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Online Reviews Matter: How Can Platforms Benefit from Online Reviews?

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Abstract: Online reviews mitigate uncertainty about product quality that is caused by information asymmetry. However, low-quality online reviews are not effective, while high-quality online reviews may raise costs. Unlike the previous studies, this paper develops a game-theoretic model to examine the feasibility and pricing of online review supervision mechanism for operators of platforms that are based on the 'network externality'. The results reveal that platforms are not always benefited from online reviews. We provide a new perspective and give some preventive suggestions for platforms with a view to furthering sustainable development.

Keywords: online reviews; information asymmetry; cross-network externalities; Bayesian theory; analytical modeling

1. Introduction

The online retailing business is increasingly becoming an important activity that can bring great convenience to everyday life. Nevertheless, with the development of online retailing, increasing amounts of sellers sell low-quality products in platforms. The results of the online trading commodities data for the second half of 2014, which were released by the Chinese State Administration for Industry and Commerce (SAIC), indicate that the product quality of online retailing is low, especially, the quality in the fertilizer and agricultural industry and the mobile phone industry are really quite low, at only 20% and 28.57%, respectively [1]. Product quality refers to the degree of excellence of a product by which differences in quality and quantity of some desired ingredient or attribute [2,3]. The proliferation and quantity of low-quality products participating seriously influence the trading environment and restricts any sustainable development for platforms. Due to the information asymmetry caused by online retailing, it exacerbates the above problems and creates a negative externality. Prior studies indicate that online reviews can mitigate uncertainty regarding product quality [4,5]. Additional information found in the online review not only strongly influences consumers' purchase decisions [6], but it also reveals the seller's reputation and product quality [7]. However, the existence of false and malicious comments reduces the quality of online reviews. Some studies mainly focus on obtaining honest online reviews by introducing moderation and voting systems (Slashdot), which provides return coins (T-Mall) or detecting fake reviews [8–10]. For instance, Xiaoying Zhang et al. propose a novel model for real rating systems to accurately characterize and debias the influence from historical ratings of online reviews [11]. Omar Besbes and Marco Scarsini explore what type of fake review generation would be more effective, and what is the best way to counter such efforts [12]. Others consider that platforms may impose large costs by offering these kinds of services [13]. There are fewer studies examine the effect of online reviews for platforms while considering both quality and cost.

As a consequence, this paper regards online reviews as a supervision mechanism to examine the feasibility and pricing of online reviews for the operators of platforms. According to Bar Ifrach et al., they establish a link between binary reviews and the product quality by using Bayesian updating to study the seller's pricing problem [14]. Similarly, we use Bayesian model to bridge a connection between product quality and the quality of online reviews, but we mainly focus on studying the effect for platforms. Next, we assume that platforms can choose either offering low-quality or high-quality online reviews services. The choice of the service for a platform affects potential sellers' and buyers' participation [15,16]. We develop a game-theoretic model to explore the results while using this framework. Finally, we found that platforms do not always benefit from online reviews. We provide a new sight and reference for platforms to make optimal choices, and also give some other preventive measures for platforms to develop.

The rest of this paper is organized as follows. We review the related literature in the next section. In Section 3, we present the description of the model, in Section 4 we perform the equilibrium analysis, and, in Section 5, we compare the two models. In the final section, we present our conclusions.

2. Literature Review

The scope of this paper is closely related to three streams, including two-sided markets, online reviews, and mechanism design.

The first related studies look at matching in typical two-sided markets due to the fact that e-commerce platform belongs to a form of two-sided markets. Lots of researchers indicate that two-sided markets refer to the existence of two or more different user groups in a market and the existence of 'cross-network externalities' among each other, that is, the scale of user participation in one side will affect the motivation of the other user to access the platform [15–18]. For instance, Jean-Charles Rochet and Jean Tirole maintain platforms in industries must get both sides of the market on board and devote much attention to their business model on each side when making money. Afterwards, they built a model of platform competition with two-sided markets to unveil the determinants of price allocation and the end-user surplus for different governance structures [16]. Jianqing Chen et al. establish a brokerage model and an advertising model that is based on network externalities, and they compared the two revenues model to study how the chosen revenue model affected a platform [17]. Jullien considers two-sided markets that provide a matching service to facilitate interaction and the operation of exchanges between two types of trading partners [18]. However, the information asymmetry in two-sided markets causes a lot of problems. For instance, consumers can only see the text and photos that are described by the sellers, so they cannot actually check the quality of products before buying. It is impossible to accurately identify whether the product is a high-quality or not [19]. Meanwhile, in the e-commerce environment, the sales of low-quality products are concealed, the operating costs are low, and the hazard are wide. These all increase the difficulty of supervision [20]. Similar to other papers on two-sided markets, our paper uses the 'network externalities' to construct models, but we consider the price and profit of products from the point of view of sellers and the value of products from the point of view of buyers to construct analytical models for both low-quality and high-quality online reviews.

The second related discussion involves online reviews that can mitigate uncertainty that is caused by information asymmetry [4–7,21,22], such as Zhu and Zhang, who consider online reviews as a new type of word-of-mouth information, which can effectively reduce product quality uncertainty [21]. Meng Y et al. believe that word-of-mouth information affects the purchase and decision-making for consumers and, at the same time, it affects the reputation and sales for sellers [22]. Several studies analyze how online reviews affect the parties in two side of a platform [5,23–27]. For example, Young Kwark et al. study the effect of online product reviews in a channel structure and investigate how reviews affect the upstream competition between not only manufacturers, but also retailers [23]. Chen and Xie develop a normative model that enables consumers to identify products matching their needs, and to show when and how the seller should adjust its own marketing communication strategy in response to online reviews [5]. Amir Ajoriou et al. study the word-of mouth effect on optimal dynamic pricing for a monopolist selling a product to consumers in a social network [27]. While the false and malicious comments reduce the quality of online reviews, there are lots of studies that focus on the detection of spam online reviews [28] and reputation models [29], and how to obtain more honest online reviews [30]. For example, Zhao et al. design a wage-based incentive mechanism to obtain honest feedback, and an extensive simulation evaluation demonstrated that the mechanism is effective [30]. Recent studies also examine specific aspects of online reviews, such as Hong et al. regarding the role of online reviews as a new measure of product types [31]; Lin et al. consider the quality of online reviews can be controlled during the process of eliciting, aggregating, and distributing [32]. Our study differs from theirs, in that we compare the low-quality online reviews model and high-quality online reviews model in order to study how can online reviews affects the number of participating members, trading numbers, so as to the quality and revenues of platforms.

The third related research is the growing literature on mechanism design in the information system. Researchers in information systems not only design and evaluate new systems in a business context [33], but also focus on how to price the services of the mechanisms [13,34–41]. For instance, Amit Basu et al. examine the online and authentication services for matching-seekers, and study how the matching platform should price its search and authentication services [13]. Guowei Dou et al. consider that the investment for one side would affect the utility of users on two sides, thus affecting the demand and profit of the platform, and then they investigate one-side value-added services investment and pricing strategies for a two-sided platform [36]. Guillaume Roger and Luis Vasconcelos introduce a pricing mechanism, whereby a two-sided platform charges transaction and registration fees to overcome the moral hazard on the sellers' side [37]. Ricardo Flores-Fillol et al. propose a model to study the optimal price strategy of an airport platform that can generate revenues both from traditional aeronautical and non-aviation activities [41]. In contrast to these papers, we regard online reviews as a supervision mechanism, and examine the feasible and pricing of this mechanism for operators of e-commerce platforms in information systems.

3. Model Description

We consider that an online platform P has multiple sellers, including high-quality and low-quality sellers on one side and multiple buyers on the other. It provides online reviews services for facilitating transactions between sellers and buyers [21]. The platform charges sellers a commission for using this service. Let $\tau(\tau < \frac{1}{2})$ be the commission rate for each sale. Buyers can participate without any cost.

A mass of high-quality sellers with measure 1 and a mass of low-quality sellers with measure 1 may sell their products in the platform. Each seller is seen as selling different products and each product is divided into high-quality product that is sold by high-quality seller and low-quality product sold by low-quality seller. Specifically, high-quality products refer to products that perform well, are reliable, conform to standards, are durable, and cost more because of the higher quality and greater quantity of materials and ingredients used [2]. Low-quality products refer to unauthorized products that mimic certain characteristics of high-quality products, but use lower quality and less materials and ingredients that are found in high-quality products. An example might be fake products that cannot be distinguished in their essential aspects from high-quality products [10,42]. Buyers prefer high quality to low quality, with everything else being equal. As a consequence, sellers have different fixed costs of providing their products through the platform: high-quality sellers' fixed cost is c_1, low-quality sellers' fixed cost is c_2; the value of high-quality and low-quality products that are evaluated by buyers are v_1 and v_2, respectively. Assume that products price is p, the average variable cost difference between high-quality and low-quality products is ε. A mass of buyers with measure 1 might participate in the platform. Buyers accessing the platform involves different opportunity cost μ. We assume that c_1, c_2, and μ satisfy the uniform distributions with support $[0, 1]$.

Relying on their costs and revenues, some high-quality sellers, low-quality sellers, and buyers participate in the platform and others do not. We denote m_1 as the mass of high-quality sellers, m_2 as the mass of low-quality sellers, and n as the mass of buyers participating in the platform. Notice that m_1, m_2, and n are endogenous and their values vary under different conditions. We assume that each product has one buyer. A buyer's probability of finding his trading partner on the platform relies on whether his selling partner is on the platform. According to these papers [43,44], the number of participating sellers indicates the likelihood that a buyer's trading partner is in the platform: as the number of sellers participating in the platform increases, so does the likelihood that a buyer's trading partner is in the platform. We denote that sellers with measure 1 will participate in the platform, so the number of sellers participating in the platform is $\frac{m_1+m_2}{2}$. Subsequently, we assume that the probability that a buyer's trading partner is on the platform is equal to the mass of participating sellers is $\frac{m_1+m_2}{2}$. Similarly, buyer's participation decisions are affected by the number of participating sellers [15], so the probability that a seller's trading partner is on the platform that equals the mass of participating buyers is n.

3.1. Platform With Low-quality Online Reviews

In the low-quality online reviews scenario, we assume that the quality of online reviews is lower than the natural selection rate, and the platform suffers no cost in stimulating buyers to generate real comments. In other words, buyers make purchase decision by their own natural selection instead of online reviews. Currently, we assume the natural selection rate is q, the proportion of high-quality and low-quality products in the platform are $\frac{\hat{m}_1}{\hat{m}_1+\hat{m}_2}$ and $\frac{\hat{m}_2}{\hat{m}_1+\hat{m}_2}$. Buyers make purchase decisions when they participate in the platform, so the probability that a buyer buys a high-quality product or a low-quality is $\frac{\hat{m}_1}{\hat{m}_1+\hat{m}_2}$ and $\frac{\hat{m}_2}{\hat{m}_1+\hat{m}_2}$ respectively.

We denote $\hat{\pi}_1$ and $\hat{\pi}_2$ as the revenue that a high-quality or a low-quality seller derives from trading, $\hat{s}$ as the expected surplus that a buyer derives from trading, and $\hat{\pi}_p$ as the revenue that the platform derives from the trading. A high-quality seller's revenue from participating in the platform is

$$\hat{\pi}_1 = \hat{n}[p(1-\hat{\tau}) - c_1] \tag{1}$$

According to the average cost difference between high-quality and low-quality product, we assume $\varepsilon(1-\hat{\tau})$ is the profit difference between high-quality and low-quality products. A low-quality seller's revenue from participating in the platform is

$$\hat{\pi}_2 = \hat{n}[(p+\varepsilon)(1-\hat{\tau}) - c_2] \tag{2}$$

A buyer's expected surplus from participating in the platform is

$$\hat{s} = \frac{\hat{m}_1+\hat{m}_2}{2} q\left[\frac{\hat{m}_1}{\hat{m}_1+\hat{m}_2} v_1 + \frac{\hat{m}_2}{\hat{m}_1+\hat{m}_2} v_2 - p\right] - \mu \tag{3}$$

The platform's revenue from trading is

$$\hat{\pi}_p = \frac{\hat{m}_1+\hat{m}_2}{2} \hat{n} q p \hat{\tau} \tag{4}$$

3.2. Platform With High-quality Online Reviews

In the high-quality online reviews scenario, we assume that the quality of online reviews is higher than the natural selection rate and a platform can moderate the quality of online reviews by paying some costs L. The quality of online reviews is described by the accuracy rate of online reviews β, where the platform can be determined by technical means. We denote a high-quality product is A and a low-quality product is B. When buyers make comments to the product that is a high-quality product, it provides a signal A'; if buyers make comments to the product that is a low-quality product, it provides signal B' [13]. The probabilities of real comment on high-quality and low-quality products are $P(A'|A) = \beta_1$ and $P(B'|B) = \beta_2$. Based on the Lirong Chen et al., they indicate that low-quality sellers are more likely to manipulate the accuracy rate of online reviews than high-quality sellers, so we derive $\beta_1 > \beta_2$ [9]. According to the proportion of high-quality products and low-quality products in the platform, we get $P(A) = \frac{m_1}{m_1+m_2}$ and $P(B) = \frac{m_2}{m_1+m_2}$; then, the probability of products commented as high-quality and low-quality products are $P(A') = \frac{m_2-m_2\beta_2+m_1\beta_1}{m_1+m_2}$ and $P(B') = \frac{m_1-m_1\beta+_1m_2\beta_2}{m_1+m_2}$. Based on Bayesian updating, the probabilities that a product of high-quality or of low-quality is conditional on the signal, as follows:

$$\begin{aligned} P(A|A') &= \tfrac{m_1\beta_1}{m_2-m_2\beta_2+m_1\beta_1} \\ P(B|A') &= \tfrac{m_2-m_2\beta_2}{m_2-m_2\beta_2+m_1\beta_1} \\ P(A|B') &= \tfrac{m_1-m_1\beta_1}{m_1-m_1\beta_1+m_2\beta_2} \\ P(B|B') &= \tfrac{m_2\beta_2}{m_1-m_1\beta_1+m_2\beta_2} \end{aligned} \tag{5}$$

Buyers make purchasing decisions according to the online reviews. When online reviews indicate that the product is a high-quality product, the buyer will buy.

We denote π_1 and π_2 as the revenue that a high-quality and a low-quality seller derive from the trading, s as the expected surplus that a buyer derives from trading and π_p as the revenue that the platform derives from the trading. A high-quality seller's revenue from participating in the platform is

$$\pi_1 = n[P(A'|A)p(1-\tau) - c_1] \tag{6}$$

We assume $(1-\beta_2)\varepsilon(1-\tau)$ is the profit difference between a high-quality and a low-quality product. A low-quality seller's revenue from participating in the platform is

$$\pi_2 = n[P(A'|B)(p+\varepsilon)(1-\tau) - c_2] \tag{7}$$

A buyer's expected surplus from participating in the platform is

$$s = \frac{m_1+m_2}{2}P(A')[P(A\big|A')v_1 + P(B\big|A')v_2 - p] - \mu \tag{8}$$

The platform's revenue from trading is

$$\pi_p = \frac{m_1+m_2}{2}P(A')np\tau - L \tag{9}$$

In each setting, a seller has to decide whether to participate in the platform. A buyer has to decide (1) whether to participate in the platform and (2) whether to buy the product in the platform. From the platform's perspective, the decision is whether to improve the quality of online reviews. The sequence of events in the game is as follows. Firstly, the platform owner announces its commission rate and the quality of online reviews. Secondly, the potential high-quality sellers, the low-quality sellers, and buyers decided whether to simultaneously participate in the platform. Finally, transactions take place between sellers and buyers. Table 1 summarizes the main notation that was used in the paper.

Table 1. Summary of Notation.

Notation	Definition and Comments
τ	commission rate for each sale
q	natural selection rate
β	accuracy rate of online reviews
L	cost of moderating quality of online reviews
m_1.	mass of participating high-quality sellers
m_2	mass of participating low-quality sellers
n	mass of participating buyers
v_1	value of high-quality product
v_2	value of low-quality product
p	product price
ε	average cost difference between high-quality and low-quality product
c_1	high-quality seller's fixed cost of providing a product
c_2	low-quality seller's fixed cost of providing a product
μ	buyer's opportunity cost
π_1	high-quality sellers' revenue from trading
π_2	low-quality sellers' revenue from trading
s	buyers' expected surplus from trading
π_p	platform' s revenue from trading

4. Equilibrium Analysis

4.1. Equilibrium for Platform With Low-quality Online Reviews

Looking at the different products, we study the optimal revenues and commission rates for a platform with low-quality online reviews; this is done by categorizing different conditions. All proofs are in the Appendix A.

Case 1. When the price satisfies $p + \varepsilon < 1$ and $1 < p < 2$, we obtain $0 < p(1 - \hat{\tau}) < 1$ and $0 < (p + \varepsilon)(1 - \hat{\tau}) < 1$**.** Based on the monotonicity that, if a seller (buyer) with a certain cost participates in the platform, those sellers (buyers) with lower costs also participate; we can characterize the marginal fixed cost of high-quality sellers and low-quality sellers who is indifferent regarding participating [17]. We denote $\hat{c}_{1\max}$ as the cost of the marginal high-quality seller and $\hat{c}_{2\max}$ as the cost of the marginal low-quality seller. The mass of participating high-quality sellers is $\hat{m}_1 = p(1 - \hat{\tau}) = \hat{c}_{1\max}$ and the mass of participating low-quality sellers is $\hat{m}_2 = (p + \varepsilon)(1 - \hat{\tau}) = \hat{c}_{2\max}$ because we assume that the costs of sellers are uniformly distributed over [0,1]. According to (3), we get

$$\hat{s} = \frac{1}{2}(1 - \hat{\tau})q[p(v_1 - p) + (p + \varepsilon)(v_2 - p)] - \mu \tag{10}$$

Without loss of generality, we assume $0 < v_1 - p < 1$. Otherwise, if $v_1 - p > 1$, all of the buyers participate in the platform and the mass of participating buyers is simply 1. Similarly, we denote $\hat{\mu}_{max}$ as the marginal opportunity cost of a buyer who is indifferent about participating, and the opportunity cost $\hat{\mu}$ is uniformly distributed over $[0, 1]$, thus the mass of participating buyers is

$$\hat{n} = \frac{1}{2}(1 - \hat{\tau})q[p(v_1 - p) + (p + \varepsilon)(v_2 - p)] = \hat{\mu}_{\max} \tag{11}$$

From these observations, we can derive the platform maximizes its revenue as

$$\hat{\pi}_p = \frac{1}{4}q^2 p(2p + \varepsilon)(1 - \hat{\tau})^2 \hat{\tau}[p(v_1 - p) + (p + \varepsilon)(v_2 - p)] \tag{12}$$

Table 2 shows the optimal commission rates.

Table 2. The optimal commission rates for platform with low-quality online reviews in Case 1.

Conditions		Optimal commission rates
$p+\varepsilon<1$	$v_2-p>\hat{\xi}_1(p-v_1)$	$\hat{\tau}^*=\frac{1}{3}$
$p+\varepsilon<2$	$v_2-p>\hat{\xi}_2(p-v_1)$ $\hat{\xi}_4(p-v_1)>v_2-p>\hat{\xi}_1(p-v_1)$	If $1<p+\varepsilon<\frac{3}{2}, \hat{\tau}^*=\frac{1}{3}$ If $\frac{3}{2}<p+\varepsilon<2, \hat{\tau}^*=\hat{\tau}_2$
	$\hat{\xi}_2(p-v_1)>v_2-p>\hat{\xi}_4(p-v_1)$	If $1<p+\varepsilon<\frac{3}{2}$, $\hat{\tau}^*=\underset{\widetilde{\tau},\frac{1}{3}}{\operatorname{argmax}}\{\hat{\pi}_p(\widetilde{\tau}),\hat{\pi}_p(\frac{1}{3})\}$ If $\frac{3}{2}<p+\varepsilon<2, \hat{\tau}^*=\underset{\widetilde{\tau},\hat{\tau}_2}{\operatorname{argmax}}\{\hat{\pi}_p(\widetilde{\tau}),\hat{\pi}_p(\hat{\tau}_2)\}$
	$\hat{\xi}_1(p-v_1)>v_2-p>p-v_1$	$\hat{\tau}^*=\overline{\tau}$
$1<p<2$ $p+\varepsilon>2$	$v_2-p>\hat{\xi}_3(p-v_1)$	$\hat{\tau}^*=\frac{1}{2}$
	$\hat{\xi}_3(p-v_1)>v_2-p>$ $\hat{\xi}_4(p-v_1)\hat{\xi}_3<\hat{\xi}_4$	If $1<p<\frac{5}{3}, \hat{\tau}^*=\hat{\tau}_1$ If $\frac{5}{3}<p<2$, $\hat{\tau}^*=\underset{\widetilde{\tau},\hat{\tau}_1}{\operatorname{argmax}}\{\hat{\pi}_p(\widetilde{\tau}),\hat{\pi}_p(\hat{\tau}_1)\}$
	$\hat{\xi}_4 p(p-v_1)>v_2-p>\frac{1}{2}p(p-v_1)$	$\hat{\tau}^*=\hat{\tau}_1$
	$\frac{1}{2}p(p-v_1)>v_2-p>p-v_1$	$\hat{\tau}^*=\overline{\tau}$
	$v_2-p<p-v_1$	0

Case 2. When the price satisfies $p<1$ and $p+\varepsilon>2$, we obtain $0<p(1-\hat{\tau})<1$ and $(p+\varepsilon)(1-\hat{\tau})>1$. The mass of participating high-quality sellers is $\hat{m}_1=p(1-\hat{\tau})=\hat{c}_{1\max}$ and the mass of participating low-quality sellers is $\hat{m}_2=1=\hat{c}_{2\max}$. According to (3), we get

$$\hat{s}=\frac{1}{2}q[p(1-\hat{\tau})(v_1-p)+(v_2-p)]-\mu \tag{13}$$

The mass of participating buyers is

$$\hat{n}=\frac{1}{2}q[p(1-\hat{\tau})(v_1-p)+(v_2-p)]=\hat{\mu}_{\max} \tag{14}$$

From these observations, we can derive the platform maximizes its revenue as

$$\hat{\pi}_p=\frac{1}{4}q^2p\Big[p^2(v_1-p)(1-\hat{\tau})^2\hat{\tau}+p(v_1+v_2-2p)(1-\hat{\tau})\hat{\tau}+(v_2-p)\hat{\tau}\Big] \tag{15}$$

Table 3 shows the optimal commission rates.

Table 3. The optimal commission rates for platform with low-quality online reviews in Case 2.

Conditions	Optimal commission rates
$v_2-p>\hat{\xi}_3(p-v_1)$	$\hat{\tau}^*=\frac{1}{2}$
$\hat{\xi}_3(p-v_1)>v_2-p>\frac{1}{2}p(p-v_1)$	$\hat{\tau}^*=\widetilde{\tau}$
$\frac{1}{2}p(p-v_1)>v_2-p>p(p-v_1)$	$\hat{\tau}^*=\overline{\tau}$
$v_2-p<p(p-v_1)$	0

Case 3. When the price satisfies $p>2$, we obtain $p(1-\hat{\tau})>1$ and $(p+\varepsilon)(1-\hat{\tau})>1$. The mass of participating high-quality sellers is $\hat{m}_1=1=\hat{c}_{1\max}$ and the low-quality sellers is $\hat{m}_2=1=\hat{c}_{2\max}$. According to (3), we get

$$\hat{s}=\frac{1}{2}q[(v_1-p)+(v_2-p)]-\mu \tag{16}$$

The mass of participating buyers is

$$\hat{n} = \frac{1}{2}q[(v_1 - p) + (v_2 - p)] = \hat{\mu}_{\max} \tag{17}$$

From these observations, we can derive the platform maximizes its revenue as

$$\hat{\pi}_p = \frac{1}{4}q^2 p\hat{\tau}[(v_1 - p) + (v_2 - p)] \tag{18}$$

Table 4 shows the optimal commission rates.

Table 4. The optimal commission rates for platform with low-quality online reviews in Case 3.

Conditions	Optimal commission rates
$v_2 - p > \hat{\xi}_1(p - v_1)$	$\hat{\tau}^* = \frac{1}{2}$
$v_2 - p < p - v_1$	0

4.2. Equilibrium for Platform with High-quality Online Reviews

We assume that when the platform's technical means can stimulate the buyers' comments, both at a high-quality level and at higher rate, not only than the natural selection rate but also higher than a certain rate

$$\beta_1 > \beta_2 > \max\{q, \frac{p+\varepsilon}{2p+\varepsilon}\} \tag{19}$$

It means that the high-quality online reviews can protect the revenue of buyers: the probability of a buyer purchasing a high-quality product from the platform with high-quality online reviews is higher than purchasing from the platform with low-quality online reviews.

Looking at the different products, we study the optimal revenues and commission rates for a platform with high-quality online reviews; this is done by categorizing different conditions. All proofs are in the Appendix A.

Case 1. When the price satisfies $p < \frac{2}{\beta_1}$ and $p + \varepsilon < \frac{2}{1-\beta_2}$, we obtain $0 < \beta_1 p(1-\tau) < 1$ and $0 < (1-\beta_2)(p+\varepsilon)(1-\tau) < 1$. We denote c_{1max} as the cost of the marginal high-quality seller and c_{2max} as the cost of the marginal low-quality seller. The mass of participating high-quality sellers is $m_1 = \beta_1 p(1-\tau) = c_{1\max}$ and the mass of participating low-quality sellers is $m_2 = (1-\beta_2)(p+\varepsilon)(1-\tau) = c_{2\max}$ because we assume that the costs of sellers are uniformly distributed over [0,1]. According to (8), we get

$$s = \frac{1}{2}\left[\beta_1{}^2 p(1-\tau)(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(1-\tau)(v_2 - p)\right] - \mu \tag{20}$$

The mass of participating buyers is

$$n = \frac{1}{2}\left[\beta_1{}^2 p(1-\tau)(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(1-\tau)(v_2 - p)\right] = \mu_{\max} \tag{21}$$

From these observations, we can derive the platform maximizes its revenue as

$$\pi_p = \frac{1}{4}[\beta_1 p + (1-\beta_2)(p+\varepsilon)]\left[\beta_1{}^2 p(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(v_2 - p)\right]p(1-\tau)^2\tau - L \tag{22}$$

Table 5 shows the optimal commission rates.

Table 5. The optimal commission rates for platform with high-quality online reviews in Case 1.

Conditions		Optimal commission rates
$p < \frac{3}{2\beta_1}, p+\varepsilon < \frac{2}{1-\beta_2}$	$v_2 - p > \xi_1(p - v_1)$	$\tau^* = \frac{1}{3}$
$\frac{3}{2\beta_1} < p < \frac{2}{\beta_1}, p+\varepsilon < \frac{2}{1-\beta_2}$	$v_2 - p > \xi_1(p - v_1)$	$\tau^* = \tau_2$
$v_2 - p < \xi_1(p - v_1)$		0

Case 2. When the price satisfies $p > \frac{2}{\beta_1}$ and $p + \varepsilon < \frac{2}{1-\beta_2}$, we obtain $\beta_1 p(1-\tau) > 1$, $0 < (1-\beta_2)(p+\varepsilon)(1-\tau) < 1$. We assume that β_1 is always sufficiently high, so that $\frac{1}{1-\beta_2} > \frac{2}{\beta_1}$, which is assumed hereafter.

The mass of participating high-quality sellers is $m_1 = 1 = c_{1max}$, the low-quality sellers is $m_2 = (1-\beta_2)(p+\varepsilon)(1-\tau) = c_{2\max}$. According to (8), we get

$$s = \frac{1}{2}\Big[\beta_1(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(1-\tau)(v_2 - p)\Big] - \mu \tag{23}$$

The mass of participating buyers is

$$n = \frac{1}{2}\Big[\beta_1(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(1-\tau)(v_2 - p)\Big] = \mu_{\max} \tag{24}$$

From these observations, we can derive the platform maximizes its revenue as

$$\pi_p = \frac{1}{4}\tau p\left[\begin{array}{c} \beta_1(1-\beta_2)^2(p+\varepsilon)(v_1-p)(1-\tau) + (1-\beta_2)^4(p+\varepsilon)^2(v_2-p)(1-\tau)^2 + \\ \beta_1(1-\beta_2)^2(p+\varepsilon)(v_2-p)(1-\tau) + \beta_1{}^2(v_1-p) \end{array}\right] - L \tag{25}$$

Table 6 shows the optimal commission rates.

Table 6. The optimal commission rates for platform with high-quality online reviews in Case 2.

Conditions	Optimal commission rates
$v_2 - p > \xi_1(p - v_1)$	$\tau^* = \frac{1}{2}$
$v_2 - p < \xi_1(p - v_1)$	0

Case 3. When the price satisfies $p > \frac{2}{\beta_1}$ and $p + \varepsilon > \frac{2}{1-\beta_2}$, we obtain $\beta_1 p(1-\tau) > 1, (1-\beta_2)(p+\varepsilon)(1-\tau) > 1$. The mass of participating high-quality sellers is $m_1 = 1 = c_{1max}$ and the mass of participating low-quality sellers is $m_2 = 1 = c_{2max}$. According to (8), we get

$$s = \frac{1}{2}[\beta_1(v_1 - p) + (1-\beta_2)(v_2 - p)] - \mu \tag{26}$$

The mass of participating buyers is

$$n = \frac{1}{2}[\beta_1(v_1 - p) + (1-\beta_2)(v_2 - p)] = \mu_{\max} \tag{27}$$

From these observations, we can derive the platform maximizes its revenue as

$$\pi_p = \frac{1}{4}[\beta_1(v_1 - p) + (1-\beta_2)(v_2 - p)]p\tau - L \tag{28}$$

Table 7 shows the optimal commission rates revenues.

Table 7. The optimal commission rates for platform with high-quality online reviews in Case 3.

Conditions	Optimal commission rates
$v_2 - p > \xi_1(p - v_1)$	$\tau^* = \frac{1}{2}$
$v_2 - p < \xi_1(p - v_1)$	0

5. Results and Discussion

In this section, we compare the equilibrium quantities in Proposition 1 to Proposition 3 to study the effect of the online reviews. Lemma 1 indicates how to price the services and how the accuracy rate of online reviews impacts the platform. All proofs are in the Appendix A.

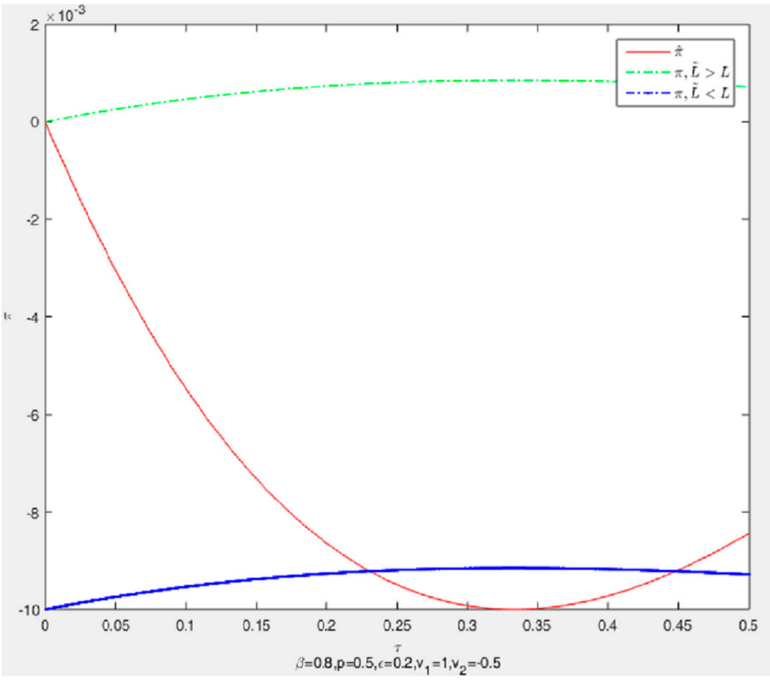
Proposition 1. *When the value satisfies $v_2 - p < \xi_1(p - v_1)$, the platform cannot benefit from online reviews.*

In this condition, the platform cannot benefit from online reviews. As low-quality products cause destructive damage to buyers, it is a disaster for a platform resulting in no buyer entering. As a consequence, whatever the quality of online reviews is, the platform will go bankrupt. For instance, assume that a buyer has bought a low-quality product (health product) through the platform, which is harmful to the buyer's healthy. Whether the platform has high-quality online reviews or not, the long-term result is that there will be no buyer entering. For a platform, online reviews cannot act as an effective means to improve the quality and revenue. The platform should establish other policies and rules (eg: punitive measures, reputation management) that strengthen the supervision and control of this kind of low-quality products [45–48]. It also needs other supervisory departments to enhance the sense of responsibilities for government and severely punish those who undertake illegal activities.

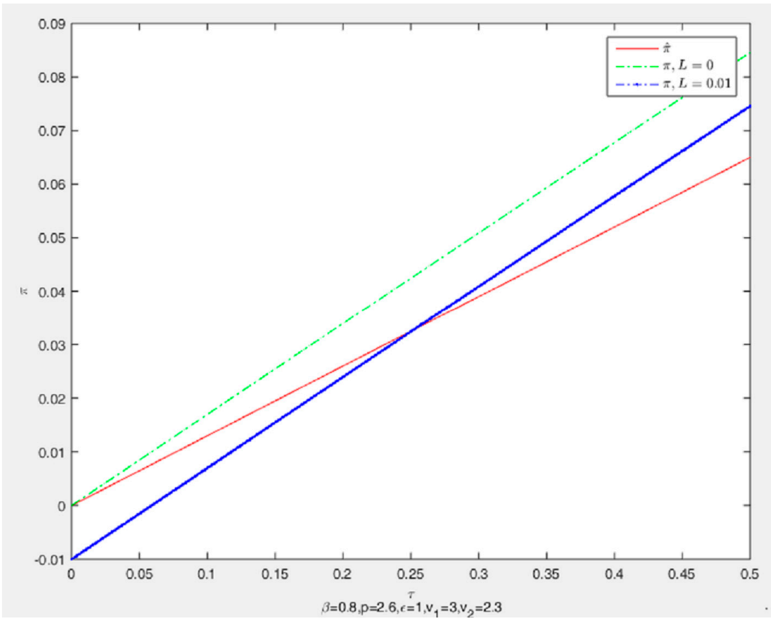
Proposition 2. *When the value satisfies $\xi_1(p - v_1) < v_2 - p < \hat{\xi}_1(p - v_1)$, there exists a threshold $\overline{L}$, the equilibrium behavior of the platform will be as follows:*

(i) *if $\overline{L} < L$, then $\pi_p{}^* < \hat{\pi}_p{}^*$, the platform cannot benefit from online reviews; and,*
(ii) *if $\overline{L} > L$, then $\hat{\pi}_p{}^* < \pi_p{}^*$, the platform can benefit from online reviews.*

The above results indicate that if the cost of moderating the quality of online reviews above a threshold $\overline{L}$, the platform cannot benefit from online reviews. When the platform with low-quality online reviews, this kind of low-quality products causes destructive damage; it leads to no buyer participating. When the platform improves the online reviews to high-quality, the platform will charge high commissions and benefit from the increasing trading number of high-quality products. However, the benefits that are generated by the improving quality of online reviews and the charge of high commission are lower than the damages that are caused by low-quality products and the costs that are generated by the improving quality of online reviews. Conversely, if the cost of moderating the quality of online reviews below the threshold $\overline{L}$, the benefit that high-quality online reviews generate overwhelms the damage that low-quality products bring, as can be seen in Figure 1. For example, assume that a buyer has bought a low-quality product (a pair of sports shoes) through the platform that is subsequently deemed not comfortable and durable. When the platform has low-quality online reviews, it will lead the trading number of low-quality increasing. Too many buyers bought low-quality products directly cause the buyer to lose trust in the platform. Over time, the platform cannot survive due to no further buyer entering. When the platform has high-quality online reviews and the cost of moderating the quality of online reviews is below the threshold, buyers suffer some damage by the inclusion of low-quality products, but they can comment on the products, which can expose the product quality and indicate the other buyers. In this way, the platform can survive due to the increase of trading number of high-quality products. In this condition, the effect of online reviews is strong.



(a)

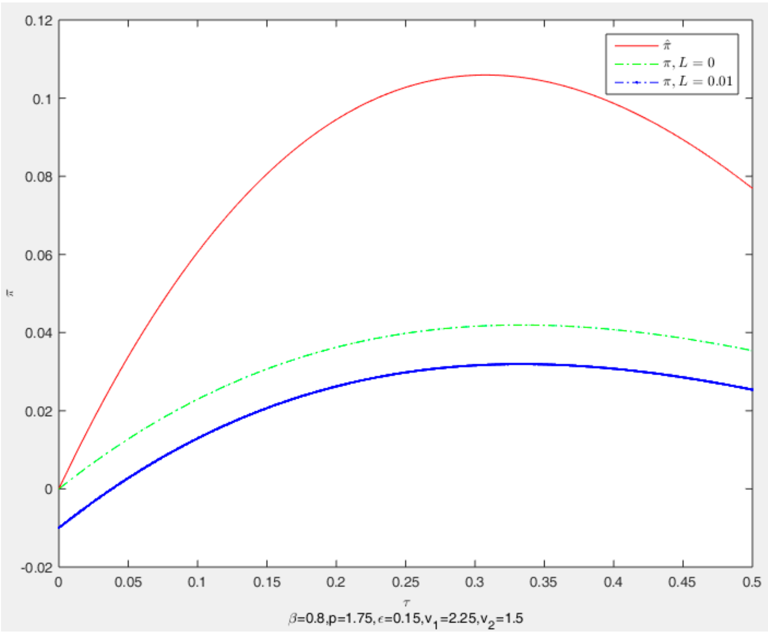


(b)

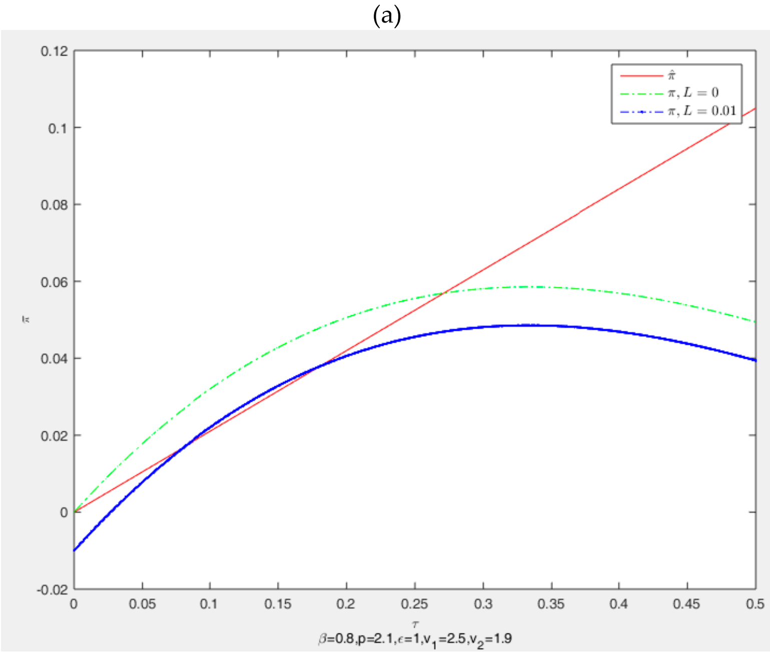
Figure 1. Equilibrium comparison in Proposition 2.

Proposition 3. *When the value satisfies $p + \varepsilon < 2$ and $\hat{\xi}_1(p - v_1) < v_2 - p < v_1 - p$, then $\pi_p{}^* < \hat{\pi}_p{}^*$, the platform cannot benefit from online reviews.*

This finding indicates that, when the price is below a certain threshold, the difference between the two profits for sellers is small, but the profit for low-quality seller is above a certain threshold, the platform cannot benefit from online reviews. Figure 2 shows this. We can indicate that, when the value of low-quality products and high-quality products is close, low-quality products do not cause too many participating buyers to withdraw. However, improving the low-quality to high-quality of online reviews will decrease the trading number of all products. The benefits are lower than the losses that high-quality online reviews generate. For instance, if a buyer buys a low-quality product (say, a roller ball pen) through the platform that has less material but does not affect the usage, because the low-quality roller ball pen is cheap and it has no effect on its use, buyers will not lose trust in the platform, whatever the quality of online reviews might be. In this condition, the platform with low-quality online reviews generates more revenue than the platform with high-quality online reviews. As a consequence, the effect of online reviews is weak and an online reviews mechanism is not necessary. The reason is that the value of low-quality products and high-quality products is close. From the perspective of the sustainable development of marketing environment, it needs the platform to establish other policies and rules (eg: punitive measures, reputation management) that strengthen the supervision and control of the low-quality sellers and products.



(a)



(b)

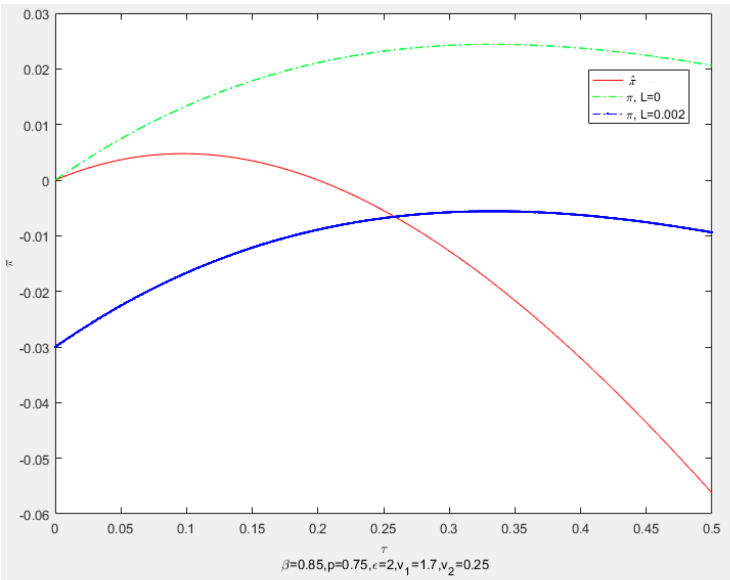
Figure 2. Equilibrium comparison in Proposition 3

Lemma 1. *If the platform can benefit from online reviews ($\hat{\pi}_p{}^* < \pi_p{}^*$):*

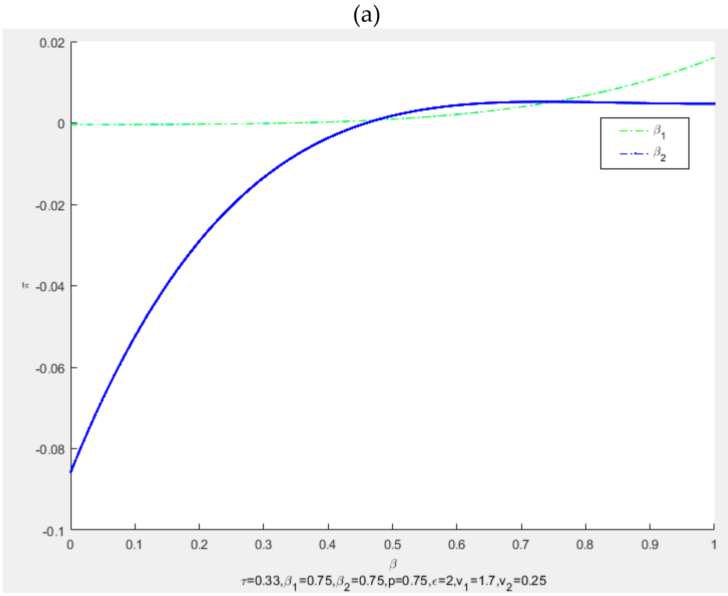
(i) the optimal commission rate is high; that is, $\hat{\tau}^* \leq \tau^*$;

(ii) $\pi_p{}^*$ is increasing in β_1 and β_2.

Lemma 4(i) implies that a platform with high-quality online reviews service not always charges high commissions and the platform does not always benefit from high commissions. Comparing Figures 1–3-(a) can indicate it. In other words, the high-quality online reviews service charges high commission, it leads to the mass of participating buyers and sellers decrease, but the trading number of high-quality products increase. As a consequence, the benefits are not always higher than the losses. Lemma 4(ii) shows that platforms benefit from an improvement in the accuracy rate of online reviews from both high-quality products and low-quality products, as shown in Figure 3-(b). It indicates that, when the quality of online reviews is above the natural selection rate, improving the accuracy rate of online reviews not only improves the quality, but it also improves the revenue for the platform.



(a)



(b)

Figure 3. Equilibrium comparison in Lemma 1.

6. Conclusions

6.1. Implications

As we all know, online reviews are important, but under what conditions they can make a big difference for platforms. Our paper has important implications for online retailing. First, our paper reveals that, not under all conditions a platform can benefit from online reviews, and whether the platform can benefit from online reviews depends on the moderating cost, the value, and the price of products. When the value of low-quality products causes destructive damage to buyers, and when the price is below a threshold and the difference between the two profits for sellers is small, the benefits are lower than the losses that high-quality online reviews generate, so the platform cannot benefit from online reviews. When the value of low-quality products for buyers is below a certain threshold, but above the threshold that can cause destructive damage, if the cost is above a threshold, the online reviews supervision mechanism does not work. If the cost is below a threshold, the benefit that high-quality online reviews generate overwhelms the damage that low-quality products bring, so the platform can benefit from online reviews. This suggests that our conclusion is not totally in contradiction with the mechanism that was suggested by Young Kwark et al. [23]

Second, we also examine the effect of online reviews on the pricing strategy of the platform. Amit Basu et al. consider that high-quality authentication services may not always justify higher authentication fee for online matching platforms [13]. Our study is the same as the result suggested by Amit Basu et al.; we find the platform that offers high-quality online reviews service not always charges high commission. Different from their result, we find the platform not always benefit from the high commission, because of the mass of participating members decreasing.

Last, we give some other preventive measures for platforms to develop. On the one hand, if the platform cannot benefit from online reviews, they can establish other policies and rules, just like punitive measures and reputation management, to strengthen supervision and control of low-quality products. On the other hand, if the platform can benefit from online reviews, we conclude the result that the platform always benefits from an improvement in accuracy rates of online reviews is same with the result that was suggested by Young Kwark et al. [49]. They can improve the accuracy rate of online reviews to improve their quality and revenue by improving the quality of the voting system and the reputation system or more often providing cash back and return coins. We indicate the platform can take different measures according to different conditions.

6.2. Limitations

Our results also point to some directions for future research. Firstly, we assume that the accuracy rate of online review's β is a parameter. Actually, the effect of different accuracy rates of online reviews for different qualities of products may be worth exploring.

Secondly, the main body of online retailing includes buyers, sellers, and platforms. This paper mainly focuses on the effect of online reviews on the revenue of platforms. Studying how exactly the online reviews affect the revenue of sellers and buyers and the social welfare implications will complement this work.

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Appendix A

Proof of Equilibrium for low-quality online reviews-Case 1

We denote $\hat{\tau}_1 = 1 - \frac{1}{p}$, $\hat{\tau}_2 = 1 - \frac{1}{p+\varepsilon}$, $\overline{\tau} = 1 - \frac{p-v_2}{p(v_1-p)}$, $\hat{\xi}_1 = \frac{p}{p+\varepsilon}$, $\hat{\xi}_2 = \frac{3p^2-2p^2(p+\varepsilon)-p(p+\varepsilon)^2+2p(p+\varepsilon)}{2p(p+\varepsilon)-p(p+\varepsilon)^2+(p+\varepsilon)^2}$, $\hat{\xi}_3 = -\frac{1}{4}p$ and $\hat{\xi}_4 = \frac{5-3p}{3-p}$.

(A) $p + \varepsilon < 1$.

From Equation (12), we obtain the first-order condition and the second-order condition are as follow

$$\frac{d\pi_p}{d\tau} = \frac{1}{4}\left(1 - 4\tau + 3\tau^2\right)pq^2(2p+\varepsilon)[p(v_1-p) + (p+\varepsilon)(v_2-p)]$$

$$\frac{d^2\pi_p}{d\tau^2} = \frac{1}{4}(-4+6\tau)pq^2(2p+\varepsilon)[p(v_1-p) + (p+\varepsilon)(v_2-p)]$$

① If $v_2 - p > \hat{\xi}_1(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(0) > 0$ and $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$, its first-order derivative is positive over $(0, \frac{1}{3})$ and is negative over $(\frac{1}{3}, \frac{1}{2})$. The objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{3}$.

② If $\hat{\xi}_1(p - v_1) > v_2 - p$, then $n = 0$.

(B) $p < 1$ and $1 < p + \varepsilon < 2$.

(a) When $\tau \in (0, \hat{\tau}_2)$, we obtain the first-order condition and the second-order condition are as follow

$$\frac{d\pi_p}{d\tau} = \frac{1}{4}pq^2\left[p^2(v_1-p)\left(1-4\tau+3\tau^2\right) + p(v_1+v_2-2p)(1-2\tau) + (v_2-p)\right]$$

$$\frac{d^2\pi_p}{d\tau^2} = \frac{1}{4}pq^2\left[p^2(v_1-p)(-4+6\tau) - 2p(v_1+v_2-2p)\right]$$

① If $v_2 - p > \hat{\xi}_2(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$; $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_2$.

② If $\hat{\xi}_2(p - v_1) > v_2 - p > \hat{\xi}_1(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(0) > 0$ and $\frac{d\pi_p}{d\tau}(\hat{\tau}_2) < 0$, its first-order derivative is positive over $(0, \widetilde{\tau})$ and is negative over $(\widetilde{\tau}, \hat{\tau}_2)$. The objective function reaches the maximum at $\hat{\tau}^* = \widetilde{\tau}$.

③ If $\hat{\xi}_1(p - v_1) > v_2 - p > p(p - v_1)$ and $\tau \in (0, \overline{\tau})$, then $n \in (0, 1)$, we obtain $\frac{d^2\pi_p}{d\tau^2}(\overline{\tau}) < 0$, $\frac{d\pi_p}{d\tau}(\overline{\tau}) > 0$, so $\frac{d^2\pi_p}{d\tau^2} < 0$ and $\frac{d\pi_p}{d\tau} > 0$, the objective function reaches the maximum at $\hat{\tau}^* = \overline{\tau}$; if $\tau \in (\overline{\tau}, \hat{\tau}_2)$, then $n = 0$.

④ If $p(p - v_1) > v_2 - p$, then $n = 0$.

(b) When $\tau \in (\tau_2, \frac{1}{2})$,

① If $v_2 - p > \hat{\xi}_1(p - v_1)$, $1 < p + \varepsilon < \frac{3}{2}$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(\hat{\tau}_2) > 0$, $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$. The objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{3}$; if $v_2 - p > \hat{\xi}_1(p - v_1)$, $\frac{3}{2} < p + \varepsilon < 2$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} < 0$, the objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_2$.

② If $\hat{\xi}_1(p - v_1) > v_2 - p$, then $n = 0$.

(C) $1 < p < 2$ and $1 < p + \varepsilon < 2$.

(a) When $\tau \in (0, \hat{\tau}_1)$, we obtain:

① If $v_2 - p > p - v_1$, then $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_1$.

② If $p - v_1 > v_2 - p$, then $n = 0$.

(b) When $\tau \in (\hat{\tau}_1, \hat{\tau}_2)$,

① If $v_2 - p > \hat{\xi}_2(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_2$.

② If $\hat{\xi}_2(p - v_1) > v_2 - p > \hat{\xi}_4(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(\hat{\tau}_1) > 0$, $\frac{d\pi_p}{d\tau}(\hat{\tau}_2) < 0$, its first-order derivative is positive over $(\hat{\tau}_1, \widetilde{\tau})$ and is negative over $(\widetilde{\tau}, \hat{\tau}_2)$. The objective function reaches the maximum at $\hat{\tau}^* = \widetilde{\tau}$.

③ If $\hat{\xi}_4(p-v_1) > v_2 - p > \hat{\xi}_1(p-v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} < 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_1$.

④ If $\hat{\xi}_1(p-v_1) > v_2 - p > p - v_1$ and $\tau \in (\hat{\tau}_1, \overline{\tau})$, $n \in (0,1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \overline{\tau}$; if $\tau \in (\overline{\tau}, \hat{\tau}_2)$, then $n = 0$.

⑤ If $p - v_1 > v_2 - p$, then $n = 0$.

(c) When $\tau \in (\hat{\tau}_2, \frac{1}{2})$, we obtain:

① If $v_2 - p > \hat{\xi}_1(p-v_1)$ and $1 < p + \varepsilon < \frac{3}{2}$, then $\frac{d^2\pi_p}{d\tau^2} < 0$,$\frac{d\pi_p}{d\tau}(\hat{\tau}_2) > 0$, $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$, its first-order derivative is positive over $(\hat{\tau}_2, \frac{1}{3})$ and is negative over $(\frac{1}{3}, \frac{1}{2})$. The objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{3}$.

② If $v_2 - p > \hat{\xi}_1(p-v_1)$ and $\frac{3}{2} < p + \varepsilon < 2$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} < 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_2$.

③ If $\hat{\xi}_1(p-v_1) > v_2 - p$, then $n = 0$.

(D)$1 < p < 2$ and $p + \varepsilon > 2$.

(a) The proof is same as Proof of Equilibrium for low-quality online reviews-Case 1-(C)-(a).

(b) When $\tau \in (\hat{\tau}_1, \frac{1}{2})$, we obtain:

① If $v_2 - p > \hat{\xi}_4(p-v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$,$\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{2}$.

② If $\hat{\xi}_3(p-v_1) > v_2 - p > \hat{\xi}_4(p-v_1)$ $(\hat{\xi}_4 < \hat{\xi}_3)$ and $1 < p < \frac{5}{3}$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} < 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_1$; if $\hat{\xi}_3(p-v_1) > v_2 - p > \hat{\xi}_4(p-v_1)$ and $\frac{5}{3} < p < 2$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(\hat{\tau}_1) > 0$, $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$, its first-order derivative is positive over $(0, \widetilde{\tau})$ and is negative over $(\widetilde{\tau}, \frac{1}{2})$. The objective function reaches the maximum at $\hat{\tau}^* = \widetilde{\tau}$.

③ If $\hat{\xi}_4(p-v_1) > v_2 - p > \frac{1}{2}p(p-v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$,$\frac{d\pi_p}{d\tau} < 0$. The objective function reaches the maximum at $\hat{\tau}^* = \hat{\tau}_1$.

④ If $\frac{1}{2}p(p-v_1) > v_2 - p > p - v_1$ and $\tau \in (\hat{\tau}_1, \overline{\tau})$,$n \in (0,1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \overline{\tau}$; if $\tau \in (\overline{\tau}, \frac{1}{2})$, then $n = 0$.

⑤ If $p - v_1 > v_2 - p$, then $n = 0$.

Proof of Equilibrium for low-quality online reviews-Case 2

① If $v_2 - p > \hat{\xi}_3(p-v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$,$\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{2}$.

② If $\hat{\xi}_3(p-v_1) > v_2 - p > \frac{1}{2}p(p-v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(0) > 0$, $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$, its first-order derivative is positive over $(0, \widetilde{\tau})$ and is negative over $(\widetilde{\tau}, \frac{1}{2})$. The objective function reaches the maximum at $\hat{\tau}^* = \widetilde{\tau}$.

③ If $\frac{1}{2}p(p-v_1) > v_2 - p > p - v_1$ and $\tau \in (0, \overline{\tau})$, $n \in (0,1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\hat{\tau}^* = \overline{\tau}$; if $\tau \in (\overline{\tau}, \frac{1}{2})$, then $n = 0$.

④ If $p(p-v_1) > v_2 - p$, then $n = 0$.

Proof of Equilibrium for low-quality online reviews-Case 3

① If $v_2 - p > p - v_1$, then $\frac{d\pi_p}{d\tau} > 0$, the objective function reaches the maximum at $\hat{\tau}^* = \frac{1}{2}$.

② If $p - v_1 > v_2 - p$, then $n = 0$.

Proof of Equilibrium for high-quality online reviews-Case 1

We denote

$$\tau_1 = 1 - \frac{1}{(1-\beta_2)(p+\varepsilon)},\ \tau_2 = 1 - \frac{1}{\beta_1 p},\ \underline{\tau} = 1 - \frac{\beta_1(p-v_1)}{(1-\beta_2)^2(p+\varepsilon)(v_2-p)}$$

$$\underline{\tau} = 1 - \frac{\beta_1(p-v_1)}{(1-\beta_2)^2(p+\varepsilon)(v_2-p)},\ \xi_1 = \frac{{\beta_1}^2 p}{(1-\beta_2)^2(p+\varepsilon)},\ \xi_2 = \frac{\beta_1}{(1-\beta_2)^2(p+\varepsilon)},$$
$$\xi_3 = \frac{\beta_1}{2(1-\beta_2)^2(p+\varepsilon)+\beta_1},\ \xi_4 = \frac{\beta_1}{\frac{1}{2}(1-\beta_2)^2(p+\varepsilon)+\beta_1}$$

(A) $p < \frac{1}{\beta_1}$. To simplify the mathematical expression, we denote:

$$\lambda = \frac{1}{4}p\left[{\beta_1}^2 p + (1-\beta_2)^2(p+\varepsilon)\right]\left[{\beta_1}^2 p(v_1 - p) + (1-\beta_2)^2(p+\varepsilon)(v_2 - p)\right]$$

We obtain the first-order condition and the second-order condition are $\frac{d\pi_p}{d\tau} = \lambda(1 - 4\tau + 3\tau^2)$, $\frac{d^2\pi_p}{d\tau^2} = \lambda(-4 + 6\tau)$. We derive $\frac{d^2\pi_p}{d\tau^2} < 0$; $\frac{d\pi_p}{d\tau}(0) > 0$, $\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$. Its first-order derivative is positive over $(0, \frac{1}{3})$ and is negative over $(\frac{1}{3}, \frac{1}{2})$. The objective function reaches the maximum at $\tau^* = \frac{1}{3}$.

(B) $\frac{1}{\beta_1} < p < \frac{2}{\beta_1}, p + \varepsilon < \frac{1}{1-\beta_2}$.

(a) When $\tau \in (0, \tau_2)$, from Equation (42), we obtain the first-order condition and the second-order condition are as follow

$$\begin{aligned}\frac{d\pi_p}{d\tau} &= \tfrac{1}{4}p\Big[-2\beta_1(1-\beta_2)^2(p+\varepsilon)(v_1+v_2-2p)\tau - 4(1-\beta_2)^4(p+\varepsilon)^2(v_2-p)\tau + {\beta_1}^2(v_1-p) \\ &\quad +3(1-\beta_2)^4(p+\varepsilon)^2(v_2-p)\tau^2 + \beta_1(1-\beta_2)^2(p+\varepsilon)(v_1+v_2-2p) + (1-\beta_2)^4(p+\varepsilon)^2(v_2-p)\Big]\end{aligned}$$

$$\frac{d^2\pi_p}{d\tau^2} = \frac{1}{2}p\left[-\beta_1(1-\beta_2)^2(p+\varepsilon)(v_1+v_2-2p) - 2(1-\beta_2)^4(p+\varepsilon)^2(v_2-p) + 3(1-\beta_2)^4(p+\varepsilon)^2(v_2-p)\tau\right]$$

① If $v_2 - p > \xi_3(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

② If $\xi_3(p - v_1) > p - v_1$, then $\frac{d^2\pi_p}{d\tau^2}(0) > 0$, $\frac{d^2\pi_p}{d\tau^2}(\tau_2) < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

③ If $p - v_1 > v_2 - p > \xi_2(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} > 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

④ If $\xi_2(p - v_1) > v_2 - p > \xi_1(p - v_1)$ and $\tau \in (0, \underline{\tau})$, then $n = 0$; if $\tau \in (\underline{\tau}, \tau_2)$, we obtain $n \in (0, 1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

⑤ If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

(b) When $\tau \in (\tau_2, \frac{1}{2})$,we obtain:

① If $v_2 - p > \xi_1(p - v_1)$ and $\frac{1}{\beta_1} < p < \frac{3}{2\beta_1}$ then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau}(\tau_2) > 0$,$\frac{d\pi_p}{d\tau}(\frac{1}{2}) < 0$, its first-order derivative is positive over $(\tau_2, \frac{1}{3})$ and is negative over $(\frac{1}{3}, \frac{1}{2})$. The objective function reaches the maximum at $\tau^* = \frac{1}{3}$; if $v_2 - p > \xi_1(p - v_1)$ and $\frac{3}{2\beta_1} < p < \frac{2}{\beta_1}$. , then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} < 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

② If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

(C) $\frac{1}{\beta_1} < p < \frac{2}{\beta_1}$, $\frac{1}{1-\beta_2} < p + \varepsilon < \frac{2}{1-\beta_2}$.

(a) When $\tau \in (0, \tau_1)$, we obtain:

① If $v_2 - p > \xi_1(p - v_1)$, the objective function reaches the maximum at $\tau^* = \tau_1$.

② If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

(b) When $\tau \in (\tau_1, \tau_2)$, we obtain:

① If $v_2 - p > p - v_1$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

② If $p - v_1 > v_2 - p > \xi_2(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} > 0$; $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \tau_2$.

③ If $\xi_2(p - v_1) > v_2 - p > \xi_1(p - v_1)$ and $\tau \in (\tau_1, \underline{\tau})$, then $n = 0$;f $\tau \in (\underline{\tau}, \tau_2)$. , then $n \in (0, 1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$, the objective function reaches the maximum at $\tau^* = \tau_2$.

④ If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

(c) The proof is same as Proof of Equilibrium for high-quality online reviews-Case 1-(B)-(b).

Proof of Equilibrium for high-quality online reviews-Case 2

(A) $p > \frac{2}{\beta_1}$ and $p + \varepsilon < \frac{1}{1-\beta_2}$.

We obtain: ① If $v_2 - p > \xi_3(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$.② If $\xi_3(p - v_1) > v_2 - p > \xi_4(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2}(0) > 0$, $\frac{d^2\pi_p}{d\tau^2}(\frac{1}{2}) < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$. ③ If $\xi_4(p - v_1) > v_2 - p > \xi_1(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} > 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$. ④ If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.
(B) $p > \frac{2}{\beta_1}$ and $\frac{1}{1-\beta_2} < p + \varepsilon < \frac{2}{1-\beta_2}$.
(a) The proof is same as Proof of Equilibrium for high-quality online reviews-Case 1-(C)-(a).
(b) When $\tau \in (\tau_1, \frac{1}{2})$, we obtain:

① If $v_2 - p > \xi_4(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$.

② If $\xi_4(p - v_1) > v_2 - p > \xi_2(p - v_1)$, then $\frac{d^2\pi_p}{d\tau^2}(\tau_1) > 0$, $\frac{d^2\pi_p}{d\tau^2}(\frac{1}{2}) < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$.

③ If $\xi_2(p - v_1) > v_2 - p > \xi_1(p - v_1)$ and $\tau \in (\tau_1, \underline{\tau})$, then $n = 0$; if $\tau \in (\underline{\tau}, \frac{1}{2})$, then $n \in (0, 1)$, $\frac{d^2\pi_p}{d\tau^2} < 0$, $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$.

④ If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

Proof of Case 3 from equilibrium for high-quality online reviews

① If $v_2 - p > \xi_1(p - v_1)$, then $\frac{d\pi_p}{d\tau} > 0$. The objective function reaches the maximum at $\tau^* = \frac{1}{2}$.

② If $\xi_1(p - v_1) > v_2 - p$, then $n = 0$.

Proof of Proposition 1

When $\xi_1(p - v_1) > v_2 - p$, we obtain:

$$\hat{s} = \frac{\hat{m}_1 + \hat{m}_2}{2} q \left[\frac{\hat{m}_1}{\hat{m}_1 + \hat{m}_2} v_1 + \frac{\hat{m}_2}{\hat{m}_1 + \hat{m}_2} v_2 - p \right] - \mu < 0$$

$$s = \frac{m + m}{2} P(A') \left[P(A|A') v_1 + P(B|A') v_2 - p \right] - \mu < 0$$

There is no buyer participating in the platform due to the quite low value of low-quality products.

Proof of Proposition 2

When $\xi_1(p - v_1) < v_2 - p < \hat{\xi}_1(p - v_1)$, the threshold is $\overline{L} = \frac{m_1+m_2}{2} nP(A')p\tau - \frac{\hat{m}_1+\hat{m}_2}{2} \hat{n}qp\hat{\tau}$. If $\overline{L} < L$, the platform with low-quality online reviews generates more revenue; if $\overline{L} > L$, the platform with high-quality online reviews generates more revenue.

Proof of Proposition 3

When $p + \varepsilon < 2$ *and* $v_1 - p > v_2 - p > \hat{\xi}_1(p - v_1)$, we obtain:

$$\frac{m_1 + m_2}{2} nP(A')p\tau - L < \frac{\hat{m}_1 + \hat{m}_2}{2} \hat{n}qp\hat{\tau}$$

The revenue in the low-quality of online reviews always higher than the revenue in the high-quality of online reviews.

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