



# Article A Simultaneous Optimization Model for Airport Network Slot Allocation under Uncertain Capacity

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**Abstract:** Serious congestion and delay problems exist in most of the busiest airports worldwide because of imbalance between scarce airport slot resources and increasing traffic demand. Various factors, especially weather conditions, exacerbate the demand–capacity imbalance. This paper presents a robust model for simultaneous slot allocation on an airport network in multiple calendar days, considering airport capacity uncertainty. The idea of robust optimization is conducive to sustainable and stable decision-making. Robustness is represented through reducing the potential scheduling conflicts in the worst case. Then the model links the strategic decisions and pre-tactical decisions in air traffic management (ATM) through the tradeoff between strategic discrepancy cost and operational congestion cost. Under the support of the Cplex solver, numerical analyses are taken to validate the characteristics and effectiveness of the proposed model. The results show that the proposed model effectively eliminates the existing and potential scheduling conflicts, and makes effective tradeoffs between airline preference and potential airport congestion risk.

Keywords: air traffic flow management; slot allocation; uncertain capacity; robustness

## 1. Introduction

In recent years, rapid growth of air traffic demand brings frequent and serious congestion and delay problems all over the world. The resulting flight delays, rescheduling, standby, and fuel consumption cause huge economic losses. Various factors, especially bad weather, lead to capacity reduction and exacerbate the imbalance between scarce airport slot resources and increasing traffic demand. In addition, the uncertainty of future weather increases the risk and instability of the initial flight plan. For instance, the 7.20 rainstorm attacking Beijing, China in 2016 caused large-scale flight delays in Beijing Capital International Airport (BCIA), and up to 212 of the daily planned 1715 flights had been cancelled by 14:00 [1]. The 1.17 snowstorm attacking USA in 2018 caused nearly 1800 flights to be cancelled and 5300 flights postponed nationwide, and more than 700 flights, accounting for nearly 25% of the daily flights, at the Hartsfield-Jackson Atlanta International Airport (ATL) were cancelled [2].

In the absence of long-term improvements in capacity through airport runway expansion or air traffic control facility upgrades, air traffic flow management (ATFM) is the most effective solution to mitigate airport congestion risk and ensure flight safety. Slot allocation is a key ATFM technology for achieving flow-capacity balance and relieving airport congestion. Slot allocation problems mainly aim to allocate requested departure and arrival movements to corresponding time slot (the main resources of airport) under airport declared capacity constraints. Here, a slot means the permission given to an airline to use the airport infrastructure to operate an aircraft take-off or landing movement "on a specific day and time". The entire process of ATFM can generally be divided into three phases of strategy,

pre-tactics, and operation/tactics in the time dimension. The strategic slot scheduling is a long-term plan for the timing of flight execution. Before each season, the air traffic control agency will work with the airlines to generate the initial schedule of flight time. The pre-tactical phase refers to the period before the flight take-off on the flight operation day. At this phase, various uncertainties gradually become clear. The strategic plan will be adjusted based on the actual situation of the operation day. The operational/tactical phase refers to the period from flight take-off to flight completion. This phase mainly deals with the issue of aerial conflict resolution during the flight. The airport slot allocation work is mainly carried out in the first two phases. Generally, the whole slot allocation processes can be briefly summarized as: (1) The airlines submit slot requests, (2) the traffic management department allocates slots to corresponding flights or airlines according to a certain mechanism, thereby forming an initial flight time slot plan, (3) according to environmental changes, the traffic management department performs secondary slot allocation to adjust time slot plan in real time. This slot allocation work has been running through the long-term strategic phase (step (1), (2)) as well as the short-term pre-tactical phase (step (3)) before the flight take-off. In current, in most practical management or academic researches, the slot allocation problems of the two phases are handled independently. This type of processing ignores the reliability and robustness of the initial slot allocation plan, at the same time increases the rescheduling costs of both the airline companies and the traffic management department, and the potential congestion risk. The examples of the Beijing 7.20 rainstorm and USA 1.17 snowstorm show that extremely bad weather causes airport capacity to reduce drastically, which leads to the complete failure of slot scheduling decision in the strategic phase. Therefore, robust slot scheduling methods not only are the urgent need of practical operations, but also the focus of future research [3].

Research shows that a minute of tactical delay cost is approximately three times higher than a minute of its strategic counterpart [4]. Thus, reducing potential tactical slot capacity violations through appropriately increasing strategic schedule discrepancies is both cost and efficiency beneficial. In this work we develop a robust optimization model for a multi-airport multi-calendar day slot allocation problem, which aims to improve decision robustness by linking the decisions of the strategic phase and the pre-tactical phase. This robust optimization idea is also a method that meets the sustainability requirements of airports. Wan et al. (2020) [5] identify four airport sustainability dimensions of economy, environment, society, and operation. Obviously, the delay and congestion issues optimized in this paper are related to the economic, environmental, and social considerations of sustainability requirements. Firstly, improving the accuracy and feasibility of slot schedule can reduce the cost and resource waste caused by rearrangement, which is economically and environmentally beneficial. Secondly, reducing flight delays and airport congestion not only improves travelers' satisfaction, but also reduces airports' workload, which is socially beneficial. Most importantly, the robust optimization method guarantees the sustainability of decision-making from an operational perspective. It is well known that robust optimization is a method of anti-interference and risk avoidance. The urgent adjustment of the initial flight scheduling caused by uncertainty is the main reason for airport congestion. Taking into account the unexpected situations that may occur on the operation day in the strategic decision-making phase, will greatly reduce the future congestion risk. Therefore, from an operational perspective, the robust optimization method provides a more stable and sustainable initial flight plan.

In traditional slot allocation research and practice, satisfying the airlines' preferences (namely, to ensure the allocation plan as consistent as possible with the airline's initial request on the premise of meeting the declared capacity) is the main optimization objective. The difference between our model and traditional research/practice is that we added the robustness consideration. The first part of the optimization objective of our model is the airlines' preference consideration, which consists of misallocation cost (namely, flight cancellation cost), movement displacement cost and flight duration displacement cost. The three costs all reflect the degree of difference between the scheduling plan and the airlines' initial request, which are not costs in money measured and borne by a certain stakeholder Therefore, they are together named as the strategic discrepancy cost in this paper. The second part of our optimization objective is to guarantee the decision robustness. Uncertain airport capacity is

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represented by a set of scenarios, which can be simulated based on historical data and weather forecasts. The probability of each scenario is unknown. Using the min-max method, minimizing the potential congestion of the worst scenario is adopted to express robustness. Then the paper aims to embody a trade-off between airlines' preference and decision robustness by simultaneously minimizing the strategic discrepancy cost and potential congestion. This robust consideration is especially beneficial for periods or regions with bad and changeable weather. In addition, intertwined capacity constraints, turnaround time and flight duration constraints are included in the model. Then, the linear conversion of the model makes it possible to apply the model to practical-size problems.

The remainder of the paper unfolds as follows. In Section 2 we review relevant research in the literature. In Section 3 we give the problem description. In Section 4 we present the robust model and corresponding notations. In Section 5, we present the model transformation. In Section 6, numerical analyses are taken to verify the proposed model. Finally, Section 7 gives the conclusions and prospects.

#### 2. Literature Review

Air traffic flow management (ATFM) refers to the safe, orderly, and efficient circulation of air traffic to ensure the maximum use of the capacity of air traffic control services and to meet the standards and capacities published by the relevant air traffic service authorities. Many topics have been studied in ATFM area to guarantee the safety and efficiency of aviation network operations. These studies focus on the optimization of air conflict resolution [6], airspace or sector traffic and capacity management [7], runway scheduling [8–10], ground-delay program [11], etc. Airport slot allocation is an important one of these topics in ATFM. The Worldwide Airport Slot Guidelines (WASG) [12] is published to provide standards for airport slot management by Airports Council International (ACI), the International Air Transport Association (IATA) and the Worldwide Airport Coordinators Group (WWACG). Several principles are included in the document to guide the slot allocation research and practice, such as grandfather right (priority rules of slot allocation), use it or lose it rule, etc. Most research about airport slot allocation follows the IATA standards. The existing research can be divided into single-airport and network-based slot allocation problems. Zografos et al. (2012) [13] formulate a deterministic integer linear program for the single-airport slot allocation problem. The objective of the model is to minimize the difference between airlines' slot request and actual allocated slot, and existing European Union (EU) and International Air Transport Association (IATA) regulations are implemented to the model. Jacquillat et al. (2015) [14] propose an integrated model that jointly optimizes strategic flight scheduling and tactical airport capacity utilization. Ribeiro et al. (2018) [15] propose a priority-based slot allocation model to optimize slot allocation decisions, which complies with the many priority requirements presented by IATA.

Single-airport models have many restrictions due to the mis-consideration of slot interdependencies of origin-destination (O-D) airports. Increasing research identify simultaneous slot scheduling problems at an airport network. Vranas et al. (1996) [16] present a static network-based integer programming model, which is adapted to the needs of Central Flow Management Unit (CFMU). Pellegrini et al. (2017) [17] propose an integer linear programming model to deal with slot allocation problem on the European scale, applying existing regulations. Aircraft rotations are considered by turnaround time constraints. Bolić et al. (2017) [18] present an integer programming model for network-based flight scheduling problem. Apart from the assigning departure time to each flight, the proposed model also gives flight route decisions. Besides, both airports and sectors capacity constraints are considered. Ivanov et al. (2017) [19] formulate a two-level mixed-integer model to simultaneously reduce propagated delay and improve airport slot adherence.

However, uncertain capacity is not considered in the above models. Corolli et al. (2014) [20] propose a two-stage framework for slot allocation problem over airports network under scenario-based uncertain airport capacity. The model aims to provide insights for slot allocation through trade-off between schedule displacement cost and operational delay cost. As well, the sample average approximation (SAA) method is adopted to deal with uncertainty of the problem. Corolli et al. (2015) [21] present a stochastic model for air traffic flow management considering capacity uncertainty both at airports and air sectors. Furthermore, a new heuristic algorithm is designed for the proposed model. The stochastic information of the problem is expressed based on an expected value method.

Robust programming, first presented by Mulvey et al. (1995) [22], a risk-averse stochastic programming processing technique, has gradually been introduced into various issues of air traffic management. Some studies focus on the robust aircraft routing problem [23,24]. Heidt et al. (2016) [25] study a robust airport runway scheduling problem under uncertain conditions. Sidiropoulos et al. (2017) [26] apply the robust optimization method to identification of air traffic flow patterns in the terminal area. Unfortunately, few studies pay attention to robust airport slot allocation problems.

Through the above analysis, we conclude that airport slot allocation research considering uncertainty are still lacking, especially robust models capturing dynamic nature and decision stability.

#### 3. Problem Description

A slot is the permission given to an airline to use the airport infrastructure to operate aircraft take-off or landing movement "on a specific day and time". Furthermore, a slot has a corresponding duration (or time interval), for instance 15 min (min), associated to it. For example, if a slot is granted to an airline at 9:00 of a specific day, a flight of the airline has the right to depart (or arrive) during time interval from 9:00–9:15. One day of 24 h is subdivided into equal size time intervals in one specific airport. Different airports may adopt different size of time interval, such as, 5 min, 10 min and 15 min. However, at all airports a time interval must begin at 0 min, 5 min, 15 min … , the duration of which is multiple of 5. Thus, we can define the coincide of different sizes of time intervals as subinterval, for instance 5. For simplification and without losing generality, we assume that all time intervals in the airport network have the same size (which can take the length of the subinterval) in our problem. In addition, the airlines' requests are reflected by series of slots, namely, one specific flight movement is requested on several calendar days at the same operation time.

At the strategic level, slot allocation aims to reduce the potential demand-capacity imbalance and further guarantee the safety and efficiency of aviation network operation. In practice, see Figure 1, airlines submit slot requests to slot coordinator. As well, the coordinator gives the original slot allocation outcome based on airport declared capacity and other operational constraints. The final slot allocation plans are determined after rounds of revisions through bilateral communications between slot coordinator and airlines. However, various factors, especially meteorological factors, have significantly impact on airport capacity. Thus, the strategic decisions of slot allocation may be ineffective on day of operations. With the continuous updating of information, the decisions in the strategic stage and the pre-tactical stage, considering early knowledge of the future situation based on historical data, the stability of decisions in the strategic level can be improved and the subsequent rescheduling cost can be reduced.



Figure 1. Slot allocation process workflow.

Based on the above interpretation, we present a robust programming model, which aims to minimize the strategic discrepancy cost and potential operational congestion cost. As Figure 2 shows, the problem deals with the trade-off between strategic discrepancy cost and potential operational congestion cost. The strategic discrepancy cost consists of three items: misallocation cost, movement displacement cost and flight duration displacement cost. Misallocation of one specific movement refers that this movement is not allocated to any time point. Namely, the flight related to the movement is cancelled. Therefore, misallocation cost is the cost of flight cancellation. The movement displacement cost means the cost caused by allocating a flight movement to a time point different from the requested time. For example, a movement is requested to be executed on time point 10. If the execution time allocated to this movement is 15, a cost of 5 is caused by the allocation deviation from the initial request. For movement pairs connected by a same flight, flight duration displacement cost may occur. For example, if a flight's requested duration is 20, but the duration after scheduling is 23. A flight duration displacement cost of 3 time units is generated. All the three costs represent the difference between the scheduling plan and the initial request. Therefore, we together name them discrepancy cost. Potential operational congestion is described as the maximum possible exceeded allocated number on the operational day. This consideration not only ensures the robustness of decisions but also avoids the problem of lacking scenario probability. Then, the model formulation process will be stated in the following section.



Figure 2. Cost structure.

## 4. The Model

In this paper, we focus on slot allocation optimization of several calendar days. Furthermore, one day is divided into equal size time intervals, such as 00:00–00:15, 00:05–00:20, 00:10–00:25..., each of which has a 15-min length associated to it. Then, the model is expressed as follows (The interpretations of parameters and variables refer to the Notation section at the end of the article):

$$F = \min\left\{\sum_{r \in R} mc_r \cdot y_r \cdot n_r + n_r \cdot \sum_{r \in R, \neg \exists r': (r, r') \in F \cup (r', r) \in F} \sum_{tm \in TM} dc_r(tm) \cdot x_{r,tm} + n_r \cdot \sum_{(r,r') \in F} \sum_{tm \in TM} [dc_r(tm) \cdot x_{r,tm} + fc_{r,r'}(tm, tm') \cdot x_{r,tm} \cdot x_{r',tm'}] + \tau \cdot \max_{\omega \in \Omega} \left(\sum_{d \in D} \sum_{a \in A} \sum_{t \in T} n_{t,a,d}^{\omega}\right)\right\}$$

$$(1)$$

Subject to:

$$y_r = y_{r'}, \qquad \forall (r, r') \in P \cup F$$
 (2)

$$\sum_{tm\in TM} x_{r,tm} = 1 - y_r, \qquad \forall r \in R$$
(3)

$$\sum_{r \in R_a^1} \sum_{t \in T, tm \lor t} x_{r, tm} \cdot r_d \le \alpha_a^{d, t}, \qquad \forall a \in A, d \in D$$
(4)

$$\sum_{r \in R_a^2} \sum_{t \in T, tm \lor t} x_{r, tm} \cdot r_d \le \beta_a^{d, t}, \qquad \forall a \in A, d \in D$$
(5)

$$\sum_{r \in R_a} \sum_{t \in T, tm \lor t} x_{r, tm} \cdot r_d \le \gamma_a^{d, t}, \qquad \forall a \in A, d \in D$$
(6)

$$\sum_{tm\in TM} x_{r',tm} \cdot tm - \sum_{tm\in TM} x_{r,tm} \cdot tm + y_r \cdot M \ge ft_{r,r'}^{\min}, \qquad (r,r') \in F$$
(7)

$$\sum_{tm\in TM} x_{r',tm} \cdot tm - \sum_{tm\in TM} x_{r,tm} \cdot tm - y_r \cdot M \le ft_{r,r'}^{\max}, \qquad (r,r') \in F$$
(8)

$$\sum_{tm\in TM} x_{r',tm} \cdot tm - \sum_{tm\in TM} x_{r,tm} \cdot tm - y_r \cdot M \ge t_{r,r'}, \qquad (r,r') \in P$$
(9)

$$n_{a,t,d}^{\omega} = \left[\sum_{r \in R_{a,d}^{1}} \sum_{t \in T, tm \lor t} x_{r,tm} - \alpha_{a}^{d,t}(\omega)\right]^{+} + \left[\sum_{r \in R_{a,d}^{2}} \sum_{t \in T, tm \lor t} x_{r,tm} - \beta_{a}^{d,t}(\omega)\right]^{+} + \left[\sum_{r \in R_{a,d}} \sum_{t \in T, tm \lor t} x_{r,tm} - \gamma_{a}^{d,t}(\omega)\right]^{+}, \quad \forall a \in A, d \in D, \omega \in \Omega$$

$$(10)$$

In objective (1), the first three terms respectively represent misallocation cost, displacement cost of requests not belonging to coupled slot set and displacement cost of coupled slots. Cost of coupled slots is represented individually because the displacement cost of coupled slots consists of movement displacement cost and flight duration displacement cost. The above costs are together called discrepancy cost in this paper. Constraints (2) guarantee that a pair of coupled slots or turnaround slots are either both allocated, or neither is allocated. Constraints (3) make sure that one slot is finally allocated to one time interval unless it is not allocated. For example, if  $y_r = 1$ , then all values of  $x_{tm}$  equals to 0. This means that if a movement is cancelled, it is not allocated to any time point. Constraints (4)–(6) together express airport capacity restrictions, in which  $tm \lor t$  denotes that time point tm belongs to time interval *t*. The three constraints represent the capacity limits of departure, arrival, and total movements respectively. Constraints (7)–(8) indicate the duration of one specific flight should fall into a feasible interval. Constraints (9) denote the turnaround time requirements Namely, a certain preparation time should be satisfied between the departure and arrival movement of the connecting flights. Constraints (10) denote the future excess allocated amount in different scenarios, which reflects the potential congestion risk.

#### 5. Model Transformation

The proposed model is a nonlinear 0–1 integer programming model. The nonlinearity of the model is mainly reflected in Constraints (10). Next, we will do some transformations on the model.

**Theorem 1**. For variables X, Y, Z and parameters a, b, if  $X = [a - b]^+$ ,  $Z \ge a - b$ ,  $Z \ge 0$ , and Y = minZ, then X = Y.

**Proof.** (1) If  $a \le b$ , then  $X = [a - b]^+ = 0$ ,

$$\begin{cases} a-b \le 0\\ Z \ge a-b \\ Z \ge 0 \end{cases} \Rightarrow Z \ge 0, \begin{cases} Z \ge 0\\ Y = \min Z \end{cases} \Rightarrow Y = 0;$$

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Then X = Y. (2) If a > b, then  $X = [a - b]^+ = a - b$ ,

$$\begin{array}{l} a-b>0\\ Z \ge a-b\\ Z \ge 0 \end{array} \Rightarrow Z \ge a-b, \begin{cases} Z \ge a-b\\ Y=\min Z \end{array} \Rightarrow Y=a-b; \end{cases}$$

Then X = Y.  $\Box$ 

Based on Theorem 1, the nonlinear characteristics caused by  $[...]^+$  can be linearized. We introduce non-negative auxiliary variables  $nd^{\omega}_{a,t,d}$ ,  $na^{\omega}_{a,t,d}$ ,  $nt^{\omega}_{a,t,d'}$  let

$$nd_{a,t,d}^{\omega} \ge \sum_{r \in R_{a,d}^1} \sum_{t \in T, tm \lor t} x_{r,tm} - \alpha_a^{d,t}(\omega)$$
(11)

$$na_{a,t,d}^{\omega} \ge \sum_{r \in \mathbb{R}^2_{a,d}} \sum_{t \in T, tm \lor t} x_{r,tm} - \beta_a^{d,t}(\omega)$$
(12)

$$nt_{a,t,d}^{\omega} \ge \sum_{r \in R_{a,d}} \sum_{t \in T, tm \lor t} x_{r,tm} - \gamma_a^{d,t}(\omega)$$
(13)

Then, the original model can be written as,

$$\min\left\{\sum_{r\in R} mc_r \cdot y_r \cdot n_r + n_r \cdot \sum_{r\in R, \neg \exists r': (r,r')\in F \cup (r',r)\in F} \sum_{tm\in TM} dc_r(tm) \cdot x_{r,tm} + n_r \cdot \sum_{(r,r')\in F} \sum_{tm\in TM} [dc_r(tm) \cdot x_{r,tm} + fc_{r,r'}(tm,tm') \cdot x_{r,tm} \cdot x_{r',tm'}] + \tau \cdot rob\right\}$$

*s.t.* Formulas (2)–(9), Formulas (11)–(13),

$$\begin{split} rob \geq \sum_{d \in D} \sum_{a \in A} \sum_{t \in T} \left( nd_{a,t,d}^{\omega} + na_{a,t,d}^{\omega} + nt_{a,t,d}^{\omega} \right), \forall \omega \in \Omega, \\ nd_{a,t,d}^{\omega} \geq 0, na_{a,t,d}^{\omega} \geq 0, nt_{a,t,d}^{\omega} \geq 0. \end{split}$$

where, *rob* is another auxiliary variable introduced. After the above transformation, the original model is transformed into an equivalent 0–1 linear programming model, which can be easily solved by many mainstream commercial software.

#### 6. Numerical Analysis

In this section, we present a practical scale case to verify the characteristics and effectiveness of the proposed model. We take the Level 3 airports in the South China airport network as the case. According to the IATA's definition, Level 3 airports are those where "it is necessary for all airlines and other aircraft operators to have a slot allocated by a coordinator in order to arrive or depart at the airport during the periods when slot allocation occurs". Generally, the slot allocation process only focuses on airports that are at capacity, namely Level 3 airports. The South China airport network includes Guangzhou Baiyun International Airport (IATA: CAN), Shenzhen Bao'an International Airport (IATA: SZX) and Sanya Phoenix International Airport (IATA: SYX) three Level 3 airports. The data of the numerical analysis includes real data (initial request and airport network), as well as hypothetical/simulated data (capacity data). We take 3317 slot request series (1 slot series includes several slot requests distributed on different calendar days) over 7 calendar days at the three airports as the original slot demand, which consist of slot pairs, turnaround slots and independent slots. Each calendar day is subdivided into 286 15-min time

intervals, such as 00:00–00:15, 00:05–00:20, 00:10–00:25, and so on. In each time interval, departure capacity constraints, arrival capacity constraints and total capacity constraints should be satisfied. Each time point, such as 00:00, 00:05, 00:10, is taken as an allocation candidate for the requests, and each day includes 288 such time points. At different airports, we generate the declared capacity basing on the slot demand of each time interval. Then, based on the declared capacity, we generate 20 capacity reduction scenarios. In practice, the capacity reduction scenarios can be generated basing on historical weather conditions. The number of scenarios does not affect the computational characteristics of the model since, in robust optimization, we only need to consider the worst case of the many scenarios. Before running the model, we perform initial data processing on the uncertain scenario set and filter out the worst case by calculating the conflict number between different capacity scenarios and the initial request. Based on the above method, we set a tight capacity constraint in our numerical experiment (as Columns 2–3 in Table 1 show, 1366 strategic conflicts and 3079 potential tactical conflicts exist in the original setting). The reason for this setting is that we want to show the model's ability to eliminate existing and potential conflicts. In addition, we assume all the slot series are executed on each day of the planning period, and each calendar day has the same capacity setting. This assumption is purely for the simplicity of data collection and does not affect the model characteristics and the calculation complexity.

We solve the 0–1 linear programming model under the support of the Cplex solver. All the test instances are solved on a computer with AMD Ryzen 7 3700U CPU with 16 GB of RAM. In our case,  $mc_r$  and  $\delta$  are set as 30 and 5 respectively, which indicates that misallocation is not preferred and unit flight duration difference cost is larger than unit movement displacement cost.

First, we use 1 as the step size to generate 30 sets of two-stage weight ( $\tau$ ) from 1 to 30, which almost cover the entire effective weight value interval. The total calculation time of the 30 sets of cases is 3.9 h. Namely, the average calculation time of one case is about 467 s, which is completely acceptable for a strategic decision. The results are shown in Figure 3. We see that as the weight for the second-stage cost increases, the strategic discrepancy cost increases, and the potential congestion quantity decreases. This reflects the effectiveness of the two-stage cost trade-offs emphasized by the model. Figure 3 also shows that  $\tau$  takes values in different intervals will lead to different cost change rates due to tradeoffs. Decision makers can choose appropriate weight values from different intervals based on their decision preferences. For example, in Figure 3, when the weight value increases from 1 to 7, the potential congestion cost decreases by 430.1% with a 52.6% increase of discrepancy cost. This reflects the advantages of the proposed model compared to traditional models. In practical slot management, such an interval (1–7 in this experiment) is a good weight value interval. By setting the weight value in this interval, we can obtain a significant reduction of congestion risk with a slight increase of discrepancy cost. Namely, a stable and robust decision can be obtained without deviating significantly from the airlines' initial request. However, the decision maker can also choose bigger weight values if they want to reduce future congestion risk more. We take three representative values—5, 10, 15 from different intervals of Figure 3 as the weight values of the subsequent numerical analyses.

We take the first calendar day's flight schedule as an example to show the optimization of the proposed model on the airlines' initial requests. The results are shown in Table 1. The second column (C2) of Table 1 denotes the number of conflicts between initial request and declared capacity. The third column denotes the maximum potential number of conflicts of the initial request. The 4th column denotes the maximum potential conflict number of the optimized scheduling. The 5th column denotes the potential conflict reduction of the proposed model to the initial request. The 6–7th columns denote the flight cancellation rate and the total deviation time of the optimized scheduling, respectively. In Table 1, when the two-stage weight ( $\tau$ ) is equal to 5, the 1366 strategic conflicts are completely eliminated and the potential conflicts are reduced by nearly half (45.6%) only by canceling a small number of flights and making some movement time adjustments. If we increase the weight value, the potential conflicts will be further reduced while increasing the number of flight cancellations and movement deviation. In summary, the proposed model can effectively eliminate the existing strategic

imbalance between airlines' requests and declared capacity, and significantly reduces the potential conflict number and congestion risk.



Figure 3. Tradeoffs between the two-stage costs.

Table 1. Comparative results with initial requests.

Weight (C1)	Initial Request		<b>Results of the Proposed Model</b>			
	Stra.no (C2)	Oper.no (C3)	Oper.no (C4)	Oper.d (C5)	Miss.r (C6)	Discr (C7)
5			1674	45.6%	0.7%	4082
10	1366	3079	594	80.7%	10.9%	13,889
15			43	98.6%	16.5%	19,618

Note: C1-C7 denote the column numbers.

In Table 2, we present the comparative results between the solution obtained by the proposed model and the decision only considering nominal capacity (without considering robustness, namely  $\tau = 0$ ). The second and third columns denote the strategic discrepancy cost and potential congestion cost when we only consider declared capacity and ignore decision robustness. Columns 4–5 respectively denote the added strategic cost and saved congestion cost by the proposed model. The last column presents the overall savings for the total costs. We can see that the proposed model aims to increase the strategic discrepancy cost to reduce the potential operational congestion cost. As well, and as the weight increases, which indicates the importance of mitigating potential congestion increases, the benefits of the proposed model increase. In summary, the model can effectively achieve the balance between airline preference and potential airport congestion in the slot allocation problem.

Table 2. Comparative results with nominal capacity.

Weight (C1)	Nominal Capacity		Results of the Proposed Model			
	Stra.Cost (C2)	Oper.Cost (C3)	Stra.Add. (C4)	Oper.Saved. (C5)	Global Gain (C6)	
5	2824	11,930	1258	3560	15.6%	
10	2824	23,860	11,065	17,920	25.7%	
15	2824	35,790	16,794	35,145	47.5%	

Note: C1–C6 denote the column numbers.

We take the first calendar day as an example to show the delay rates of the initial request, decision without robustness and robust decisions. On the operational day, flight movements that exceed the capacity will be delayed or cancelled. In this paper, we name the ratio of these delayed or cancelled movements to the total movements delay rate. Then, the worst-case delay rates of the above decisions

are presented in Figure 4. We see that the proposed model is significantly better than the model without robustness in reducing flight delay rate. If a larger weight value is selected, the flight delay rate will be reduced to a very low level.



Figure 4. Delay rate comparison.

Further, taking the departure flight plan of Baiyun Airport on the first day as an example, we present the slot allocation decisions of several different methods in Figure 5. In Figure 5, the standard deviation of the four curves is 7.62 (initial), 6.54 ( $\tau = 0$ ), 6.08 ( $\tau = 5$ ) and 5.50 ( $\tau = 15$ ) We see that the decisions obtained by the proposed model have smoother distributions over different time periods. Along with the increase of the weight ( $\tau$ ), the curve becomes smoother. This indicates that the proposed model transfers some slot requests of busy time intervals to other intervals, which makes the flight scheduling smooth and uniform in the time dimension. The smooth distribution of slot requests not only ensures the full and reasonable utilization of airport time slot resources, but also reflects the anti-disturbance ability of the decision and indicates the robustness of the proposed model.



Figure 5. Departure flight distribution on the first calendar day of Baiyun Airport.

#### 7. Conclusions

Today, slot scheduling practice suffers significant inefficiency for lacking effective slot allocation optimization applications. Strategic slot scheduling plans vary frequently and significantly due to various factors, especially meteorological factors. In this paper, we propose a robust optimization model for a network-based strategic slot scheduling problem, which contributes to obtain sustainable and stable decisions. The model simultaneously considers strategic scheduling discrepancy cost and potential airport congestion to guarantee the robustness of decisions. The strategic scheduling discrepancy cost are expressed with an exhaustive cost structure, which consists of misallocation cost, flight duration displacement cost and slot displacement cost. Through numerical analyses, we conclude that:

- (1) The proposed model can be equivalently converted into a linear model, which can be solved efficiently using mainstream commercial software like Cplex. This guarantees the applicability of the model in practical large-scale problems;
- (2) The proposed model can effectively eliminate the existing and potential scheduling conflicts;
- (3) The proposed model can significantly reduce the flight delay rate through tradeoffs between airline preference and potential airport congestion in the slot allocation problem;
- (4) The proposed model transfers slot requests of busy time intervals to other intervals, which makes slot allocation smooth and uniform in the time dimension. This not only ensures the full use of airport time slot resources, but also improves the anti-disturbance ability and robustness of the decisions.

The proposed model provides another way of thinking. It is a risk-averse method sacrificing some airline preferences to get more reliable decisions. This may be different from most existing practical applications. However, we think the robust method makes sense in theory, enriches the current literature, and has application prospects. In further directions, more researches are needed to deal with slot allocation problems under uncertain environment, including model construction and solution algorithm design. Complex system theory is recognized to be introduced to characterize decision stability and system resilience. In addition, further work considering multi-stakeholder collaboration and gaming will be an interesting and meaningful topic.

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# Abbreviations

Parameters	
D	set of scheduled days, indexed by d
Α	set of airports, indexed by a
Т	set of time intervals of one day indexed by $t, t = 1, 2, 286$
TM	set of time points on which movements are allocated indexed by $tm$ , $tm = 1, 2, 288$ , any two adjacent time points are separated by 5 min
R	set of movements indexed by <i>r</i> , which is reflected by series of slots
n <sub>r</sub>	the requested times of movement <i>r</i> during the whole planning period
r <sub>d</sub>	whether movement <i>r</i> is requested on calendar day <i>d</i> , if requested, $r_d = 1$ , else, $r_d = 0$
$R_a$	set of requested movements at airport <i>a</i>
$R^1_a$	set of requested departure movements at airport <i>a</i>
$R_a^2$	set of requested arrival movements at airport <i>a</i>
Р	set of pairs of requested turnaround slots $(r, r')$
F	set of requested coupled slots $(r, r')$ executed by one flight
$t_r^*$	requested slot time for requested movement <i>r</i>
$t_{r,r'}$	the minimum turnaround time between movements $r$ and $r'$ , $\forall (r, r') \in P$
$ft_{r,r'}, ft_{r,r'}^{min}, ft_{r,r'}^{max}$	requested, minimum feasible and maximum feasible flight time for coupled requested slots $(r, r') \in F$
mc <sub>r</sub>	cost for the missed allocation of requested slot $r \in R$
$dc_r(tm)$	cost for allocating requested movement $r \in R$ to time <i>tm</i> which is expressed by $ tm - t_r^* $ in our research
$fc_{r,r'}(tm,tm')$	the cost of difference between actual flight time and requested flight time, denoted by $\delta \cdot  tm' - tm - (t_{r'}^* - t_r^*) $ . Here, $\delta > 1$ , which represents that the cost of airborne delay is greater than ground delay.
$\alpha_a^{d,t}, \beta_a^{d,t}, \gamma_a^{d,t}$	the declared departure, arrival and total capacity at airport <i>a</i> during time interval <i>t</i> on calendar day <i>d</i>
τ	weight for the displacement cost and potential delay cost
Ω	scenario sets, indexed by $\omega$
$\alpha_a^{d,t}(\omega), \beta_a^{d,t}(\omega), \gamma_a^{d,t}(\omega)$	the departure, arrival and total capacity at airport a during time interval t on calendar day d under scenario $\omega$
$n_{a+d}^{\omega}$	excess allocated amount at airport a during time interval t on day d under scenario $\omega$
M	a big number
Decision variables	
<i>Yr</i>	0-1variable, if movement <i>r</i> is allocated, $y_r = 1$ , else, $y_r = 0$
<i>x<sub>r,tm</sub></i>	0-1variable, if movement <i>r</i> is allocated to time <i>tm</i> , $x_{r,tm} = 1$ , else $x_{r,tm} = 0$
	Parameters D A T T TM R $n_r$ $r_d$ $R_a$ $R_a^1$ $R_a^2$ P F $t_r^*$ $t_{r,r'}$ $ft_{r,r'}, ft_{r,r'}^{min}, ft_{r,r'}^{max}$ $mc_r$ $dc_r(tm)$ $fc_{r,r'}(tm, tm')$ $\alpha_a^{d,t}, \beta_a^{d,t}, \gamma_a^{d,t}$ $\tau$ $\Omega$ $\alpha_a^{d,t}(\omega), \beta_a^{d,t}(\omega), \gamma_a^{d,t}(\omega)$ $n_{a,t,d}^{\omega}$ M Decision variables $y_r$ $x_{r,tm}$

# References

- 1. Available online: http://news.carnoc.com/list/355/355227.html (accessed on 3 April 2020).
- 2. Available online: https://www.cifnews.com/article/32299 (accessed on 3 April 2020).
- 3. Zografos, K.G.; Madas, M.A.; Androutsopoulos, K.N. Increasing airport capacity utilisation through optimum slot scheduling: Review of current developments and identification of future needs. *J. Sched.* **2017**, *20*, 3–24. [CrossRef]
- 4. Cook, A.J.; Tanner, G. European Airline Delay Cost Reference Values. Performance Review Unit, EUROCONTROL, Brussels. 2011. Available online: https://pdfs.semanticscholar.org/d623/ad3a438429eac1c9a691d253580ecb79f116. pdf?\_ga=2.192787946.1936941888.1594116465-1195491553.1594116465 (accessed on 8 July 2020).
- Wan, L.; Peng, Q.; Wang, J.; Tian, Y.; Xu, C. Evaluation of airport sustainability by the synthetic evaluation method: A case study of guangzhou baiyun international airport, china, from 2008 to 2017. *Sustainability* 2020, 12, 3334. [CrossRef]
- Lehouillier, T.; Allignol, C.; Jérémy, O.; François, S. Interactions between operations and planning in air traffic control. In Proceedings of the 6th International Conference on Research in Air Transportation, Istanbul, Turkey, 26–30 May 2014.
- Ruiz, S.; Kadour, H.; Choroba, p. A novel air traffic flow management model to optimise network delay. In Proceedings of the Thirteenth USA/Europe Air Traffic Management Research and Development Seminar (ATM2019), Vienna, Austria, 17–21 June 2019.
- 8. Balakrishnan, H.; Chandran, B. Scheduling aircraft landings under constrained position shifting. In Proceedings of the AIAA Guidance, Navigation, & Control Conference & Exhibit, Keystone, CO, USA, 21–24 August 2006.
- 9. Grunewald, E. Incentive-based slot allocation for airports. Transp. Res. Procedia 2016, 14, 3761–3770. [CrossRef]
- 10. Grunewald, E.; Knabe, F.; Rudolph, F.; Schultz, M. Priority rules as a concept for the usage of scarce airport capacity. *Transp. Res. Procedia* **2017**, *27*, 1146–1153. [CrossRef]
- 11. Chang, K.; Howard, K.; Oiesen, R.; Shisler, L.; Tanino, M.; Wambsganss, M.C. Enhancements to the FAA ground-delay program under collaborative decision making. *Interfaces* **2001**, *31*, 57–76. [CrossRef]
- 12. Worldwide Airport Slot Guidelines. Airports Council International, International Air Transport Association, Worldwide Airport Coordinators Group. 2020. Available online: https://www.iata.org/ (accessed on 28 June 2020).
- 13. Zografos, K.G.; Salouras, Y.; Madas, M.A. Dealing with the efficient allocation of scarce resources at congested airports. *Transp. Res. Part C* 2012, *21*, 244–256. [CrossRef]
- 14. Jacquillat, A.; Odoni, A.R. An integrated scheduling and operations approach to airport congestion mitigation. *Oper. Res.* **2015**, *63*, 1390–1410. [CrossRef]
- 15. Ribeiro, N.A.; Jacquillat, A.; António, P.A.; Odoni, A.R.; Pita, J.P. An optimization approach for airport slot allocation under IATA guidelines. *Transp. Res. Part B Methodol.* **2018**, *112*, 132–156. [CrossRef]
- 16. Vranas, P.B.M. Optimal slot allocation for European air traffic flow management. *Air Traffic Control Q.* **1996**, *4*, 249–280. [CrossRef]
- 17. Pellegrini, P.; Bolić, T.; Castelli, L.; Pesenti, R. SOSTA: An effective model for the simultaneous optimisation of airport slot allocation. *Transp. Res. Part E Logist. Transp. Rev.* **2017**, *99*, 34–53. [CrossRef]
- Bolić, T.; Castelli, L.; Corolli, L.; Rigonat, D. Reducing ATFM delays through strategic flight planning. *Transp. Res. Part E Logist. Transp. Rev.* 2017, 98, 42–59. [CrossRef]
- Ivanov, N.; Netjasov, F.; Jovanović, R.; Starita, S.; Strauss, A. Air traffic flow management slot allocation to minimize propagated delay and improve airport slot adherence. *Transp. Res. Part A Policy Pract.* 2017, 95, 183–197. [CrossRef]
- 20. Corolli, L.; Lulli, G.; Ntaimo, L. The time slot allocation problem under uncertain capacity. *Transp. Res. Part C Emerg. Technol.* **2014**, *46*, 16–29. [CrossRef]
- 21. Corolli, L.; Lulli, G.; Ntaimo, L.; Venkatachalam, S. A two-stage stochastic integer programming model for air traffic flow management. *IMA J. Manag. Math.* **2017**, *28*, 19–40. [CrossRef]

- 22. Mulvey, J.M.; Vanderbei, R.J.; Zenios, S.A. Robust optimization of large-scale systems. *Oper. Res.* **1995**, *43*, 264–281. [CrossRef]
- 23. Aloulou, M.A.; Haouari, M.; Mansour, F.Z. Robust aircraft routing and flight retiming. *Electron. Notes Discret. Math.* **2010**, *36*, 367–374. [CrossRef]
- 24. Lonzius, M.C.; Lange, A. Robust scheduling: An empirical study of its impact on air traffic delays. *Transp. Res. Part E Logist. Transp. Rev.* **2017**, *100*, 98–114. [CrossRef]
- 25. Heidt, A.; Helmke, H.; Kapolke, M.; Liers, F.; Martin, A. Robust runway scheduling under uncertain conditions. *J. Air Transp. Manag.* 2016, *56*, 28–37. [CrossRef]
- Sidiropoulos, S.; Han, K.; Majumdar, A.; Ochieng, W.Y. Robust identification of air traffic flow patterns in metroplex terminal areas under demand uncertainty. *Transp. Res. Part C Emerg. Technol.* 2017, 75, 212–227. [CrossRef]



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