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# A Variance Maximization Based Weight Optimization Method for Railway Transportation Safety Performance Measurement

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**Abstract:** Based on the idea of maximizing variances, a weight optimization method is proposed in this research to improve railway transportation safety evaluation. Firstly, the main evaluation indicators that can reflect the safety of railway transportation are selected as the independent variables. Secondly, in order to avoid the influence of experts' empowerment on the evaluation results of railway transport safety, fuzzy set theory is introduced to generate the variation range of the weights of each evaluation index, which is used as the constraint of weight optimization model. Then, the weight optimization model for railway transportation safety performance measurement is established based on the principle of maximum variance. The structure of the optimization model shows the characteristics of the quadratic programming model. Therefore, the optimal weight is calculated by using the branch bounded algorithm, which is one of the quadratic programming model solution algorithms. Finally, the empirical analysis of the safety performance measurement for 18 railway bureaus shows that using the optimized index weight for safety performance measurement can not only make full use of prior information but also ensure that 18 railway bureaus can be distinguished to the maximum extent.

**Keywords:** railway transportation; safety performance measurement; weight optimization; variance maximization

## 1. Introduction

The length of China's railway in operation has reached 127 thousand kilometers until the end of 2017, including 25 thousand kilometers of high-speed railways. As the main mode of passenger and freight transportation in China, railway transportation safety is of vital importance to the development of the national economy and society. However, the research on railway transportation safety performance measurement is still in its infancy in China. At present, some progress has been made in theoretical exploration and concrete practice in recent years to gain a better understanding of railway transportation safety performance measurement, including pre-risk analysis methods, checklist methods, risk matrix methods [1,2], tree-based assessment methods (such as event trees, fault trees and decision trees) [3–5] and probability-based risk probability estimation methods [6,7]. These methods make a significant contribution to railway transportation safety performance measurement in a complicated decision-making environment. However, most of these studies only consider the single factor, such as the number of railway crashes, as the main decision-making variable that

influences the railway transportation safety performance [8,9]. In fact, there are many other factors that affect the railway transportation safety performance, such as the personnel risk factors, equipment risk factors and management risk factors [10]. Moreover, the effects of different factors on the railway transportation safety are different. In general, in the process of railway transportation safety performance measurement, the weights of different factors which are used to distinguish the impact of different factors on the railway transportation safety are usually determined by expert scoring method [11,12] and different weights have a great influence on the assessment results. Therefore, how to determine the index weight reasonably becomes the key problem of railway transportation safety performance measurement. However, due to the complexity of railway transportation safety performance measurement and the limitations of railway transportation safety statistics, it is difficult to get the accurate index weights for railway transportation safety performance measurement evaluating the safety management level of the railway transportation company. In fact, the nature of railway transportation safety performance measurement actually belongs to the issues of multi-attribute decision-making. There are three kinds of reliable methods to solve the problem of weight optimization in multi-attribute decision making. First, one is subjective weighting methods, such as expert scoring, Delphi and analytic hierarchy process (AHP) [13]. The second type is the objective weighting method, such as entropy weight method [14], variation coefficient method, correlation coefficient method, weighted average planning method [15] and TOPSIS method [16,17]. The other type is the combination method of subjectivity and objectivity [18–20], such as ELECTRE method, fuzzy comprehensive evaluation method [21,22] and PROMETHEE and so forth [23]. Moreover, two weight optimization methods based on variance maximization are proposed in a previous research [24], which discusses the multi-index comprehensive evaluation model considering two different conditions respectively (for example with prior information and without prior information). It is obviously known that the based-variance-maximization method with initial weight information can not only make full use of prior information but also satisfy the normalized constraints and standardized evaluation requirements. Considering these advantages of this method, therefore, the weight optimization method of railway transportation safety performance measurement based on maximizing variance is proposed in this research. Firstly, based on the initial weights given by experts, the weight change interval is calculated by the method of expert group decision-making [25,26] based on fuzzy theory [27,28]. Thereafter, constrained by the fuzzy weighted interval and the principle of weight normalization, the optimal weight of each index of the railway transportation safety performance measurement is obtained by solving the quadratic programming model using the iterative optimization methods.

## 2. Study Area and Data Survey

This paper mainly focuses on the index weight optimization of railway transport safety performance measurement. Reasonable evaluation index which directly related to the accuracy of the evaluation results is the basis of railway transportation safety performance measurement and most of the existing researches evaluate the safety performance of the railway transportation bureaus in terms of human, equipment, environment and management and so forth [29] but most of the existing evaluation indexes for railway safety performance measurement are qualitative evaluation indexes, such as safety education of staff and workers, safe operating environment risk and Safety awareness of employees [10]. In order to ensure the objectivity of the safety performance measurement for railway transportation bureaus and the availability of the statistical data, four related quantitative indexes are proposed in this study. The equivalent incident rate is proposed to characterize the safety management effect of the railway transportation bureau. The proportion of security managers in all employees is introduced to explain the personal risk of railway transportation safety performance. The rate of the locomotive maintenance and repair is put forward to indicate the equipment risk of railway transportation safety performance. The safety complaint rate is introduced to characterize the management risk of the railway transportation safety performance in this research. It is important to note that all the statistical data required for the calculation of these quantitative indexes are from the

railway statistical yearbook, which ensures the objectivity of railway transportation safety evaluation to some extent. The specific meaning and calculation method of each evaluation index are as follows.

The million train-kilometer incident rate and one million passenger death rates are the two internationally accepted railway safety performance evaluation indexes proposed by the International Union of Railways (UIC) [10]. Combining with the advantages of such relative indexes facilitate horizontal comparison, the rate of equivalent incident (such as the equivalent number of incidents per unit of transport turnover) is put forward to indicate the safety operation effect of the railway transportation bureaus. The calculation method for this evaluation index is shown in Formula (1).

$$REI = \frac{\sum(N_i \times f_i)}{N_{AT}} \quad (1)$$

where  $REI$  is the rate of equivalent incident,  $N_i$  indicates the number of the  $i$ th level railway incident,  $f_i$  denotes the equivalent conversion factor of the  $i$ th level railway incident and the value of  $f_i$  refers to Table 1 and  $N_{AT}$  refers to the annual number of railway transport turnover.

**Table 1.** The equivalent incident conversion factor.

Levels of Railway Incident ( $i$ )	Equivalent Conversion Factor ( $f_i$ )
Special major incident	100
Major incident	15
Large incident	5
General type A incident	2
General type B incident	1
General type C incident	0.5
General type D incident	0.2

In addition to the rate of equivalent incident that characterizes the safety performance of railway transportation bureaus, the other three indexes were proposed in this study. The proportion of security managers in all employees is proposed to demonstrate the personnel risk of railway transportation bureaus. The rate of locomotive maintenance and repair is used to characterize the equipment risk of railway transportation bureaus. In addition, the complaint rate is proposed to interpret the safety management risk of railway transportation bureaus. The calculation method for the three evaluation indexes is shown in Formulas (2)–(4).

$$PSM = \frac{N_{SM}}{N_{TE}} \times 100\% \quad (2)$$

where  $PSM$  is the proportion of the security managers in all employees of the railway transportation bureaus,  $N_{SM}$  is the number of security managers of the railway transportation bureaus and  $N_{TE}$  is the total number of employees of the railway transportation bureaus.

$$RLM = \frac{N_{AM}}{N_{AD}} \times 100\% \quad (3)$$

where  $RLM$  is the rate of the locomotive maintenance and repair,  $N_{AM}$  is the annual number of locomotives repaired and  $N_{AD}$  is the annual number of disposable locomotives.

$$SCR = \frac{N_{AC}}{N_{AT}} \times 100\% \quad (4)$$

where  $SCR$  is the safety complaint rate of the railway transportation bureaus,  $N_{AC}$  is the annual number of complaints and  $N_{AT}$  is the annual number of railway transport turnover.

### 3. Weight Optimization Method for Railway Transportation Safety Performance Measurement

The railway transportation safety performance measurement is actually the comparison of the safety performance of multiple railway bureaus. Obviously, it is easy to obtain the evaluation results based on the comprehensive evaluation value with the explicitly given weight vector. However, due to the complexity of railway transportation safety performance measurement and the limitation of expert empowerment, direct application of expert weight information often results in a large gap between the evaluation results and the actual safety performance of railway transportation bureaus. In order to solve the problem of the randomness of the expert empowerment for railway transportation safety performance measurement, the expert group decision-making technology is applied in this study to process the initial weights in a fuzzy manner and the generated fuzzy weight intervals are introduced to weight optimization model as the restrictive. Finally, by using of the idea of variance maximization, the weight optimization model is established to optimize the weights of the railway transportation safety performance measurement.

#### 3.1. Weight Interval Division

One of the main reasons for the expert preference problem is the lack of assignment reference. However, rational weight interval division is conducive to improve the comparability and reasonability of the expert empowerment. Therefore, in order to guarantee the objective and rationality of expert empowerment, the weights of railway transportation safety performance measurement are divided into 10 levels according to the fuzzy hierarchy theory [30] and represented by triangular fuzzy numbers, as shown in Table 2.

**Table 2.** Fuzzy number representation of weight interval level.

Weight Interval Level	Fuzzy Numbers		
	$a_h$	$c_h$	$b_h$
1	0.00	0.05	0.10
2	0.10	0.15	0.20
3	0.20	0.25	0.30
4	0.30	0.35	0.40
5	0.40	0.45	0.50
6	0.50	0.55	0.60
7	0.6	0.65	0.70
8	0.70	0.75	0.80
9	0.80	0.85	0.90
10	0.90	0.95	1.00

#### 3.2. Expert Reliability Index Calculation

In order to reduce the influence of expert background information on the preference of experts, expert confidence index is introduced to improve the credibility of the initial weight of experts. In this research, experts' educational background, working years and professional title compose the expert confidence index, which is calculated by the Equation (5).

$$\omega_i = \sum_{i=1}^3 \alpha_i q_i \quad (5)$$

where  $\omega_i$  indicates the  $i$ th expert's confidence index,  $q_i$  is the credibility of the  $i$ th expert as shown in Table 3 and  $\alpha_i$  refers to weight values of the three influencing factors.

According to the importance of the different background information, the weights of the three influencing factors are respectively assigned. Here the weight value of the professional qualifications is 0.5, the weight value of the years of work experience is 0.4 and the weight value of the education background is 0.1.

**Table 3.** Fuzzy number representation of weight interval level.

Educational Background	Working Years	Professional Level	$q_i$
Graduate degree or above	more than 30 years	Senior engineers	1.0
Bachelor’s degree	20–30 years	Engineers	0.9
Associate degree	10–20 years	Assistant engineers	0.8
High school graduate	5–10 years	Skilled worker	0.7
Less than high school	1–5 years	Ordinary worker	0.6

3.3. Calculation of Weight Interval Limits

It is generally believed that the expert empowerment information conforms to the Gaussian distribution [31]. In other words, the membership probability of the weight will decrease with the increase of the distance away from the weight level given by expert. Then the probability that each index weight belongs to the  $h$ th weight interval can be obtained by using the following equation:

$$P_h = \begin{cases} \frac{(c_s - c_{s-h})}{\sum_{l=1}^{s-1} (c_s - c_l)} \times \frac{1-\omega}{2}, & 1 \leq h \leq s - 1 \\ \omega, & h = s \\ \frac{(c_{1+s-h} - c_k)}{\sum_{l=s+1}^{10} (c_l - c_s)} \times \frac{1-\omega}{2}, & s + 1 \leq h \leq 10 \end{cases} \tag{6}$$

where  $p_h$  indicates the probability which one expert considers one index lying in the  $h$ th weight interval;  $\omega$  represents the expert confidence indicator;  $c_s, c_l, c_{s-h}$  and  $c_{1+s-h}$  refer to the median values of each weight intervals as shown in Table 1;  $h, l$  and  $s$  refer to the level of the weight interval.

The membership function for each index weight can be obtained with weighted average of the multiple experts’ initial weighting information. Thereafter, the mean and standard deviation of the weight of each indicator can be obtained according to Formulas (7)–(9) [31]. It is important to note that the mean and variance obtained by using the expert group decision making method directly affect the weight change interval. In order to ensure the rationality of the weight variation interval, special attention should be paid to the calculation process of the weight interval. Accordingly, according to the principle of  $\sigma$  criterion, the upper and lower limits of each weight change interval can be calculated by the following equation 10.

$$P_h = \sum_{j=1}^n P_{hj} / n \tag{7}$$

$$m = E(P) = \sum_{h=1}^{10} (c_h \times P_h) \tag{8}$$

$$\sigma = \sqrt{D(P)} = \sqrt{\sum_{h=1}^{10} [(c_h - E(P))^2] \times P_h} \tag{9}$$

$$a = m - \sigma, b = m + \sigma \tag{10}$$

where  $P_h$  refers to the probability which one index weight lying in the  $h$ th weight interval,  $j$  and  $n$  represent the number of the experts,  $c_h$  indicates the intermediate value of the  $h$ th weight interval as shown in Table 2,  $m$  and  $\sigma$  denote the mean and the standard deviation of the weight probability distribution,  $a$  and  $b$  refer to the upper and lower limits of the weight change interval.

3.4. Weight Optimization Model Based on Variance Maximization

As mentioned above, the railway transportation safety performance measurement in this research is actually the comparison and sorting based on the safety performance of multiple railway bureaus. This issue belongs to the category of the multiple attribute decision problems. On the basis of making full use of expert experience and knowledge, the initial weight intervals are calculated as the constraint

of the weight optimization model. Moreover, when ranking the evaluation objects, the evaluation index with larger variance should be given a greater weight, because the index is more beneficial to evaluation and ranking. In other words, with the condition that the initial weight constraint and the normalized principle constraint are satisfied, the optimal weight should maximize the total variance of all evaluation indicators for all evaluation objects. In view of this principle, the weight optimization model of railway transportation safety performance measurement is proposed in this article.

Let  $R = \{r_1, r_2, \dots, r_n\}$  represent the evaluation object collection,  $G = \{g_1, g_2, \dots, g_m\}$  represent the evaluation index collection and  $W = \{w_1, w_2, \dots, w_m\}$  denote the index weight vector, which satisfies  $w_i \geq 0$  and  $\sum_{i=1}^m w_i = 1$ .

$$\begin{aligned} \max D(w) &= \max \sum_{i=1}^m D_i(w) = \max \sum_{i=1}^m \sum_{j=1}^n D_{ij}(w) = \max \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^n (x_{ij} - x_{ik})^2 w_i^2 \\ \text{s.t. } &0 \leq a_i \leq w_i \leq b_i, \sum_{i=1}^m w_i = 1 \end{aligned} \quad (11)$$

where  $D_{ij}(w)$  indicates that for the evaluation attribute  $g_i$ , the deviation of the object  $r_j$  and all the other evaluation objects.  $D_i(w)$  indicates that for the evaluation attribute  $g_i$ , the variance of all evaluation objects and all the other evaluation objects.  $D(w)$  indicates that for the all attributes, the total variance of all evaluation objects and other evaluation objects.  $w_i$  indicates the  $i$ th evaluation index weight.  $i = (1, 2, \dots, m)$  is the number of evaluation index,  $j = (1, 2, \dots, n)$  and  $k = (1, 2, \dots, n)$  is the number of the evaluation object. The  $a_i$  and  $b_i$  is the upper and lower limits of the interval. In addition,  $x_{ij} = g_i(r_j)$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) represents the evaluation value of object  $r_j$  in evaluation attribute  $g_i$ , so the comprehensive evaluation value of object  $r_j$  is  $x_j = \sum_{i=1}^m w_i x_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ).

The value of the railway safety evaluation indicator ( $x_{ij}$ ) can be calculated by Formulas (1)–(4). Judging from the model form, it belongs to a conventional quadratic programming model, while the coefficient matrix of this model is a positive definite or semi-definite matrix. The active set algorithm is correspondingly proposed to solve this convex quadratic programming model [32,33]. However, while the coefficient matrix is a negative definite matrix, the model is a non-convex quadratic programming problem. Moreover, it is difficult to obtain a global optimal solution using traditional numerical optimization methods [34]. A branch and bound algorithm [35] is introduced to solve this non-convex quadratic programming problem to obtain the optimal weight vector.

In summary, the weight optimization algorithm for railway transportation safety evaluation is established as following:

- Step 1: according to the calculation Formulas (1)–(4) of the evaluation index and the basic data from the china railway yearbook, the evaluation index value of each evaluation object is obtained and the decision matrix of the evaluation index value is established.
- Step 2: according to the specific meaning of the evaluation indicators, the infinitude no dimension method is used to obtain a standard decision matrix
- Step 3: the fuzzy decision-making method is proposed to calculate the weight change interval (such as the value of  $a_i$  and  $b_i$ ) based on the initial weight given by expert scoring. Moreover, this interval can be used as the constraint of the weight optimization model.
- Step 4: the numerical optimization algorithm is correspondingly proposed to solve the weight optimization model. Finally, the optimal weight vector is obtained for railway transportation safety performance measurement.

#### 4. Case Study

In order to illustrate the effectiveness of the weight optimization method, based on the statistical data of 18 railway bureaus in 2015 published by the China Railway Corporation [36], a case study was conducted in this section.

##### 4.1. Calculate the Value of Evaluation Index

According to the calculation Formulas (1)–(4) of railway transportation safety evaluation indexes in Section 2, the evaluation index values of 18 railway bureaus can be obtained by using the Equations (1)–(4), which are shown in Table 4. It should be noted that the basic data required for the calculation of all evaluation indicators come from the China Railway Yearbook in 2015 [36], which mainly includes the annual number of incidents of various types, the annual turnover volume, the number of employees in the company, the number of security managers of the company and the number of security complaints.

**Table 4.** The evaluation indexes values of 18 railway bureaus.

Railway Bureau	The Rate of Equivalent Incident(1/million ton*kilometer)	The Rate of the Locomotive Maintenance (%)	The Proportion of the Security Managers (%)	The Complaint Rate(1/million ton*kilometer)
Harbin Railway Bureau	0.26	6.4	13.20	1.81
Shenyang Railway Bureau	0.37	11.8	13.97	0.97
Beijing Railway Bureau	0.18	5.4	16.40	3.13
Taiyuan Railway Bureau	0.33	9.1	20.57	1.18
Hohhot Railway Bureau	0.23	4.8	18.06	0.58
Zhengzhou Railway Bureau	0.07	9.3	15.69	1.26
Wuhan Railway Bureau	0.17	7.1	14.59	0.76
Xian Railway Bureau	0.32	8.6	16.49	0.38
Jinan Railway Bureau	0.05	8.0	19.70	1.19
Shanghai Railway Bureau	0.01	6.8	15.13	1.00
Nanchang Railway Bureau	0.33	6.9	19.12	8.04
Guangzhou Railway Group	0.12	11.0	10.52	0.76
Nanning Railway Bureau	0.47	10.8	15.20	1.60
Chengdu Railway Bureau	0.39	8.4	14.63	2.21
Kunming Railway Bureau	0.24	7.5	23.49	1.42
Lanzhou Railway Bureau	0.17	10.0	16.05	1.12
Urumqi Railway Bureau	0.25	10.1	25.89	0.74
Qingzang Railway Bureau	0.16	8.2	19.09	3.83

##### 4.2. Index Dimensionless Processing

In order to eliminate the influence of indices' dimensions on the evaluation result, the infinitude dimensionless method is applied to make the indices being dimensionless and the results are shown in Table 5. According to the definition of the evaluation indexes, the four evaluation indicators are all fixed-value evaluation indicators. In other words, the closer the indicator value is to a certain fixed value, the better the evaluation result. Specifically, the more the equivalent incident rate and safety complaint rate indicators are close to 0, the better the safety management performance of the railway company is. The more the locomotive maintenance rate and personnel composition ratio are close to 100, the higher the safety and security level of the railway transportation company is.

**Table 5.** The results of index dimensionless.

Railway Bureaus	The Rate of Equivalent Incident	The Rate of the Locomotive Maintenance	The Proportion of the Security Managers	The Complaint Rate
Harbin Railway Bureau	0.45	0.93	0.85	0.77
Shenyang Railway Bureau	0.21	0.88	0.84	0.88
Beijing Railway Bureau	0.62	0.94	0.82	0.61
Taiyuan Railway Bureau	0.30	0.90	0.77	0.85
Hohhot Railway Bureau	0.51	0.95	0.80	0.93
Zhengzhou Railway Bureau	0.85	0.90	0.82	0.84
Wuhan Railway Bureau	0.64	0.93	0.84	0.91
Xian Railway Bureau	0.32	0.91	0.82	0.95
Jinan Railway Bureau	0.89	0.92	0.78	0.85
Shanghai Railway Bureau	0.98	0.93	0.83	0.88
Nanchang Railway Bureau	0.30	0.93	0.79	0.00
Guangzhou Railway Group	0.74	0.88	0.88	0.91
Nanning Railway Bureau	0.00	0.89	0.83	0.80
Chengdu Railway Bureau	0.17	0.91	0.84	0.73
Kunming Railway Bureau	0.49	0.92	0.74	0.82
Lanzhou Railway Bureau	0.64	0.89	0.82	0.86
Urumqi Railway Bureau	0.47	0.89	0.71	0.91
Qingzang Railway Bureau	0.66	0.91	0.79	0.52

#### 4.3. Index Weight Constraints

According to the weight interval calculation method, four railway transportation safety experts are invited to assign the initial weight of the four evaluation indicators. The weight interval that used as the restriction in the optimization model are given by the Formulas (5)–(10), the values are shown in Table 6.

**Table 6.** The upper and lower value of the weight interval.

Evaluation Index	Weight Interval $[a_i, b_i]$
The rate of equivalent incident	[0.3563, 0.5465]
The rate of the locomotive maintenance	[0.1861, 0.3256]
The proportion of the security managers	[0.0838, 0.2326]
The complaint rate	[0.0980, 0.2672]

#### 4.4. Establishment of Weight Optimization Model

The weight variation interval and index dimensionless value are incorporated in the weight optimization model. According to the Formula (11), the following quadratic programming model is established as shown in Equation (12). It is obvious that the weight optimization model for railway transportation safety performance measurement is a non-convex quadratic programming model with four inequality constraints and one equality constraint. Applying the branch and bound algorithm which is one of the numerical optimization methods to solve this non-convex quadratic programming model, the optimized index weight for rail transportation safety performance measurement are obtained as shown in the third column of Table 7.

$$\begin{aligned}
 \max D(w) &= 40.69w_1^2 + 14.80w_2^2 + 13.52w_3^2 + 42.27w_4^2 \\
 \text{s.t.} & 0.3563 \leq w_1 \leq 0.5465, 0.1861 \leq w_2 \leq 0.3256, \\
 & 0.0838 \leq w_3 \leq 0.2326, 0.0980 \leq w_4 \leq 0.2672, \\
 & w_1 + w_2 + w_3 + w_4 = 1
 \end{aligned} \tag{12}$$

#### 4.5. Results and Discussions

In terms of the optimized weight results, the optimized weight value of the equivalent incident rate index is the upper limit of the weight intervals. However, the optimized weight value of the locomotive maintenance rate index is the lower limit of the weight interval and the remaining two indicators (such as the proportion of the security managers and the complaint rate) obtain the intermediate value of the interval. The results of such optimization weights indicate that the equivalent incident rate index is the best one to distinguish the safety performance of railway transportation bureau, while the third evaluation index (the proportion of the security managers) is the weakest one of the four indexes to distinguish the safety performance of railway transportation bureau. One of the reasons for this result is applied the weight intervals which calculated based on expert prior knowledge as the constraint condition. Meanwhile, the other more important reason is due to the influence of the optimization ideas based on variance maximization. Simultaneously, in terms of the specific meaning of each indicator, only the first evaluation index (the rate of equivalent incident) is used to characterize the safety management effect of the railway transportation bureaus, while the others are used to interpret the safety management risk of the railway transportation bureaus. In other words, the first evaluation index (the rate of equivalent incident) can more effectively characterize the safety performance of railway transportation bureaus than the other three indexes (the rate of the locomotive maintenance, the proportion of the security managers and the complaint rate). Therefore, compared with the average of the weight intervals, the optimized weights can more reasonably illustrate the contribution of different indexes to the evaluation target (the safety performance of different railway transportation bureaus).

**Table 7.** Comparison between optimized weights with the initial weights.

Evaluation Index	The Average of the Weight Intervals	Optimized Weights
The rate of equivalent incident	0.4450	0.5465
The rate of the locomotive maintenance	0.2550	0.1861
The proportion of the security managers	0.1625	0.0838
The complaint rate	0.1375	0.1836

In order to illustrate the effect of the optimization weights on the assessment results, a comparative test by separately using initial weights mean and optimized weights is conducted in this study. Accordingly, the safety performance measurement results of the 18 railway transportation bureaus

are shown as Table 8. Judging from the ranking results, although there is no significant change except for Beijing railway bureau and Hohhot railway bureau. The comprehensive evaluation values of different railway bureaus with the optimized weights show a more distant distribution, it will be more conducive to differentiating the safety performance of different railway bureaus. Moreover, by comparing the actual safety management and operation of these different railway administrations, obviously the safety performance of Beijing railway bureau is better than that of the Hohhot railway bureau. Meanwhile, due to the equivalent accident rate of Guangxi railway bureau was higher than that of Nanchang railway bureau. the sorting result with the optimized weights shows the more actual safety performance compared the result with the initial weights.

**Table 8.** Comparison of weight optimization results.

Railway Transportation Company	The Assessment Results with Initial Weights		The Assessment Results with Optimized Weights	
	Comprehensive Evaluation Value	Sorting Results	Comprehensive Evaluation value	Sorting Results
Harbin Railway Bureau	0.684	12	0.6315	12
Shenyang Railway Bureau	0.586	15	0.5115	15
Beijing Railway Bureau	0.726	8	0.6933	7
Taiyuan Railway Bureau	0.614	14	0.5523	14
Hohhot Railway Bureau	0.734	7	0.6930	8
Zhengzhou Railway Bureau	0.858	3	0.8570	3
Wuhan Railway Bureau	0.785	5	0.7574	5
Xian Railway Bureau	0.649	13	0.5870	13
Jinan Railway Bureau	0.875	2	0.8806	2
Shanghai Railway Bureau	0.926	1	0.9381	1
Nanchang Railway Bureau	0.473	18	0.4013	17
Guangzhou Railway Group	0.828	4	0.8118	4
Nanning Railway Bureau	0.481	17	0.3816	18
Chengdu Railway Bureau	0.549	16	0.4659	16
Kunming Railway Bureau	0.690	10	0.6518	10
Lanzhou Railway Bureau	0.767	6	0.7422	6
Urumqi Railway Bureau	0.685	11	0.6484	11
Qingzang Railway Bureau	0.717	9	0.6926	9

Therefore, it can be seen that the assessment results with the optimized weights is closer to the actual safety performance of the railway transportation bureaus. The main reason is that the initial weight is a simple average of the expert's empowerment and the assessment results completely depend on the expert's prior knowledge, ignoring the bias caused by the expert's preference. However, the expert group decision-making technique is introduced to quantify the credibility of expert information

through the expert confidence index in this paper. Moreover, the fuzzy theory is applied to fuzzy the initial weights from expert scoring and the fuzzy weight intervals are simultaneously used as the restrictions in the weight optimization model. Then based on the idea of variance maximization, the index weights optimization model is established from the perspective of satisfying the norm of railway transportation safety performance measurement, which can not only make full use of the initial weight information from expert experience but also satisfy the normalized constraints of multi-index comprehensive evaluation.

## 5. Conclusions

In order to improve the reliability of the railway transportation safety performance measurement, a weight optimization method, based on the variance maximization, is proposed in this research. The information preference problem generated by the expert initial empowerment process is overcome by introducing the expert group decision-making technology and the deviation caused by the subjectivity of the decision expert is minimized. Then based on the idea of maximizing variance, the index weights optimization model is established and the optimal weight is calculated based on the prior information of standardized evaluation and normalized constraints. In the end, an empirical analysis of the safety performance measurement for the 18 railway bureaus under the China National Railway Corporation is conducted. It is verified that the application of optimized index weights for safety assessment not only makes full use of the prior information of experts but also can objectively reveal the safety management level of railway enterprises.

The essence of the method is to solve the weight optimization problem in multiple attribute decision making problems. Using the maximum variance of the evaluation result as the optimization objective function, the optimal weights are obtained by searching in the initial weight change intervals which are calculated by using the group decision making method. According to the measurement and sorting results based on the actual safety performance of the railway transportation bureaus, it is helpful to establish a targeted reward and punishment management strategy to improve the safety management level for the railway transportation safety management departments. However, this method is limited to the comparative analysis for the multiple evaluation objects (i.e., the safety management level of several railway transportation bureaus). It is not suitable for a railway transportation enterprise or certain railway transport lines to conduct a separate evaluation. In addition, the branch and bound algorithm, which is one of the numerical optimization methods, is applied to solve the weight optimization model in this study and the result is positive and interpretable but the comprehensive evaluation result, which is obtained by using the optimal weights, is not much better than the comprehensive evaluation results are calculated with the initial weights. There may be two reasons for this result. On the one hand, because the unquantifiable values of the evaluation indexes are calculated with the statistical data of the 2015 railway Yearbook, there may be some unobserved regularity leading to the optimization result tending to a local optimal solution. On the other hand, the numerical optimization algorithm exhibits better performance in solving the convex quadratic programming problems but the conventional branch and bound algorithm fails to reflect the characteristics for solving the non-convex quadratic programming model. So, the future direction for this research is to find a more effective and precise algorithm to solve the local convergence problem of the weight optimization model.

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