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An Empirical Study on the Comprehensive Optimization Method of a Train Diagram of the China High Speed Railway Express

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Abstract: The rapid and stable development of China's economy has driven the increasing demand for express transportation. Based on network operation, China Railway Corporation of High-speed Railway launched high-speed rail products, which have attracted wide attention from all walks of life. With the application of high-speed express trains, the market structure of express transportation in China will change dramatically, from highways as the main mode of transportation to high-speed railway transportation relying on a high-speed railway network, which will effectively reduce the environmental pollution caused by express transportation and further improve the sustainable development of the economy and the logistics industry. At present, the freight Electric Multiple Units (EMU) has been successfully developed and has entered the final test stage. In the last paper, we have introduced the theory and method of the high-speed rail express train operation plan. In addition, a train diagram is an important foundation of railway transportation organization. In order to ensure the sustainable development of high-speed rail express trains after they are put into use, based on the operation plan of high-speed rail express trains, this paper establishes a comprehensive compilation model of a high-speed rail express train diagram, considering train running time, freight flow distribution scheme, and the operation plan of freight multiple units, and an exact solution algorithm based on the Lagrange relaxation algorithm is designed. The computational results are encouraging and demonstrate the effectiveness of the model and solution method.

Keywords: high-speed rail express; train diagram; Lagrange relaxation algorithm; sustainable transportation; sustainability

1. Introduction

In recent years, with the rapid development of China's economy and the rapid development of China's logistics industry, the demand for express freight transportation has increased rapidly, which brings high profit space to the express transport market. At the same time, it also raises increasing demands and challenges in the supply of transport products. How to ensure the rational use of transport resources has become a key issue to meet the demand of express transport. Fortunately, China's high-speed railway has realized network operation. Relying on this complete network, China Railway Corporation launched the high-speed railway express business, effectively utilizing the advantages of a high-speed railway being fast, economic, and green, as shown in Figure 1.

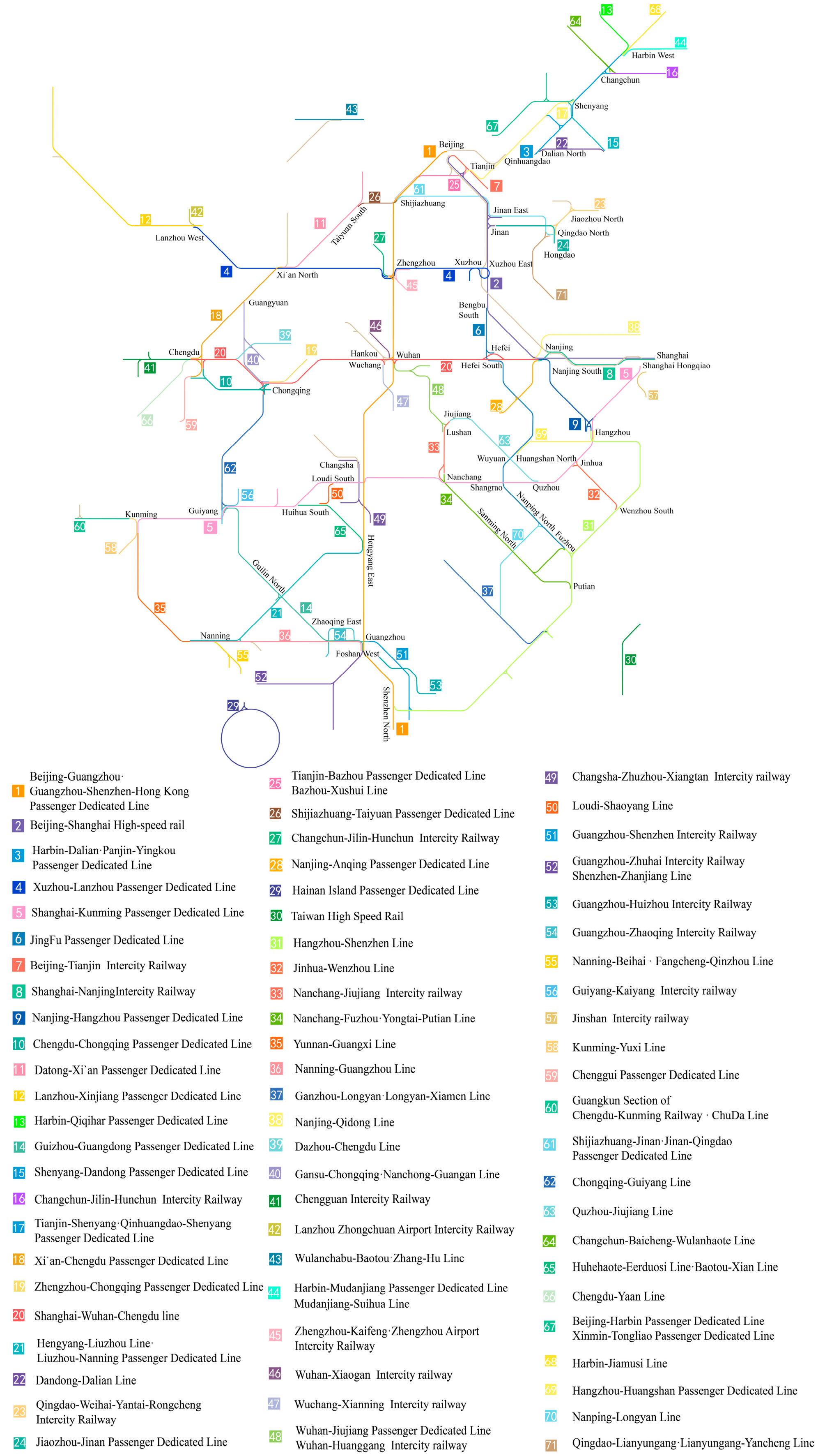

Figure 1. Structural Chart of China High Speed Railway Network. Source: The China Railway.

As a green transportation mode, a high-speed railway has advantages that other transportation modes do not have in express transportation [1]. At the same time, in order to further enhance the economic and social benefits of a high-speed rail transport, the high-speed rail transport business has is greatly importance to both transport demanders and transport suppliers in the context of the growing demand for high-speed rail transport and the continuous improvement of the high-speed rail service network. The high-speed rail express service market has attracted wide attention from all walks of life. However, as it is limited by the existing transport organization mode, network capacity, node operation capacity, equipment carrying capacity, and other factors, while high-speed rail express has the advantages of timeliness, high security, green and environmental protection, it is still not the main mode of transportation of fast freight. Lack of effective supply to meet market demand and a scientific transportation decision-making system has become an important reason for restricting the development of the high-speed rail express.

As a new force in the express market, breaking up the traditional market pattern and opening up the market situation have become key issues for its development. Based on this, we should establish a scientific decision-making mechanism for the operation of high-speed rail express transportation. Starting by improving the quality of product supply, we should promote the structural adjustment of transport products, rationally allocate transport resources, expand effective supply, and improve the flexible adaptability of the supply structure to changes in demand, so as to better meet the growing demand of the express transportation market.

At present, China Railway Corporation, relying on the high-speed railway network, has launched four major transport products, which are delivered on the same day, the next morning, the next night, and the third day, in order to meet the needs of different customers for express transportation.

In the last article [2], we studied the operation scheme of high-speed railway express trains, and determined the operation interval, stopping scheme, and frequency of high-speed railway freight trains, which is the framework of train transport organization. However, such a framework cannot support the operation of freight trains. We need to know the specific train operation times and stop times in order to make rational use of freight trains. In order to ensure the operation of freight trains, it is necessary to consider the constraints of railway network capacity and relations with other trains, and to determine a scheme that includes train running time and stopping time. This is what we are going to do in this article. The high-speed rail express train diagram is the basis to ensure that the high-speed rail express train can take on the task of freight transportation. Unlike the general train diagram study, the freight train diagram also needs to take into account the following two factors: product arrival time limit and EMU routing plan. Different kinds of transport products have different requirements for the running time of trains, and the operation plan of EMU will also affect the running time of freight trains. Specific impacts will be discussed in the description of the model. This is also the advantage of comprehensive optimization, which can take into account the impact of multiple factors on the objectives to ensure the feasibility of the train diagram.

Under the conditions of a high-speed railway network operation, the high-speed train not only enriches the railway transportation product system, but also increases the pressure for high-speed railway network operation. Therefore, in view of the early stage of the operation of freight EMUs, this paper optimizes the allocation of spatial-temporal resources of a high-speed railway without affecting the passenger transport of the high-speed railway, and draws the operation diagram of high-speed railway express trains, considering the operation plan of EMUs and the distribution of product and freight flow.

The main contributions of this paper are as follows. At present, there is no research on the transport organization of high-speed railway freight trains. We take the organization scheme of high-speed railway freight trains as the research object, and design the high-speed railway freight train diagram based on the timetable of high-speed railway passenger trains by applying the theory of space-time network construction. As mentioned above, a comprehensive optimization scheme, including train operation scheme, freight flow distribution scheme, and EMU routing plan, can effectively support the development decision-making of a high-speed railway freight transport business. In order to obtain such a comprehensive optimization scheme, we construct a comprehensive optimization model considering these three factors, and design an algorithm to solve the model. The final comprehensive optimization scheme can effectively provide decision-making for train operation.

The remainder of this paper is organized as follows. Section 2 introduces the related research review. Section 3 establishes a comprehensive optimization model for the compilation of a high-speed rail transit train diagram, which aims at the lowest transportation cost, the shortest Spatial-temporal path of freight flow, the least number of EMUs, and the least number of overhauls. Section 4 designs a Lagrange relaxation algorithm to solve the model. The original problem is decomposed into "the spatial-temporal shortest path sub-problem with additional constraints" and "the shortest path sub-problem with resource constraints", which are solved by labeling method, a classical algorithm of operational research dynamic programming. Section 5 takes the Harbin-Dalian high-speed railway as an example. Based on the operation plan of a high-speed train, a comprehensive optimization scheme is designed, which includes the schedule of a high-speed freight train, train assignment scheme, and EMU operation plan. The validity of the model and the algorithm is verified.

2. Literature Review

China is the country with the largest scale of high-speed railway construction and operation in the world. It is also the country with the most complex railway operation scenario and external environment. For the research of high-speed railway engineering, we can refer to previous studies [3–5].

The research object of this paper includes train time-space path planning, freight flow allocation and EMU operation planning. Therefore, the following three aspects will be analyzed separately in the existing literature.

2.1. Theory and Method of Train Diagram Compilation

The optimization problem of train diagram compilation is mainly solved by the mathematical programming method in operation research. From our last paper's summary of foreign train operation plan compilation methods, we can see that the theory of train operation plan compilation mainly focuses on customer-oriented and conflict-oriented directions. Because the train operation plan is the basis of train operation plan compilation, similarly, in the study of a train operation plan, it is also divided into the conflict-oriented train diagram compilation method and the resource-oriented train diagram compilation method. The classical papers in these two directions are Carey M. [6] and Caprara A. [7]. Most of the follow-up studies follow the ideas of these two papers. In order to reduce the scale of the problem, Carey M. [6] proposes an intelligent optimization algorithm to optimize the remaining 0–1 variables and the continuous variables in the model, including passenger flow and train-to-departure events, while keeping part of the 0–1 variables unchanged. The principle of planning is to ensure the optimality of all operation charts by guaranteeing the optimality of each train. Caprara A. [7] based on the idea of operational research graph theory, a train diagram is compiled, and train arrival and departure events are described in the form of directed graph points. Based on this, an optimization model is constructed, and a Lagrange relaxation algorithm is designed to improve the efficiency of the solution. Furthermore, in the following research, the author considers more realistic scenarios, such as blocking interval, node operation capability, maintenance skylight, and new train lines on the existing train diagram, which makes the model more realistic.

Periodic train charts are widely used in developed countries, while non-periodic train diagrams are mainly used in China.

2.1.1. Compiling a Periodic Train Diagram Model

The model of the periodic train diagram compilation can be divided into two kinds of background models: the periodic event scheduling model and cyclic periodic model. Serafini [8] first proposed the mathematical model of Periodic Event Scheduling Problem, PESP and its optimization idea, and looked forward to its application in the field of transportation. The background model of a train diagram compilation based on PESP can be described as:

Objective:

$$\min F(\pi) \tag{1}$$

Subject to:

$$l_{ij} \leq \pi_j - \pi_i + T \cdot p_{ij} \leq k_{ij} \tag{2}$$

$$0 \leq \pi_i, \pi_j \leq T \tag{3}$$

$$p_{ij} \in Z \tag{4}$$

Among them, the objective function is about π, π represents the arrival and departure time of train, p_{ij} is 0–1 variable, T is the period, l_{ij}, k_{ij} represents the time window. From the background model of PESP, we can see that it mainly compiles the periodic operation chart by restraining the arrival and departure time of trains, so as to achieve the objectives of minimizing the number of trains and the travel cost. On the basis of PESP model, Peeters [9] proposed a cycle model based on the interval between arrival and departure events. Its background model is as follows:

Objective:

$$\min Z = F(\pi) \tag{5}$$

Subject to:

$$l_a \leq x_a \leq k_a \tag{6}$$

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = Tq_C \tag{7}$$

$$q_C \in Z \tag{8}$$

Among them, x_a represents the interval variable (continuous variable), q represents the loop variable (integer variable), l_{ij}, k_{ij} represents the time window.

Subsequent research on the establishment of a periodic train diagram is mostly based on the above two modeling ideas. The optimization objectives of the periodic train diagram compilation include train operation cost [10], passenger travel time [11], transfer time [12], and robustness of the diagram [13]. The constraints of compiling a model of a periodic operation diagram mainly include train operation safety constraints, stopping time and number constraints, undercarriage connection constraints, among others [14].

2.1.2. Non-Periodic Train Diagram Model

Cacchiani [15] compares and analyses the efficiency and quality of a precise solution strategy and a heuristic algorithm solution strategy to the problem of train diagram compilation, and systematically describes the robustness of the train diagram and the strategy of late adjustment of a train diagram, and constructs an integer programming model of train diagram compilation based on the shortage of capacity of railway transportation network, and designs heuristic algorithm pairs. The solution of the model achieves a fast solution to the problem of train diagram compilation. Xuesong Zhou [16,17] applied the integer programming method of operations research and the Lagrange relaxation algorithm to solve the train diagram compilation model in single-track and double-track scenarios, and optimized the quality evaluation indices of passenger waiting time, passenger travel time, and vehicle waiting time. Yixiang Yue [18] built a mixed integer programming model based on customer requirement satisfaction, aimed at maximizing the utilization rate of operation diagram capability, designed a column generation algorithm to solve the model, and finally optimized the train operation plan with the Beijing-Shanghai high-speed railway as the background. Cordeau [19] pointed out that the constraints such as train arrival and departure interval constraints and capacity constraints are numerous and complex, which leads to the fact that most of the existing literature can only optimize one aspect and ignores the overall integrity. In addition, the actual road network is mostly a multi-scenario complex network, and most of the existing case studies only focus on a simple road network.

2.2. Cargo Flow Assignment Problem

At present, there are few literature studies directly studying the problem of freight flow distribution, and most of them are from the perspective of the distribution of freight flow; that is to say, the distribution of freight flow formed by a train as the carrier. In the field, Haghani [20] has established a comprehensive planning optimization model considering linear train content, flow path, and empty allocation, and a heuristic algorithm to solve the model of design. Holmbergab [21] transformed the traffic assignment problem into a multi-commodity flow problem, designed a Lagrange relaxation algorithm to solve the multi-commodity flow model, discussed the influence of spatial-temporal constraints and cargo priority on the railway empty car allocation plan, and constructed a mixed integer programming model aimed at the shortest distance of empty car allocation. The problem of empty car allocation is effectively solved by applying the double sub-gradient optimization rule.

2.3. Research on the Application of EMU

The operation and maintenance plan of EMUs has attracted wide attention from scholars, both at home and abroad. It mainly focuses on the assignment of EMUs in a definite train diagram or operation plan. In order to reduce the number and duration of EMUs, the consideration of train connection relationship is mainly strengthened. Therefore, the existing studies mostly focus on the number of EMUs, the duration of EMUs, and the minimum number of maintenance times. The objective is to construct the optimization model with the constraints of traffic organization, network capability, and other factors. In the design of the algorithm, it is the same as the general optimization problem-solving strategy. According to the characteristics of the model, the exact solution strategy can be chosen to solve, or the heuristic algorithm can be designed to solve the problem.

In terms of model construction, Sun [22] constructed an optimization model of a maintenance plan based on train type and maintenance grade, taking into account the influence of maintenance mileage, time, and location constraints on maintenance plan. Lingaya [23] constructed an optimization model for EMU operation based on the problem of EMU assignment, which aimed to achieve the shortest train connection time. The constraints of operation organization, such as safety interval and arrival-departure interval, are considered. On this basis, Fioole [24] studied the train connection relationship under the conditions of railway networking and the assignment of multiple units under the operation mode of non-fixed multiple units. Considering the relationships between cost and benefit, service level, and other factors, an optimization model of multiple unit operation is constructed. Abbink [25] considered the special situation of EMU operation when the demand for railway transportation is high and the transport capacity is tense, and constructed an optimization model for EMU operation aimed at minimizing the shortage of EMUs. Alfieri [26] accounted for the operation characteristics of EMU based on the fluctuation of transport demand. Taking the multi-commodity flow model as the background model, the optimization model of EMU operation is constructed.

The main features of the optimization model for the use of EMUs constructed in the existing literature are shown in Table 1.

Table 1. Research characteristics of EMU operation optimization problem.

Research object	① Optimization of train formation and undercarriage connection ② Application of EMU in rush hour ③ Establishment of maintenance plan for EMU ④ EMU operation plan ⑤ Route Planning of EMU
Model characteristics	① Integer Programming Model ② Multi-commodity flow model ③ Transition Model and Interchange Model ④ Neural Network Model
Objective function	① Minimum cost of EMUs ② Optimum number and mileage of EMUs ③ Minimizing seat shortages ④ Minimizing shunting operations
Constraint condition	① Flow equilibrium constraints ② Coupling constraints between operation plan and operation diagram ③ Shunting sequence constraints ④ Conservation of Number of EMUs

In addition to the separate optimization of the EMU operation plan, many scholars at home and abroad have considered it with other problems and formed a comprehensive optimization scheme, which guarantees the comprehensiveness and feasibility of the scheme, such as the comprehensive optimization of the EMU operation plan and passenger service plan [27], the EMU operation plan and operation chart [28], the EMU undercarriage operation, and maintenance plan [29].

3. Construction of Comprehensive Optimizing Model for a High-Speed Railway Express Train Diagram

3.1. Problem Description

The essence of high-speed railway train diagram compilation is the allocation of spatial-temporal resources in a high-speed railway network. How to coordinate the relationship between passenger and freight transportation in a high-speed railway has become the key to compiling a high-speed railway train diagram. The further coupling relationship between freight flow and vehicle flow has become the main influencing factor of freight cost and revenue in a high-speed railway. The application and maintenance scheme of the bottom of the high-speed rail express train guarantees the rational use of the freight train set; that is, the fixed cost of purchasing the high-speed rail freight train set is controlled and the profit is guaranteed.

The comprehensive optimization problem of high-speed railway freight train diagram compilation can be divided into three sub-problems: the high-speed railway freight train diagram compilation, high-speed rail freight flow distribution, and high-speed rail express train operation plan, which correspond to three sub-networks respectively: freight train Spatial-temporal service network, high-speed rail physical transport network, and freight train connection network.

3.1.1. Compilation of High-Speed Railway Express Train Diagram: Freight Train Spatial-Temporal Service Network

A high-speed railway express train spatial-temporal network is composed of service nodes and spatial-temporal arcs with time and space constraints. The service path of one or more trains is the spatial-temporal path based on the Origin-Destination (OD) requirements and time limit requirements of high-speed rail products. In a spatial-temporal network, train types can be distinguished because of the different spatial-temporal resources occupied.

Considering the constraints of spatial-temporal resources in a train diagram, based on the theory of spatial-temporal networks, the network flow method is applied to describe the situation of a train occupying spatial-temporal resources, as shown in Figure 2.

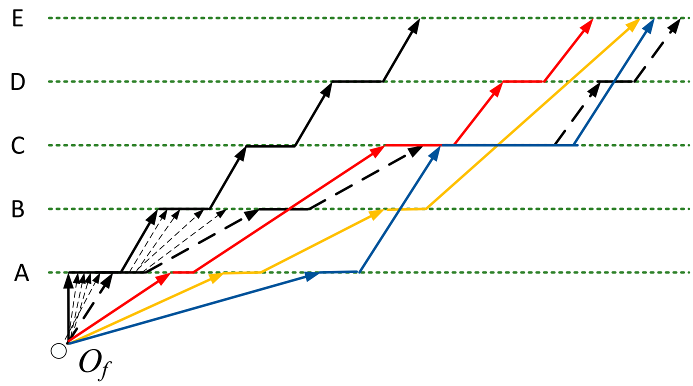

Figure 2. The train path in a spatial-temporal network.

In Figure 2, each node is constrained by two dimensions of time and space to describe the running process and state of the train. The running line of train 1 from station A to E represents the corresponding spatial-temporal path of train 1. Different grades of trains have different spatial-temporal arcs, and the spatial-temporal path is constrained by the relationship between trains. That is, if there is only one spatial-temporal path, there is no restriction. When drawing the second spatial-temporal path, the relationship with the first spatial-temporal path should be considered. By analogy, the whole spatial-temporal network considering the connection of trains can be constructed.

Therefore, the problem of high-speed railway freight train operation drawing can be expressed as the problem of finding the shortest spatial-temporal path in OD time in a given spatial-temporal service network. In order to reduce the impact on passenger transport, this paper, based on the schemes of a high-speed rail express train generated in Chapter IV, and account for the constraints of safe departure time interval, prohibition of overtaking, and the connection times of freight trains, draws up a high-speed rail freight train diagram in order to keep the timetable of a passenger train unchanged.

3.1.2. Distribution of Freight Flow in High-Speed Railway Express Transportation: Physical Transportation Network of a High-Speed Railway

Due to the differing punctuality of different transport products, this will have an impact on matching high-speed rail express trains; that is, whether the stop times and arrival times of high-speed rail express trains can meet the product demand. At the same time, the different OD demands of different products will have an impact on the stoppages of high-speed rail express trains and the allocation of transportation resources. Under the parallel organization mode of high-speed railway freight trains and passenger trains, based on the characteristics of four products, i.e., the day arrival, the morning arrival, the next arrival, and the next arrival, the coupling of freight flow and passenger flow in a high-speed railway should be realized by considering the constraints of train loading capacity, the time limit for freight arrival, and the balance of node flow, so as to ensure the economic benefits of a railway. Specifically, it can be understood as the distribution of cargo flow between different transport products and different transport organization modes; that is, the matching of transport products and vehicles constrained by time and carrying capacity. We use the compilation method of the train operation plan of the high-speed railway express in the previous paper to determine a basic framework, including train stopping, running frequency, and running interval. However, this basic framework skill describes the total load of a train, and the specific freight flow allocation scheme is introduced in this paper.

3.1.3. Operation Planning of Freight EMU-Train Connection Network

Since the use of passenger train sets has been taken into account in the compilation of the passenger train diagram, the train connection problem studied in this paper only considers freight trains.

The time between the completion of the first freight task and the start of the next freight task is called the continuation time. During this period, EMUs need to complete the necessary freight handling operations and technical operations, such as return, maintenance, and so on. Therefore, when determining the continuity relationship of multiple freight units, it is necessary to give a minimum continuation time to meet the above requirements. However, in order to improve the efficiency of EMUs, the actual connection time of freight EMUs should not be much longer than the minimum one.

In order to facilitate the management of EMUs, China allocates depots to designated EMUs, which are responsible for the operation and maintenance of EMUs. At present, the overhaul of EMUs in our country is mainly preventive, which is divided into five levels, of which the first level overhaul and the second level overhaul are completed in the EMU depot. This paper only considers the influence of the first-level overhaul on the operation plan of the car bottom.

In the early stage of operation, due to the limitations of market scale, transportation organization mode, and line network capacity, considering the relationship between cost and revenue, the purchase of freight EMUs should be minimized. Therefore, a reasonable undercarriage route should be delineated to improve the undercarriage operation efficiency of freight EMUs. In addition, it is necessary to take into account the overhaul of freight train units and ensure that empty and idle freight train units are minimized. The EMU routing diagram is shown in Figure 3.

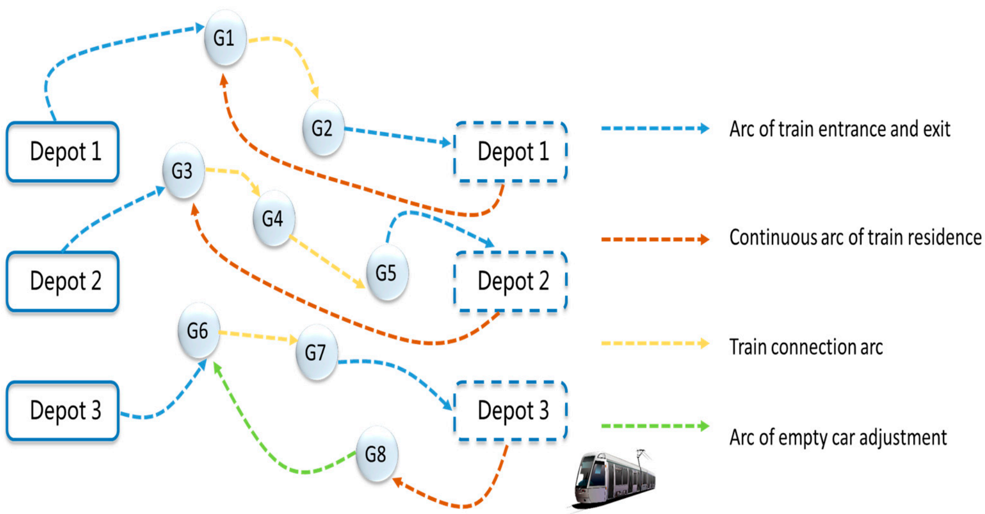

Figure 3. Schematic diagram of EMU routing.

In summary, based on the above three sub-problems, this paper constructs a comprehensive optimization model of a high-speed railway freight train diagram to generate a high-speed railway freight train diagram, which takes into account the timetable of high-speed railway passenger trains and the maintenance plan of EMU, including the undercarriage route and freight flow distribution scheme.

3.2. Model Hypothesis

In order to solve the problem of synthetic optimization of high-speed railway freight train diagram compilation, appropriate abstractions and assumptions are made.

3.2.1. Dual-Line Operation

It is assumed that the high-speed rail express train runs on the double-track railway without considering the use of arrival and departure lines and the arrangement of routes.

3.2.2. Maintenance Assumptions

According to the operation rules of EMUs in China, the overhaul of EMUs can be divided into five grades, among which the first-level overhaul is a routine maintenance. After a certain mileage, the EMUs need to enter the garage for testing and performance testing. Compared with other maintenance grades, the first-level overhaul is more frequent and has the greatest impact on the use of the car bottom. Therefore, this paper only considers the first-level overhaul of EMUs.

3.2.3. Train Operation Hypothesis

Assuming that the traffic block mode is an automatic block, the train tracking interval is a fixed value, and the pure running time of the train in the interval is fixed, without considering the additional time when the train starts and stops.

3.3. Notation

3.3.1. Sets

The set definition is shown in Table 2.

Table 2. Set definition table.

N	Set of stations, indexed by i, j, n, where $i, j, n \in N$.
W	Category set of high-speed rail express products, indexed by $w_1, \ldots, w_4$, where $w_1, \ldots, w_4 \in W$.
F	The set of trains in the operation plan, F_K denotes the set of passenger trains, F_W denotes the set of freight trains, $F = F_K \cup F_W$, f denotes one of the trains.
R_{FW}	Spatial-temporal path set of freight trains;
R_{WEMU}	Route set of freight EMU.
S, T	The set of Spatial-temporal node, s, t denotes the time when the train arrives and departs, where $(i, t) \in ST$.
A	The set of the spatial-temporal arc (i, j, s, t, f).
F_{con}	The set of train connection relation.
$F_{YX} = \{< f, f' >\}$	The set of train pairs prohibited from overtaking.

3.3.2. Parameters

The parametric interpretation is shown in Table 3.

Table 3. Parametric Interpretation Table.

$td_{i,j,s,t,f}$	Time of departure of train f from station i through time-space arc (i, j, s, t).
$ta_{i,j,s,t,f}$	Time of arrive of train f from station i through time-space arc (i, j, s, t).
$t_{i,j,s,t,f}$	Running time cost of spatial-temporal arc (i, j, s, t, f).
$t_{con\_\min}$	Minimum continue time for trains.
CS_n	Number of arrival and departure lines at station n.
$IDD_i^{l(f)l(f')}$	Interval time of train departure at station i with speed grades $l(f)$ and $l(f')$ respectively.
$IAA_j^{l(f)l(f')}$	Interval time of train arriving at station j with speed grades $l(f)$ and $l(f')$, respectively.
c_{r1}	Number of freight EMUs.
c_{r2}	Daily maintenance number of freight trains.

3.3.3. Decision Variables

The variable interpretation is shown in Table 4.

Table 4. Variable Interpretation Table.

$q_{i,j,s,t,f,w}$	The volume of train f chooses spatial-temporal arc (i,j,s,t) to transport product w.
$x_{r,f}$	Equals 1 if the time-space path of freight train is selected; 0 otherwise.
$y_{r,f}$	Equals 1 if the route r of freight trains f is selected; 0 otherwise.
$z_j^{ff'}$	Equals 1 if at the same station j, the order f' always precedes f'; 0 otherwise.
$u_{i,j,s,t,f}$	Equals 1 if train f carries cargo w on the Spatial-temporal path i,j,s,t; 0 otherwise.

3.4. Model Formulation

Based on the principles of spatio-temporal networks and multi-commodity flow, we construct a comprehensive optimization model of a high-speed railway freight train diagram. Previous papers have also solved the problem using this method [30,31].

3.4.1. Objective Function

The timeliness of the high-speed rail express is its core competitiveness. Therefore, it is necessary to ensure the shortest spatial-temporal path of the high-speed rail express train operation. Secondly, in order to balance the relationship between cost and profit, it is necessary to ensure that under the condition that all the demands are met, the running cost of the train is the smallest, which can be divided into two aspects: transportation cost and related cost of train operation. Therefore, the objective function of the comprehensive optimization problem of train operation planning for a high-speed railway can be expressed as follows:

Objective:

$$\min Z_1 = W_1 \cdot \sum_{f \in F_W} \sum_{r \in R_{FW}} c_{r,f} x_{r,f} = \sum_{i,j,s,t,f \in A} t_{i,j,s,t,f} u_{i,j,s,t,f} \tag{9}$$

$$\min Z_2 = W_2 \cdot \sum_{i,j,s,t,f \in A} c_{i,j,s,t,f,w} q_{i,j,s,t,f,w} + \sum_{i,j,s,t,f \in A} c_{i,j,s,t,f} a_{i,j,s,t,f} \tag{10}$$

$$\min Z_3 = \sum_{r \in R_{WEMU}} c_r y_{r,f} = W_3 \cdot \sum_{r \in R_{WEMU}} (c_{r1} + c_{r2}) y_{r,f} \tag{11}$$

Among them, W_1 W_2, and W_3 represent the weight values of the three objective functions, respectively. Formula (9) indicates the shortest time-space path of train operation to ensure the timeliness of the high-speed rail express products. Formula (10) indicates the lowest operating cost of the high-speed rail express trains. Formula (11) indicates the least number of freight train units and the least number of maintenance times applied in a high-speed rail express business.

3.4.2. Constraint Condition

(a) Assignment constraints of freight trains' spatial-temporal paths:

$$\sum_{r \in R_{FW}} x_{r,f} = 1 \, \forall f \in F_W \tag{12}$$

Formula (12) indicates that for any given freight train, there is only one Spatial-temporal path to ensure the uniqueness of the same spatial-temporal path.

(b) The node operating capacity constraints:

$$\sum_{f \in F_{WEMU}} x_{rf} \leq CS_n \ \forall n \in N \tag{13}$$

Formula (13) indicates that the loading and unloading capacity of the train operation station is constrained, and the station is influenced by the track structure. It is necessary to arrange the loading and unloading operation plan reasonably to meet the loading and unloading operation requirements of all trains and to avoid conflict with passenger trains.

(c) The mapping constraints of auxiliary variables:

$$u_{i,j,s,t,f} \leq \sum_{w} q_{i,j,s,t,f,w} \ \forall f \in F \tag{14}$$

$$\sum_{w} q_{i,j,s,t,f,w} \leq u_{i,j,s,t,f}\boldsymbol{M} \ \forall f \in F \tag{15}$$

$$x_{r,f} = \sum_{i}\sum_{j} u_{i,j,s,t,f} \ \forall f \in F \ \forall r \in R_f \tag{16}$$

Formulas (14) and (15) denote the constraints of the mapping relationship of auxiliary variables, i.e., the decision variable $q_{i,j,s,t,f,w}$ is changed to 0–1 variable, which denotes the relationship between train spatial-temporal path and traffic; that is, if a certain spatial-temporal path is selected, there must be traffic, and vice versa. Formula (16) denotes the relationship between auxiliary variables and train spatial-temporal path; that is, whether a train's spatial-temporal path is selected is related to whether the transport arc is selected in each section.

(d) The constraints of train safety interval time.
In order to ensure traffic safety and make the most effective use of the passing capacity of the section, the train should abide by the relevant regulations and technical operation time standards of the station, and meet the constraints of safe departure and arrival time intervals. The relationship between them is shown in Figure 4.

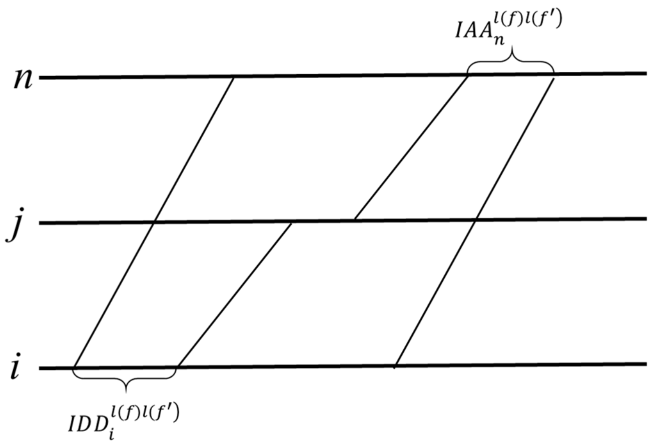

Figure 4. Schematic diagram of safe interval between arrival and departure of train.

(d_1) Safe departure interval constraint:

$$IDD_i^{l(f)l(f')} - (1 - z_i^{ff'})M \leq td_{i,j,s,t,f}u_{i,j,s,t,f} - td_{i',j',s,t,f'}u_{i',j',s,t,f'} \tag{17}$$

$$IDD_i^{l(f)l(f')} - Mz_i^{ff'} \leq td_{i',j',s,t,f'}u_{i',j',s,t,f'} - td_{i,j,s,t,f}u_{i,j,s,t,f} \ \forall f \in F_W \ \forall f' \in F \tag{18}$$

(d_2) Safe arrival interval constraint:

$$ta_{i,j,s,t,f}u_{i,j,s,t,f} - ta_{i',j',s,t,f'}u_{i',j',s,t,f'} \geq IAA_j^{l(f)l(f')} - M(1 - z_j^{ff'}) \ \forall f, f' \in F \ f \neq f' \tag{19}$$

$$ta_{i',j',s,t,f'}u_{i',j',s,t,f'} - ta_{i,j,s,t,f}u_{i,j,s,t,f} \geq IAA_j^{l(f)l(f')} - Mz_j^{ff'} \ \forall f, f' \in F \ f \neq f' \tag{20}$$

In order to ensure the safety of the train in the departure throat area of the station, the minimum train tracking time is set. Formulas (17) and (18) represent the constraints of the continuous train departure interval. Formulas (19) and (20) indicate that the train must meet the constraints of the minimum arrival tracking interval.

(d_3) Train relationship constraints.

In order to describe the relationship between trains reasonably, the constraint of prohibiting overtaking should also be considered, as shown in Figure 5.
From Figure 5, it can be seen that although the train guarantees the minimum safe interval between arrival and departure, the two trains in interval (i, j) show unreasonable overtaking due to the different speed grades. Therefore, in order to avoid the occurrence of such an unreasonable situation, it is necessary to define the relationship between the two trains; that is, within the interval, no overtaking can occur.

$$z_i^{ff'} = z_j^{ff'} \ \forall f \in F_W \ \forall f' \in F \tag{21}$$

Formula (21) denotes the running relationship of trains; that is, the relationship between train f and f' remains unchanged from beginning to end, so as to ensure that f and f' do not cross each other.

(e) Train connection time constraints.
The train connection relationship is coupled; that is, the arrival and departure sequence and the connection time between the two trains are interrelated. The connection time between the two trains should meet the technical operation time of the nodes, as shown in Figure 6.

$$ta_{i,d,s,t,f}u_{i,d,s,t,f} - td_{i',d',s,t,f'}u_{o',j',s,t,f'} \geq t_{con\_min} \ \forall f, f' \in F_{con} \tag{22}$$

Formula (22) indicates that the difference between the train departure time and the train arrival time should satisfy the minimum connecting time.

(f) Time window constraints for freight products.
At present, there are clear arrival time requirements for today's, next morning's, next day's, and third day's high-speed rail express products. Therefore, different kinds of transport products should meet different time window constraints.

$$ta_{i,d,s,t,f,w}u_{i,d,s,t,f,w} \leq T_w \tag{23}$$

Formula (23) indicates that the arrival time of the goods is earlier than the latest arrival time of the transport product.

(g) Flow balance constraints:

$$\sum_w \sum_{i,j,s,t,f \in A} q_{i,j,s,t,f,w} = \sum_w \sum_{j,k,s,t,f \in A} q_{j,k,s,t,f,w} \ \forall f \in F_W \ \forall j \in S \tag{24}$$

$$\sum_w \sum_{i,d,s,t,f \in A_d} q_{i,d,s,t,f,w} = Q_d \ \forall f \in F_W \tag{25}$$

$$\sum_w \sum_{o,j,s,t,f \in A_o} q_{o,j,s,t,f,w} = Q_o \ \forall f \in F_W \tag{26}$$

Formula (24) denotes flow equilibrium constraints, i.e., for two adjacent intervals, the inflow volume equals the outflow volume. Formulas (25) and (26) indicate that the requirements of the initial node and the end-to-end node are equal to the amount of traffic sent or that has arrived, respectively.

(h) Train Loading Capacity Constraints:

$$\eta C_f \leq \sum_f q_{i,j,s,t,f,w} \ \forall f \in F_W \tag{27}$$

Formula (27) expresses the constraint of full-load rate of trains. It is necessary to run freight trains only when the freight flow reaches a certain amount, so as to ensure economic benefits.

$$\sum_w q_{i,j,s,t,f,w} \leq C_f \ \forall f \in F_W \tag{28}$$

Form (28) denotes the capacity constraints of passenger or freight trains. The total amount of goods loaded by trains should not exceed the maximum carrying capacity of trains.

(i) Routing constraints of freight EMUs:

$$\sum_{r \in R_{WEMU}} y_{rf} = 1 \ \forall f \in \boldsymbol{F}_W \tag{29}$$

$$\sum_{r \in R_{WEMU}} \alpha_{depot} y_{rf} \leq MC_{depot} \ \forall depot \in \boldsymbol{Depot} \tag{30}$$

$$\sum_{r \in REMU} \eta_r y_{rf} \leq N_{\min} \tag{31}$$

Formula (29) means that a given freight train task must be undertaken by a certain freight train set; that is, all tasks need to be performed by the EMU. Formula (30) means that the operation of the train cannot exceed the maintenance capacity of the EMU depot. Formula (31) means that all freight train sets cannot exceed the given number of EMUs.

(j) Range Constraints of Decision Variables:

$$y_{r,f} = \{0,1\} \ \forall r \in R_f \ \forall f \in F_W \tag{32}$$

$$u_{i,j,s,t,f} = \{0,1\} \ \forall (i,j,s,t,f) \in A \tag{33}$$

$$z_j^{ff'} = \{0,1\} \ \forall f,f' \in F \ f \neq f' \ \forall j \in N \tag{34}$$

Formulas (32)–(34) denotes the range constraints of decision variables.

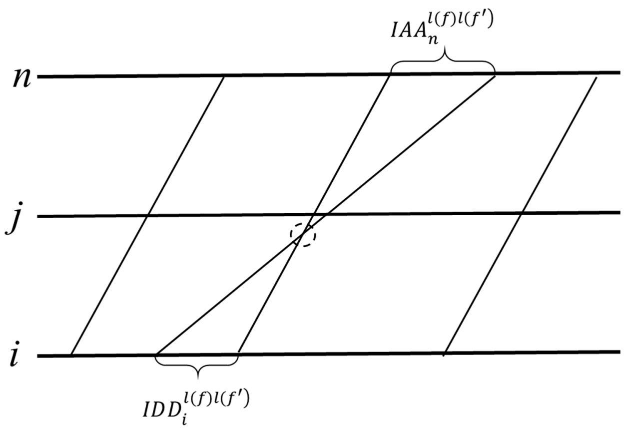

Figure 5. Illegal overtaking of trains.

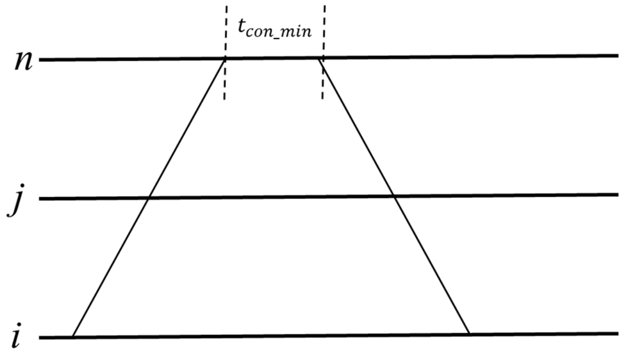

Figure 6. Schematic diagram of train connection relationship.

4. Algorithm Design

The essence of train diagram problem is the allocation of limited spatial-temporal resources. For this kind of problem, the Lagrange relaxation algorithm has a good solution effect.

4.1. Algorithm Principle and Low

The basic principle of the Lagrangian relaxation algorithm is to relax some complex constraints in the form of a penalty function into the objective function through the Lagrangian multipliers. By solving the newly constructed Lagrangian relaxation problem, the upper and lower boundaries of the solution of the original problem can be obtained. Then, through special optimization rules, new Lagrangian multipliers can be generated iteratively, so that the optimal solution of the relaxation problem keeps approaching the optimal solution of the original problem. The basic flow of the Lagrange algorithm is shown in Figure 7.

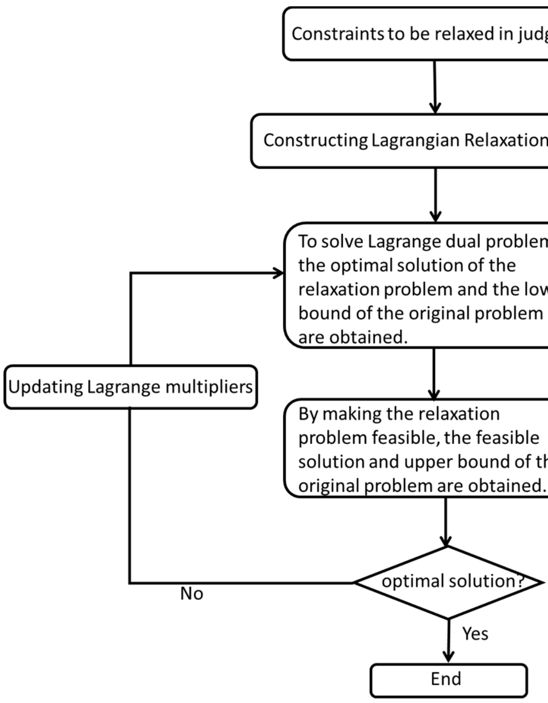

Figure 7. Basic flow chart of Lagrange algorithm.

As can be seen from Figure 7, judging which constraints need to be relaxed is the basis of Lagrange's algorithm. Usually, in the optimization of a railway operation diagram, the constraints related to capacity will lead to the formation of a combination relationship between trains, resulting in difficulties in solving the problem. Therefore, this paper will build a Lagrangian relaxation model based on capacity-related constraint relaxation.

4.2. Algorithm Steps

Based on the framework flow of the Langrangian relaxation algorithm, π_n represents the Lagrangian multiplier of formulas (13), $\rho^{\alpha}_{i,j,s,t,f'}$, $\rho^{\beta}_{i,j,s,t,f'}$, $\rho^{\gamma}_{i,j,s,t,f'}$, $\rho^{\lambda}_{i,j,s,t,f'}$, $\rho^{\lambda}_{f,f'}$ represent Lagrange multipliers of formulas (17), (18), (19), (20), and (21), $\mu_{ff'}$ represents the Lagrangian multiplier of formulas (22). After relaxing the above formulas, the following objective functions can be obtained:

$$\begin{aligned}\min Z = Z_1 + Z_2 + Z_3 &+ \sum_{i,j,s,t,f' \in A} \rho^{\alpha}_{i,j,s,t,f'} \left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f} u_{i,j,s,t,f} - td_{i,j,s,t,f'} u_{i,j,s,t,f'} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\beta}_{i,j,s,t,f'} \left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f'} x_{i,j,s,t,f'} - td_{i,j,s,t,f} x_{i,j,s,t,f} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\gamma}_{i,j,s,t,f'} \left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f} x_{i,j,s,t,f} - ta_{i,j,s,t,f'} x_{i,j,s,t,f'} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\lambda}_{i,j,s,t,f'} \left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f'} x_{i,j,s,t,f'} - ta_{i,j,s,t,f} x_{i,j,s,t,f} \right) \right] \\ &+ \sum_{ff'} \rho_{ff'} (z_i^{ff'} - z_j^{ff'}) + \sum_n \pi_n (\sum_{f \in F_{WEMU}} x_{rf} - CS_n) \\ &+ \sum \mu_{ff'} [t_{con\_min} - (ta_{i,d,s,t,f} u_{i,d,s,t,f} - td_{i',d',s,t,f'} u_{o',j',s,t,f'})]\end{aligned} \quad (35)$$

Taking a constraint of $\sum_{i,j,s,t,f' \in A} \rho^{\alpha}_{i,j,s,t,f'} \left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f} u_{i,j,s,t,f} - td_{i,j,s,t,f'} u_{i,j,s,t,f'} \right) \right]$ in Equation (35) as an example, the influence of relaxed constraint conditions on the objective function is analyzed.

When initializing $\rho^{\alpha}_{i,j,s,t,f'} = 0$, the departure interval constraint has no effect on the objective function. When $\rho^{\alpha}_{i,j,s,t,f'} > 0$ is applied, based on the sub-gradient optimization rule, departure interval constraint $IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f} u_{i,j,s,t,f} - td_{i,j,s,t,f'} u_{i,j,s,t,f'} \right)$ will gradually affect the objective function. In order to ensure that the objective function can be minimized, the difference between departure interval and arrival time of the actual two trains is required to satisfy the departure interval constraint. Similarly, arrival interval constraints, train overtaking constraints, capacity constraints, and connection time constraints also affect the objective function.

In order to facilitate the solution of the model, the above problems can be decomposed into two sub-problems, as follows.

Sub Problem 1: Spatial-Temporal Shortest Path Problem with Additional Constraints

The essence of sub-problem 1 is how to find the shortest path in the spatial-temporal network to ensure the transport efficiency of high-speed railway freight trains. However, unlike the shortest path in general, the constraints of train relationships and network capacity need to be taken into account. Therefore, the model can be expressed as follows:

Objective:

$$\begin{aligned}\min Z = Z_1 + Z_2 + Z_3 &+ \sum_{i,j,s,t,f' \in A} \rho^{\alpha}_{i,j,s,t,f'} \left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f} u_{i,j,s,t,f} - td_{i,j,s,t,f'} u_{i,j,s,t,f'} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\beta}_{i,j,s,t,f'} \left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f'} x_{i,j,s,t,f'} - td_{i,j,s,t,f} x_{i,j,s,t,f} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\gamma}_{i,j,s,t,f'} \left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f} x_{i,j,s,t,f} - ta_{i,j,s,t,f'} x_{i,j,s,t,f'} \right) \right] \\ &+ \sum_{i,j,s,t,f' \in A} \rho^{\lambda}_{i,j,s,t,f'} \left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f'} x_{i,j,s,t,f'} - ta_{i,j,s,t,f} x_{i,j,s,t,f} \right) \right] \\ &+ \sum_{ff'} \rho_{ff'} (z_i^{ff'} - z_j^{ff'}) + \sum_n \pi_n (\sum_{f \in F_{WEMU}} x_{rf} - CS_n) \\ &+ \sum \mu_{ff'} [t_{con\_min} - (ta_{i,d,s,t,f} u_{i,d,s,t,f} - td_{i',d',s,t,f'} u_{o',j',s,t,f'})]\end{aligned} \quad (36)$$

Subject to:
Constraints (12), (14)–(16), (23)–(28).

Sub Problem 2: Shortest Path Problem with Resource Constraints

The essence of sub-problem 2 is also the shortest path problem. However, unlike the traditional shortest path problem, the resource constraints, including the number of EMUs, the number of overhauls, and other factors, should be taken into account. Therefore, the model can be expressed as follows:

Objective:

$$\min \sum_{r \in R_{WEMU}} c_r y_{r,f} = \sum_{r \in R_{WEMU}} (c_{r1} + c_{r2}) y_{r,f} \tag{37}$$

Subject to:
Constraints (29)–(31).

Therefore, based on the idea of decomposing the comprehensive optimization problem of a high-speed railway express train diagram into two sub-problems, the overall solution algorithm flow is shown in Figure 8.

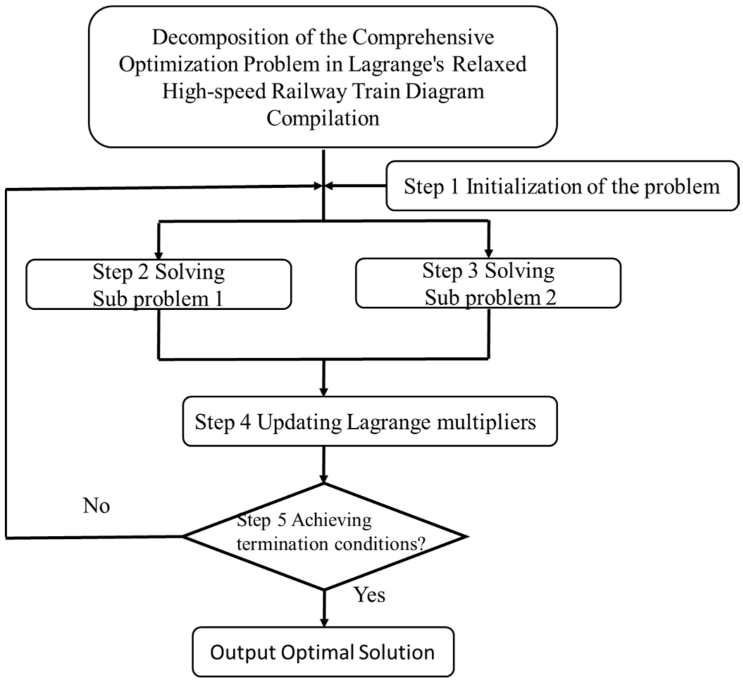

Figure 8. Flow chart of the Lagrange algorithm.

Based on the Lagrangian relaxation algorithm, the specific steps of solving the comprehensive optimization problem of the high-speed railway freight train diagram compilation are as follows:

Step 1: Problem Initialization
Set the number of iterations $n = 0$; Initialization of Lagrange multipliers; Initial sub-gradient iteration step size θ.

Step 2: Solving Sub-problem 1
The dynamic programming algorithm is used to search the shortest path with additional constraints.

Step 3: Solving Sub-problem 2
The dynamic programming algorithm is used to search the shortest path with resource constraints.

Step 4: Updating Lagrange multipliers with the sub-gradient optimization process

① Computing Lagrange multipliers:

$$\nabla \rho^{\alpha}_{i,j,s,t,f'} = IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f}u_{i,j,s,t,f} - td_{i,j,s,t,f'}u_{i,j,s,t,f'}\right) \tag{38}$$

$$\nabla \rho^{\beta}_{i,j,s,t,f'} = IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f'}x_{i,j,s,t,f'} - td_{i,j,s,t,f}x_{i,j,s,t,f}\right) \tag{39}$$

$$\nabla \rho^{\gamma}_{i,j,s,t,f'} = IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f}x_{i,j,s,t,f} - ta_{i,j,s,t,f'}x_{i,j,s,t,f'}\right) \tag{40}$$

$$\nabla \rho^{\lambda}_{i,j,s,t,f'} = IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f'}x_{i,j,s,t,f'} - ta_{i,j,s,t,f}x_{i,j,s,t,f}\right) \tag{41}$$

$$\nabla \rho_{ff'} = z_i^{ff'} - z_j^{ff'} \tag{42}$$

$$\nabla \pi_n = \sum_{f \in F_{WEMU}} x_{rf} - CS_n \tag{43}$$

$$\nabla \mu_{ff'} = t_{con\_min} - (ta_{i,d,s,t,f}u_{i,d,s,t,f} - td_{i',d',s,t,f'}u_{o',j',s,t,f'}) \tag{44}$$

② Updating Lagrange multipliers:

$$\rho^{\alpha(n+1)}_{i,j,s,t,f'} = \max\left\{0, \rho^{\alpha(n)}_{i,j,s,t,f'} + \theta_n\left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f}u_{i,j,s,t,f} - td_{i,j,s,t,f'}u_{i,j,s,t,f'}\right)\right]\right\} \tag{45}$$

$$\rho^{\beta(n+1)}_{i,j,s,t,f'} = \max\left\{0, \rho^{\beta n}_{i,j,s,t,f'} + \theta_n\left[IDD_i^{l(f)l(f')} - \left(td_{i,j,s,t,f'}x_{i,j,s,t,f'} - td_{i,j,s,t,f}x_{i,j,s,t,f}\right)\right]\right\} \tag{46}$$

$$\rho^{\gamma(n+1)}_{i,j,s,t,f'} = \max\left\{0, \rho^{\gamma(n)}_{i,j,s,t,f'} + \theta_n\left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f}x_{i,j,s,t,f} - ta_{i,j,s,t,f'}x_{i,j,s,t,f'}\right)\right]\right\} \tag{47}$$

$$\rho^{\lambda(n+1)}_{i,j,s,t,f'} = \max\left\{0, \rho^{\lambda(n)}_{i,j,s,t,f'} + \theta_n\left[IAA_j^{l(f)l(f')} - \left(ta_{i,j,s,t,f'}x_{i,j,s,t,f'} - ta_{i,j,s,t,f}x_{i,j,s,t,f}\right)\right]\right\} \tag{48}$$

$$\rho^{(n+1)}_{ff'} = \max\left\{0, \rho^{(n)}_{ff'} + \theta_n\left[z_i^{ff'} - z_j^{ff'}\right]\right\} \tag{49}$$

$$\pi_n^{(n+1)} = \max\left\{0, \pi_n^{(n)} + \theta_n\left[\sum_{f \in F_{WEMU}} x_{rf} - CS_n\right]\right\} \tag{50}$$

$$\mu^{(n+1)}_{ff'} = \max\left\{0, \mu^{(n)}_{ff'} + \theta_n\left[t_{con\_min} - (ta_{i,d,s,t,f}u_{i,d,s,t,f} - td_{i',d',s,t,f'}u_{o',j',s,t,f'})\right]\right\} \tag{51}$$

Step 5: Closing condition

If the maximum number of iterations or the value of the objective function does not change, the algorithm ends and outputs the optimal lower bound; otherwise, $n = n + 1$, the algorithm is back to Step 2.

4.3. The Solution and Computation of Sub Problems

4.3.1. Solution of Sub Problem 1

Since the essence of sub-problem 1 is the shortest path problem, it can be solved by the dynamic programming method in operational research. The dynamic programming method is based on the optimization principle proposed by Bellman et al.; that is, the optimal strategy of a process has the following properties. Regardless of the initial state or the initial decision, for the state formed by the previous decision, all subsequent decisions must constitute the optimal strategy. Based on the idea of dynamic programming, in order to achieve the shortest space-time path of the train, it is required that every space-time arc of the train is the shortest path.

In order to accurately express the process of the dynamic programming labeling method, the new parameter symbols are defined as shown in Table 5.

Table 5. Table of parameter variables.

s	The starting point of the train
r	The terminal of the train
$\Psi(i,t)$	Used to obtain the corresponding number of (i,t) station spatio-temporal node
$\lambda_s(i,t)$	Minimum path cost from (s,t) to (j,t')
$\zeta(i,t)$	The spatio-temporal node (i,t) of the station with minimum cost and precedence of node (j,t')
$\vartheta(i,j,s,t)$	Cost of occupying arc (i,j,s,t)
$\Gamma(i,t)$	Subsequent node set of node (i,t)

The classical dynamic programming algorithm of operational research is applied to solve the spatio-temporal shortest path problem with additional constraints, as follows (Algorithm 1):

Algorithm 1 Dynamic programming labeling method:

Input: Spatio-temporal network G, Starting point s, Terminal point r, Resource cost vector, Initial value of Lagrange multiplier.
Output: Minimum cost path from start s to terminal r.
Step 1: Parameter initialization
Create an empty list of node collections SE, set $\lambda_s(i,t) = +\infty$, $\zeta(i,t) = \phi$, Insert (s,t) into the collection list SE.
Step 2: Update the node label
When SE is a nonempty set do
  Remove a node (i,t) from SE
  For node $(j,e') \in \Gamma(i,t)$ do
    If $\Psi(i,t) = r$ Then
      The cost of setting a new label alternate node is:
      $\lambda'_s(j,t) = \lambda_s(i,t) + \vartheta(i,j,s,t)$
    End If
    If $\lambda'_s(j,t) < \lambda_s(j,t)$ Then
      The labeling cost of the recording node is $\lambda_s(j,t) = \lambda'_s(j,t)$
        Record (j,t) as the precursor node of (i,t)
    End If // Updating the cost label of the node   End For
   Delete node (i,t) from SE
 End do
Step 3: Get the shortest spatio-temporal path
  Step 3.1 Find the minimum cost node (j,t) corresponding to the end point, and set (j,t) as the current node (k,t)
  Step 3.2 Nodes backtracking r to s
    While Current node (k,t) is not s
      (1) Find the precursor (i,t') of the current (k,t) node
      (2) Update the current node (i,t') as the current node (k,t)
    End
  Output Minimum Cost Path
  Step 3.4 Algorithm termination

4.3.2. Solution of Sub Problem 2

Sub problem 2 is a multi-objective problem, which not only ensures the minimum number of EMUs applied, but also ensures the longest maintenance mileage of freight EMUs. The core problem of EMU routing is to solve the shortest path problem based on resource constraints. The number of EMUs and the number of overhauls can be regarded as a limited resource. The dynamic programming labeling method with resource constraints can also be used to solve the shortest path with resource constraints. The optimization process can be transformed into a process of labeling generation and deletion on nodes along the directed arc associated with the nodes, according to the given constraints.

By describing the process of solving sub-problem 1, it can be seen that the operation process is based on nodes, and the parallel computation of sub-tasks can be realized only by establishing the data structure for storing labels at each node and designing the corresponding labeling process.

In order to accurately express the labeling process of dynamic programming for solving sub-problem 2, the new parameter symbols are defined as shown in Table 6:

Table 6. Table of parameter variables

$L(i)$	Effective label set of node i
$l_i = (RC_i, AD_i, AT_i, P_i)$	A label for node i, for each label $l_i^h \in L(i)$, represents a route from d of the starting EMU depot to node i
RC_i	Cumulative cost savings from start node to node i
AT_i	Cumulative time from the end of last overhaul to node i
AD_i	Accumulated train mileage from the end of last overhaul to node i
P_i	Boolean vectors $P_i = (\theta_1, \theta_2, \ldots, \theta_n, \ldots, \theta_{\lvert N \rvert})$ are used to describe the visited nodes, where $\theta_{\lvert N \rvert}$ represents the number of trains, N represents the number of elements contained in the set, and P_i represents the coupling relationship between EMUs and trains

H is used to represent the upper limit of the number of labels that can be stored in $L(i)$. When the number of labels generated does not meet the need of optimization, H can be dynamically adjusted according to the need. With the increase of H, the solution time will increase significantly. Every time a new label is generated, if the number of existing labels has reached the upper limit of storage, it should be compared with the label in the set. If the newly generated label is better than the existing label, then it is inserted into the set, and the old label, which is worse than the new label, is deleted, otherwise the new label will be deleted directly.

Then the dynamic programming labeling algorithm is applied to solve the shortest path problem with resource constraints, as Algorithm 2.

Algorithm 2. dynamic programming algorithm.

Setting the initial state: $l_d^1 = (RC_d^1, AD_d^1, AT_d^1, p_d^1)$, $RC_d^1 = 0$, $AD_d^1 = 0$, $AT_d^1 = 0$, $p_d^1 = (0, \ldots, 0)$;
$\boldsymbol{L}(d) = l_d^1 \ \forall d \in \boldsymbol{D}$
$\boldsymbol{L}(i) = \phi \ \forall i \in \boldsymbol{F}$
Set: $LN = \boldsymbol{d}$
While $LN \neq \varnothing$
Select a node i from LN
For all $l_i^h = (RC_i^h, AD_i^h, AT_i^h, p_i^h) \in L(i)$ do
For all $j \in \boldsymbol{FS}(i) \& \theta_j^h < 1,\ \theta_j^h \in P_i^h$ do
$l_j \leftarrow Extend(l_i^h, j)$
If l_j is not empty and l_j is better than the existing label in $L(j)$ then
Set: $L(j) = L(j) \cup \left\{ l_j \right\}$
Delete the label eliminated by l_j from $L(i)$
Add j to LN only when j is not in LN
End If
End
Output final label.

5. Case Analysis

5.1. Parameter Value

On 1 December 2012, the Harbin-Dalian High-speed Railway was officially opened and operated with a total length of 921 km. There are 22 stations along the route. The designed speed of passenger trains is 350 km/h and the operating speed is 300 km/h. It occupies an important position in China's high-speed railway network. The Harbin EMU depot and Dalian EMU depot have redundant maintenance capacity, with a daily maintenance capacity of about 40 rows. The structure of the Harbin-Dalian high-speed railway and its related road network is shown in Figure 9.

Figure 9. Network structure of the Harbin-Dalian high speed railway.

As can be seen from Figure 9, besides the local trains of the Harbin-Dalian high-speed railway, there are still a large number of cross-line trains, such as the Harbin-Beijing, Harbin-Shanghai, Harbin-Hunchun, and other trains, which also occupy the capacity of the network. Therefore, it should be fully considered when constructing the train space-time network.

In this paper, we use d, c, r, and s to represent four kinds of transportation products. The daily average demand for transport products between major cities of the Harbin-Dalian high-speed railway is shown in Table 7.

Table 7. Daily average demand of transportation products in major cities of the Harbin-Dalian high-speed railway.

O \ D	Harbin			Changchun			Shenyang			Anshan			Yingkou			Dalian		
Harbin		——		d	2.96	33	d	5.84	65	d	1.08	12	d	1.17	13	d	7.27	81
				c	12.67		c	24.95		c	4.61		c	4.99		c	31.10	
				r	10.85		r	21.38		r	3.95		r	4.28		r	26.64	
				s	6.51		s	12.83		s	2.37		s	2.57		s	15.99	
Changchun	d	4.85	54		——		d	3.23	36	d	0.72	8	d	0.81	9	d	4.13	46
	c	20.73					c	13.82		c	3.07		c	3.46		c	17.66	
	r	17.76					r	11.84		r	2.63		r	2.96		r	15.13	
	s	10.66					s	7.11		s	1.58		s	1.78		s	9.08	
Shenyang	d	6.82	76	d	4.67	52		——		d	1.44	16	d	1.17	13	d	7.81	87
	c	29.18		c	19.96					c	6.14		c	4.99		c	33.40	
	r	25.00		r	17.10					r	5.26		r	4.28		r	28.61	
	s	15.00		s	10.26					s	3.16		s	2.57		s	17.17	
Anshan	d	0.90	10	d	0.63	7	d	1.17	13		——		d	0.90	10	d	2.42	27
	c	3.84		c	2.69		c	4.99					c	3.84		c	10.37	
	r	3.29		r	2.30		r	4.28					r	3.29		r	8.88	
	s	1.97		s	1.38		s	2.57					s	1.97		s	5.33	
Yingkou	d	0.72	8	d	0.63	7	d	1.17	13	d	0.63	7		——		d	2.60	29
	c	3.07		c	2.69		c	4.99		c	2.69					c	11.13	
	r	2.63		r	2.30		r	4.28		r	2.30					r	9.54	
	s	1.58		s	1.38		s	2.57		s	1.38					s	5.72	
Dalian	d	7.99	89	d	6.74	75	d	8.71	97	d	2.60	29	d	3.05	34		——	
	c	34.17		c	28.79		c	37.24		c	11.13		c	13.05				
	r	29.27		r	24.67		r	31.90		r	9.54		r	11.18				
	s	17.57		s	14.81		s	19.15		s	5.72		s	6.71				

Based on the compilation theory of the train operation plan in the previous article, we can calculate the freight train operation plan of the Harbin-Dalian high-speed railway as shown in Figures 10 and 11.

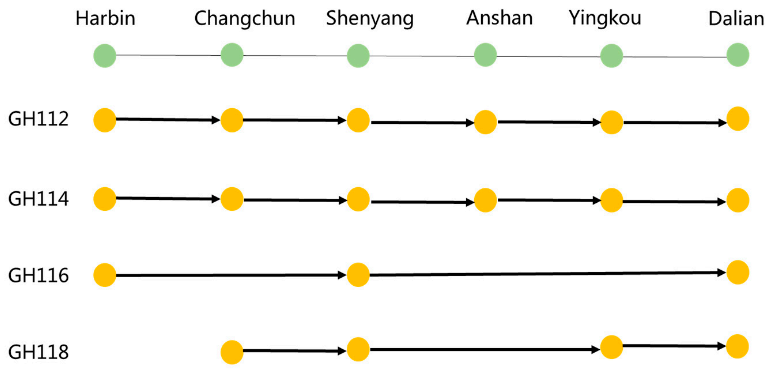

Figure 10. The high-speed rail express train plan for the up direction.

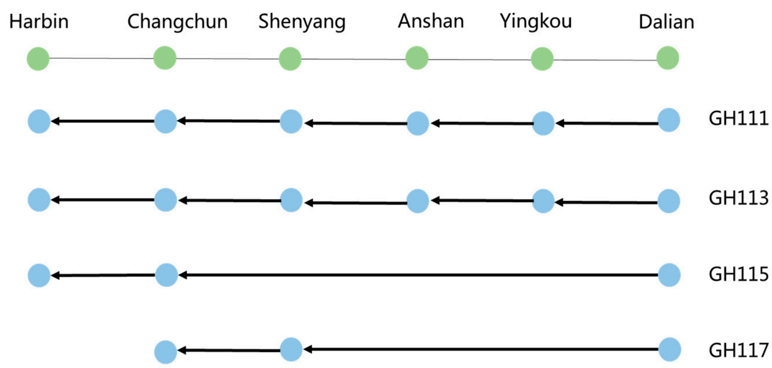

Figure 11. The high-speed rail express train plan for the down direction.

The train arrival interval is 5 min; the train departure interval is 5 min; according to the "High-speed Railway Express Business Manual", the train connection time should not be less than 40 min; the loading and unloading operation time should not be less than 10 min; the maximum speed of the high-speed freight train set is 250 km/h; the first-class repair course should be 4400 km.

The distance between major cities of the Harbin-Dalian high-speed railway is shown in Table 8.

Table 8. The distance between major cities.

	Harbin	Changchun	Shenyang	Anshan	Yingkou	Dalian
Harbin	-	234	543	638	715	921
Changchun	234	-	309	404	481	687
Shenyang	543	309	-	95	172	378
Anshan	638	404	95	-	77	283
Yingkou	715	481	172	77	-	206
Dalian	921	687	378	283	206	-

5.2. High-Speed Railway Express Train Diagram

In this paper, the Cplex 12.7.1 engine is programmed by Python language, and the Lagrange relaxation algorithm is implemented on a computer configured with an Intel Core i53235M 2.60GHz CPU and 4 GB RAM. The train diagram of the Harbin-Dalian high-speed railway was generated using red lines to represent passenger trains and blue lines to represent freight trains, as shown in Figure 12.

Because the timetable of high-speed railway passenger trains is fixed, the space-time paths left for high-speed railway passenger trains are very limited. Therefore, the solution efficiency is high, and the calculation can be completed in only one minute.

Figure 12 shows only six main nodes of the Ha-Da high-speed railway, so the technical operation of passenger trains in small and medium-sized stations along the line is not reflected. Shenyang North and Shenyang Railway Station, and Changchun Railway Station and Changchun West Railway Station can receive and send high-speed trains, so some trains do not meet the arrival and departure interval constraints in the diagram.

The timetable of express trains of the Harbin-Dalian High Speed Railway is shown in Table 9.

The routing plan of the high-speed railway express train group is shown in Figure 13.

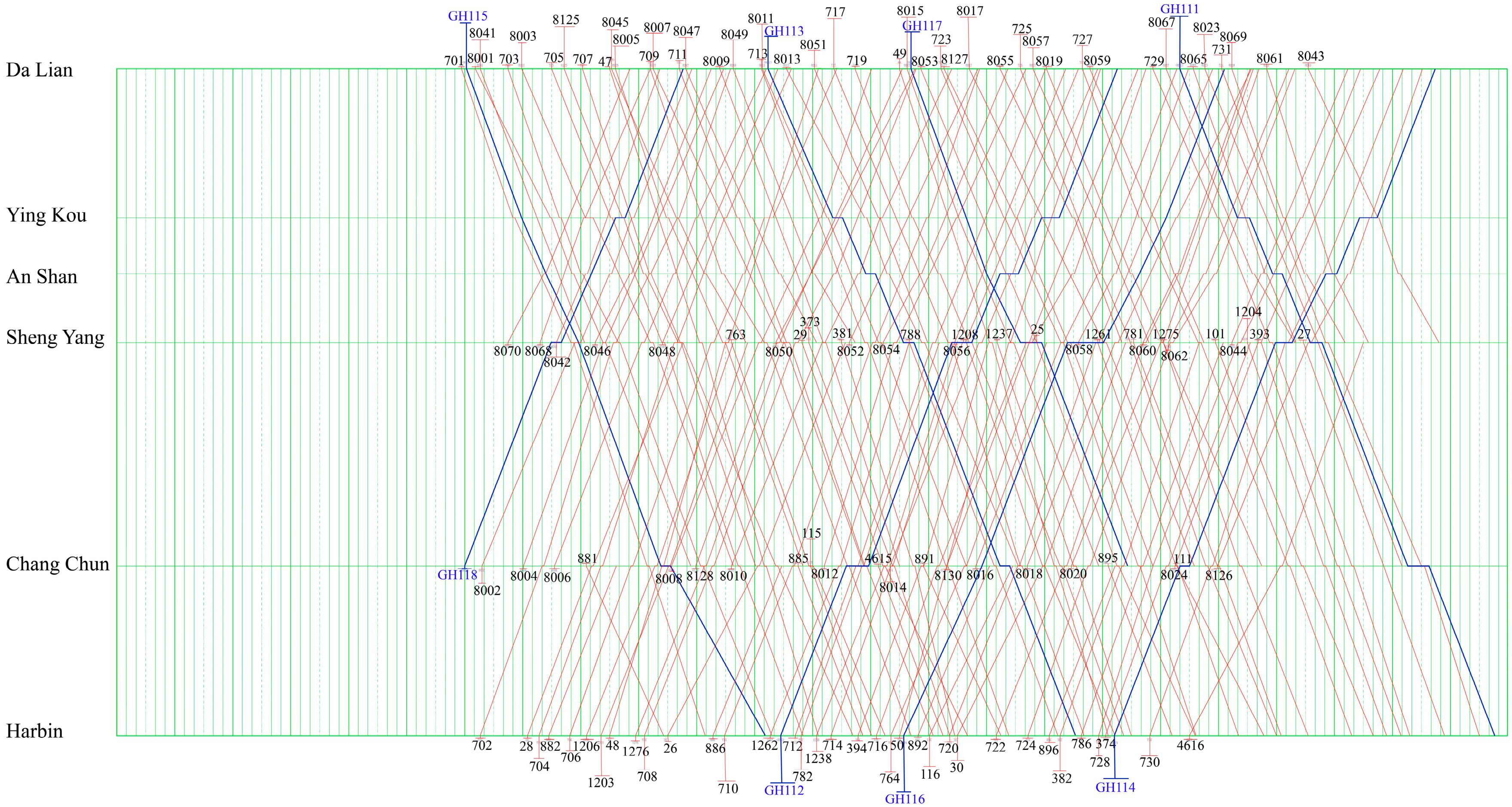

Figure 12. Harbin-Dalian high-speed railway express train operation diagram.

Table 9. Harbin-Dalian high speed railway express train timetable.

Train Number	Station	Arrival Time	Departure Time	Operation Time (min)	Train Number	Station	Arrival Time	Departure Time	Operation Time (min)
GH111	Dalian	—-	18.20	—	GH112	Harbin	—-	11.27	—
	Yingkou	19.20	19.32	12		Changchun	12.35	12.58	23
	Anshan	19.56	20.06	10		Shenyang	14.24	14.45	21
	Shenyang	20.35	20.47	12		Anshan	15.14	15.33	19
	Changchun	22.16	22.38	22		Yingkou	15.57	16.15	18
	Harbin	23.46	—-	—		Dalian	17.15	—-	—
GH113	Dalian	—-	11.14	—	GH114	Harbin	—-	17.12	—
	Yingkou	12.21	12.31	10		Changchun	18.20	18.30	10
	Anshan	12.55	13.05	10		Shenyang	19.59	20.16	17
	Shenyang	13.34	13.45	12		Anshan	20.51	21.02	11
	Changchun	15.14	15.24	10		Yingkou	21.26	21.44	18
	Harbin	16.32	—-	—		Dalian	22.44	—-	—
GH115	Dalian	—-	6.02	—	GH116	Harbin	—-	13.34	—
	Changchun	9.23	9.33	10		Shenyang	16.24	17.01	37
	Harbin	11.11	—-	—		Dalian	19.06	—-	—
GH117	Dalian	—-	13.42	—	GH118	Changchun	—-	6.00	—
	Shenyang	15.35	15.57	22		Shenyang	7.30	7.40	10
	Changchun	17.26	—-	—		Yingkou	8.36	8.46	10
						Dalian	9.46	—-	—

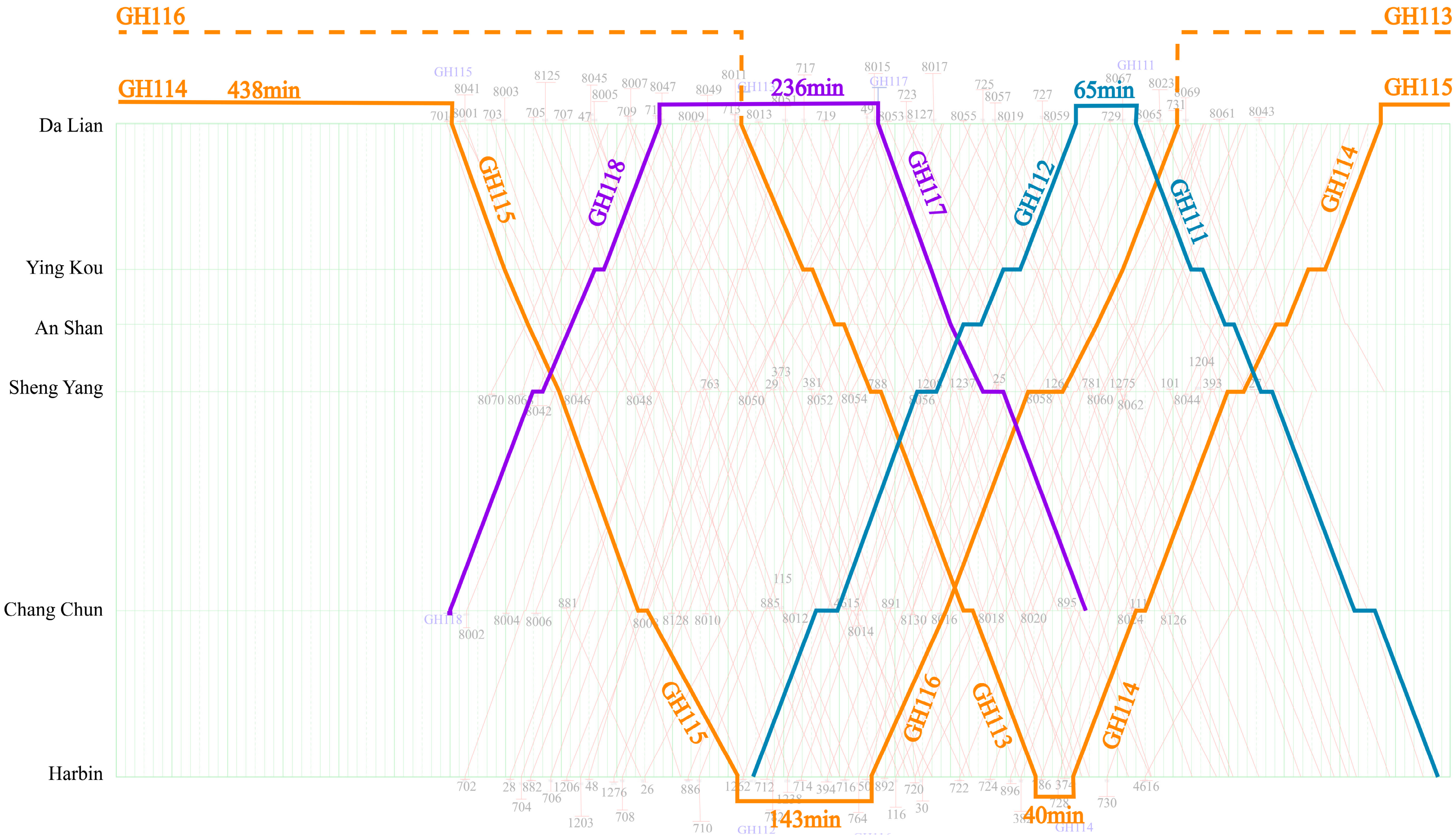

Figure 13. The route plan of freight EMUs.

5.3. Flow Distribution Scheme for High-Speed Railway Express Trains

The freight flow distribution scheme of Harbin-Dalian high-speed railway express trains is shown in Figures 14 and 15.

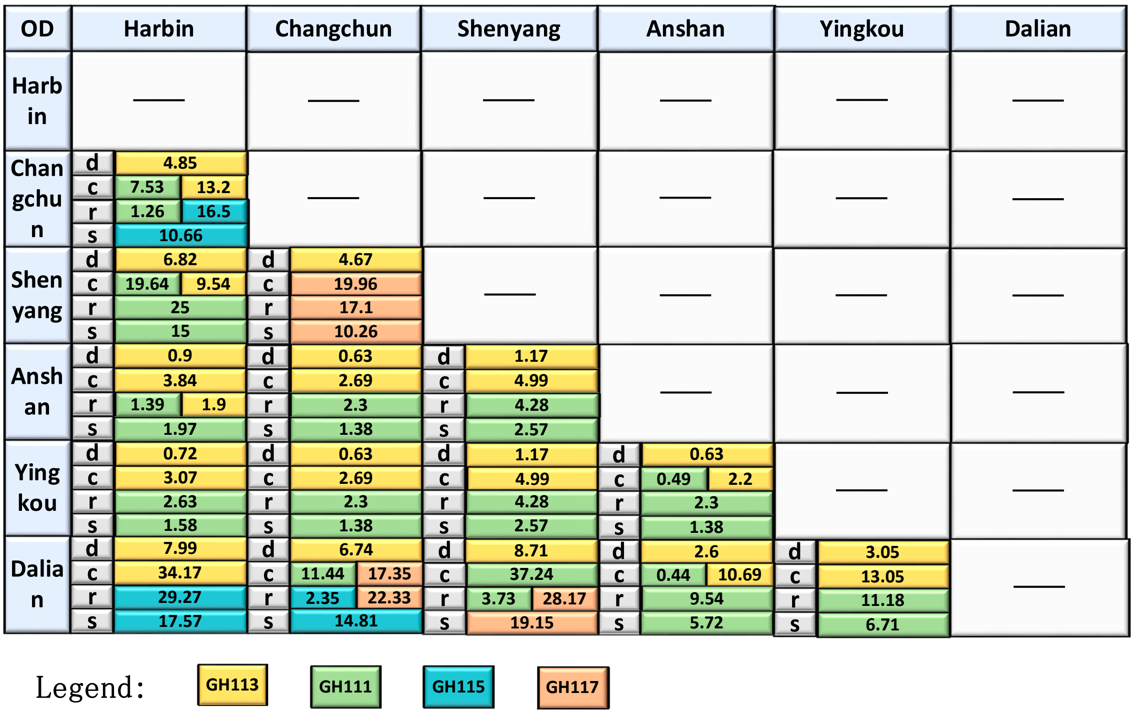

Figure 14. The cargo flows undertaken by the trains between the major cities of the Harbin-Dalian high-speed railway in the down direction.

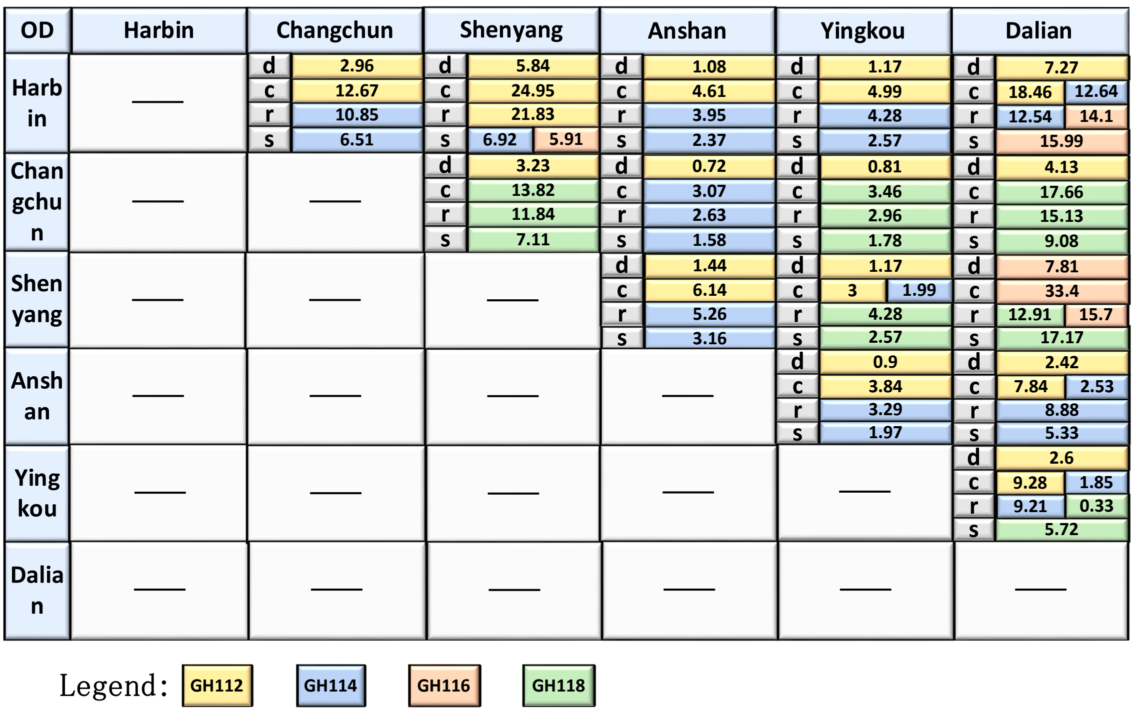

Figure 15. The cargo flows undertaken by the trains between the major cities of the Harbin-Dalian high-speed railway in the up direction.

From Figures 14 and 15, it can be seen that the high-speed railway express train effectively meets the market demand. Different kinds of transport products have different arrival times, which affects the running time of high-speed railway express trains and the operation plan of the EMUs. Therefore, it is of great significance to compile the operation chart for the high-speed railway, considering the cargo flow distribution scheme.

6. Conclusions

This paper studies the transportation organization optimization of freight trains on a high-speed railway. At present, there are few related studies in China. This manuscript enriches the organization theory of China's high-speed railway transportation and a provides necessary reference to further improve the capacity of the high-speed railway. Specific main research contents include:

(1) A comprehensive optimization model for the compilation of a high-speed freight train diagram is constructed.

In order to ensure the reasonable operation of high-speed railway trains, this paper analyzes the influence of the high-speed rail express train line drawing, cargo flow distribution, and operation plan of multiple units on the high-speed rail express train operation drawing. Considering the constraints of physical transport network capacity, train operation, and product demand, and aiming to achieve the shortest time-space path of the freight train, the lowest transport cost of physical network, and the highest efficiency of multiple units, a comprehensive optimization model of high-speed railway passenger train operation diagram is constructed based on the timetable of high-speed railway passenger trains, which can reasonably describe the coupling relationship between cargo flow and train flow, and a generate comprehensive optimization scheme.

(2) A Lagrange relaxation algorithm is designed to solve the model.

Based on the Lagrangian relaxation principle, a new Lagrangian relaxation model is formed by relaxing the capacity constraints that lead to the complex relationships between trains. In addition, the Lagrangian relaxation model is decomposed into the spatial-temporal shortest path problem with additional constraints and the shortest path problem with resource constraints. The classical dynamic programming labeling method of operational research is applied to solve the two sub-problems separately. This algorithm can realize the solution strategies of decomposition and integration, and obtain the global optimal solution efficiently.

(3) An empirical case of a comprehensive optimization scheme is generated.

Taking the Harbin-Dalian high-speed railway as an example, a comprehensive optimization scheme, including train operation plan, freight flow distribution, routing plan, and maintenance plan, is obtained, which verifies the effectiveness of the relevant models and algorithms. This comprehensive optimization scheme can provide a scientific reference for railway departments to organize and operate high-speed railway freight trains.

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