



# Article Differential Pricing Strategies of High Speed Railway Based on Prospect Theory: An Empirical Study from China

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**Abstract:** Based on the single pricing method of the high-speed railway (HSR) in China, a pricing strategy without flexibility leads to the problem of extreme fluctuations in passenger flow and difficulty in increasing revenue. In order to achieve sustainable development of the HSR from the perspective of pricing, in this study, we divided the passenger market according to the different factors affecting passengers' choice behavior, maximized ticket sales revenue with expected travel cost as the reference point, and used prospect theory to construct a differentiated pricing model under elastic demand. A simulated annealing algorithm was used to solve this model under two passenger flow intensities. Taking the Beijing–Shanghai corridor as an example for analysis, the results show that differential pricing can be implemented on the basis of passenger decision-making, and price reductions at off-peak periods will attract passenger flow which will increase ticket sales revenue by 10.41%. During the peak period, prices can be increased to maintain passenger flow, and ticket sales revenue will increase by 7.98%. We also found that increasing passenger expectations have a greater impact on ticket sales. This study provides theoretical and methodological support for the sustainable development of the HSR.

**Keywords:** differential pricing; price discrimination; passenger expectation; prospect theory; simulated annealing

# 1. Introduction

With the continuous expansion of the high-speed railway (HSR) in China [1], the formulation of passenger ticket strategies has attracted more attention. The single pricing policy of the HSR is facing flexible and diverse price competition from other modes of transportation, which leads to unbalanced capacity utilization with fluctuations in passenger flow. Since most of the HSR in China was in a state of loss for a long time, it is important to study the pricing strategy to change the revenue problem [2]. At the same time, HSR pricing is also key to achieving sustainable development. From the perspective of improving the quality of people's lives [3], we can meet more people's travel needs and ensure smooth and sustainable HSR lines. Differential pricing is a method of setting prices with the willingness to pay. Due to the strong substitutability of trains in the same origin–destination (OD) between the same lines, passengers can be subdivided into several parts with different price elasticity [4]. That is, the differential pricing concept can be applied in this situation. In order to develop a better pricing strategy for the HSR to meet more travel needs, prospect theory is introduced for the first time to analyze the psychology of passengers when they are purchasing tickets, with the aim of adjusting the ticket price for each train to meet their expectations, so as to achieve the goal of balancing passenger flow and improving revenue.

This paper has four points. First, we would like to understand the travel choice behavior of different passengers by analyzing the data of their ticket purchasing. By doing this, we can divide the

passenger market into several groups to make sure that they all have the same or similar travel choices. Second, we confirm the effectiveness of differentiated pricing for the HSR passenger market in China, which has led to a significant increase in ticket revenue. Third, we use prospect theory to simulate the real decision-making process of passengers and prove the feasibility of its application to the HSR pricing model. Fourth, we provide a theoretical basis for the sustainable development of the HSR to achieve the goal of balanced passenger flow and market regulation.

The differences in people's travel choices motivate our interest in developing an empirical analysis to understand how passenger expectations affect decision-making. Our hypothesis is that the psychology of passengers when they purchase tickets can be used to achieve HSR price discrimination. We use a real dataset to address the research question, which includes all ticket purchase records of HSR passengers from 2016 to 2017 in China. From these data, we analyze the differences in passengers' decision-making behaviors and classify them into several categories to apply price discrimination strategies for different passenger markets.

Our results confirm the applicability of price discrimination in the HSR market and the impact of passenger expectations on ticket revenue. By formulating different prices for passenger markets, revenue will be greatly improved regardless of the density of passenger flow, which will have a positive effect on guiding and regulating passenger flow.

This paper is organized as follows. In Section 2 we survey relevant studies in the literature. In Section 3 we analyze the differential pricing problem of the HSR and Section 4 proposes the model. In Section 5 we build the algorithm, and Section 6 provides examples and results. We offer our discussion and conclusions in Section 7.

#### 2. Literature Review

HSR pricing is a critical factor in the competitiveness of transportation operations, which helps to maximize revenue and achieve sustainable development [5]. Following the main points of this paper, we start with a review of studies that analyzed the development and application of price discrimination. Then, we focus on research that studied the application of differentiated pricing in the transportation industry. Finally, we discuss the feasibility of applying prospect theory to HSR pricing.

#### 2.1. Development and Application of Price Discrimination

Dupuit [6] explained in 1894 that price discrimination refers to companies requesting different prices from customers according to different needs when selling identical or differentiated products of the same type. Then, in 1932 Pigou [7] categorized it into three types according to the degree of discrimination. Price discrimination, which has been extensively studied in various fields such as airlines [8,9], retail [10] and so on, is shown to have a positive effect on earnings to increase profitability [11]. Both theoretical and empirical studies show that the premise of price discrimination is to have a flexible market that can be segmented. Asplund [12] confirmed the authenticity of price discrimination in an oligopoly from regional newspapers. Puller [13] analyzed the price discrimination adopted by airlines based on the time of ticket purchase. In order to maximize revenue, operators use this method to achieve price discrimination for different markets.

Among the three levels, second-degree and third-degree price discrimination are more practically applied to commodity pricing [14]. In first-degree discrimination, the price of a product equals the buyer's maximum willingness to pay, which is hard to realize. For second-degree discrimination, which has been applied in the mobile communication market, the price depends on the number of units to be purchased. David [15] used examples from Japanese newspapers to confirm that second-degree price discrimination increased social welfare. Third-degree discrimination reflects that pricing policy, according to the price elasticities of demand in different markets, is changed to adapt to relevant segments of the market. Holmes [16] analyzed the third-degree price discrimination effect in an oligopoly as early as 1989. In general, while a series of pricing strategies through price discrimination have a strong impact on revenue, price discrimination is used less in the HSR according to existing research.

#### 2.2. Development and Research of HSR Pricing

Since its existence, the HSR has brought great convenience for people's travel, and has gradually become the main means of long-distance travel, with the advantages of comfort, speed, and security [17]. As a result, it's pricing policy has attracted more attention. HSR pricing strategies vary from country to country, mostly based on floating prices, including base fares and additional fares. Taniguchi [18] studied the pricing model of the Japanese Shinkansen. Voss [19] discussed pricing models for different types of fares, such as student tickets. Zhou [20] proposed a pricing model of railway passenger transportation under competition. Although the HSR developed rapidly in China, the research on fares started late. It is still in the exploratory stage of learning from the experience of airline pricing. The research mainly includes dynamic pricing and differentiated pricing [21].

HSR pricing draws on the concept of revenue management in airline pricing, including research on differentiated pricing, demand forecasting, and seat allocation. Bitran [22] reviewed the pricing model of revenue management. Sibdari [23] studied the application of revenue management in practice and confirmed the feasibility of the model. Jiang [24] proposed a demand forecasting model for HSR, which provides a basis for effective railway revenue management. In some studies, the trains are priced according to the rules of passenger ticket purchasing, and the fares are determined by considering the change in demand under different passenger flow intensity [25,26]. Some have studied the impact of advanced time for ticket purchasing, with reference to the floating fares of airlines [27]. Other studies include setting several types of fares according to different seat levels, adjusting prices based on remaining seats [28], and so on. Although demand forecasting and pricing issues have been studied, the reasons for changes in passenger flow and pricing are still unclear, with a lack of research that combines passenger analysis with the market.

#### 2.3. Applicability of Prospect Theory to HSR Pricing

In order to explain the real decision-making process, Kahneman and Tversky [29] first proposed a famous prospect theory through a series of psychological analysis and found that people have reference dependence in actual decision-making. When faced with choices, the decision maker will pre-set a reference point to weigh the effectiveness of each option to judge its gains or losses [30], and the reference point reflects the person's psychological expectations [31]. Prospect theory provides a good description of the decision characteristics of people under uncertain conditions, which can be used to analyze the characteristics of travelers' choice behavior in various environments [32,33].

Schwanen and Ettema [34] proved that the characteristics of travel behavior in a transportation system are consistent with the theory of cumulative prospects, which has certain applicability. Based on prospect theory, one study established a travel selection model for travelers [35]. Another confirmed the influence of prospect theory on the departure time selection mechanism of passenger groups [36]. Theoretical research confirms the volatility of utility and loss aversion in prospect theory, which can be used to judge passenger travel decisions [37]. Although prospect theory can be used to study the psychology of passengers, the research is still only on the study of passengers and does not apply to pricing.

From the existing literature we see the limitations of HSR pricing and a lack of research on the combination of passenger analysis and marketization. Therefore, we propose, for the first time, applying the analysis of passengers' psychological activities and behaviors when purchasing tickets to the pricing system. Through the combination of prospect theory and differentiated pricing, we can achieve the goal of balancing passenger flow and improving profit, which proposes a new direction for future research.

## 3. Problem Analysis

The factors affecting the choice behaviors of HSR passengers, can be divided into subjective and objective [38]. Objective factors affect passengers' choice of travel modes and subjective factors determine the choice behaviors of passengers as individuals. Due to individual differences,

passengers will focus on significantly different ticketing decisions. Passenger groups with the same or similar subjective factors tend to exhibit similar travel choice behaviors, and there are differences in choices among multiple groups. Prospect theory holds that people's judgments of gains and losses have reference dependency, which means the choice of reference points has a greater impact on decision-making. A passenger's evaluation of the trip is relative to the change of the reference point. Therefore, applying prospect theory to passenger travel choices requires solving two problems: passenger classification and the selection of reference points.

## 3.1. Passenger Classification

Passenger classification divides passengers who exhibit the same or similar travel choice behaviors into the same group. In addition to the impact of fares, time is also an important factor affecting passenger travel choices. According to sensitivity to fare and time, passengers are divided into three types in this paper, classified as class *I*. The characteristics of class i(i = 1, 2, ..., I) passengers are shown in Table 1.

Passenger Time Price Elastic i Elasticity Choice Type Sensitivity Sensitivity Coefficient Economy 1 Weak Strong >1 Price priority Large Middle 2 =1 Medium Medium Single Comprehensive **Business** 3 Weak Small <1 Strong Time priority

Table 1. Passenger categories by price and time sensitivity.

## 3.2. Selection of Reference Point

According to the differentiated pricing of trains, differences in prices directly affects passengers' choice behavior. After learning information about train fares, departure times, and travel time, passengers will imagine an expected price based on their travel experience, that is, as a reference point for this travel decision. For different groups of passengers, the differences between reference points should be reflected in the importance of fares, departure times, and travel time. In this paper, passenger flow history data are used. Statistically in travel history, the proportion of passengers on various trains reflects the passengers' preference for particular departure times, and the theoretical expectation after differential pricing is calculated accordingly. Combined with travel costs history and subjective deviations, this forms the reference point for the travel decision. The difference between the actual travel cost and the reference point can be described by the value function, reflecting the passenger's lack of aversion to the loss.

In this paper, the subjective probability weight function is used to weight the value function to determine the passenger's choice behavior. A person will choose the option of maximum utility, which makes the research of prospect theory closer to the selection behavior of HSR passengers. At the same time, combined with the expected travel cost as a reference point, it is more in line with the passenger's actual decision-making process, as shown in Figure 1. After the passenger conceives of a reference point, that is, the expected travel cost, compared with the actual travel cost of the selected train, he or she will purchase ticket if the actual cost is lower, otherwise, some passengers will decline.

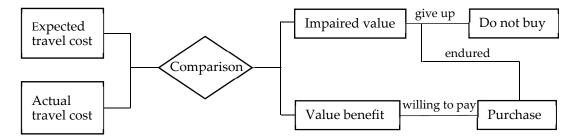


Figure 1. Passenger decision-making process when purchasing tickets.

#### 4. Methodology

For a clear analysis of the problem, we suppose that train tickets are sold in a fixed amount, regardless of over-sale, changes or refunds. Furthermore, regardless of the influence of external factors such as delays, the train will be sent to the station according to the schedule.

Suppose we set the EMU (Electric Multiple Unit) trains on a HSR line to *H*. For any train  $h \in H$ , the train capacity is C(h). The train running line includes *k* stations,  $W = \{(r, s) | r = 1, 2, ..., k - 1, s = r + 1, 2, ..., k\}$  means the origin–destination (OD) pair set from *r* stations to *s* stations, and the train set serving the OD pair (r, s) is expressed by  $H_{rs}$ .

In the point pair (r, s), for any train  $h \in H_{rs}$ , the actual travel cost of the class *i* passengers on the train *h* can be expressed in terms of a generalized cost  $c_{rs}^{hi}$ , which is affected by many factors, mainly the fare, travel time, and departure time [39]:

$$c_{rs}^{hi} = p_{rs}^{h} + v_{rs}^{hi} t_{rs}^{h} + m_{rs}^{hi}$$
(1)

Among them, for any pair of points (r, s),  $p_{rs}^h$  is the fare for the train h,  $v_{rs}^{hi}$  is the value of class i passengers in terms of travel time,  $t_{rs}^h$  is the time taken by the passengers of train h, and  $m_{rs}^{hi}$  represents the conversion cost of the departure time for class i passenger on train h.

Since different types of passengers have different travel time preferences, the expected departure time is usually fixed, indicated by  $\theta_{rs}^i = \{\theta_{rs}^1, \dots, \theta_{rs}^n, n \in N\}$  between point pair (r, s). The departure time of train h is  $\theta_{rs}^h$ , and the conversion cost  $m_{rs}^{hi}$  is a function of the degree of deviation between the departure time and the passenger's preferred travel time; the larger the deviation in time, the faster the conversion cost increases, which is in line with the exponential function rising trend:

$$m_{rs}^{hi} = v_{rs}^{hi} \exp\left\{\mu_{rs}^{i} * \min\left|\theta_{rs}^{h} - \forall \theta_{rs}^{i}\right|\right\}$$
(2)

Here,  $\mu_{rs}^i$  is the adjustment coefficient, and the sensitivity difference of various passengers to time can be realized by adjusting the coefficient. The closer the departure time is to the passenger's expectation, the lower the ticket cost.

Based on travel history data, the estimated number of tickets distributed by train *h* to pair (r, s) can be represented by  $n_{rs}^{hi}$ , wherein the expected number of tickets for class *i* passenger is  $n_{rs}^{hi}$ . The statistics of each train can be used to get a more accurate average of fares and time. The average price  $\overline{p}_{rs}$  of all trains between the pair (r, s), the average travel time value  $\overline{v}_{rs}^i$  of the class *i* passengers, and the average conversion cost  $\overline{m}_{rs}^i$  of trains at that time are expressed as

$$\overline{p}_{rs} = \frac{\sum_{h \in H_{rs}} n_{rs}^h p_{rs}^h}{\sum_{h \in H_{rs}} n_{rs}^h}$$
(3)

$$\overline{v}_{rs}^{i} = \frac{\sum_{h \in H_{rs}} n_{rs}^{hi} v_{rs}^{hi} t_{rs}^{h}}{\sum_{h \in H_{rs}} n_{rs}^{hi}}$$
(4)

$$\overline{m}_{rs}^{i} = \frac{\sum_{h \in H_{rs}} n_{rs}^{hi} m_{rs}^{hi}}{\sum_{h \in H_{rs}} n_{rs}^{hi}}$$
(5)

Therefore, for class *i* passengers, the average travel cost for all trains serving point pair (r, s) is

$$\bar{c}_{rs}^i = \bar{p}_{rs} + \bar{v}_{rs}^i + \bar{m}_{rs}^i \tag{6}$$

Based on the generalized cost, the travel cost expected by a passenger is

$$E_{rs}^{i} = \rho \overline{c}_{rs}^{i} + (1 - \rho) \widetilde{p}_{rs}^{i} + \xi_{rs}^{i}$$

$$\tag{7}$$

where  $\rho$  is the weighting factor,  $\tilde{p}_{rs}^{i}$  represents the generalized cost of passengers' travel history, and  $\xi_{rs}^{i}$  is the deviation value.

Taking the passenger's expected travel cost as the reference point for travel choice,  $\Delta x_{rs}^{hi}$  can be used to indicate the deviation between actual and expected travel cost after class *i* passengers choose train *h*:

$$\Delta x_{rs}^{hi} = -n_{rs}^{hi} (c_{rs}^{hi} - E_{rs}^i) \tag{8}$$

When the actual travel cost is higher than expected,  $\Delta x_{rs}^{hi}$  is negative, which means passengers purchase the low-value product at a high price, which is not good value. When the actual travel cost is lower than expected,  $\Delta x_{rs}^{hi}$  is positive, and passengers enjoy the service at a lower price, which is a benefit.

Since travel decision makers tend to be more sensitive to increments than to losses, the value function  $V(\Delta x_{rs}^{hi})$  can be used to indicate the impact of cost deviations on travel decisions:

$$V(\Delta x_{rs}^{hi}) = \begin{cases} \left(\Delta x_{rs}^{hi}\right)^{\alpha} & \Delta x_{rs}^{hi} \ge 0\\ -\lambda \left(-\Delta x_{rs}^{hi}\right)^{\beta} & \Delta x_{rs}^{hi} \le 0 \end{cases}$$
(9)

Among them,  $\alpha$ ,  $\beta$  ( $0 < \alpha \le 1$ ,  $0 < \beta \le 1$ ) measures the degree of reduced sensitivity away from the reference point. Larger  $\alpha$ ,  $\beta$  indicates that the traveler is more sensitive to risk,  $\lambda$  indicates a loss avoidance coefficient, and  $\lambda > 1$  always holds, reflecting that the individual is more sensitive to loss. The subjective probability weight function  $\pi(f_{rs}^{hi})$  is used to describe people's responses to objective risk probability.

When decision makers face benefits, it is calculated as follows:

$$\pi^{+}(f_{rs}^{hi}) = \frac{(f_{rs}^{hi})^{\gamma}}{\left[(f_{rs}^{hi})^{\gamma} + (1 - f_{rs}^{hi})^{\gamma}\right]^{1/\gamma}}$$
(10)

When decision makers face loss, it is calculated as follows:

$$\pi^{-}(f_{rs}^{hi}) = \frac{(f_{rs}^{hi})^{\circ}}{\left[(f_{rs}^{hi})^{\delta} + (1 - f_{rs}^{hi})^{\delta}\right]^{1/\delta}}$$
(11)

where  $f_{rs}^{hi}$  is the probability of class *i* passengers selecting train *h*,  $\pi^+(f_{rs}^{hi})$ ,  $\pi^-(f_{rs}^{hi})$  represent the subjective perception probability when facing benefits and loss, respectively. Parameters  $\gamma$ ,  $\delta$  determine the curvature of the weight function, and the smaller the corresponding value, the greater the degree of curvature.

According to the weight function and the value function, the utility of selecting train *h* can be obtained as follows:

$$U_{rs}^{h} = \sum_{i \in I} \pi(f_{rs}^{hi}) * V(\Delta x_{rs}^{hi})$$
(12)

$$U_{rs} = \sum_{h \in H_{rs}} U_{rs}^h \tag{13}$$

where,  $U_{rs}^h$  represents the utility of each train, and  $U_{rs}$  represents the sum of the utility of all trains, that is, the utility of passengers choosing HSR as a mode of travel.

Due to the strong substitutability of different trains in the same OD pair, the demand for flexibility of the HSR is generally an elastic demand for all railway passenger trains serving the same point pair.  $U_{rs}^0 = \sum_{h \in H_{rs}} |U_{rs}^h|$  is defined as the sum of the absolute values of all trains' utility. As the utility reflects

passengers' willingness to choose HSR and take a train, the elastic demand of HSR passengers between point pair (r, s) can be described as a function of the utility [40]:

$$q_{rs}(U_{rs}) = q_{rs}^{0} \exp(\eta_{rs}^{0} * \frac{\sum_{h \in H_{rs}} U_{rs}^{h}}{\sum_{h \in H_{rs}} |U_{rs}^{h}|}) = q_{rs}^{0} \exp(\eta_{rs}^{0} * \frac{U_{rs}}{U_{rs}^{0}})$$
(14)

Between the point pair (r, s),  $q_{rs}^0$  represents the demand corresponding to the generalized travel cost  $\bar{c}_{rs}^0$ , and  $\eta_{rs}^0$  is the elastic coefficient. As above, when  $U_{rs}$  is positive, the utility is positive, that is, the selection of HSR showing the overall value of passengers increases, prompting more passengers to choose HSR and increasing the passenger flow. If it is negative, the overall value of passengers is reduced, and some passenger flow is lost.

For train  $h \in H_{rs}$  serving the same pair (r, s), the function of passenger travel demand can be obtained by decomposing the elastic demand  $q_{rs}(U_{rs})$  of all trains. According to the different utility of each train, the elastic demand  $q_{rs}^h(U_{rs}^h)$  of train  $h \in H_{rs}$  is divided by using logit distribution, which is a discrete selection method used in passenger flow distribution:

$$q_{rs}^{h}(U_{rs}^{h}) = q_{rs}(U_{rs})A(h, U_{rs}) = q_{rs}(U_{rs})\frac{\exp(\omega U_{rs}^{h})}{\sum_{j \in H_{rs}} \exp(\omega U_{rs}^{j})}$$
$$= q_{rs}^{0}\exp(\eta_{rs}^{0} * \frac{U_{rs}}{U_{rs}^{0}}) * \frac{\exp(\omega U_{rs}^{h})}{\sum_{j \in H_{rs}} \exp(\omega U_{rs}^{j})}$$
$$= q_{rs}^{0}\frac{\exp(\eta_{rs}^{0} * U_{rs}/U_{rs}^{0} + \omega U_{rs}^{h})}{\sum_{j \in H_{rs}} \exp(\omega U_{rs}^{j})}$$
(15)

where  $\omega$  is the adjustment factor. The passenger flow of each train not only depends on its own utility, but also is also affected by the overall utility of all trains on the HSR.

The main purpose of considering a differentiated pricing strategy is to realize revenue management for the HSR. Therefore, it is realistic to optimize ticket sales revenue. The differentiated pricing strategy is reflected in the fact that each train is regarded as a product. Appropriately raising and lowering the price of certain tickets creates a price difference, and the appropriate tickets are sold to passengers who need them more to improve the overall passenger load rate and maximize profit. The ticket sales income is expressed by R(p). The optimization model is as follows:

$$\max R(p) = \sum_{(r,s) \in W} \sum_{h \in H_{rs}} p_{rs}^{h} * q_{rs}^{h}(U_{rs}^{h})$$
(16)

s.t.

$$\sum_{(r,s)\in W} q_{rs}(U_{rs}^h) \le C(h) \quad (r,s)\in W, h\in H_{rs}$$
(17)

$$\widehat{p}_{rs}^{h} \ge p_{rs}^{h} \ge \widetilde{p}_{rs}^{h} \quad (r,s) \in W, h \in H_{rs}$$
(18)

$$q_{rs}^h(U_{rs}^h) \ge 0 \quad (r,s) \in W, h \in H_{rs}$$
<sup>(19)</sup>

$$p_{ij}^h > p_{mn}^h \quad \forall (i,j), (m,n) \in W, h \in H_{rs}, l_{ij} > l_{mn}$$
 (20)

$$p_{ij}^{h} + p_{jk}^{h} \ge p_{ik}^{h} \quad \forall (i,j), (j,k) \in W, h \in H_{rs}$$
 (21)

$$p_{rs}^{h} \in N \quad (r,s) \in W, h \in H_{rs}$$

$$\tag{22}$$

Among them, Formula (17) is the capacity constraint, and it represents that the total ticket sales of all OD pairs in any section shall not exceed the maximum transport capacity of the train. Formula (18) is a fare constraint, which indicates that the fare of each section shall not exceed the published fare, nor shall it be lower than the lowest limit of the fare, guaranteeing reasonable HSR ticket prices. Formula (19) indicates that the demand is not negative, and Formula (20) means the fare is increased

with the OD distance. Formula (21) is an upside down constraint, showing that the sum of fares of each section on an OD should be no less than the OD fare, and Formula (22) is an integer constraint of the fare.

## 5. Solution Algorithm

Since the model of this paper is large in scale and has certain randomness, a heuristic algorithm is needed to obtain the optimal solution rather than the exact solution. Simulated annealing (SA) is a kind of optimization algorithm based on the Monte Carlo iterative solution strategy. It uses the probability jump feature to randomly find the global optimal solution of the objective function in the solution space, that is, it can jump probabilistically when it falls into the local optimum and tends toward global optimality eventually [41].

#### 5.1. Generate Initial Solution

The SA algorithm is based on the initial solution for optimal iterative calculation. Therefore, the first feasible solution to the problem needs to be obtained when using the algorithm. The initial solution of the problem can be generated stochastically within the upper and lower limits of the fare  $(\vec{p}_{rs}, \vec{p}_{rs})$ , and each OD fare  $p_{rs}^h$  for each train is generated freely, which satisfies Formulas (20)–(22), so fare combination *p* of the initial solution is obtained, as shown in Figure 2. The elastic passenger flow of each OD is calculated according to Formulas (1)–(14), and logit distribution is performed according to Formula (15) to judge whether each train meets the capacity limit of Formula (17). If the passenger flow exceeds the capacity of the train, the excess is unloaded and reloaded onto other trains until the capacity constraints are met. Finally, through Formula (16), the ticket sales revenue R(p) is obtained as the initial solution.

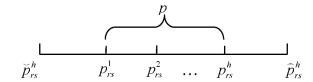


Figure 2. Initial fare for each origin-destination (OD) train.

# 5.2. Neighborhood Structure

In order to improve the search efficiency of the SA algorithm, it is necessary to design an efficient neighborhood construction method so that each search is carried out as far as possible toward the optimal solution. In terms of fares, a new range of fares can be constructed based on the current fare combination to find a better one. Focusing on the current solution p, we reduce or increase the OD fare of each train and construct a neighborhood ( $p - \Delta p, p + \Delta p$ ) that satisfies the constraint condition. The ticket price of each train changes slightly, and a neighborhood solution  $p^{\circ}$  of the current solution p is obtained, as shown in Figure 3.

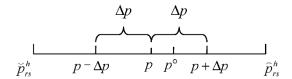


Figure 3. Neighborhood structure of high-speed rail fare.

## 5.3. Specific Steps of SA Algorithm

In addition to initial solution and neighborhood construction, the SA algorithm also includes cyclic iteration and algorithm termination. The detailed steps of the SA algorithm are as in Algorithm 1.

Input: initial temperature  $T_0$ , temperature drop ratio  $\alpha$ , final temperature t, price range  $(p_{rs}^h, p_{rs}^h)$ Output:  $R(\overline{p}), \overline{p}$ for  $T_{i+1} = T_0$  to  $T_{i+1} = t$  do Set  $R(\overline{p}) \leftarrow 0$ ; for K = 1 to K = L do Generate initial solution  $R(p) \leftarrow p$ ; Construct a neighborhood solution of the current solution  $p^\circ \rightarrow R(p^\circ)$ ; Update the global optimal solution: if  $R(p) \ge R(\overline{p}), \overline{p} \leftarrow p^\circ, R(\overline{p}) \leftarrow R(p^\circ), p \leftarrow p^\circ, R(p) \leftarrow R(p^\circ)$ ; if  $R(p) < R(\overline{p})$ , acceptance probability  $P(A) = \exp(-\Delta R/T_i)$ , generates a random number  $\zeta$  from (0,1), satisfy  $\zeta < P(A), p \leftarrow p^\circ, R(p) \leftarrow R(p^\circ)$ ;  $T_{i+1} = \kappa \cdot T_i$ 

## 6. Examples and Results

The model proposed in this paper can be applicable to the pricing systems of all HSRs, and the Beijing–Shanghai corridor in China is only used as an example to prove the validity and practicability of the model. In order to study the impact of departure time on differentiated pricing, eight trains of the G2, G4, G6, G8, G12, G14, G16, and G18 lines were selected as the research objects. As shown in Figure 4, stations including Shanghai, Changzhou, Nanjing, Xuzhou, Jinan, and Beijing generated 15 OD pairs in total. The same stations for each train include Shanghai, Nanjing, and Beijing. The departure times of the stops, from morning to night, are shown in Table 2. The maximum number of people in all trains was 1005.

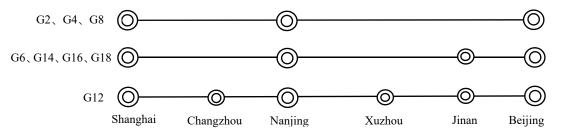


Figure 4. Stopping scheme for several trains.

Tal	ble	2.	Depar	ture	times	of	trains.	
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Starting Station	Shanghai	Changzhou	Nanjing	Xuzhou	Jinan
G2	09:00		10:09		
G4	14:00		15:09		
G6	07:00		08:09		10:23
G8	19:00		20:09		
G12	08:00	08:42	09:16	10:33	11:44
G14	10:00		11:09		13:23
G16	11:00		12:09		14:23
G18	15:00		16:09		18:23

Regarding the parameter values, according to many experiments, the general values of the parameters in prospect theory are  $\alpha = \beta = 0.88$ ,  $\lambda = 2.25$ ,  $\gamma = 0.61$ , and  $\delta = 0.69$ . Based on the classification of passengers, the time sensitivity adjustment coefficient is set as  $\mu_{rs}^1 = 0.8$ ,  $\mu_{rs}^2 = 1.0$ ,  $\mu_{rs}^3 = 1.2$ . Since there are many economy tourists, accounting for 80% of the passenger market, the elasticity coefficient is taken as  $\eta_{rs}^0 = 1.33$  and the passengers' imagined price is assumed to be a rational expectation (i.e.,  $\xi_{rs}^i = 0$ ).

#### 6.1. Differential Pricing

The effect of travel history on passenger travel choices is represented by a weighting factor  $\rho$ , which impacts travelers differently. Therefore, the influence of the coefficient on the yield of HSR in different periods is studied, as shown in Figure 5. An increase in the coefficient means that passengers' travel history has a greater impact, and their judgment on the expected travel cost is more dependent on past experience. As can be seen from the figure, as the coefficient increases, ticket sales revenue shows a downward trend in peak periods. Due to the increased ticket sales during peak hours, passengers are affected by past experience, and high fares tend to curb their desire to purchase tickets. Increased ticket sales revenue during the off-peak period with the increased coefficient is due to the decrease in fares stimulating passengers' purchasing desire. However, in general, total revenue is almost a straight line, and there is no significant change with increased coefficient. It can also be said that the coefficient has little effect on total revenue.

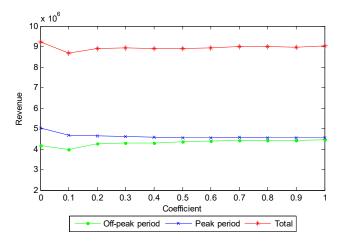


Figure 5. The impact of historical travel on revenue.

Algorithm 1 is used to optimize the fares of each train and OD. By reducing price in off-peak periods to attract passenger flow and increasing them in peak periods, the integrated passenger flow from Beijing to Shanghai can be significantly improved, while also maximizing ticket sales revenue. On the basis of statistical data, peak and off-peak periods of passenger flow were optimized at the same time. The optimized fares after differential pricing are shown in Tables 3 and 4. It can be seen that at the peak period, except for G18, the fares of other trains increased. The largest increase is for G12, which is 18.9% higher than the initial price. After the implementation of differentiated pricing, the difference between the highest and lowest price is more than 20%, and the difference in fare between trains is obvious. By raising train fares, HSR revenue increased by 7.98%, which is double the 3–5% of earnings in previous studies.

As shown in Table 4, during the off-peak period, the train fares (except for G6) were reduced in order to attract more passengers. Among them, G14 has the lowest fare, which is 16.4% lower than the initial price, and 20.5% lower than the highest price after differential pricing. Through the price reduction, income can be increased by 10.41%, and increased attendance and balanced passenger flow can be achieved. At the same time, lower fares can promote more travel demand and achieve sustainable development of the HSR.

Starting Station	Terminal	G2	G4	G6	G8	G12	G14	G16	G18	Initial Price
Shanghai	Changzhou					89				74.5
0	0	144	133	_ 147	 137	160	154	 142	132	134.5
Shanghai	Nanjing	144	133	147	137		154	142	132	
Shanghai	Xuzhou	-	-	-	-	332	-	-	-	279
Shanghai	Jinan	-	-	435	-	474	458	422	392	398.5
Shanghai	Beijing	593	547	604	561	658	635	585	544	553.5
Changzhou	Nanjing	-	-	_	_	71	-	-	_	59.5
Changzhou	Xuzhou	_	-	-	-	249	-	-	-	209
Changzhou	Jinan	-	-	-	-	397	-	-	-	334
Changzhou	Beijing	-	-	-	-	587	-	-	-	493.5
Nanjing	Xuzhou	-	-	-	-	178	-	-	-	150
Nanjing	Jinan	-	-	305	-	332	320	295	274	279
Nanjing	Beijing	475	439	484	450	527	509	469	436	443.5
Xuzhou	Jinan	_	-	-	-	154	-	-	-	129.5
Xuzhou	Beijing	-	-	-	-	368	-	-	-	309
Jinan	Beijing	-	-	201	-	219	211	195	181	184.5

Table 3. Train fares during the peak period, in RMB.

Note: - indicates that the train does not stop at this station.

Table 4. Train fares during the off-peak period, in RMB.

Starting Station	Terminal	G2	G4	G6	<b>G8</b>	G12	G14	G16	G18	<b>Initial Price</b>
Shanghai	Changzhou	-	-	-	-	68	-	-	-	74.5
Shanghai	Nanjing	126	118	136	116	122	113	133	123	134.5
Shanghai	Xuzhou	-	-	-	-	253	-	-	-	279
Shanghai	Jinan	-	-	402	-	361	333	393	363	398.5
Shanghai	Beijing	517	485	558	478	502	463	545	504	553.5
Changzhou	Nanjing	-	-	-	-	54	-	-	-	59.5
Changzhou	Xuzhou	-	-	-	-	190	-	-	-	209
Changzhou	Jinan	-	-	-	-	303	-	-	-	334
Changzhou	Beijing	-	-	-	-	448	-	-	-	493.5
Nanjing	Xuzhou	-	-	-	-	136	-	-	-	150
Nanjing	Jinan	-	-	281	-	253	233	275	254	279
Nanjing	Beijing	415	389	447	383	402	371	437	404	443.5
Xuzhou	Jinan	-	-	-	-	117	0			129.5
Xuzhou	Beijing	-	-	-	-	280	0			309
Jinan	Beijing	-	-	186	-	167	154	182	168	184.5

Note: - indicates that the train does not stop at this station.

#### 6.2. Elastic Passenger Flow

Due to differential pricing, the passenger flow of each OD changes accordingly. A comparison between peak and off-peak passenger flow before and after optimization is shown in Table 5. Regarding the OD pairs in Changzhou and Xuzhou, since only G12 operates without any competitions, it is regarded as just needed, that is, the utility is 0, which means the passenger flow remains stable. In other ODs, since the original off-peak passenger flow is small, the price reduction greatly increases the OD passenger flow, so that the utility is positive. The growth of passenger flow not only compensates for the loss of fares, but also increases ticket sales revenue by 10.41% compared with fixed fares. During the peak period, the original passenger flow is close to the limit of each train's capacity, so the fluctuation of OD passenger flow is small. In this case, ticket sales revenue increased by 7.98% by raising the prices.

At the peak period, there was no significant change in total elastic passenger flow relative to fixed fare. However, due to differentiated pricing, passengers are still affected by the choice of trains. The changes between elastic and initial passenger flow of ODs are shown in Figure 6. As can be seen in the figure, the peak hours of passenger flow among ODs are not the same: elastic passenger flow is closer to the peak time of the initial passenger flow. The farthest OD, Shanghai–Beijing, has the least fluctuation in passenger flow, and a higher fare in the morning with fewer passengers in the

distribution of passenger flow in one day. The change trend of passenger flow from Nanjing to Jinan and Jinan to Beijing, which have the second and third shortest distances, is the biggest. Generally speaking, increased ticket prices in the peak period have little influence on the choice of travel time expected by passengers.

Starting	Terminal _	<b>Off-Peak Period</b>			Peak Period			
Station		$q_{rs}^0$	$q_{rs}(U_{rs})$	$U_{rs}/U_{rs}^0$	$q_{rs}^0$	$q_{rs}(U_{rs})$	$U_{rs}/U_{rs}^0$	
Shanghai	Changzhou	161	161	0	183	183	0	
Shanghai	Nanjing	1485	1536(+51)	0.0606	1858	1904(+46)	0.0392	
Shanghai	Xuzhou	50	50	0	55	55	0	
Shanghai	Jinan	404	509(+105)	0.3968	504	468(-36)	-0.0979	
Shanghai	Beijing	4151	5109(+958)	0.3475	5187	5143(-44)	-0.0114	
Changzhou	Nanjing	37	37	0	41	41	0	
Changzhou	Xuzhou	17	17	0	19	19	0	
Changzhou	Jinan	13	13	0	14	14	0	
Changzhou	Beijing	102	102	0	113	113	0	
Nanjing	Xuzhou	23	23	0	23	23	0	
Nanjing	Jinan	95	101(+6)	0.1392	117	98(-19)	-0.2001	
Nanjing	Beijing	1443	1758(+315)	0.3339	1811	1993(+182)	0.1448	
Xuzhou	Jinan	10	10	0	11	11	0	
Xuzhou	Beijing	51	51	0	57	57	0	
Jinan	Beijing	377	469(+92)	0.3713	463	537(+74)	0.2239	
Original income			3,499,017			4,359,156		
Current	income	3,863,277			4,706,952			
Growt	h ratio		10.41%			7.98%		

Table 5. Changes in passenger flow of ODs during off-peak and peak periods.

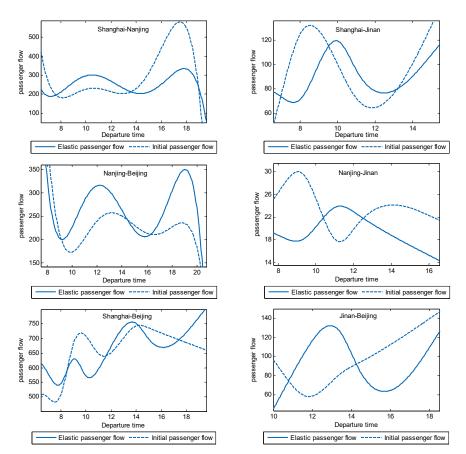


Figure 6. Changes in passenger flow of ODs at peak periods.

Compared with the peak period, passenger flow in the off-peak period greatly increases, and the distribution of each period is more in line with the situation of differentiated pricing. Most of the time, when ticket prices are relatively cheap, passenger flow increases significantly, as can be seen from Figure 7. The two longest ODs, Shanghai to Beijing and Nanjing to Beijing, have the largest passenger growth because of the great benefits of lowering prices. The shortest OD, Shanghai to Nanjing, cannot attract more passengers due to the low initial fare, despite the price reduction. The passenger flow of other ODs changes little, and the time distribution of passenger travel selection also change little compared with the initial passenger flow. In general, lowering fares has a great positive impact on increasing off-peak passenger flow, so differentiated pricing should be adopted to deal with the bleak season of ticket sales.

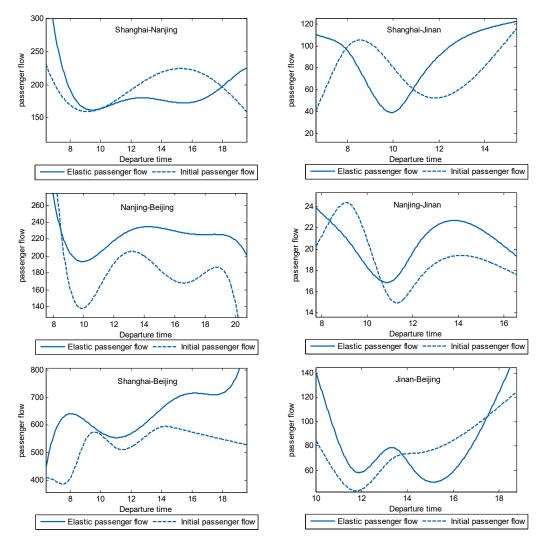


Figure 7. Changes in passenger flow of ODs at off-peak period.

From the perspective of sustainable development, passengers are diverted through differential pricing, achieving the goals of balanced passenger flow and increased attendance on the HSR. While attracting more people to travel on the HSR, it also limits the outbreak of passenger flow to a certain extent. In the long run, the price strategy will use the market adjustment mechanism to continue to meet the travel needs of more people and ease the burden at peak periods.

## 6.3. Impact of Passenger Expectations on HSR Revenue

In addition to increased HSR revenues from differentiated pricing based on prospect theory, other factors, such as passenger expectations, can work together to further increase revenue. Passengers expect travel costs to be affected by a variety of factors, including prices for previous travel, expected travel time, service experience, and perception of HSR's corporate image. This paper considers the impact of the most important factors in passenger expectations: fares and time. It is assumed that passengers fully understand market information, and reasonable travel costs for various types of passenger are obtained from the general and historical travel cost of each train. Although the rational passenger expectation in the study is a fixed value, to a large extent it is susceptible to rapid changes due to external factors. If passengers' acceptable expenses for travel improves their perception based on known rational expectations, which means  $\xi_{rs}^i > 0$ , that is, they are willing to pay higher prices and have a better understanding of the value of traveling by HSR, this will further increase HSR income, as shown in Table 6.

			Expect	ed Travel Cost	
		$\xi_{rs}^i = 0$	Increase by 1%	Increase by 2%	Increase by 3%
	Original income		3	3,499,017	
Off-peak period	Current income	3,863,277	4,241,360	4,521,127	4,807,496
	Growth ratio	10.41%	21.22%	29.21%	37.40%
	Original income		4	1,359,156	
Peak period	Current income	4,706,952	4,850,718	5,004,958	5,145,175
	Growth ratio	7.98%	11.28%	14.81%	18.03%

Table 6. Impact on	revenue by increas	ing passengers'	' expected travel	costs, in RMB.

As can be seen from Table 6, during the off-peak period, when passengers' expected travel costs increased by 1%, ticket sales revenue increased by 9%. At the peak, when passengers accepted a 1% increase, the HSR had a 4% increase in ticket sales revenue. This is expected to be significant for the realization of profitability management of HSR. By cultivating a good corporate image and improving service quality, the HSR can gain the loyalty and travel intention of passengers and can further enhance passengers' expectations and achieve continuous growth to realize revenue management. At the same time, figuring out how to improve passengers' expectations and studying changes in expectations are future research directions and key concerns for the sustainable development of HSR.

## 7. Discussion and Conclusions

This paper studied the differential pricing problem of HSR. Considering the impact of passengers' expected travel cost on their selection behavior, a fare optimization model is established based on prospect theory and uses the SA algorithm to solve it. The Beijing–Shanghai corridor in China is taken as the example for analysis, and some valuable research results were obtained. The results show that by applying prospect theory to differentiated pricing strategies, the goal of balancing passenger flow and improving revenue can be achieved.

First, the results indicate that based on optimizing the prices of trains during peak and off-peak periods with passenger flow data, there is a trend of lower prices during off-peak and higher prices during peak periods. The differences of fares among the trains make the distribution of passenger flow more balanced, and revenue increases of 7.98% and 10.41% under peak and off-peak passenger flow intensity, respectively, are achieved, which is double the 3–5% of earnings in previous studies. From the pricing of each train, G6 and G12 have higher prices, indicating that passengers prefer to travel between 7 and 8 am. That is to say, for HSR pricing, closer to passengers' expected price rather than the lowest price, HSR revenue can be maximized.

Second, in terms of passenger flow, not only does the passenger flow at off-peak periods increase under the stimulation of low prices, but also the decrease due to rising fares at peak periods is not obvious, and even some segments of passenger flow are seen to rise. This shows that passengers are greatly affected by low prices, and increased fares are still acceptable. On the whole, although the passenger flow fluctuates under the differentiated pricing, the overall trend is guaranteed to be stable, and a large number of demands are stimulated during the off-peak period, thus solving the problem of wasted energy.

Third, by studying the impact of passengers' expectations on ticket sales, it is found that if passengers expect a 1% increase, ticket sales revenue can increase by 4% during peak periods and reach 9% during off-peak periods from the example of the Beijing–Shanghai corridor. Since passengers' expectations have a huge impact on revenue, how to improve their expectations is a future research direction.

These results have important implications for HSR pricing strategies in the future. Actually, from our study, we see the impact of passengers' expectations on passenger flow and revenue, that is, increased expectations will directly increase HSR revenue. Research on passengers' expectations should be included in the formulation of pricing strategies, especially in the implementation of differentiated pricing. Differentiated pricing is a shortcut to achieve HSR revenue management and is the only way for the HSR to achieve sustainable development in China and the world. Therefore, the study in this paper offers some new thinking on future research directions. On the one hand, the in-depth study of passenger ticketing behavior and psychology should continue to develop a more comprehensive HSR pricing policy. On the other hand, research on how to evaluate and enhance passenger expectations is of great significance to the sustainable development of the HSR.

The study first proposes applying prospect theory to the HSR pricing system, and examples indicate that it achieved good results, but there are still some limitations in the study of passenger travel costs. This paper mainly considers the two important factors of time and fares, and other factors such as refunds and services are not considered. These factors should also be fully considered in future research.

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