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# Pricing Decision for Reverse Logistics System under Cross-Competitive Take-Back Mode Based on Game Theory

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**Abstract:** Considering the reverse logistics system composed of two manufacturers and two recyclers under the cross-competitive take-back mode, which is influenced by multiple factors (industry competition, economies of scale, government subsidies, remanufacturing rate, etc.), a model for remanufacturing reverse logistics system based on Stackelberg game is established. Then, the Nash equilibrium solution of decision variables is solved to obtain the best profit of all participants under the cross-competitive take-back mode. Furthermore, the parameter constraint analysis is carried out, and the monopolistic take-back mode is introduced for comparative analysis. Then, the sensitivity analysis of the model is carried out. At last, a case analysis is carried out based on the current situation of waste electrical and electronic equipment (WEEE) recycling in China. The results show that the cross-competitive take-back mode is more advantageous to all participants in the reverse logistics system than the monopolistic take-back mode. Recyclers should actively sign contracts with multiple manufacturers to recycle waste products, making full use of the advantages of cross-competitive take-back mode to maximize the profits of all participants, so as to encourage them to recycle waste products and achieve sustainable development.

**Keywords:** remanufacturing; reverse logistics; take-back modes; Stackelberg game

## 1. Introduction

The rapid development of science and technology, significant improvement of living standards and increasingly changing consumption concept make people's demand for products more diversified and personalized, especially electronic products. Therefore, the life cycle of electronic products is getting shorter and shorter, and more and more electronic products are eliminated, resulting in a large amount of waste electrical and electronic equipment (WEEE). If WEEE is discarded directly, it will not only waste resources, but also pollute the environment. According to Apple's annual environmental report released on April 15, 2016, more than 600,000 pounds of steel, aluminum and other metals were recycled from waste products for the remanufacturing of new products, which not only saved resources and achieved the goal of sustainable development, but also saved nearly 40 million dollars of production costs for Apple [1]. It can be seen that the reverse logistics system can achieve both economic and social benefits. Therefore, the construction of the reverse logistics system for recycling and reuse has become a rational choice for manufacturers to solve cost and environmental problems.

Stock [2], for the first time, pointed out that the research on reverse logistics should involve the recovery of waste products, reducing the waste of resources, environmental protection, as well as waste disposal, recycling, reuse, repair and remanufacturing. So far, many scholars have made in-depth research on the reverse logistics system.

However, few researchers considered a reverse logistics system with multiple manufacturers and recyclers when they are modeling. Most scholars only consider the game among participants of different levels, such as the game between the government and manufacturers, manufacturers and retailers, retailers and third parties, respectively. On the contrary, few researchers established the game among participants of the same level, such as the game among different manufacturers or the game among different recyclers. Even so, they either failed to add the remanufacturing factor of resource reuse or researched the reverse logistic system under the circumstance when the recycling amount is complete symmetry.

Therefore, this study aims to fill the above gap in the current literature by considering a reverse logistics system composed of two manufacturers and two recyclers, where the game for pricing decision is conducted among participants of the same level and different levels, respectively. The main contributions of this paper are as follows:

1. The one-to-one cooperation between manufacturers and recyclers in the competitive take-back mode is extended to multi-party cooperation, which enriches the situation that recyclers only cooperate with a single manufacturer in previous studies.
2. The base allocation rate is introduced into the multi-party cooperation between manufacturers and recyclers, and the impact of the manufacturer's cooperation preference on the pricing and profit of all participants is analyzed, so as to consider the complexity of the market environment further.

The rest of the paper is organized as follows. Section 2 illustrates the literature review. Section 3 presents model construction and its notations and assumptions under cross-competitive take-back mode and monopolistic take-back mode. Section 4 derives the model under the two take-back modes. Then in Section 5, we carry out the parameter constraint analysis and comparative analysis of two take-back modes. In Section 6, the sensitivity analysis of the model is carried out. Section 7 gives the case results and analysis. Section 8 concludes the paper.

## 2. Literature Review

In recent years, many scholars have done much research in the take-back modes of reverse logistics. Savaskan et al. [3] established a model in which manufacturers recycle waste products for remanufacturing in three different channels (directly from consumers, through existing retailers and third parties) and studied the efficiency of these channels separately. Later, Savaskan and Waseenhove et al. [4] extended the former modal by researching the scene with multiple retailers. Östlin et al. [5] studied the importance of reverse logistics relationship for products remanufacturing, and came up with seven different types of relationship models. Based on enterprise social responsibility, Wen et al. [6] studied the selection of reverse logistics recycling mode. Dobos et al. [7] established an extended production-recycling model with fixed demand and recycling rate, and analyzed optimal problems of pure strategies and mixed strategies. Based on the earlier model, Dobos et al. [8] added the quality factor into it, and analyzed the two different kinds of recycling modes. Kim et al. [9] established a supply planning model for the remanufacturing system in reverse logistics environment and verified the validity of the model and algorithm by numerical analysis. Miao et al. [10] constructed a recycling mode dominated by the third party of reverse logistics, and carried out empirical analysis by the example of Midea Corp. Georgiadis et al. [11] studied reverse channel capacity planning of closed-loop supply chains by developing a system dynamics (SD) model for two sequential types of products under two different scenarios regarding the consumer preferences over the products. Bal et al. [12] considered the triple-bottom-line approach and multi-objective programming to achieve the maximization of economic, social and environmental goals at the same time, proposing a multi-equipment, multi-product and multi-cycle reverse logistics recycling model. Tosarkani et al. [13] applied fuzzy analysis network process (FANP) to convert environmental quality factors into quantitative parameters, and adopted multi-objective mixed-integer linear (MILP) programming model to minimize the defect rate and maximize the total profit, green practices and on-time delivery in reverse logistics. He et al. [14] analyzed the recycling efficiency of a two-echelon decentralized model composed of a manufacturer and a retailer under the inconvenience-perception in the collection and compared its performance with a centralized model, and finally proposed two coordination mechanisms (contract mechanism and authorization mechanism) to improve the decentralized model. Kuşakcı et al. [15] assumed that the end-of-life vehicles (ELVs) supply in the reverse logistics network is uncertain, developing a fuzzy mixed integer location-allocation model for a reverse logistic network of ELVs conforming to the existing directives in Turkey. Duman et al. [16] used grey models to estimate the amount of WEEE to help decision makers plan more efficient reverse logistics infrastructure to ensure the correct collection, recycling and disposal of WEEE. Huang et al. [17] considered a closed-loop supply chain consisting of a manufacturer and a third party in which the manufacturer licenses the third party to undertake remanufacturing activities in the presence of strategic consumers. Feng et al. [18] considered the preference of green consumers for remanufactured products and constructed a dual-sale-channel supply chain model with government non-intervention, government remanufacturing subsidy policy, and carbon tax policy, respectively. Ding et al. [19] discussed the coopetition relationships between a manufacturer and a collector in the collection, to examine the evolution mechanism and the optimal reward-penalty mechanism (RPM) for their collection strategies. Li et al. [20] used entropy weight method and Borda selection model to select and evaluate three recycling modes (self-operation recycling, joint recycling and third-party recycling) of reverse logistics of packaging waste. On the basis of the extended producer responsibility system, Wu [21] made a comparative analysis of the producer's self-recovery mode, the third-party recovery mode and the joint operation recovery mode. Ni et al. [22] took WEEE under uncertain market demand as example, constructed three closed-loop supply chain models under the dual-channel recycling model, compared the optimal solution of recycling pricing, recycling volume and profit under the three recycling models, and analyzed the influence of the competition coefficient of dual-channel recycling on the optimal solution under different models. Kai et al. [23] applied Internet thinking to establish a reverse logistics recycling model of waste home appliances based on O2O, aiming at problems, such as disordered recycling channels, weak awareness of recycling and environmental protection, so as to improve the utilization rate of waste home appliances recycling and reduce the logistics cost. However, to the author's knowledge, most of the above studies on reverse logistics recycling mode only consider single manufacturer, retailer and third party, and little attention is paid to other influencing factors, such as government intervention, economies of scale and industry competition.

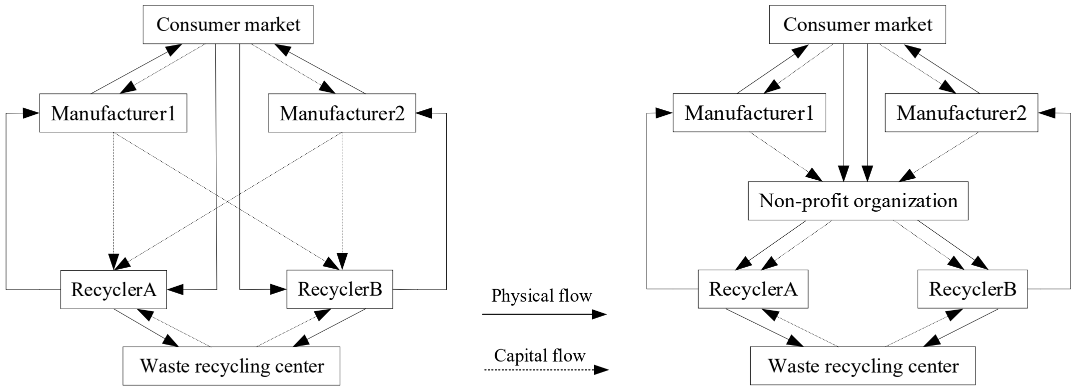
With the in-depth study of reverse logistics by domestic and foreign scholars, more and more people apply game theory to the research of reverse logistics. Choi et al. [24] investigated a reverse logistics system composed of a retailer, a recycler, and a manufacturer, examining the efficiency of different reverse logistics system, in which the retailer, the recycler, and the manufacturer act as the channel leader (Stackelberg leader). Biswas et al. [25] established a complete information game model for designing supply chain coordination with triple bottom line objectives. By applying game theory and fuzzy theory, Wei et al. [26] studied the impact of the three different used-product recycling modes to the decisions of the manufacturer, the retailer, and the third party. Arshad et al. [27] established a dual-channel sales reverse logistics game model based on manufacturer's direct recovery and third-party recovery, and studied the impact of offline fairness concerns of offline retailers on the pricing and profit of decision-making subjects in the reverse logistics system. Feng et al. [28] studied the reverse logistics system with two-level recycling channels and established a game model with recyclable dealers as the leader and recyclers as the followers in the Stackelberg game. Taking manufacturers as leaders, Zhang et al. [29] considered a reverse supply chain composed of a supplier and a retailer and formulated how consumers make their choices in the face of new and remanufactured products using game theory. Zand et al. [30] constructed a game model of reverse logistics in which direct online sales of manufacturers coexist with offline sales through retailers and recycling through retailers in the case of government intervention. Tan et al. [31] considered the impact of various government policies on the reverse logistics system when the recovery quality was uncertain. Chang et al. [32] established a two-stage recovery game model composed of the government, a monopoly manufacturer and a recycler, and studied the influence of the joint tax-subsidy mechanism on the decision-making of manufacturers and recyclers. Liang et al. [33] established three alternative mixed recovery models with Stackelberg game theory, and the influence of substitution coefficient is analyzed with numerical examples. Jian et al. [34] established an evolutionary game model between the government and the producer in the process of waste mobile phone recycling, so as to seek ways for both parties to achieve common benefits. From the perspective of game theory, Xia et al. [35] set up a reverse logistics recovery model led by manufacturers and retailers, respectively, to solve the problem of how to reduce the logistics cost of reverse packaging for electromechanical products. Huang et al. [36] built a reverse logistics system using Stacklberg game where the manufacturer is behaving like the leader, and the retailer and the third party are followers. In addition, there is a non-cooperate game between the retailer and the third party. It can be seen from the above literature that in the application of game theory to the modeling of reverse logistics system, many current studies consider the game among participants of different levels, such as the game between the government and manufacturers, manufacturers and retailers, retailers and third parties, respectively. There is little information available in the literature about establishing the game among participants of the same level, such as the game among different manufacturers and the game among different retailers. Toyasaki et al. [37] considered the game among participants of the same level, but failed to add the remanufacturing factor of resource reuse into the model. Although Ding et al. [38] added remanufacturing factors on this basis, it established a model in the case when recycling amount is complete symmetry, and each manufacturer only signed one contract with a single recycler for recycling and remanufacturing, which has limitations in the current changeable market environment.

## 3. Model Construction of Reverse Logistics System Based on Game Theory

### *3.1. Model Description and Basic Assumptions*

The reverse logistics system model considered in this paper is composed of two manufacturers and two recyclers. Each recycler is allowed to sign contracts with two manufacturers at the same time. Recyclers contract to recycle waste products corresponding to the two manufacturers from the consumer market. Thus, it forms a dual competitive pattern in which both manufacturers directly compete by selling electronic products and recyclers directly compete by recycling waste products. As there is a crossover of capital flows between manufacturers and recyclers, hereinafter referred to as cross-competitive take-back mode, as shown in Figure 1a.

At the same time, the monopolistic take-back mode is introduced for comparison, as shown in Figure 1b. The non-profit organization (NPO) collects waste products directly from the consumer market on behalf of the manufacturers. It charges recycling fees to the manufacturers according to the market share of their product, and signs contracts with the recyclers to distribute waste products and recycling fees to the two recyclers. Manufacturers directly compete with each other through the sales price of new products, and recyclers indirectly compete with each other through the sales price of manufacturers, market share and the recycling amount of waste products. In these two take-back modes, the recyclers are responsible for classifying waste products, returning the useful ones to the manufacturer for remanufacturing, and sending the unused ones to the waste recycling center for unified treatment.




**(a)** Cross-competitive take-back mode **(b)** monopolistic take-back mode

**Figure 1.** Reverse Logistics Recovery Model.

In these modes, let two recyclers as leaders of Stackelberg game. They formulate the unit recycling fees to maximize their own interests. Then the manufacturers, as the followers, formulate the product price. This game is a two-stage dynamic game:

Stage 1: Each recycler first sets the unit recycling fees of waste products, and there is competition between the two recyclers for the recycling fees, taking into account the response of the manufacturers to the recycling fees at the same time.

Stage 2: According to the recycling fees of the recyclers or the NPO, each manufacturer sets the product price to maximize its own profit, and there is competition between the two manufacturers for the product price, meanwhile taking into account the recycling fees charged by the recyclers (cross-competitive take-back mode) or the NPO (monopolistic take-back mode).

The specific assumptions are described below.

- 1. Manufacturers have the responsibility and obligation to bear the corresponding costs of waste products recycling and pay the recyclers (or NPO), which will inevitably increase their costs and indirectly affect product pricing, so the manufacturer's product pricing will be affected by the recyclers' decisions.
- 2. Whether the waste products collected by the recyclers are returned to the manufacturer for remanufacturing, they will get subsidies from the government.
- 3. The operating cost of the NPO is provided by the fund, and all the recycling fees collected from the manufacturers are distributed to the recyclers. The NPO does not generate extra profits by itself. In other words, it does not affect the competition between the two recyclers.
- 4. Ignore the recycling fees that recyclers (or NPO) pay to consumers when they recycle waste products from the consumer market.
- 5. The remanufactured product is identical with the new product in terms of performance, price and market recognition, and the quality of the recycled waste products for remanufacturing is constant, that is, the unit cost of the remanufactured product is constant.
- 6. The demand function of the market is a linear function.

### 3.2. Model Construction

Based on the above model description and assumptions, this paper establishes a Stackelberg master-slave game model with recyclers as leaders and manufacturers as followers. The market demand function of Manufacturer j can be expressed as follows:

$$d_j^k = \alpha_j - p_j^k + \beta p_{3-j}^k \; j = 1,2 \; k = c, m. \tag{1}$$

Here, $j = 1$ represents Manufacturer 1 and $j = 2$ represents Manufacturer 2; $k$ is the type of take-back mode, $c$ represents the cross-competitive take-back mode, and $m$ represents the monopolistic take-back mode. $\alpha_j(\alpha_j>0)$ represents the market capacity of manufacturer $j$; $\beta(0<\beta<1)$ represents the elasticity of market demand (that is, the degree of substitutability of the manufacturer's products). Here, $0<\beta<1$ indicates that the substitutability of products of two manufacturers has a certain impact on market demand, but it has a smaller impact on market demand than the manufacturers' product pricing.

Under the cross-competitive take-back mode, the optimal profit of Manufacturer 1 and Manufacturer 2 are:

$$\max_{p_1^c}\Pi_1^c = \left(p_1^c - c_m\right)(1-\sigma)\left(\alpha_1 - p_1^c + \beta p_2^c\right) + \left(p_1^c - c_r\right)\sigma\left(\alpha_1 - p_1^c + \beta p_2^c\right) - t_A^c\delta\tau\left(\alpha_1 - p_1^c + \beta p_2^c\right) - t_B^c(1-\delta)\tau\left(\alpha_1 - p_1^c + \beta p_2^c\right) = \left[p_1^c - c_m + \sigma(c_m - c_r) - t_A^c\delta\tau - t_B^c(1-\delta)\tau\right]\left(\alpha_1 - p_1^c + \beta p_2^c\right) \tag{2}$$

$$\max_{p_2^c}\Pi_2^c = \left(p_2^c - c_m\right)(1-\sigma)\left(\alpha_2 - p_2^c + \beta p_1^c\right) + \left(p_2^c - c_r\right)\sigma\left(\alpha_2 - p_2^c + \beta p_1^c\right) - t_A^c(1-\delta)\tau\left(\alpha_2 - p_2^c + \beta p_1^c\right) - t_B^c\delta\tau\left(\alpha_2 - p_2^c + \beta p_1^c\right) = \left[p_2^c - c_m + \sigma(c_m - c_r) - t_B^c\delta\tau - t_A^c(1-\delta)\tau\right]\left(\alpha_2 - p_2^c + \beta p_1^c\right). \tag{3}$$

Here, $\Pi_j^c$ represents the profit function of Manufacturer $j$ under the cross-competitive take-back mode, $p_j^c$ represents the product pricing of Manufacturer $j$ under the cross-competitive take-back mode, $c_m$ represents the unit cost of new product; $c_r(c_m>c_r)$ represents the unit cost of the remanufactured product; $\sigma$ is the remanufacturing rate ($0\le\sigma\le1$). $\tau$ represents the recycling rate ($0<\tau\le1$), $t_A^c$ represents unit recycling fees charged by Recycler A to the manufacturers, $t_B^c$ represents unit recycling fees charged by Recycler B to the manufacturers. In practice, manufacturers will consider many other factors besides cost when signing contracts with their strategic partners. This indicates that even if two recyclers charge the same fee, a manufacturer may have a stronger preference for one particular recycler because of practical factors not involved in our model (e.g., closeness, long-term collaboration potential, etc.). Specifically, suppose that Manufacturer 1 is more inclined to Recycler A and allocates its $\delta\times100\%$ waste products in the consumer market to it ($0.5\le\delta\le1$), and the remaining $(1-\delta)\times100\%$ waste products is distributed to recycler B as the base allocation rate. Manufacturer 2 is in the same way.

Under the monopolistic take-back mode, the best profit of Manufacturer $j$ is:

$$\max_{p_j^m}\Pi_j^m = \left(p_j^m - c_m\right)(1-\sigma)\left(\alpha_j - p_j^m + \beta p_{3-j}^m\right) + \left(p_j^m - c_r\right)\left(\alpha_j - p_j^m + \beta p_{3-j}^m\right) - t^m\tau\left(\alpha_j - p_j^m + \beta p_{3-j}^m\right) = \left[p_j^m - c_m + \sigma(c_m - c_r) - t^m\tau\right]\left(\alpha_j - p_j^m + \beta p_{3-j}^m\right) = 1, 2. \tag{4}$$

Here, $\Pi_j^m$ represents the profit function of Manufacturer $j$ under the monopolistic take-back mode; $p_j^m$ represents the product pricing of Manufacturer $j$ under the monopolistic take-back mode. Under the monopolistic take-back mode, NPO allocates recycled waste products to two recyclers at a fixed rate. Specifically, the ratio of recycling amount that Recycler A being distributed to total recycling amount is $\lambda$ ($0\le\sigma<\lambda\le1$), then the ratio of recycling amount that Recycler B being distributed to total recycling amount is $1-\lambda$. Here, $\sigma<\lambda$ stands for that the number of remanufactured products must less than that of the original waste products. Since the recycling fees that the NPO collects from the manufacturers are all distributed to the recyclers, it makes no additional profit for its own. Therefore, the recycling fees that charged by NPO to manufacturers is: $t^m = \lambda t_A^m + (1-\lambda)\, t_B^m$.

Under the cross-competitive take-back mode, since each recycler has signed contracts with two manufacturers, each recycler can recycle waste products of both manufacturers from the consumer market simultaneously. The recycling amount of waste products of Recycler A and Recycler B are as follows:

$$\omega_A^c\left(t_A^c, t_B^c\right) = \delta\tau d_1^c\left(p_1^c, p_2^c\right) + (1-\delta)\,\tau d_2^c\left(p_1^c, p_2^c\right), \tag{5}$$

$$\omega_B^c\left(t_A^c, t_B^c\right) = \delta\tau d_2^c\left(p_1^c, p_2^c\right) + (1-\delta)\,\tau d_1^c\left(p_1^c, p_2^c\right). \tag{6}$$

Under the monopolistic take-back mode, the NPO's distribution of waste products to Recycler A and Recycler B are as follows:

$$\omega_A^m\left(t_A^m,\ t_B^m\right) \ = \ \lambda\tau\left[d_1^m\left(p_1^m\ ,\ p_2^m\right) + d_1^m\left(p_1^m\ ,\ p_2^m\right)\right], \tag{7}$$

$$\omega_B^m\left(t_A^m,\ t_B^m\right) \ = \ (1-\lambda)\,\tau\left[d_1^m\left(p_1^m\ ,\ p_2^m\right) + d_1^m\left(p_1^m,\ p_2^m\right)\right]. \tag{8}$$

Under the two take-back modes, recyclers will generate recycling costs on, such as classification, treatment and transportation of waste products. Bohr [39] analyzed the relationship among recycling cost, recycling amount and economies of scale: $\eta_i\omega_i^k - \theta_i\left(\omega_i^k\right)^2$. Here, $\eta_i$ represents the cost of Recycler $i$ ($i$ = A, B) to recycle unit waste products, and $\theta_i$ represents the economies of the scale factor. Therefore, the best profit of Recycler $i$ is:

$$\max_{t_i^m}\Pi_i^k = \left(t_i^k + \mathrm{r}\right)\,\omega_i^k\left(t_i^k\right) - \left[\ \eta_i\omega_i^k\left(t_i^k\right) - \theta_i\left(\omega_i^k\left(t_i^k\right)\right)^2\right]. \tag{9}$$

Here, $\Pi_i^k$ represents the profit function of Recycler $i$ under the take-back mode of $k$ ($k\ =\ c,\ m$); $r$ stands for government subsidies that recyclers could get when they recycle unit waste products.

## 4. Model Derivation

In this section, we derive the recycling fees and product prices under both take-back modes. In order to reasonably compare the two take-back modes and simplify the derivation of the model, we consider a symmetric scenario, which makes us compare the two take-back modes on an equal footing. In the symmetric case, the two manufacturers have the same market demand $\alpha_1 = \alpha_2 = \alpha$, and recyclers have the same cost factor $\eta_A = \eta_B = \eta$, $\theta_A = \theta_B = \theta$.

### *4.1. Cross-Competitive Take-Back Mode*

The two-stage dynamic game can be analyzed by using backward induction method. In each stage, we first derive the reaction functions of the participants and then solve for the Nash equilibrium.

In the second stage of the game, each manufacturer prices its own product, and there is competition between the two manufacturers for the product price. Given a set of recycling fees ($t_A^c$, $t_B^c$) charged by recyclers to manufacturers, whatever the price of Manufacturer 2's product is, Manufacturer 1 can always find a price $p_1^c$ that maximizes its profit, so the reaction function of Manufacturer 1's product pricing is:

$$p_1^c\left(p_2^c\right) \ = \ \frac{\alpha + \beta p_2^c + c_m - \sigma\left(\left(c_m - c_r\right)\right) + t_A^c\delta\tau + t_B^c(1-\delta)\,\tau}{2}. \tag{10}$$

Similarly, the reaction function of Manufacturer 2's product pricing is:

$$p_2^c\left(p_1^c\right) \ = \ \frac{\alpha + \beta p_1^c + c_m - \sigma\left(\left(c_m - c_r\right)\right) + t_B^c\delta\tau + t_A^c(1-\delta)\,\tau}{2}. \tag{11}$$

Nash equilibrium solution of product price can be obtained by combining Equations (10) and (11):

$$p_1^c\left(t_A^c,\ t_B^c\right) \ = \ \frac{(2+\beta)\left[\alpha + c_m - \sigma(c_m - c_r)\right] + \left\{2\left[t_A^c\delta + t_B^c(1-\delta)\right] + \beta\left[t_B^c\delta + t_A^c(1-\delta)\right]\right\}\tau}{4-\beta^2}, \tag{12}$$

$$p_2^c\left(t_A^c,\ t_B^c\right) \ = \ \frac{(2+\beta)\left[\alpha + c_m - \sigma(c_m - c_r)\right] + \left\{\beta\left[t_A^c\delta + t_B^c(1-\delta)\right] + 2\left[t_B^c\delta + t_A^c(1-\delta)\right]\right\}\tau}{4-\beta^2}. \tag{13}$$

In the game of the first stage, the recycling amount of Recycler A and Recycler B are:

$$\omega_A^c = \ \delta\tau d_1^c\left(p_1^c, p_2^c\right) + (1-\delta)\,\tau d_2^c\left(p_1^c, p_2^c\right), \tag{14}$$

$$\omega_B^c = \delta\tau d_2^c\left(p_1^c, p_2^c\right) + (1-\delta)\,\tau d_1^c\left(p_1^c, p_2^c\right). \tag{15}$$

No matter how much Recycler B's recycling fees is, Recycler A can always find a recycling fee $t_A^c$ that maximizes its profit, so the reaction function of Recycler A 's recycling fees is:

$$t_A^c\left(t_B^c\right) = \frac{-(2+\beta)\left\{\left(4-\beta^2+2\theta\tau^2 X\right)\left[\alpha+\left[c_m-\sigma(c_m-c_r)\right](\beta-1)\right]+(2-\beta)\,(r-\eta)\,\tau X\right\}-\left[\left(4-\beta^2\right)\tau Y+2\theta\tau^3 XY\right]t_B^c}{2\tau\left(4-\beta^2\right)X+2\theta\tau^3 X^2}. \tag{16}$$

Similarly, the reaction function of Recycler B's recycling fees is:

$$t_B^c\left(t_A^c\right) = \frac{-(2+\beta)\left\{\left(4-\beta^2+2\theta\tau^2 X\right)\left[\alpha+\left[c_m-\sigma(c_m-c_r)\right](\beta-1)\right]+(2-\beta)\,(r-\eta)\,\tau X\right\}-\left[\left(4-\beta^2\right)\tau Y+2\theta\tau^3 XY\right]t_A^c}{2\tau\left(4-\beta^2\right)X+2\theta\tau^3 X^2}. \tag{17}$$

We can obtain Nash equilibrium solution of recycling fees by combining Equations (16) and (17):

$$t_A^{*c} = t_B^{*c} = \frac{\left(\beta^2-4-2\theta\tau^2 X\right)\left[\alpha+\left[c_m-\sigma(c_m-c_r)\right](\beta-1)\right]+(2-\beta)\,(r-\eta)\,\tau X}{\tau\left[(2-\beta)\,(2X+Y)\,+2\theta\tau^2(\beta-1)\,X\right]}. \tag{18}$$

As the formula in this section is of highly complex, in order to facilitate writing, let:

$$X = 2\delta^2\beta^2 - 2\delta^2\beta - 2\delta\beta^2 + 2\delta\beta - 4\delta^2 + \beta^2 + 4\delta - 2, \tag{19}$$

$$Y = -2\delta^2\beta^2 + 2\delta^2\beta + 2\delta\beta^2 - 2\delta\beta + 4\delta^2 - 4\delta + \beta. \tag{20}$$

Substituting equilibrium recycling fees $t_A^{*c}$ and $t_B^{*c}$ into Equations (12) and (13), we obtain the manufacturers' equilibrium prices:

$$p_1^{*c} = p_2^{*c} = \frac{X\left[c_m-\sigma(c_m-c_r)-(r-\eta)\,\tau\right]+\alpha\left(X+\beta^2-4-2\theta\tau^2 X\right)}{(2-\beta)\,(2X+Y)\,+2\theta\tau^2(\beta-1)\,X}. \tag{21}$$

Then, substituting Equations (18) and (21) into Equations (2), (3) and (9), manufacturers' best profit can be obtained as follows:

$$\Pi_1^{*c} = \Pi_2^{*c} = \left(\frac{X\left[\alpha+(\beta-1)\left[c_m-\sigma(c_m-c_r)-(r-\eta)\,\tau\right]\right]}{(2-\beta)\,(2X+Y)\,+2\theta\tau^2(\beta-1)\,X}\right)^2. \tag{22}$$

The best profits of recyclers are:

$$\Pi_A^{*c} = \Pi_B^{*c} = \frac{X\left(\beta^2-4-2\theta\tau^2 X\right)\left[\alpha+(\beta-1)\left[c_m-\sigma(c_m-c_r)-(r-\eta)\,\tau\right]\right]^2}{\left[(2-\beta)\,(2X+Y)\,+2\theta\tau^2(\beta-1)\,X\right]^2}. \tag{23}$$

### *4.2. Monopolistic Take-Back Mode*

In the same way, the equilibrium solution can be obtained under the monopolistic take-back mode. Here, the derivation process is not repeated, but only a few important derivation results are as shown below.

The reaction function of two manufacturers are:

$$p_j^m = p_{3-j}^m = \frac{\alpha+c_m-\sigma(c_m-c_r)+\beta p_{3-j}^m+\left[\lambda t_A^m+(1-\lambda)\,t_B^m\right]\tau}{2}. \tag{24}$$

Nash equilibrium solution of manufacturers' product price are:

$$p_1^m\left(t_A^m, t_B^m\right) = p_2^m\left(t_A^m, t_B^m\right) = \frac{\alpha+c_m-\sigma(c_m-c_r)+\left[\lambda t_A^m+(1-\lambda)\,t_B^m\right]\tau}{2-\beta}. \tag{25}$$

Because the two recyclers have the same status, in order not to lose generality, it can be assumed that the NPO distribute waste products equally between the two recyclers, so we set $\lambda$ = 1/2. Therefore, the recycling amount of Recycler A and Recycler B are:

$$\omega_A^m = \omega_B^m = \frac{1}{2}\tau\left(d_1^m\left(p_1^m, p_2^m\right) + d_2^m\left(p_1^m, p_2^m\right)\right). \tag{26}$$

Nash equilibrium solution of two recyclers' recycling fees are:

$$t_A^{*m} = t_B^{*m} = \frac{2(\beta-2)\,[\alpha + [c_m - \sigma c_m - c_r](\beta-1)] + (r-\eta)\,(\beta-2)\,(\beta-1)\,\tau - 2\theta\tau^2[\alpha + [c_m - \sigma(c_m - c_r)](\beta-1)\,](\beta-1)}{\tau(\beta-1)\,[6 - 3\beta + 2\theta\tau^2(\beta-1)\,]}. \tag{27}$$

Therefore, the recycling fees charged by NPO to manufacturers is:

$$t^{*m} = \frac{1}{2}\left(t_A^{*m} + t_B^{*m}\right) = \frac{2(\beta-2)\,[\alpha + [c_m - \sigma(c_m - c_r)](\beta-1)\,] + (r-\eta)\,(\beta-2)\,(\beta-1)\,\tau - 2\theta\tau^2[\alpha + [c_m - \sigma(c_m - c_r)](\beta-1)\,](\beta-1)}{\tau(\beta-1)\,[6 - 3\beta + 2\theta\tau^2(\beta-1)\,]}. \tag{28}$$

The manufacturers' product price are as follows:

$$p_1^{*m} = p_2^{*m} = \frac{(\beta-1)\left[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau\right] + \alpha\left[3\beta - 5 - 2\theta\tau^2(\beta-1)\,\right]}{(\beta-1)\,[6 - 3\beta + 2\theta\tau^2(\beta-1)\,]}. \tag{29}$$

The manufacturers' best profit is:

$$\Pi_1^{*m} = \Pi_2^{*m} = \left(\frac{\alpha + (\beta-1)\,[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau]}{6 - 3\beta + 2\theta\tau^2(\beta-1)}\right)^2. \tag{30}$$

The best profit of recyclers is:

$$\Pi_A^{*m} = \Pi_B^{*m} = \frac{\left[2\beta - 4 - \theta\tau^2(\beta-1)\,\right][\alpha + (\beta-1)\,[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau]]^2}{(\beta-1)\,[6 - 3\beta + 2\theta\tau^2(\beta-1)\,]^2}. \tag{31}$$

## 5. Model Parameter Constraint Analysis and Comparison of Two Take-Back Modes

### *5.1. Model Parameter Constraint Analysis*

In order to ensure that the solution of the model conforms to the reality, the following constraint analysis is carried out on the model parameters.

#### 5.1.1. Existence Conditions of Nash Equilibrium

As early as 1991, Fudenberg [40] pointed out that in order to ensure the existence of a unique equilibrium solution for the model, the objective function should be the quasi-convex function of the independent variable. That is, in this paper, it is necessary to ensure that the manufacturers' profit function is the quasi-convex function of product price, and the recyclers' profit function is the quasi-convex function of recycling fees. As can be seen from Equations (2), (3) and (4), the profit functions of manufacturers are all convex functions, so there is a unique equilibrium solution. For the recycler's profit function, a constraint must be established to ensure that it is a quasi-convex function with a unique equilibrium solution. It can be seen from the model assumption that the profit function of the recycler should be quasi-convex, that is, equivalent to convex. Therefore, the second partial derivative of the recycler's profit function with respect to the recycling fees should be less than or equal to 0. Since $\Pi_A^{*k} = \Pi_B^{*k} (k = c, m)$, the following analysis takes Recycler A as an example.

Under the cross-competitive take-back mode:

$$\frac{\partial^2 \Pi_A^c}{\partial \left(t_A^c\right)^2} = 2\frac{\partial \omega_A^c}{\partial t_A^c} + 2\theta \left(\frac{\partial \omega_A^c}{\partial t_A^c}\right)^2 \leq 0. \tag{32}$$

It can be found that:

$$\frac{2\tau^2 X\left[4 - \beta^2 + \theta\tau^2 X\right]}{\left(4 - \beta^2\right)^2} \leq 0. \tag{33}$$

And it can be proved that: $\tau^2 > 0$, $X < 0$, $\left(4 - \beta^2\right)^2 > 0$, so we have:

$$4 - \beta^2 + \theta\tau^2 X \geq 0. \tag{34}$$

It can be derived that the value range of $\theta$:

$$\theta \leq \frac{\beta^2 - 4}{\tau^2 X}. \tag{35}$$

Under the monopolistic take-back mode (not define the value of $\lambda$):

$$\frac{\partial^2 \Pi_A^m}{\partial \left(t_A^m\right)^2} = 2\frac{\partial \omega_A^m}{\partial t_A^m} + 2\theta \left(\frac{\partial \omega_A^m}{\partial t_A^m}\right)^2 \leq 0. \tag{36}$$

It can be found that:

$$\frac{4\lambda^2\tau^2(\beta - 1)\left[2 - \beta + 2\theta\lambda^2\tau^2(\beta - 1)\right]}{(2 - \beta)^2} \leq 0. \tag{37}$$

And it can be proved that: $(2 - \beta)^2 > 0$, $\lambda^2 > 0$, $\tau^2 > 0$, $\beta - 1 < 0$, so we have:

$$2 - \beta + 2\theta\lambda^2\tau^2(\beta - 1) \geq 0. \tag{38}$$

It can be calculated that the value range of $\theta$:

$$\theta \leq \frac{2 - \beta}{2\lambda^2\tau^2(1 - \beta)}. \tag{39}$$

Therefore, to ensure that the recycler's profit function is convex in both take-back modes, the value range of $\theta$ should satisfy the following condition:

$$\theta \leq \overline{\theta} = \min\left\{\frac{\beta^2 - 4}{\tau^2 X}, \frac{2 - \beta}{2\lambda^2\tau^2(1 - \beta)}\right\}. \tag{40}$$

5.1.2. Non-Negative Market Demand Conditions

Because the product market demand is always non-negative, the market demand function under the two take-back modes should meet the non-negative conditions, that is:

Under the cross-competitive take-back mode:

$$d_j^c = \alpha + (\beta - 1)\, p_j^c \geq 0 \quad j = 1, 2. \tag{41}$$

Substituting Equation (21) into Equation (41) and simplifying would change the equation into:

$$d_j^{*c} = \frac{X[\,\alpha + (\,\beta - 1)\,[c_m - \sigma(c_m - c_r) - (r - \eta)\,\tau]]}{(2-\beta)\,(2X+Y)\; + 2\theta\tau^2(\beta-1)\,X} \geq 0\,. \tag{42}$$

Because $0<\lambda\leq 1$, it can be known from Equation (39) that $\theta \leq \frac{2-\beta}{2\tau^2(1-\beta)}$. Thus, it can be derived that:

$$2\theta\tau^2(\beta-1)\,X \leq (\beta - 2\,)\;X. \tag{43}$$

In addition: $X+Y<0$, so we have:

$$(2-\beta)\,(2X+Y)\; + 2\theta\tau^2(\beta-1)\,X \leq \,(2-\beta)\,(2X+Y) + (\beta-2\,)\;X = \;(2-\beta)\,(X+Y)<0. \tag{44}$$

It can be verified that Equation (42) is true because

$$X<0,\; \alpha + (\,\beta - 1)\,[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau] \geq 0 \tag{45}$$

Under the monopolistic take-back mode:

$$d_j^m \;=\; \alpha + (\,\beta{-}1)\;p_j^m \geq 0\;\; j \;=\; 1,2. \tag{46}$$

Substituting Equation (28) into Equation (44):

$$d_j^{*m} = \frac{\alpha + (\,\beta - 1)\,[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau]}{6 - 3\beta + 2\theta\tau^2(\beta-1)} \geq 0, \tag{47}$$

According to Equation (38) and $0\leq\beta<1$, the denominator of Equation (47) satisfy the following inequation:

$$6 - 3\beta + 2\theta\tau^2(\beta-1) \;=\; 3(2-\beta)\; + 2\theta\tau^2(\beta-1) > 2 - \beta + 2\theta\tau^2(\beta-1)\; \geq 0. \tag{48}$$

Therefore, in order to make Equation (46) true, the following inequation shall be satisfied:

$$\alpha + (\,\beta - 1)\,[c_m - \sigma(c_m - c_r) - (r-\eta)\,\tau] \geq 0. \tag{49}$$

In conclusion, model parameter constraints need to satisfy the Nash equilibrium condition: Equation (40), and non-negative market demand condition: Equation (49).

### *5.2. Comparison of Cross-Competitive and Monopolistic Take-Back Modes*

In this sub-section, we compare the optimal profits and product prices of manufacturers, as well as the optimal profits and recycling fees of recyclers under the above two take-back modes, respectively.

After comparing Equations (21), (22) and (29), (30), we have a conclusion that although manufacturers' product prices under cross-competitive take-back mode are lower than that under monopolistic take-back mode($p_j^{*c}<p_j^{*m}$), their profits are higher than that under monopolistic take-back mode($\Pi_j^{*c}>\Pi_j^{*m}$).

Similarly, we can know that recyclers' recycling fees under cross-competitive take-back mode are lower than that under monopolistic take-back mode ($t_j^{*c}<t_j^{*m}$). Furthermore, if $\delta = 0.5$ or $0.5<\delta\leq 1$ and $0<\beta\leq \frac{-\delta^2+\delta-1+\sqrt{9\delta^4-18\delta^3+3\delta^2+6\delta+3}}{-2\delta^2+2\delta+1}$, meanwhile satisfying $\theta<\frac{(\beta-2)\,(X+2Y)}{(\beta-1)^2(\beta-2)\,\tau^2}$, recyclers' profits are higher under cross-competitive take-back mode. Otherwise, recyclers' profits are higher under monopolistic take-back mode.

In conclusion, we can sort out the take-back mode selection of manufacturers and recyclers (Table 1).

**Table 1.** Take-back Mode Selection of Manufacturers and Recyclers.

| Participant | Take-Back Mode Selection | |
|---|---|---|
| Manufacturers | $p_j^{*c} < p_j^{*m}$, $\Pi_j^{*c} > \Pi_j^{*m}$, choose cross-competitive take-back mode | |
| Recyclers | ①When $\delta = 0.5$<br>②When $0.5 < \delta \leq 1$ and $0 < \beta \leq \frac{-\delta^2+\delta-1+\sqrt{9\delta^4-18\delta^3+3\delta^2+6\delta+3}}{-2\delta^2+2\delta+1}$ | and $\theta < \frac{(\beta-2)(X+2Y)}{(\beta-1)^2(\beta-2)\tau^2}$ $t_j^{*c} < t_j^{*m}$, $\Pi_j^{*m} < \Pi_j^{*c}$, choose cross-competitive take-back mode |
| | Otherwise, choose monopolistic take-back mode | |

It can be seen from Table 1 that the cross-competitive take-back mode is more advantageous to the participants of the reverse logistics recovery system. Under this mode, both manufacturers and recyclers can achieve optimal profits, and consumers benefit from the lower product prices. However, only when the base allocation rate $\delta = 0.5$ or when $0.5 < \delta \leq 1$ and the product's elasticity of market demand $\beta$ is low ($0 < \beta \leq \frac{-\delta^2+\delta-1+\sqrt{9\delta^4-18\delta^3+3\delta^2+6\delta+3}}{-2\delta^2+2\delta+1}$) and the economies of scale factor $\theta$ is not strong ($\theta < \frac{(\beta-2)(X+2Y)}{(\beta-1)^2(\beta-2)\tau^2}$), recyclers and manufacturers will choose the cross-competitive take-back mode, so as to achieve a win-win situation for all participants.

## 6. Sensitivity Analysis

In order to investigate the impact of multi-factors on recyclers' equilibrium recycling fees and their profits, manufacturers' product prices and their profits, in this sub-section we take the parameters: Manufacturers' market capacity $\alpha$, economies of scale factor $\theta$, recyclers' unit recycling cost $\eta$, government subsidies for recyclers to recycle unit waste products $r$, remanufacturing rate $\sigma$, and base allocation rate $\delta$ under cross-competitive take-back mode for sensitivity analysis. Then we determine the influence of each parameter on the objective function by computing the first partial derivative of the recyclers' equilibrium recovery fee, the manufacturers' equilibrium product price and their optimal profit and judging whether they are positive or negative. After computing, we can get a parameter sensitivity analysis table for the equilibrium solution of recyclers and manufacturers (Table 2).

**Table 2.** Parameter Sensitivity Analysis.

| Parameter | Manufacturers ($j$ = 1,2) | | | | Recyclers ($I$ = 1,2) | | | |
|---|---|---|---|---|---|---|---|---|
| | $p_j^{*c}$ | $p_j^{*m}$ | $\Pi_j^{*c}$ | $\Pi_j^{*m}$ | $t_i^{*c}$ | $t_i^{*m}$ | $\Pi_i^{*c}$ | $\Pi_i^{*m}$ |
| $\alpha$ | $\pm^1$ | + | + | + | $\pm^2$ | + | + | + |
| $\theta$ | − | − | + | + | − | − | $\pm^3$ | + |
| $\eta$ | + | + | − | − | + | + | − | − |
| $r$ | − | − | + | + | − | − | + | + |
| $\sigma$ | − | − | + | + | $\pm^2$ | + | + | + |
| $\delta$ | − | / | + | / | − | / | $\pm^4$ | / |

(Notes: $\pm^1$: + if $\theta < \frac{X+\beta^2-4}{2X\tau^2}$, − otherwise. $\pm^2$: + if $\theta < \frac{\beta^2-4}{2X\tau^2}$, − otherwise. $\pm^3$: + if $\theta < \frac{(\beta-2)(2X+3Y)}{2(\beta-1)X\tau^2}$, − otherwise. $\pm^4$: + if $\frac{1}{2} \leq \delta < \frac{1}{2} + \frac{\sqrt{(\beta^2-2)^2-\beta^2}}{-2\beta^2+2\beta+4}$, − if $\frac{1}{2} + \frac{\sqrt{(\beta^2-2)^2-\beta^2}}{-2\beta^2+2\beta+4} \leq \delta \leq 1$.)

Here, the sign + indicates that the corresponding objective function increases with the increase of parameters, while the sign - indicates that the corresponding objective function decreases with the increase of parameters. The sign ± indicates that the corresponding objective function may increase or decrease depending on the other parameters (see notes of Table 2).

As can be seen from Table 2, when the economies of scale factor $\theta$ are relatively low, the changing trend of corresponding objective function under the monopolistic and the cross-competitive take-back modes are the same. Specifically, with the increase of market capacity $\alpha$, the product market demand increases, which makes recyclers and manufacturers raise the recycling fees and product prices and improve their respective profits. Reduction on recyclers' unit recycling cost $\eta$ and increase in government subsidy $r$ enhance the profitability of recyclers, which enable recyclers to reduce the recycling fees, while improving profits. Consequently, manufacturers also reduce product price and gain higher profits.

However, when the economies of scale factor $\theta$ grow higher and exceed some thresholds, things are different. Because the impact of recycler's decision-making is more closely related to that of manufacturer's decision-making under the cross-competitive take-back mode, when recyclers are faced with strong economies of scale, the fierce competition causes sharply reduced recycling fees, recyclers' profits also decrease accordingly. Moreover, As the growing up of market capacity $\alpha$, it does not only enhances the influence of economies of scale factor $\theta$, but also lead to more intense competition between the two manufacturers. As a result, recyclers are forced to cut their recycling fees, and manufacturers are also forced to lower their product prices.

In addition, under the cross-competitive take-back mode, with the increase of base allocation rate $\delta$, recyclers' recycling fees decrease, while their profits show the trend of increasing first and then decreasing (specific reasons will be explained in the case analysis in the following section). While the manufacturers' cost is reduced as a result of the reduction of the recycling fees charged by the recyclers, which eventually leads to the reduction of product prices and increased demand, as well as profits.

As for the effect of the elasticity of market demand $\beta$ and the recycling rate $\tau$ on the equilibrium solution, because their universal law is difficult to obtain, they cannot be determined by sensitivity analysis. However, it will be verified by specific numbers in the following section.

## 7. Case Analysis

The results derived from the model in the above section are complex and cannot be visually analyzed and compared. In order to make the conclusion clearer, we carry out case analysis in this section to discuss the sensitivity analysis of parameters and comparison of two take-back modes.

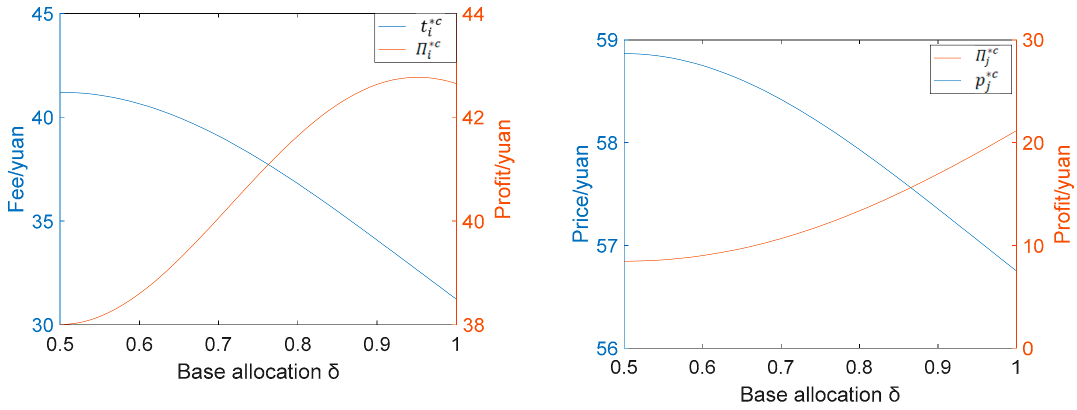
Due to the increasingly short update cycle of electronic products, a huge WEEE market is generated, especially in China. Therefore, we took the current situation of WEEE recycling in China as an example of case analysis.

Focusing on the present situation of reverse logistics recycling of WEEE in China, we collected and sorted out relevant data, which combined with the existing condition of Nash equilibrium solution of the model (40) and non-negative constraint of the demand function value (49). Thereout, we set values for each parameter in the model, as shown in Table 3. For example, for the setting of parameter $\tau$, by analyzing the collected relevant data, we found that compared with developed countries, China's WEEE treatment industry is still in its infancy and the rate of regular recycling channels is only 28% [41]. Therefore, we might as well set recycling rate $\tau$ as 0.3 approximately.

**Table 3.** Parameter Values.

| Parameter | Symbol | Value |
|---|---|---|
| Manufacturers' market capacity | $\alpha$ | 50 |
| Elasticity of market demand (substitutability of the manufacturer's products) | $\beta$ | 0.2 |
| Recycling rate of WEEE | $\tau$ | 0.3 |
| Remanufacturing rate of WEEE | $\sigma$ | 0.5 |
| Recyclers' unit recycling cost | $\eta$ | 3 |
| Economies of scale factor | $\theta$ | 0.05 |
| Government subsidy for recyclers recycle unit WEEE | $r$ | 10 |
| Unit cost of new product | $c_m$ | 50 |
| Unit cost of the remanufactured product | $c_r$ | 40 |
| Base allocation rate under cross-competitive take-back mode | $\delta$ | 1 |

By using the control variate method, the value of only one parameter is changed at a time, and the value of the corresponding objective function is calculated, so as to obtain the change of the value of the corresponding decision variable (recycling fees set by recyclers and product prices set by manufacturers) and profit function under the two take-back modes, which are shown in Figures 2–7.

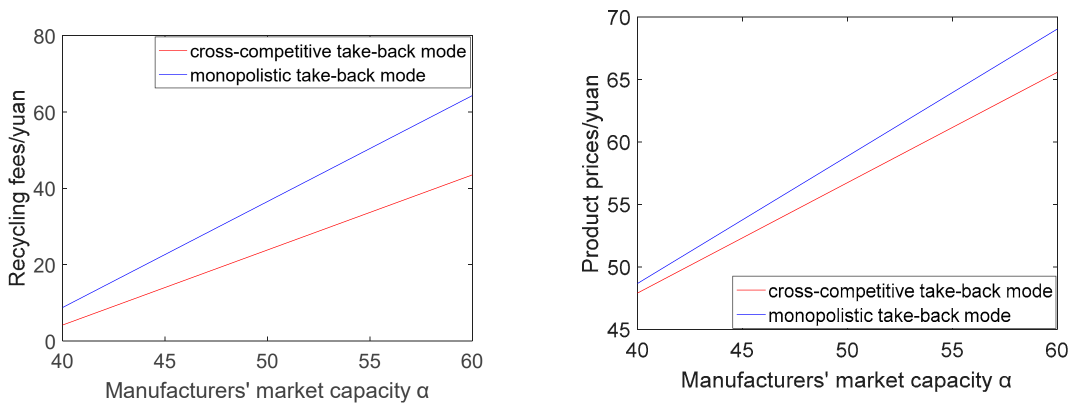



(**a**) recycling fee and recyclers' profit

(**b**) product price and manufacturers' profits under cross-competitive take-back mode.

**Figure 2.** Impact of Base Allocation $\delta$ on (**a**) and (**b**).

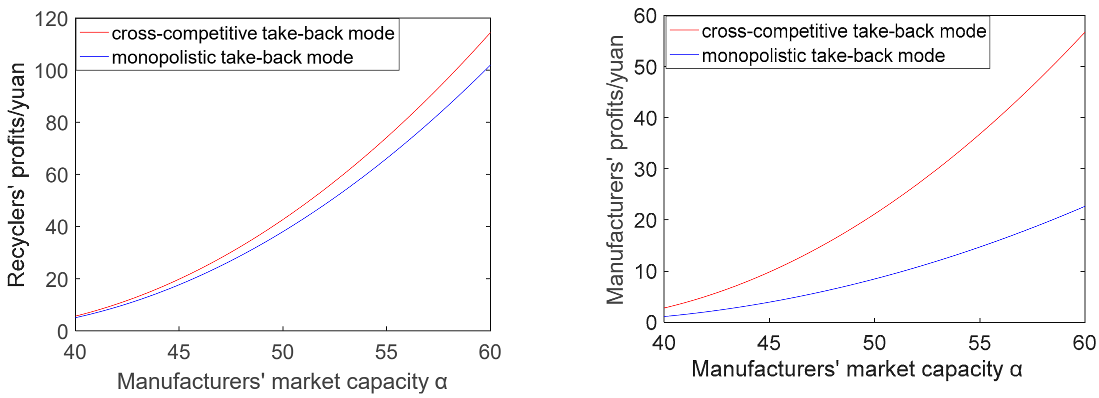
Figure 2 shows that with the increase of the base allocation rate $\delta$, the recycling fees will decrease even if the recyclers have guaranteed allocations. Therefore, recyclers become more dependent on waste products from specific manufacturers, they try to steal business from each other by influencing their decisions in the original product market, forcing them to lower their recycling fees. At first, falling recycling fees (and therefore, lower product prices) gives recyclers more waste products business and increases their profitability. However, when the recyclers' business is too dependent on a single manufacturer ($\delta$ becomes too high), the influence of the rapid decline in profit margin is greater than the effect of increased market demand and waste products, making recyclers' profits decrease eventually. Lower recycling fees lead to lower product prices and increased demand, which is the situation that manufacturers are always happy to see. Hence, manufacturers prefer to contract with a single dedicated recycler for waste products recycling in order to maximize their profit ($\delta = 1$); while it is different for recyclers that only when the base allocation rate $\delta$ equal to $\frac{1}{2}+\frac{\sqrt{(\beta^2-2)^2-\beta^2}}{-2\beta^2+2\beta+4}$ (it is 0.9513 in this case), can recyclers get the optimal profits. In other words, recyclers prefer to deal with multiple manufacturers and recycle WEEE, which is consistent with the conclusion in Table 2. It is worth noting that the competitive take-back mode in Ding et al. [38] is the special case of this paper when $\delta = 1$.




(**a**) recycling fees

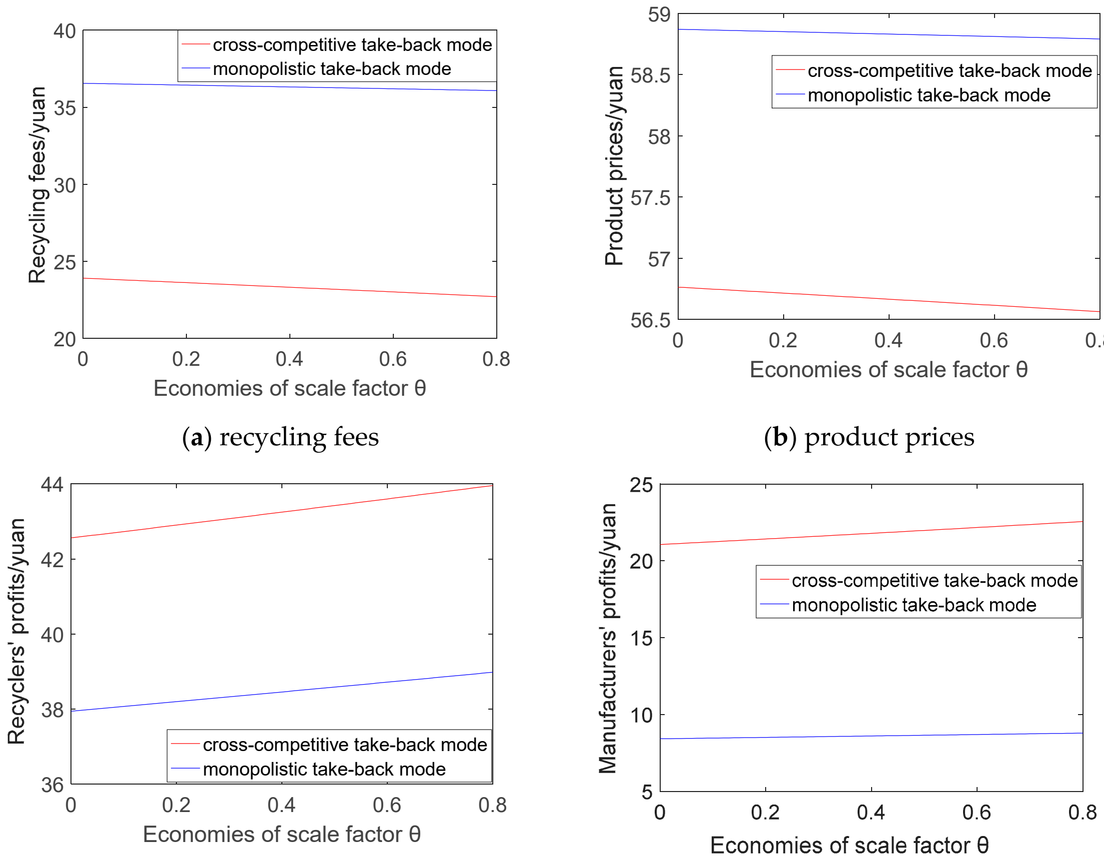
(**b**) product prices

**Figure 3.** *Cont.*




(**c**) recyclers' profits (**d**) under two take-back modes

**Figure 3.** Impact of Manufacturers' Market Capacity $\alpha$ on (**a**), (**b**), (**c**), and (**d**).




(**a**) recycling fees (**b**) product prices

(**c**) recyclers' profits (**d**) manufacturers' profits under two take-back modes.

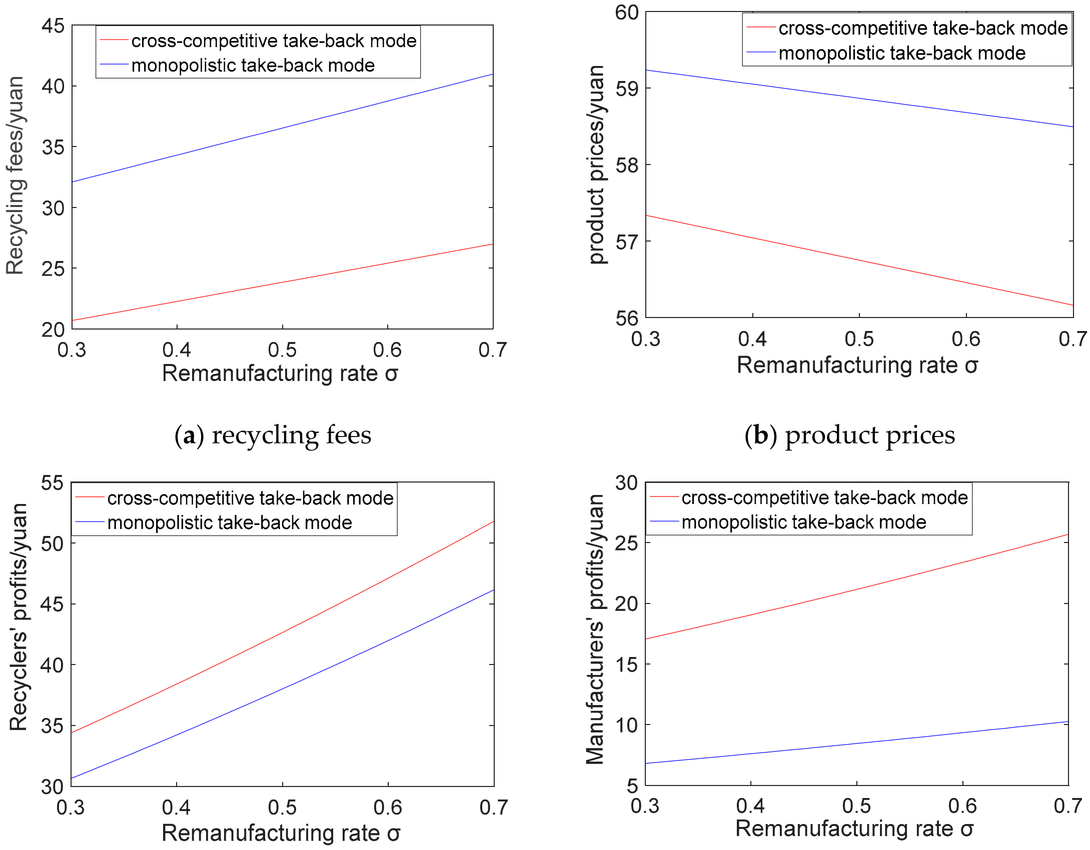
**Figure 4.** Impact of Economies of Scale Factor $\theta$ on (**a**), (**b**), (**c**), and (**d**).

It can be seen from Figure 3 that, with other parameters unchanged, as the manufacturers' market capacity $\alpha$ increases, the increase in market demand causes recyclers and manufacturers to improve recycling fees and product prices, and their profits increase accordingly. Compared with monopolistic take-back mode, it has lower recycling fees and lower product prices under the cross-competitive take-back mode, while recyclers and manufacturers are more profitable.

It can be seen from Figure 4 that when other parameters are unchanged, as the economies of scale factor $\theta$ increases, recycling fees and product prices decrease under the two take-back modes, and the profits of recyclers and manufacturers increase. Compared with monopolistic take-back mode, the recycling fees and the product prices are lower under the cross-competitive take-back mode, but the corresponding profits are higher than that under the monopolistic take-back mode.

It can be seen from Figure 5: When other parameters remain unchanged, with the increase of the remanufacturing rate $\sigma$, recycling fees, profits of recyclers and manufacturers increase under the two take-back modes, and the product prices decrease. It is because when the remanufacturing rate $\sigma$ increases, the cost of the average unit product of manufacturer falls, and the profits of manufacturers and recyclers increase. Therefore, manufacturers lower product prices and encourage the recyclers to recycle waste products. Compared with monopolistic take-back mode, the recycling fees and the product prices under the cross-competitive take-back mode are lower, but the corresponding profit is higher than that under monopolistic take-back mode.

As can be seen from Figure 6, with other parameters unchanged, as the increase of substitutability of products $\beta$, the market demand increases, leading to the increase of manufacturers' product prices, recyclers' recycling fees and their profits. Compared with monopolistic take-back mode, the recycling fees and the product prices under the cross-competitive take-back mode are lower, but the corresponding profit is higher than that under the monopolistic take-back mode. However, as can be seen from Figure 6c, when the substitutability of products $\beta$ increases to more than $\sqrt{3}-1$, the recyclers' profits under the cross-competitive take-back mode is slightly lower than that under the monopolistic take-back mode, and the profits of recyclers seems to be very close to one another in these two scenarios, which is consistent with the conclusion in Table 1.



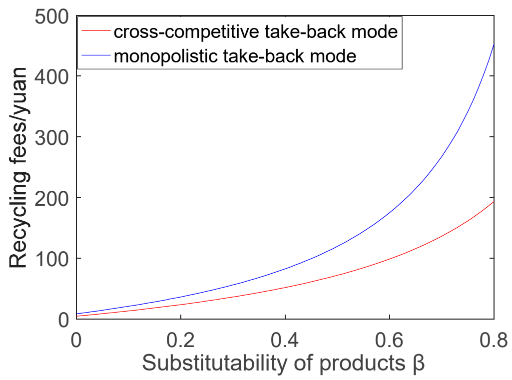

(**a**) recycling fees (**b**) product prices

(**c**) recyclers' profits (**d**) manufacturers' profits under two take-back modes

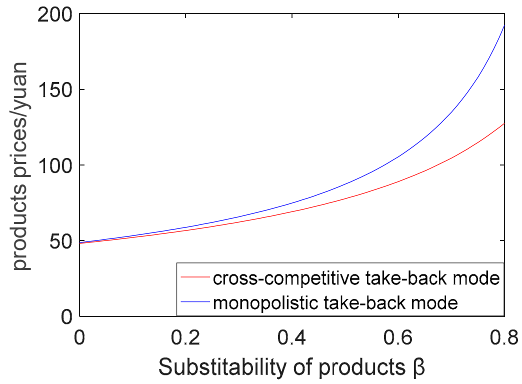
**Figure 5.** Impact of Remanufacturing Rate $\sigma$ on (**a**), (**b**), (**c**), and (**d**).

As shown in Figure 7, when other parameters are unchanged and with the increase of recycling rate $\tau$, recycling fees and product price both decrease, while their profits increase under the two take-back modes. This is because the increase in the number of recycled waste products allows recyclers to keep increasing profits even if the recycling fees are reduced, and manufacturers can produce more remanufactured products to obtain additional profits. Compared with the monopolistic take-back mode, the recycling fees and the product prices under the cross-competitive take-back mode are lower, but the corresponding profit is higher than that under the monopolistic take-back mode.

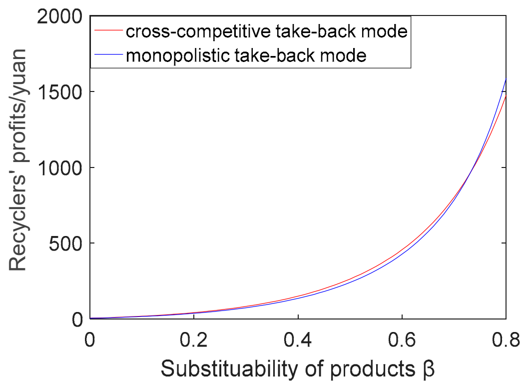
Through the analysis of Figures 2–7, cross-competitive take-back mode is more likely to maximize the profit of all participants than the monopolistic take-back mode, which further verifies the accuracy of the qualitative analysis in Section 5.2.



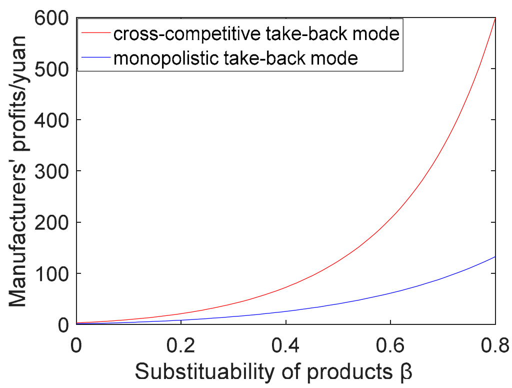

(**a**) recycling fees




(**b**) product prices

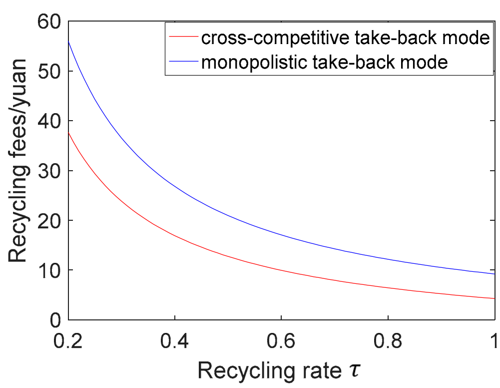



(**c**) recyclers' profits

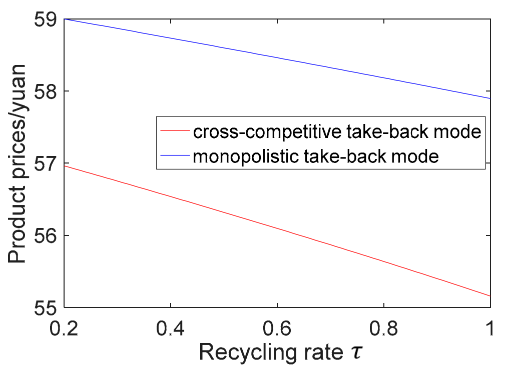



(**d**) manufacturers' profits under two take-back modes

**Figure 6.** Impact of Substitutability of Products $\beta$ on (**a**), (**b**), (**c**), and (**d**).

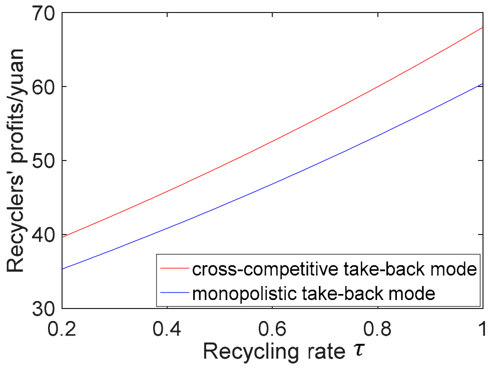



(a) recycling fees

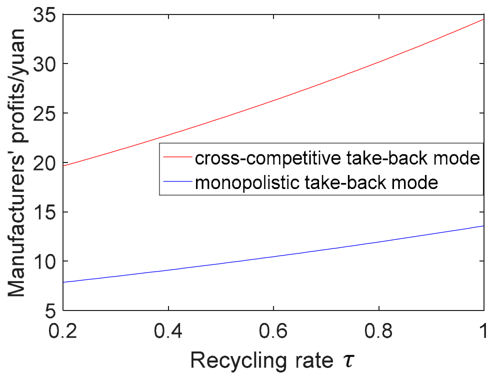



(b) product prices

**Figure 7.** *Cont.*




(c) recyclers' profits




(d) manufacturers' profits under two take-back modes.

**Figure 7.** Impact of Recycling Rate $\tau$ on (**a**), (**b**), (**c**), and (**d**).

## 8. Conclusions

In this paper, first, we took the profits maximization of recyclers and manufacturers as the goal of decision making, the recycling fees and the product prices as decision variables, and established the game model under the cross-competitive take-back mode of the reverse logistics system. Then, the optimal decisions and profits were derived based on Stackelberg game theory under the cross-competitive take-back mode. Furthermore, parameter constraint analysis was carried out, and monopolistic take-back mode was introduced for comparative analysis. In the end, the sensitivity analysis was carried out on the model. Several main conclusions are obtained as follows:

(1) The competitive take-back mode in Ding et al. [38] is the special case of cross-competitive take-back mode in this paper when the base allocation rate $\delta = 1$;

(2) On the one hand, each manufacturer under cross-competitive take-back mode prefers to choose a single special recycler to sign contract for waste products recycling in order to maximize their profit ($\delta = 1$); on the other hand, recyclers tend to choose multiple manufacturers to sign contracts at the same time when base allocation rate $\delta$ is $\frac{1}{2}+\frac{\sqrt{(\beta^2-2)^2-\beta^2}}{-2\beta^2+2\beta+4}$, at this time recyclers reach the best profit;

(3) It is true that cross-competitive take-back mode is better than monopolistic take-back mode and can achieve a win-win situation for all participants when the base allocation rate $\delta = 0.5$ under the cross-competitive take-back mode, or $0.5 < \delta \leq 1$ and substitutability of products $\beta$ is low ($0 < \beta \leq \frac{-\delta^2+\delta-1+\sqrt{9\delta^4-18\delta^3+3\delta^2+6\delta+3}}{-2\delta^2+2\delta+1}$), as well as economies of scale factor $\theta$ is small ($\theta < \frac{(\beta-2)(X+2Y)}{(\beta-1)^2(\beta-2)\tau^2}$).

For the manufacturers in China, it should try to improve the remanufacturing rate of waste products. As for recyclers, the recycling rate of waste products also should be improved. Meanwhile, recyclers should actively sign contracts with multiple manufacturers to recycle waste products, make full use of the advantages of cross-competitive take-back mode, and maximize the profits of all participants, so as to promote the recycling and the reuse of waste products and to achieve sustainable development.

It should be noted that there are only two leaders and two followers considered in the reverse logistics system in this research, and their demand is certain. In order to simplify the calculation of our model and facilitate the research, at present, we focused our efforts on studying the situation that there are only two leaders and two followers. In the future, we will try to extend our study to the situations in the more real world. Furthermore, future research could be extended to the situation when parameters for the participants are not the same, and maybe extended to consider the uncertain market demand, that is, there is a disturbance in the market demand, which is more consistent with the real world.

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