) = \text{Very low}, v_{14}(A_5) = \text{Low}\}$
Interval values	$V_{15} = \{14 \leq v_{15}(A_1) \leq 29, 19 \leq v_{15}(A_2) \leq 35, 14 \leq v_{15}(A_3) \leq 23, 16 \leq v_{15}(A_4) \leq 17, 15 \leq v_{15}(A_5) \leq 23\}$
Verbal	$V_{16} = \{v_{16}(A_1) = \text{High}, v_{16}(A_2) = \text{Very low}, v_{16}(A_3) = \text{Very high}, v_{16}(A_4) = \text{High}, v_{16}(A_5) = \text{Low}\}$
Weak preference	$V_{17} = \{1 \geq v_{17}(A_4) \geq v_{17}(A_5) \geq v_{17}(A_3) \geq v_{17}(A_2) \geq v_{17}(A_1) \geq 0\}$
Ratio preference	$V_{18} = \{1 \geq v_{18}(A_2) \geq 1.5v_{18}(A_3) = 1.5v_{18}(A_4) \geq 3v_{18}(A_1) \geq 4.5v_{18}(A_5) \geq 0\}$
Weak differences of preference	$V_{19} = \{v_{19}(A_3) - v_{19}(A_4) \geq v_{19}(A_4) - v_{19}(A_2) \geq v_{19}(A_2) - v_{19}(A_5) \geq v_{19}(A_5) - v_{19}(A_1)\}$
Weak preference	$V_{20} = \{1 \geq v_{20}(A_5) \geq v_{20}(A_4) \geq v_{20}(A_3) \geq v_{20}(A_2) \geq v_{20}(A_1) \geq 0\}$
Weak preference	$V_{21} = \{1 \geq v_{21}(A_1) \geq v_{21}(A_2) \geq v_{21}(A_3) \geq v_{21}(A_4) \geq v_{21}(A_5) \geq 0\}$
Weak preference	$V_{22} = \{1 \geq v_{22}(A_5) \geq v_{22}(A_1) \geq v_{22}(A_3) \geq v_{22}(A_2) \geq v_{22}(A_4) \geq 0\}$
Verbal	$V_{23} = \{v_{23}(A_1) = \text{High}, v_{23}(A_2) = \text{Very high}, v_{23}(A_3) = \text{Low}, v_{23}(A_4) = \text{Average}, v_{23}(A_5) = \text{Average}\}$
Weak preference	$V_{24} = \{1 \geq v_{24}(A_3) \geq v_{24}(A_5) \geq v_{24}(A_2) \geq v_{24}(A_4) \geq v_{24}(A_1) \geq 0\}$
Verbal	$V_{25} = \{v_{25}(A_1) = \text{Very low}, v_{25}(A_2) = \text{Very low}, v_{25}(A_3) = \text{High}, v_{25}(A_4) = \text{Very low}, v_{25}(A_5) = \text{Average}\}$
Weak preference	$V_{26} = \{1 \geq v_{26}(A_4) = v_{26}(A_3) \geq v_{26}(A_5) \geq v_{26}(A_2) \geq v_{26}(A_1) \geq 0\}$

a: total cost per order. Unit = \$10,000, b: maximum and minimum numbers of the quality team visit during the last ten years., c: maximum and minimum numbers of the entitled claims during the last ten years.

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