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Multiple Types of Plug-In Charging Facilities' Location-Routing Problem with Time Windows for Mobile Charging Vehicles

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Abstract: Increasing attention is being paid to the technology of battery electric vehicles (BEVs) because of their environmental friendliness. However, their short range, extended recharging times, and insufficient charging facilities hinder the improvement in the market share of BEVs. As a remedy, this paper presents a novel approach to providing a service for the battery charge replenishment of BEVs. Instead of using traditional alternative methods by only providing a charging service in a fixed location, such as battery-swapping and charging lanes, the novel charge replenishment is provided by mobile charging vehicles (MCVs), which could offer a charging service at any time and at location requested. To consider the limited running range and the opportunity to recharge from MCVs, as well as to determine the location strategy of multiple types of plug-in charging facility locations and the routing plan of the MCVs simultaneously, the location routing problem (LRP) that can integrate two decision levels, with a strategic level (location) and tactical level (routing), is applied. Then, we present the multiple types of plug-in charging facilities' location-routing problem with time windows for mobile charging vehicles (MTPCF-LRPwTW-MCVs), and formulate the MTPCF-LRPwTW-MCVs as a mixed integer linear program for the convenience of solving. To demonstrate the model, test instances are designed and computational results are presented. Furthermore, sensitivity analyses on battery capacity, recharging rate, and so on, are also examined. The results show that with the increase of the battery capacity or the improvement of the charging rate of the charging facilities, the service efficiency of the MCVs can reasonably be improved. Therefore, the proposed method could be used in real world problems.

Keywords: mobile charging services; battery electric vehicles; location-routing problem; time windows

1. Introduction

1.1. Motivation

It was shown that transportation was responsible for roughly 23% of the world-wide carbon dioxide emissions [1–4], therefore stimulating the revolution of the transportation sector as a result of global climate change. By reducing petroleum consumptions, carbon dioxide emissions, and the dependency on foreign oil, the battery electric vehicle (BEV), an emerging technology, could even reduce the direct carbon emissions by up to 60% in some cases, compared with conventional hybrid vehicles [5,6]. Furthermore, compared with conventional hybrid vehicles, BEVs have other advantages, such as energy efficiency and quiet operation, which make them ideal for urban areas [7]. Owing to their contribution to environmental protection, national governments have further popularized

BEVs through various incentives, such as incentives on purchase, free parking, and waivers on access restrictions [8,9]. Hence, the BEVs have been deployed increasingly in recent years. Lee and Han [10] estimated that sales of BEVs will maintain an annual growth rate above 25% by 2025. However, there are several barriers for people that are going to be taking a long journey preventing them from choosing BEVs, including an extended recharging time, reduced driving range, and so on. Therefore, the deployment of public charging infrastructures plays a critical role in nurturing the BEV market and promoting the adoption of BEVs [11,12]. Of course, the increasing use of BEVs and the deployment of charging stations will have an effect on the power system; Amini and Karabasoglu [5] studied the optimal operation of coupled electrified transportation and power networks.

The lack of sufficient refueling infrastructures greatly hinders the expansion of the BEV market share [13–15]. To increase the market share of the BEVs, many governments are planning to deploy charging stations in their regions, and a number of other strategies tackling their range have emerged [16]. But, it seems unrealistic to expect that range anxiety can be eliminated in the near future. Therefore, there are cases that a BEV could breakdown due to the lack of power en route. If so, rescue service should be provided for the brokendown BEVs. Compared with the traditional gasoline vehicles, it is not easy to refill the power of BEVs. Therefore, Huang et al. [17] presented a new charging service method where, according to the charging service request sent by the BEV, the mobile charging vehicle (MCV) arrives at the appointed place and provides a charging service for the car. The charging service requests can be sent via apps on smartphones, like Eyuechongdian. The request may contain information including the BEV identification, state of charge, charging time interval, location, and so on. After the requests are collected by the operators, mobile charging vehicles will be arranged for providing charging services. The detailed mobile charging service flow is shown in Appendix Figure A1. The mobile charging vehicle and charging equipment of the Eyuechongdian Company are shown in Appendix Figure A2. Appendix Figure A3 shows the mobile charging car providing a charging service for two vehicles at the same time.

If a number of charging service requests are received, there are two key problems for the operators, namely a strategic problem and a tactical problem. The strategic problem is that for the current service, operators need to know which charging stations to open to supplement the power of the mobile charging service. The tactical problem is the routing problem, which includes how to arrange the mobile charging vehicles, how to route the vehicles for more profit, which charging station should be used for power refill, and how much power should be recharged at the station, and so on. Generally, facility location and vehicle routing appear to be two interrelated decisions, so there is the danger of sub optimizing by separating the location and vehicle routing [18]. Therefore, in this paper, we formulate this problem of operators as a location-routing problem (LRP), to integrate two decision levels with strategic level (location), and tactical level (routing), and introduce the multiple types of plug-in charging facilities' location-routing problem with time windows for mobile charging vehicles (MTPCF-LRPwTW-MCVs). Following this, the related studies and our contributions are presented.

1.2. Literature Review

The relevant work can be classified into the following: (1) current battery charging methods of BEVs, (2) the introduction and related applications of LRP, (3) the one stream of LRP for locating refueling stations, and (4) another relevant stream of LRP for the vehicle routing problem with time windows. After a detailed presentation of the relevant research, we list our relevant contributions.

Until now, some alternative methods have been proposed, such as plug-in charging [16], 15 battery-swapping [19–21], dynamic wireless recharging [22], and charging lanes [23]. The most common charging method for BEVs is plug-in charging, which is static conductive charging via a cable and a vehicle connector when the BEV is parked. These chargers can be divided into different classifications according to the power rate, and then Yilmaz and Krein [24] defined three levels using the power rate.

The LRP is a research area within location analysis, with the distinguishing property of paying special attention to the underlying issues of vehicle routing [25]. Therefore, the LRP is clearly related to the location problem and the vehicle routing problem. Then, it is an NP-hard problem, as it encompasses two NP-hard problems (facility location and vehicle routing). There are many real-life applications, such as food and drink distribution [26], waste collection [27], military equipment location [28], bill delivery [29], optical network design [30], and so on.

The relevant stream of MTPCF-LRPwTW-MCVs is to locate refueling stations. For this problem, there are two mainstream approaches, including spatial approaches and flow-based approaches. Spatial approaches, such as the *p*-median approach, minimize the distances between the location of a facility and the possible customers that are served by this facility [31,32]. Furthermore, in this paper, the facility corresponds to a recharging station, and the customer to a BEV driver. In the *p*-median models, the BEV driver will visit the recharging stations before the battery is exhausted. The goal is to minimize the number of stations while maximizing the coverage, which is why it is called the spatial approach. However, due to the limited travel ranges of BEVs, complete coverage across the nation is impossible, so the median approach primarily targets urban transport planning problems [33].

The flow-based approach captures BEV traffic by inserting recharging stations on traffic routes where the traffic can be refueled, respecting the range limitations of the vehicles. One demand is assumed to be the vehicle flow between the origin–destination (OD) pair. A flow refueling location model (FRLM) has been proposed by Kuby and Lim [13], where a long route is divided into several segments, so that each of them can be traversed without recharging, and then a recharging station is deployed at the junction between the two adjacent segments. In general, the FRLM has been used in the literature to place charging stations [34,35].

In this paper, the multiple types of charging stations with different charging speeds are studied. The charging method is plug-in charging, and the multiple types are also classified according to the power rate. However, the charging station only provides a charging service for mobile charging vehicles. The multiple types of charging stations were first studied by Wang and Lin [36]. However they focused on maximizing the coverage of the BEV flows on paths, through the placement of multiple types of stations. Liu and Wang [22] were to assist the government planners on the optimal location of multiple types of BEV recharging facilities within a given budget, to minimize the public social cost. They formulated this problem as a tri-level programming.

Another relevant stream of MTPCF-LRPwTW-MCVs is the vehicle routing problem with time windows (VRPTW). VRPTW, where customer nodes have to be reached within a specified time interval [37], is an important area handling the realistic complications and generalizations of the basic routing model [38,39]. Time windows were applied in bank deliveries [7,40], postal delivers [41,42], 4 48 school bus routing [43,44], and dial-a-ride services [45,46]. Concerning the wide use of BEVs, Schneider et al. [47] 43 developed the electric vehicle routing problem with time windows (E-VRPTW) as an extension of VRPTW. In their study, two important characteristics of BEVs are focused on, which are the limited running range and the possibility to recharge at certain stations. For convenience, Schneider et al. [47] assumed that the BEVs must be fully recharged at the stations they have visited. To make the model more realistic, Keskin and Çatay [48] proposed a new model to find recharging strategies that could recharge BEVs partially and only charge the power required. Therefore, in this paper, the partial recharge strategies for the mobile charging vehicles are applied. This means that the recharging amount of the mobile charging vehicle is equal to what they need, and no more redundant power should be charged en route. A more detailed description on the partial charging strategies is shown in Section 2.2.

1.3. Contribution of This Paper

To the extent of our knowledge, until now, there has only been one study focused on the mobile charging service for BEVs, which was conducted by Huang et al. [17]. In the study, a queuing based analytical approach with a strategy called the 'nearest job next' was provided. However, our subject of this paper is totally different. In fact, the paper is regarding the optimization of minimizing the total distance of mobile charging vehicles to complete all of the charging requests from the BEV drivers at distributed locations. Then, we define the mobile charging service problem as MTPCF-LRPwTW-MCVs to determine the location strategy of multiple types of plug-in charging stations and the routing plan of mobile charging vehicles simultaneously. We formulate it as a 0–1 mixed integer linear program (MILP). In this paper, the limited running range and the possibility to recharge at certain stations are considered. Then, the time windows are considered, which are different from the Battery Swap Station for Electric Vehicles Location Routing Problem (BSS-EV-LRP) presented by Yang and Sun [4]. In their study, the time cost was not taken into consideration. Furthermore, the construction cost of the battery swap station is high, where the cost of a battery swap station is $500,000, evaluated by Yarow [49], and it has not been widely used presently. Secondly, a set of benchmark instances is designed based on the work by Schneider et al. [47]. As in reality, in this research, there are not so many rescue requests at the same time, and the largest instances include 15 customer nodes and 21 candidate recharging stations, which can be solved using commercial solvers. Then, the potential applications of the model are discussed extensively. Finally, the sensitivity analyses are conducted, such as battery capacity, recharging rate, budget, and so on.

The contributions of this paper can be summarized as follows:

(1) In order to avoid the disadvantages of the traditional charging methods, namely only providing charging service in a fixed location, the novel charge replenishment is presented, in which the charging service can be provided by the MCVs, which could offer charging service at any time and location requested.
(2) To avoid sub optimizing by separating the location and vehicle routing, we built the mobile charging service of the MCVs on the LRP to integrate two decision levels with the strategic level (location) and tactical level (routing).
(3) Using the linearization skill, we formulate the mobile charging service of MCVs as the multiple types of plug-in charging facilities' location-routing problem with time windows for mobile charging vehicles (MTPCF-LRPwTW-MCVs), and the model is formulated as a 0–1 mixed integer linear program (MILP) to ensure the accuracy of the solution.
(4) Based on the work by Schneider et al. [47], a set of benchmark instances are designed to verify the correctness of the model and the efficiency of the solution.
(5) Some reasonable sensitivity analyses are conducted to provide the operators of the MCVs with reasonable investment methods, such as the capacity of the battery of MCVs, the charging rate of the charging device, and so on.

1.4. Organization of This Paper

The remainder of this paper could be organized in the following manner. In Section 2, a detailed description of the mobile charging service is presented. The MILP formulation of the model is put forward in Section 3. As benchmarks, several instances are designed, where a network description and data preparation are given, and the numerical results for the instances with the sensitivity analyses are presented in Section 4. The conclusions and potential applications of the proposed model are included in Section 5.

2. Problem Description

2.1. Notations

For the readability of the paper, we have listed the notations used in Table 1.

Table 1. Indices, sets, parameters, and variables.

Indices	Definition
$0, N+1$	Index of depot instances and every route starts at 0 and ends at $N+1$
i, j	Index of nodes, $i, j \in V'_{0,N+1}$
(i, j)	Index of physical link between two adjacent nodes, $(i, j) \in A$
Sets	
Φ	Set of feasible types of plug-in charging station; $\varphi \in \Phi = \{1, 2, 3\}$, where 1, 2, 3 refer to the level 1, 2, and 3 charging station, respectively
F	Set of recharging station
F'	Set of dummy vertices of F
$F0'$	Set of dummy vertices of depot instance 0
F'_0	Set of recharging visits including depot instance 0
V	Set of customers
V_0	Set of customers including depot instance 0
V'	Set of customer vertices including visits to recharging stations, $V' = V \cup F'$
V'_0	Set of customers and recharging visits including depot instance 0, $V'_0 = V' \cup \{0\}$
V'_{N+1}	Set of customers and recharging visits including depot instance $N+1$, $V'_{N+1} = V' \cup \{N+1\}$
$V'_{0,N+1}$	Set of customers and recharging visits including depot instance 0 and $N+1$, $$V'_{0,N+1} = V' \cup \{0\} \cup \{N+1\}$$
Parameters	
φ	The φth type plug in charging station
B	The budget limit for building charging station
Cap	Capacity of mobile charging vehicles
d_{ij}	Distance of link $(i, j) \in A$
t_{ij}	Travel time of link $(i, j) \in A$
ε_φ	Recharging rate of type φ
b^φ	Cost of building a plug-in charging station of type φ
q_i	The power demand for one electric car in node $i \in V$
Q	Battery capacity for mobile charging service car
h	Charge consumption rate in traveling
α	The number of corresponding dummy nodes of one candidate charging station
u_i	Demand number of node $i \in V$, and $u_i = 0$ if i not belong to set V
e_i	Earliest start of service at node $i \in V$
l_i	Latest start of service at node $i \in V$, it is always a positive number of infinity
z_i	Service time at node $i \in V$
M	The sufficiently large constant
Variables	
s_i^φ	=1, if a charging station of type φ should be built in node i; otherwise = 0
τ_i	The time of arrive at node $i \in V'_{0,N+1}$
F_i	The recharging amount of electricity at node $i \in V'_{0,N+1}$
x_{ij}	$= 1$ if arc (i, j) is traveled; $= 0$ otherwise
y_i	The remaining battery on arrival at node i

2.2. Problem Description

In this paper, we study the mobile charging service problem. The problem has a single fixed depot; a set of customers and their electricity demand; a feasible set of candidate charging locations; and a homogeneous fleet of mobile charging vehicles powered by a rechargeable battery with a fixed capacity, starting from/returning to a common depot to handle the set of customers. For the set of customers, the BEVs lack power and should be served inside specific time windows, and then each customer node must be visited once. For the candidate charging station sites, we treat it as a dummy node set, whose distance from the corresponding station is zero to simply the model. In this paper, we assume that the depot could also be used as a charging station and could provide a charging service. As it is assumed that the network is congestion free, then, as the time and energy consumptions are both highly related to the travel distance, it is therefore reasonable to set the aim for routing the mobile charging vehicles as minimizing the total travel distance of the vehicles. Furthermore, it is also assumed that for the mobile charging vehicles, the state of charge will not only decrease with the traveled distance proportionally, but will also decrease at the customer nodes for providing the charging services. When the charging vehicles lack power, they may need to recharge their battery power at a charging station. In the service procedure, the mobile charging vehicles could arrive at or depart from a charging station with any state of charge. For less recharging time en route, it is reasonable to assume that the mobile charging vehicles depart from the depot fully recharged, and return to the depot with an empty battery if it has been recharged en route. This means that the recharging amount of the mobile charging vehicle is equal to what they need, and no more redundant power should be charged en route. There is a fixed construction cost associated with establishing a station at some candidate charging stations. In this paper, three types of charger can be selected. The charging rate of Level 1 is the fastest with the highest cost, and the charging rate of Level 3 is the lowest with the least cost. However, a candidate node can only build one type of charger, and the corresponding dummy node has the same charger. For the depot, because the vehicle can have a more adequate charging time, we assume that the third charger is built and it does not consume the total budget.

In order to make the description clearer, we present an example in Figure 1 with 10 BEVs charging requests at nodes $C1, C2, \cdots, C10$. As is shown, there are four candidate charging stations, $S1, S2, S3, S4$, and one depot (D). We assume that $S1$ and $S4$ build the first charging station, $S3$ builds the second charging station and the depot builds the third charging station, while there is no charging station built in $S2$ as no vehicle passes it. From Figure 1, one can confirm that all of the services could be completed by three vehicles with three different service paths, accordingly. The percentages of the state of charge for each vehicle arriving at/departing from a customer node or a charging station are presented along the links. From the figure, we can see that vehicle two and three visit charging stations en route, and that when they arrive at the depot, their states of charge have reached zero. Nevertheless, vehicle one completes the service procedure without any recharging and returns to the depot with the remaining 20% of the battery power. Note that a charging station could be visited several times by the same (see $S3$) or different vehicles, and furthermore, the candidate stations are not necessarily visited (see $S2$). To facilitate the establishment of a model, we formulate a charging station as corresponding to the dummy charging stations, whose distance from the station is zero. We assume that each dummy charging station can be visited at most once. The result of Figure 1 can be shown as Figure 2. In Figure 2, we assume that each charging station has two corresponding dummy nodes. A dummy node of the charging station can be visited at most once. Therefore, vehicle two visits charging station $S3$ twice, so we assume that vehicle two has visited its corresponding two dummy nodes, visiting node 14 for the first time, and visiting node 15 for the second time. From the small example, one still could not confirm whether the locations of the charging stations are optimal or not. In the next section, a mathematical model is proposed to find the optimal locations, and a MILP formulation is presented.

For the convenience of model building, the definition of some sets is stated here. For the depot, we set both 0 and $N+1$ to denote it, which is the same node. Note that every route starts at 0 and ends at $N+1$. Let $V = \{1, \cdots, \gamma\}$ be a set of BEVs with requests, where γ is the total number of customers. Therefore, $V = \{1, \cdots, 10\}$ and $\gamma = 10$ for the example shown in Figure 2. Let $F' = \{\gamma+1, \gamma+2, \cdots, \gamma+\alpha, \gamma+\alpha+1, \cdots, \gamma+\beta\alpha+1, \cdots, \gamma+(\beta+1)\alpha\}$ be a set of dummy nodes to permit several visits to each candidate charging station in the set $F = \{\gamma+1, \gamma+\alpha+1, \gamma+\alpha+2, \cdots, \gamma+(\beta-1)\alpha+1, 0\}$, where β is the total number of candidate charging stations and α is the number of corresponding dummy nodes of one candidate charging station. The subset $F'_0 = \{\gamma+\beta\alpha+1, \cdots, \gamma+(\beta+1)\alpha\}$ of F' is the dummy nodes of depot. In Figure 2, $\beta = 4$ and $\alpha = 2$, so $F' = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, $F = \{11, 13, 15, 17, 0\}$, and $F'_0 = \{19, 20\}$, which is not marked in Figure 2. Let $V' = V \cup F'$. For convenience, we set $F'_0 = F' \cup \{0\}$, $V'_0 = V' \cup \{0\}$, $V'_{N+1} = V' \cup \{N+1\}$, and $V'_{0,\ N+1} = V' \cup \{0\} \cup \{N+1\}$. Therefore, $N+1 = \gamma+(\beta+1)\alpha+1$. Let the set of links be $A = \{(i,j) | i,j \in V'_{0,N+1}, i \neq j\}$, and then the problem could be defined on a complete directed graph $G = (V'_{0,N+1}, A)$.

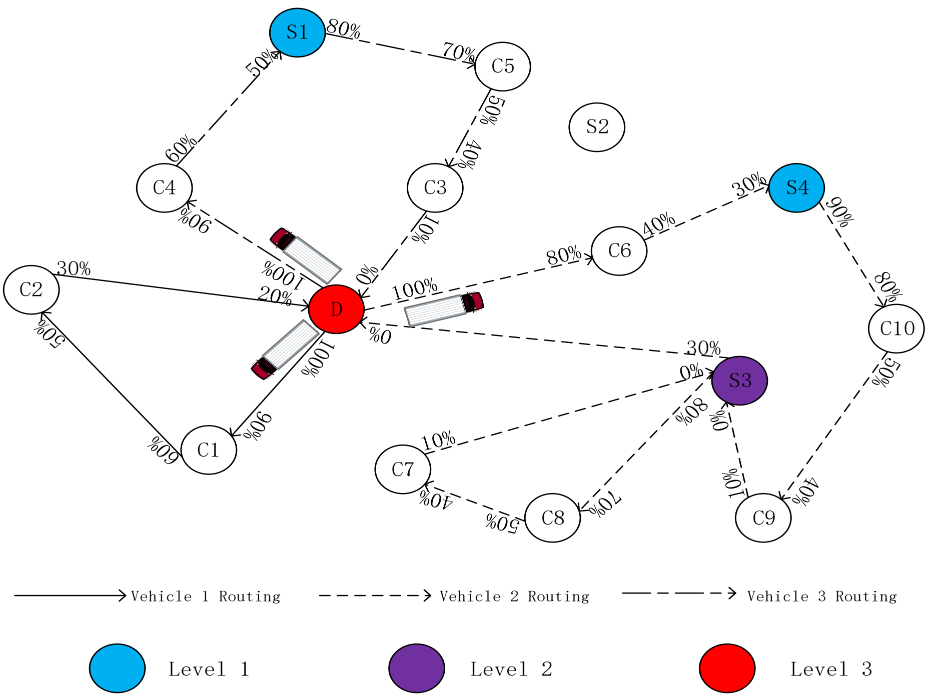

Figure 1. An illustrative example.

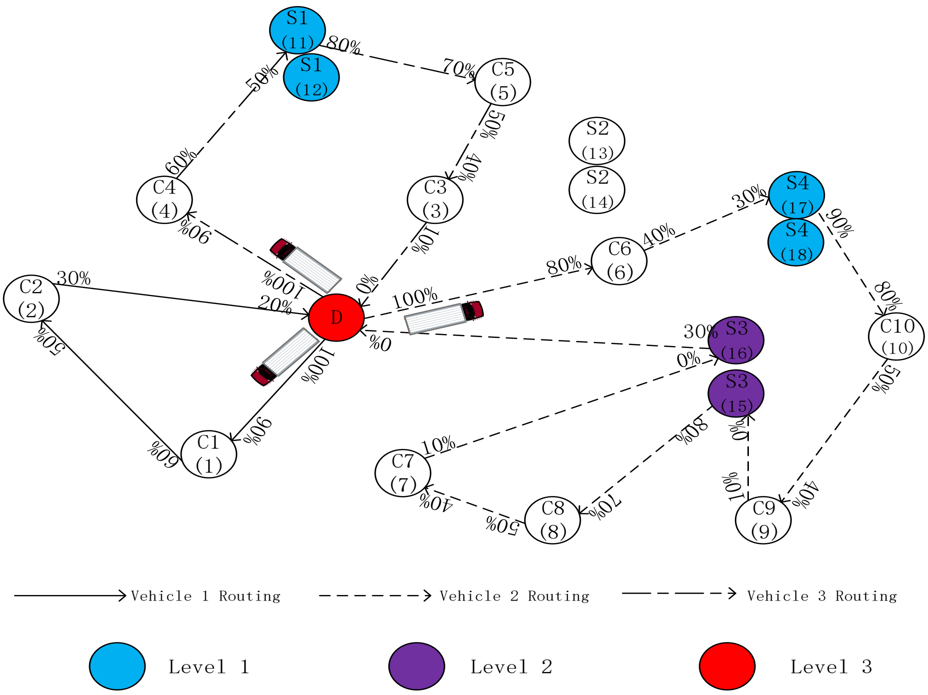

Figure 2. The transformation of Figure 1.

3. Optimization Framework

In this section, we develop a mathematical model of this problem. Two important characteristics of employing BEVs, namely a reduced operating rang and the possibility to recharge at certain stations to increase this range, are considered in this model. Some mobile charging service characteristics are simplified, where parameters such as the number of BEVs requesting the charging service, the electricity demand of each car, and service time windows in each customer node are set in advance. The partial recharge strategy is adopted in this section. For the partial recharge strategy, the MCV departs from the depot fully charged, but may depart from a charging station with any state of charge. Furthermore, the MCV returns to the depot with an empty battery when it has been recharged once during its route.

Then, we follow common Vehicle Routing Problem (VRP) modeling technologies and simplify several real-world characteristics. Firstly, a flat terrain is assumed and the case that vehicles increase their speed to fulfill time windows requirements is not in considered. Therefore, the travel speeds between every node are constant and given. Then, the recharging rate of φth charger is fixed and given as a constant, ε_φ, where $\varphi \in \Phi = \{1,2,3\}$. In the real world, the charging time increases for last 10%–20% of the battery capacity in the recharging processes [50]; for simplification reasons, the last assumption is presented. Finally, the charging services are performed by a homogeneous fleet of mobile charging vehicles with a fixed battery capacity, Q, and energy consumption rate, h. Neubauer et al. [51] found that the travel ranges of BEVs can vary by up to 60%, depending on the weather, temperature, use of in-car heater, speed, and so on. Then, a detailed travel range report was presented by Tesla, a maker of BEV, which reported that the travel range might be reduced by half when the speed is doubled [52]. Lee and Han [10] has focused on the uncertain travel range by a probabilistic consideration, which is simplified in this paper.

The mathematical model can be formulated as a mixed integer framework as follows:

Objective: Minimize the total traveled distance, as follows:

$$\min \sum_{i \in V'_0, j \in V'_{N+1}, i \neq j} d_{ij} x_{ij} \tag{1}$$

where d_{ij}, is a parameter that denotes the distance for each arc, it can be calculated by the locations of starting node and ending node of this arc. x_{ij} is a binary decision variable, and takes the value of 1 if the arc (i,j) is traveled, and 0 otherwise.

The charging station constraints are as follows:

$$\sum_{i \in F/\{0\}} \sum_{\varphi \in \Phi} s_i^\varphi b^\varphi \leq B \tag{2}$$

$$\sum\nolimits_{\varphi \in \Phi} s_i^\varphi \leq 1, \ \forall i \in F' \tag{3}$$

$$s_i^3 = 1, \ \forall i \in \mathrm{F0}' \tag{4}$$

$$s_i^\varphi = s_{i+1}^\varphi = \cdots = s_{i+\alpha-1}^\varphi, \ \forall i \in F/\{0\}, \ \forall \varphi \in \Phi \tag{5}$$

$$s_i^\varphi \in \{0,1\}, \ \forall i \in F', \ i \neq j, \ \forall \varphi \in \Phi \tag{6}$$

s_i^φ is a binary decision variable, which means whether a plug-in charging station of type, φ, should be built in the node, i. When the φth type charging station is built in the node, i, $s_i^\varphi = 1$; otherwise $s_i^\varphi = 0$. b^φ denotes the cost of building a plug-in charging station of type, φ. Then, B denotes the total budget limit for building the charging facilities. Constraint (2) is the budget limit, where $F/\{0\}$ denotes the set of candidate nodes where a plug-in charging station can be built, as the third type of charger has been built in the depot node $\{0\}$. Because all of the cars start from the depot and will return to the starting point after completing all of the tasks, therefore, a third type of charging equipment with the lowest charging rate can be built at the starting point to reduce costs, and all of the vehicles have enough time to charge. The dummy charging stations corresponding to the initial point should be the third charger station, which is limited by Constraint (4). Constraint (3) ensures that only one type of charging device can be built for each dummy node. Constraint (5) ensures that the charging type of the dummy nodes is the same as that of the corresponding charging station. Constraint (6) indicates the binary constraint for the decision variable.

The flow constraints are as follows:

$$\sum_{j \in V'_{N+1}, i \neq j} x_{ij} = 1, \ \forall i \in V \tag{7}$$

$$\sum_{j \in V'_{N+1}, i \neq j} x_{ij} \leq 1, \ \forall i \in F' \tag{8}$$

$$\sum_{i \in V'_{N+1}, i \neq j} x_{ji} - \sum_{i \in V'_{0}, i \neq j} x_{ij} = 0, \ \forall j \in V' \tag{9}$$

$$\sum_{j \in V'_{N+1}} x_{0j} \leq Cap \tag{10}$$

$$x_{ij} \in \{0,1\}, \ \forall i,j \in V'_{0,N+1}, \ i \neq j \tag{11}$$

where x_{ij} is a binary variable indicated by Constraint (11), it will equal one if an arc is traveled, and zero otherwise. Constraint (7) enforces the connectivity of the customer visits, which ensures that every customer could be visited only once by any mobile charging vehicle. Constraint (8) handles the connectivity of the visits to the dummy nodes, which guarantees the connectivity of visits to the dummy recharging stations and restricts each dummy node visited once at most by any mobile charging vehicle. Constraint (9), a flow conservation at intermediate nodes, enforces that the number of outgoing arcs equals the number of incoming arcs at each vertex. Constraint (10) controls the total number of vehicles by limiting the number outgoing from the depot $\{0\}$. In order to avoid the increase in the number of mobile charging vehicles due to an insufficient budget, we restrict the number of vehicles here.

The time constraints are as follows:

$$\tau_i + \left(t_{ij} + z_i\right)x_{ij} - l_0\left(1 - x_{ij}\right) \leq \tau_j, \ \forall i \in V_0, \ \forall j \in V'_{N+1}, \ i \neq j \tag{12}$$

$$\tau_i + t_{ij}x_{ij} + \sum_{\varphi \in \Phi} F_i s_i^{\varphi} \varepsilon_{\varphi} - (l_0 + \varepsilon_3 Q)\left(1 - x_{ij}\right) \leq \tau_j, \ \forall i \in F', \ \forall j \in V'_{N+1}, \ i \neq j \tag{13}$$

$$e_i \leq \tau_i \leq l_i, \ \forall i \in V'_{0,N+1} \tag{14}$$

As the travel speeds between every node are constant and given, and the distance d_{ij} can be calculated easily by the locations of the starting node and ending node of this arc, therefore, travel time, t_{ij}, is a predetermined parameter. For each customer node, the positive service time, z_i, and the time window is assumed to be predetermined also, and it could be noted as $[e_i, l_i]$. As a result of the constraints of the time windows, the mobile charging service cannot begin before e_i, and is not allowed to start after l_i, but it might end later. Therefore, the waiting time could be caused by it. l_0 is the end time of charging service, which equals to l_{N+1}. In order to track the arrival time of the mobile charging vehicles, a decision variable τ_i is defined for each node, i. F_i denotes the recharging amount in node i, which is obviously less than or equal to the battery capacity, Q. ε_{φ} represents the recharging rate of the φth charger.

Constraint (12) is the time feasibility constraints for the customer nodes and depot node with an instance of 0. Constraint (13) considers the recharge time instead of the service time when the previous node, i, is a recharging station. Constraint (14) enforces that every vertex is visited within the predetermined time window $[e_i, l_i]$.

The electricity constraints are as follows:

$$x_{ij}q_j u_j \leq y_j \leq y_i - hd_{ij}x_{ij} - q_i u_i x_{ij} + Q\left(1 - x_{ij}\right), \ \forall i \in V, \ \forall j \in V'_{N+1}, \ i \neq j \tag{15}$$

$$x_{ij}q_ju_j \le y_j \le y_i + F_i - hd_{ij}x_{ij} + Q\big(1 - x_{ij}\big),\ \forall i \in F',\ \forall j \in V'_{N+1},\ i \neq j \tag{16}$$

$$x_{0j}q_ju_j \le y_j \le Q - hd_{0j}x_{0j},\ \forall j \in V'_{N+1} \tag{17}$$

$$0 \le F_i \le \left(\sum_{\varphi\in\Phi} s_i^{\varphi}\right)M,\ \forall i \in F' \tag{18}$$

$$0 \le y_i + F_i \le Q,\ \forall i \in F' \tag{19}$$

$$0 \le q_iu_i \le Q,\ i \in V \tag{20}$$

For the mobile charging vehicle, the battery consumption rate is assumed to be a constant, h, so every traveled link consumes hd_{ij} of the remaining battery. Let y_i denote the remaining charge level on arrival at vertex $i \in V'_{0,N+1}$. Each vertex, $i \in V'_{0,N+1}$, is assigned a positive number of requests, u_i, and the demand of charging for each request, q_i, but both are zero when node i does not belong to set V. Hence, it is obvious that the total amount of charging could be computed as u_iq_i at node $i \in V'_{0,N+1}$. M is an infinity value.

Constraint (15), Constraint (16), and Constraint (17) keep track of the state of charge of the battery for the customer nodes, charging the dummy nodes and deport, respectively, and ensure that the battery state of charge never falls below the electric demand of the next customer node. Because, in the early assumptions, all of the vehicles are full when they leave the depot $\{0\}$, therefore Constraint (16) and Constraint (17) are different from the power when leaving the node where the electricity leaving node is equal to the sum of the remaining charge level, y_i, and the recharging amount, F_i, in Constraint (16), while equaling the capacity of battery Q in Constraint (17). Constraint (18) suggests that the MCVs can only recharge at the nodes with a charging station. Constraint (19) sets the upper and lower bounds of the battery state, which makes sure that the battery state of charge could not exceed its capacity and that the MCVs do not run out of charge. Constraint (20) represents the constraint for the demand of the customer nodes, which sets the upper and lower bounds of the demand.

However, there is a common bilinear term, $F_is_i^{\varphi}$, in Constraint (13), which is nonlinear and leads to the nonconvex property of the whole problem. To simplify the solution, the nonconvex problem here is transformed into an equivalent MILP by the reformulation-linearization technique (RLT) [53]. Therefore, commercial solvers can solve the latter problem directly and the optimal solution can be guaranteed.

Let $\theta_i^{\varphi} = F_is_i^{\varphi}$, for each $\varphi \in \Phi$ and $i \in F'$, to linearize the bilinear term. Thus, Constraint (13) can be rewritten as follows:

$$\tau_i + t_{ij}x_{ij} + \sum_{\varphi\in\Phi} \theta_i^{\varphi}\varepsilon_{\varphi} - (l_0 + \varepsilon_3Q)\big(1 - x_{ij}\big) \le \tau_j,\ \forall i \in F',\ \forall j \in V'_{N+1},\ i \neq j \tag{21}$$

Following the rules of RLT, $\theta_i^{\varphi} = F_is_i^{\varphi}$ is equivalent to the following linear constraints:

$$\theta_i^{\varphi} \ge 0 \tag{22}$$

$$\theta_i^{\varphi} - s_i^{\varphi}M \le 0 \tag{23}$$

$$\theta_i^{\varphi} - F_i \le 0 \tag{24}$$

$$\theta_i^{\varphi} - F_i + M - s_i^{\varphi}M \ge 0 \tag{25}$$

We can separately let s_i^{φ} equal to one, where the type of φth charger level is built in node i, or zero, where the φth charger level is not built in node i to prove the equivalence between these two. The plugged results are shown in following equations:

This time let $s_i^{\varphi} = 1$, then, Constraints (22)–(25) can be written as follows:

$$\theta_i^{\varphi} \ge 0 \tag{26}$$

$$\theta_i^{\varphi} - M \leq 0 \tag{27}$$

$$\theta_i^{\varphi} - F_i \leq 0 \tag{28}$$

$$\theta_i^{\varphi} - F_i \geq 0 \tag{29}$$

From Constraints (26)–(29), we can gain the result that $\theta_i^{\varphi} = F_i$, which means when the φth charger level is built in node i, θ_i^{φ} equals to the charging amount F_i. Then, as $s_i^{\varphi} = 0$, Constraints (22)–(25) can be written as follows:

$$\theta_i^{\varphi} \geq 0 \tag{30}$$

$$\theta_i^{\varphi} \leq 0 \tag{31}$$

$$\theta_i^{\varphi} - F_i \leq 0 \tag{32}$$

$$\theta_i^{\varphi} - F_i + M \geq 0 \tag{33}$$

From Constraints (30)–(33), we can get $\theta_i^{\varphi} = 0$, which means there is no a charging activity when the φth charger level is not built in node i. From the equivalence proof, $\theta_i^{\varphi} = F_i s_i^{\varphi}$ is equivalent to the linear Constraints (22)–(25).

Next, we will further deal with the muti-equality-based constraint, Constraint (5): $s_i^{\varphi} = s_{i+1}^{\varphi} = \cdots = s_{i+\alpha-1}^{\varphi}, \forall i \in F/\{0\}, \forall \varphi \in \Phi$.

In air traffic control, Constraint (5) is referred to as coupling constraints, while in stochastic programming, it is referred as non-anticipativity. To reduce the complexity of the solution constraint, the following three propositions are proposed and verified.

Proposition 1. *Constraint (5) equals essentially the following constraints:*

$$s_i^{\varphi} = s_{i+1}^{\varphi},\ s_{i+1}^{\varphi} = s_{i+2}^{\varphi},\ \cdots,\ s_{i+\alpha-2}^{\varphi} = s_{i+\alpha-1}^{\varphi},\ \forall i \in F/\{0\},\ \forall \varphi \in \Phi \tag{34}$$

Proposition 2. *Constraint (5) is equivalent to the following constraints:*

$$s_{i+d}^{\varphi} - \overline{s_i^{\varphi}} = 0,\ i \in F/\{0\},\ \varphi \in \Phi,\ d = 0, 1, \cdots, \alpha - 1 \tag{35}$$

$$\overline{s_i^{\varphi}} = \frac{1}{\alpha} \times \left(\sum_{d=0}^{\alpha-1} s_{i+d}^{\varphi} \right),\ i \in F/\{0\},\ \varphi \in \Phi \tag{36}$$

Proposition 3. *Constraint (5) is equivalent to the following inequality constraints:*

$$s_{i+d}^{\varphi} - \overline{s_i^{\varphi}} \leq 0,\ i \in F/\{0\},\ \varphi \in \Phi,\ d = 0, 1, \cdots, \alpha - 1 \tag{37}$$

Proof. As it is obvious for the necessary condition, therefore, the sufficient condition is proved in this part using contradiction. Firstly, we assume that Constraint (35) holds, while Constraint (5) is not established. Then, there must exist two nodes not equal, where $s_{i+1}^{\varphi} \neq s_{i+2}^{\varphi}$ is assumed. Therefore, one of two variables, s_{i+1}^{φ} and s_{i+2}^{φ}, has a value of zero. By Constraint (36), we can get $\overline{s_i^{\varphi}}$ is less than one strictly, and $\overline{s_i^{\varphi}} \leq \frac{\alpha-1}{\alpha}$. Let $s_{i+1}^{\varphi} = 1$, and $s_{i+2}^{\varphi} = 0$ without the loss of the generality. Through simple processing, we can obtain $s_{i+1}^{\varphi} - \overline{s_i^{\varphi}} \geq \frac{1}{\alpha} > 0$. That is all, the sufficient condition proved. □

Propositions 1 and 2 are easy to verify, so the certification processes are not stated here. Propositions 2 and 3 are simple forms of Constraint (5), and have equivalence. The corresponding constraints can be directly used in the model. So far, by applying the RLT technique and simple formula conversion, the model can be transferred into the following equivalent MILP form: Constraint (1) $\min \sum_{i \in V'_0, j \in V'_{N+1}, i \neq j} d_{ij} x_{ij}$ subject to Constraints (2)–(4), (6)–(12), (14)–(25), (35) or (37), (36).

Proposition 4. *In an optimal solution, if a mobile charging vehicle has been recharged at once and returns to the depot at the end of its route with a positive battery state, that is, $y^*_{n+1} > 0$, then, when the same cars return to the depot with empty battery, that is, $y^*_{n+1} = 0$, the solution is also optimal.*

Proof. Because there is plenty more time to charge the car, it therefore reduces the charging time so that the amount of electricity at the end of the route is exactly zero, that is, $y^*_{n+1} = 0$, and the solution is also an optimal solution. □

Corollary 1. *If one car does not consume all of the electricity at the end of the route, that is, $\overline{y}_{n+1} > 0$, and the solution is optimal, then the problem has infinite multiple optimal solutions.*

Proof. By contradiction, we assume, in the optimal solution, that one car does not consume all of the electricity at the end of the route, that is, $\overline{y}_{n+1} > 0$, while, when the car's charge is $\overline{y}_{n+1} - \varepsilon$, the solution will not be optimal, where ε is a small positive scalar. By Proposition 4, we can find in this case when the car's electricity is full consumed the solution is also optimal, while $\overline{y}_{n+1} - \varepsilon \geq 0$, so the multiple optima exist. □

4. Numerical Experiments

In this section, a number of numerical experiments are conducted to demonstrate the model. The computing device used in this research is a personal computer with Intel(R) Core (TM) i7 6700U 3.40 GHz CPU and 16.00 GB RAM, using the Microsoft Windows version 7 (Microsoft, Seattle, WA, USA) (64 bit) OS. The general purpose optimization package GAMS with solver CPLEX is employed as the modeling tool in the numerical experiment.

We are the first to study MTPCF-LRPwTW-MCVs, and therefore, there are no benchmark instances to evaluate the performance of this model. Some benchmark instances, based on the well-known VRPTW instances of Schneider et al. [47], are designed in Section 4.1. Section 4.2 presents the results of our studies on the generated instances. The sensitivity analysis to the input parameters, such as the total budget, recharging rate, and so on, is investigated in Section 4.3.

4.1. Generation of MTPCF-LRPwTW-MCVs Benchmark Instances

A set of 36 benchmark instances with 5, 10, and 15 customers and 21 candidate recharging stations per instance is created. The 36 instances include three main problem classes, where customer locations are clustered (C), randomly distributed (R), and both clustered and randomly distributed (RC) by geographical information. Furthermore, two groups are defined according to schedule horizon. Specifically, Group 1, including C1, R1, and RC1, contains the instances with short scheduling horizons, while Group 2, including C2, R2, and RC2, contains the ones with a long scheduling horizon. Therefore, for Group 1, more mobile charging vehicles will be requested to complete all of the charging services, and there are less mobile charging vehicles requested for Group 2.

Then, for the 21 candidate recharging stations, we first locate one recharging station at the depot and build the charger of Level 3 with the slowest charging rate. Because the mobile charging car has a long charging time at the depot, therefore, the above claim seems to be reasonable and the mobile charging car is fully charged when it first leaves the depot. For the remaining 20 candidate recharging stations, we adopt a random manner, which is similar to Schneider et al. [47]. Some reasonable restrictions are also applied, that is, each customer should be visited by the mobile charging vehicles depart from the depot with at most two different charging stations.

There are also other parameters to be determined, such as the power demand, q_i, of the BEV at the customer node; number of requests, u_i, at the customer node; the battery capacity, Q, of the mobile charging vehicle; and the recharging rate of the mobile charging vehicle. As we assume that the power demands of all BEVs at one customer node are all equal, the total power demand could be computed as the product of the power demand of one BEV, q_i, and the number of requests, u_i. In the research, the battery capacity of the BEVs request for charging services is to 15 kW·h. As every BEV driver's need for power may be different, here, we assume that in each instance, 90% of the power demand of the BEVs request for charging services at the customer node is set between 10 kW·h and 15 kW·h, and 10% of them is set between 5 kW·h and 10 kW·h. The specific values are randomly generated within the range. The number of requests could not exceed 10 and is generated randomly at each customer node.

As in reality, there are not too many requests simultaneously, the battery capacity of the mobile charging vehicles could be set as seven times the average value of the total power demand at the customer nodes. As the battery capacity of the mobile charging vehicle is larger, the charging rate of Level 1 is set as the average charging rate for the BEVs at the customer nodes, which could be calculated by dividing the service time by the total power demand. The charging rate of Level 2 is six times the average value, and Level 3 is twelve times the average value. The construction cost for the Levels 1, 2, and 3 plug-in charging stations are five, three, and one, respectively. The total budget cost is set as 10 and only one type of charging device can be built for each candidate charging station node. We set the total number of mobile charging vehicles to be less than four, which means $Cap = 4$. We assume that a charging station corresponds to a dummy charging station, which means a charging station can be visited at most once. Therefore, α, the number of corresponding dummy nodes of one candidate charging station, is set as one. For simplicity, we assume that there is no waste of electricity conversion from the mobile charging vehicles to the BEVs. Corresponding data of per instance is included in the Supplementary Materials, folder Section 4.2.

4.2. Performance on All Instances

Table 2 presents an overview of the baseline results. For the solutions obtained, the vehicle number, traveled distance, and the total recharging amount are given in columns #v, TD, and R_A. The location plans for the Levels 1, 2, and 3 obtained from the presented solution are given in columns $\varphi = 1$, $\varphi = 2$, and $\varphi = 3$. The total construction cost of some instances, such as $C101 - 5$, $C103 - 5$, and so on, is less than 10, the total budget, so the results are reasonable where there is no waste of construction cost.

Table 2. Baseline results.

No.	*Instance*	*#v*	*TD*	*R_A*	$\phi = 1$	$\phi = 2$	$\phi = 3$
1	*C*101–5	3	241.97	267.98	0	1	1
2	*C*103–5	2	163.77	34.83	1	0	0
3	*C*206–5	2	225.21	231.58	0	1	2
4	*C*208–5	1	162.89	282.06	1	1	0
5	*R*104–5	2	137.32	156.88	1	0	0
6	*R*105–5	2	154.4	105.74	1	0	0
7	*R*202–5	1	134.41	194.35	2	0	0
8	*R*203–5	1	178.31	153.76	1	1	1
9	*RC*105–5	2	227.19	0	0	0	0
10	*RC*108–5	2	246.54	62.94	1	0	0

Table 2. *Cont.*

No.	Instance	#v	TD	R_A	$\phi = 1$	$\phi = 2$	$\phi = 3$
11	RC204–5	1	172.49	169.06	1	0	0
12	RC208–5	1	168.21	153.85	1	0	1
13	C101–10	3	372.55	283.8	1	0	3
14	C104–10	2	271.72	249.76	1	1	1
15	C202–10	2	245.98	274.51	0	1	1
16	C205–10	2	232.92	410.85	1	1	2
17	R102–10	3	244.9	98.28	2	0	0
18	R103–10	3	202.01	116.7	2	0	0
19	R201–10	3	207.53	294.24	2	0	0
20	R203–10	1	226.4	289.04	1	1	2
21	RC102–10	3	395.18	141.88	2	0	0
22	RC108–10	3	345.35	189.61	1	1	0
23	RC201–10	2	308.59	271.78	1	1	1
24	RC205–10	3	340.51	301.86	0	3	0
25	C103–15	3	350.99	414.78	1	1	2
26	C106–15	4	301.64	229.71	1	1	2
27	C202–15	3	375.19	575.1	1	0	4
28	C208–15	2	309.62	380.09	0	1	3
29	R102–15	4	329.64	107.43	2	0	0
30	R105–15	3	292.81	216.15	2	0	0
31	R202–15	3	361.47	477.85	0	2	2
32	R209–15	2	281.41	353.83	1	1	2
33	RC103–15	4	391.34	74.01	2	0	0
34	RC108–15	3	359.98	269.52	2	0	0
35	RC202–15	4	433.09	228.75	1	1	2
36	RC204–15	2	349.82	410.29	1	1	2

Note: “$\#v$” denotes the vehicle number, “TD” the traveled distance, and “R_A” the recharging amount. “$\varphi = 1$”, “$\varphi = 2$”, and “$\varphi = 3$” denote the location plans for the Levels 1, 2, and 3, respectively.

4.3. Sensitivity Analyses

As the larger battery capacity leads to less charging time for the mobile charging vehicles and the faster charging rate makes it provide charging services more efficiently, the two factors may have an impact on the routing strategies of the mobile charging vehicles. Therefore, sensitive analyses on the battery capacity and charging rate of mobile charging vehicle are conducted in Sections 4.3.1 and 4.3.2. In the Section 4.3.3, the effect of the total budget on the results is further analyzed. If one charging station corresponds to more than one dummy node, then one charging station can be visited multiple times, which can increase the utilization efficiency of the charging station. Hence, the value of α, the number of corresponding dummy nodes of one candidate charging station, will be analyzed in Section 4.3.4.

4.3.1. The Sensitivity Analysis of Battery Capacity

For the battery capacity of the mobile charging vehicle, in this part, we set the battery capacity to be nine times the average value of the total power demand at the customer nodes. The charge rates of three level chargers do not change. (Corresponding data is included in the Supplementary Materials, folder Section 4.3.1, which is contained in folder Section 4.3). All of the sensitive analysis results for the battery capacity of the mobile charging vehicles are shown in Table 3. Column $\Delta v(\%)$ denotes the gap to the number of the employed vehicles found by Section 4.2, and $\Delta TD(\%)$ denotes the gap to the traveled distance, while columns $\#v$, TD, and R_A have the same meaning as Section 4.2.

Table 3. The sensitive analysis on different battery capacity.

No.	***Instance***	***Seven***			***Nine***			**Δv(%)**	**ΔTD(%)**
		#v	***TD***	***R_A***	***#v***	***TD***	***R_A***		
1	C101–5	3	241.97	267.98	3	234.72	127.25	0.00	3.00
2	C103–5	2	163.77	34.83	2	161.26	0	0.00	1.53
3	C206–5	2	225.21	231.58	2	219.54	47.46	0.00	2.52
4	C208–5	1	162.89	282.06	1	157.72	87.76	0.00	3.17
5	R104–5	2	137.32	156.88	2	136.45	22.72	0.00	0.63
6	R105–5	2	154.40	105.74	2	151.15	0	0.00	2.10
7	R202–5	1	134.41	194.35	1	126.83	80.17	0.00	5.64
8	R203–5	1	178.31	153.76	1	178.17	118.25	0.00	0.08
9	RC105–5	2	227.19	0	2	227.19	0	0.00	0.02
10	RC108–5	2	246.54	62.94	2	245.87	0	0.00	0.27
11	RC204–5	1	172.49	169.06	1	172.49	69.37	0.00	0.00
12	RC208–5	1	168.21	153.85	1	163.33	108.21	0.00	2.90
13	C101–10	3	372.55	283.80	3	366.5	218.68	0.00	1.62
14	C104–10	2	271.72	249.76	2	261.21	151	0.00	3.87
15	C202–10	2	245.98	274.51	2	243.19	266.12	0.00	1.13
16	C205–10	2	232.92	410.85	2	228.32	172.51	0.00	1.97
17	R102–10	3	244.90	98.28	3	242.42	47.44	0.00	1.01
18	R103–10	3	202.01	116.70	3	193.69	0	0.00	4.12
19	R201–10	3	207.53	294.24	3	206.05	59.69	0.00	0.71
20	R203–10	1	226.40	289.04	1	214.61	261.43	0.00	5.21
21	RC102–10	3	395.18	141.88	3	385.79	0	0.00	2.38
22	RC108–10	3	345.35	189.61	3	331.14	80.33	0.00	4.11
23	RC201–10	2	308.59	271.78	2	304.00	0	0.00	1.49
24	RC205–10	3	340.51	301.86	2	325.27	195.29	33.33	4.48
25	C103–15	3	350.99	414.78	3	343.47	399.38	0.00	2.14
26	C106–15	4	301.64	229.71	3	267.82	103.17	25.00	11.21
27	C202–15	3	375.19	575.10	2	362.59	359.57	33.33	3.36
28	C208–15	2	309.62	380.09	2	305.90	286.04	0.00	1.20
29	R102–15	4	329.64	107.43	4	326.01	98.50	0.00	1.10
30	R105–15	3	292.81	216.15	3	290.12	98.80	0.00	0.92
31	R202–15	3	361.47	477.85	3	358.28	340.91	0.00	0.88
32	R209–15	2	281.41	353.83	2	272.14	247.60	0.00	3.29
33	RC103–15	4	391.34	74.01	4	389.89	32.89	0.00	0.37
34	RC108–15	3	359.98	269.52	3	358.86	56.47	0.00	0.31
35	RC202–15	4	433.09	228.75	3	398.76	208.77	25.00	7.93
36	RC204–15	2	349.82	410.29	2	306.37	256.65	0.00	12.42
Average		2.42	270.65	235.36	2.31	262.70	127.85	3.24	2.75

Note: "#v" denotes the vehicle number, "TD" the traveled distance, and "R_A" the recharging amount. and "$\Delta TD(\%)$" denote the number of employed vehicles gap and the traveled distance gap to the battery capacity to be seven times the average value of the total power demand at the customer nodes, respectively.

From the column $\Delta v(\%)$, we can find that with the increment of the battery capacity, the total travel distance is decreasing, except instance RC204 $-$ 5, which is straightforward. These results can easily be observed in Figure 3. Furthermore, we can find that as the number of customer nodes increases, the difference in distance between the different battery capacities will be more obvious. Because, with the increase of the customer nodes, the difference in the vehicle service paths under different battery capacities will increase, owing to the existence of more nodes selected, which is a reasonable result. As the object function is to minimize the total distance, with a larger battery capacity, one mobile charging vehicle can service more customers without a recharging demand. Meanwhile, one may also notice that the battery capacity could leverage the number of mobile charging vehicles, such as $RC205-10$, $C106-15$, $C202-15$, and $RC202-15$, as well as the charging stations used, which can be observed through the average charge.

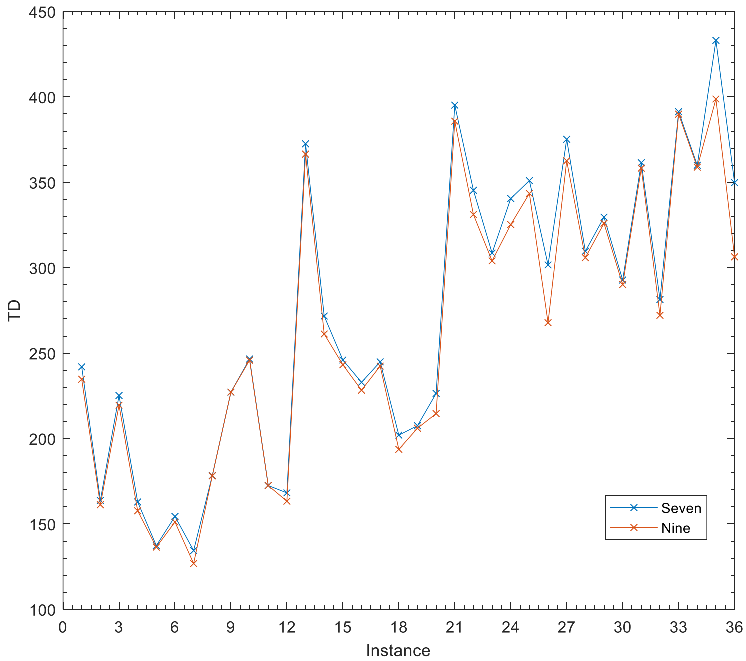

Figure 3. Numerical results for comparison with different battery capacities.

4.3.2. The Sensitivity Analysis of Recharging Rate

Next, the sensitive analysis on the recharging rate of the mobile charging vehicles is discussed. The corresponding results are shown in Table 4. The results for all 36 instances are categorized into the following three cases, which is that (I) the recharging rate values of the Levels 1, 2, and 3 plug-in charging stations shown in Section 4.1 are adopted (CR-Case I); (II) we will increase the charging rates of CR-Case I by three times (CR-Case II); (III) we further increase the charging rates of CR-Case II by three times (CR-Case III). The battery capacity of the mobile charging vehicles is seven times the average value of the total power demand at the customer nodes. (Corresponding data is included in the Supplementary Materials, folder Section 4.3.2, which is contained in folder Section 4.3).

Table 4. The sensitive analysis on different recharging rate.

No.	*Instance*	*CR–Case I*			*CR–Case II*			*CR–Case III*		
		#v	*TD*	*R_A*	*#v*	*TD*	*R_A*	*#v*	*TD*	*R_A*
1	C101–5	3	241.97	267.98	3	241.97	267.98	3	241.97	267.98
2	C103–5	2	163.77	34.83	2	163.77	34.83	2	163.77	34.83
3	C206–5	2	225.21	231.58	2	225.21	231.58	2	225.21	231.58
4	C208–5	1	162.89	282.06	**1**	**162.60**	**224.68**	**1**	**159.53**	**119.62**
5	*R*104–5	2	137.32	156.88	2	137.32	156.88	2	137.32	156.88
6	*R*105–5	2	154.4	105.74	2	154.4	105.74	2	154.4	105.74
7	*R*202–5	1	134.41	194.35	1	134.41	194.35	1	134.41	194.35
8	*R*203–5	1	178.31	153.76	1	178.31	153.76	1	178.31	153.76
9	*RC*105–5	2	227.19	0	2	227.19	0	2	227.19	0
10	*RC*108–5	2	246.54	62.94	2	246.54	62.94	2	246.54	62.94
11	*RC*204–5	1	172.49	169.06	1	172.49	169.06	1	172.49	169.06
12	*RC*208–5	1	168.21	153.85	1	168.21	153.85	1	168.21	153.85
13	C101–10	3	372.55	283.8	3	372.55	283.8	3	372.55	283.8
14	C104–10	2	271.72	249.76	**2**	**269.53**	**310.39**	**1**	**246.17**	**431.35**
15	C202–10	2	245.98	274.51	2	245.98	274.51	2	245.98	274.51
16	C205–10	2	232.92	410.85	2	232.92	410.85	2	232.92	410.85
17	*R*102–10	3	244.9	98.28	3	244.9	98.28	3	244.9	98.28
18	*R*103–10	3	202.01	116.7	3	202.01	116.7	3	202.01	116.7
19	*R*201–10	3	207.53	294.24	3	207.53	294.24	3	207.53	294.24

Table 4. *Cont.*

No.	Instance	CR–Case I			CR–Case II			CR–Case III		
		#v	TD	R_A	#v	TD	R_A	#v	TD	R_A
20	R203–10	1	226.4	289.04	**1**	**225.35**	**287.98**	**1**	**225.35**	**287.98**
21	RC102–10	3	395.18	141.88	**3**	**394.34**	**95.66**	**3**	**394.34**	**95.66**
22	RC108–10	3	345.35	189.61	**2**	**330.72**	**239.73**	**2**	**330.72**	**239.73**
23	RC201–10	2	308.59	271.78	2	308.59	271.78	2	308.59	271.78
24	RC205–10	3	340.51	301.86	**2**	**326.67**	**303.52**	**2**	**326.67**	**303.52**
25	C103–15	3	350.99	414.78	3	350.99	414.78	3	350.99	414.78
26	C106–15	4	301.64	229.71	**3**	**296.45**	**381.75**	**3**	**296.45**	**381.75**
27	C202–15	3	375.19	575.1	**2**	**370.69**	**531.94**	**2**	**370.44**	**557.37**
28	C208–15	2	309.62	380.09	2	309.62	380.09	2	309.62	380.09
29	R102–15	4	329.64	107.43	**4**	**323.71**	**162.38**	**4**	**323.71**	**162.38**
30	R105–15	3	292.81	216.15	**3**	**292.73**	**238.02**	**3**	**292.73**	**238.02**
31	R202–15	3	361.47	477.85	3	361.47	477.85	3	361.47	477.85
32	R209–15	2	281.41	353.83	**2**	**275.56**	**457.28**	**2**	**275.56**	**457.28**
33	RC103–15	4	391.34	74.01	4	391.34	74.01	4	391.34	74.01
34	RC108–15	3	359.98	269.52	3	359.98	269.52	3	359.98	269.52
35	RC202–15	4	433.09	228.75	**3**	**407.65**	**386.68**	**2**	**397.56**	**695.98**
36	RC204–15	2	349.82	410.29	**2**	**306.44**	**424.81**	**2**	**306.40**	**421.71**
Average		2.42	270.65	235.36	2.28	267.23	248.39	2.22	266.21	258.05

Note: "#v" denotes the vehicle number, "TD" the traveled distance, and "R_A" the recharging amount.

Comparing the #v, TD, and R_A columns for the three cases, we can find that with the increment of the charging rate, the number of mobile charging vehicles used, the total distance traveled, and the recharging amount are decreasing in the 12 instances that have been marked in bold. The traveled distance results of three cases can easily be observed in Figure 4. This is even more noticeable when the number of customer nodes is relatively large, which is similar to Section 4.3.1. Because of the higher recharging rate, the mobile charging vehicle can recharge more power at one charging station ensuring the time windows of customer nodes, thereby avoiding more round trips, reducing the number of used vehicles, the total traveled distance, and further reducing the total recharging amount.

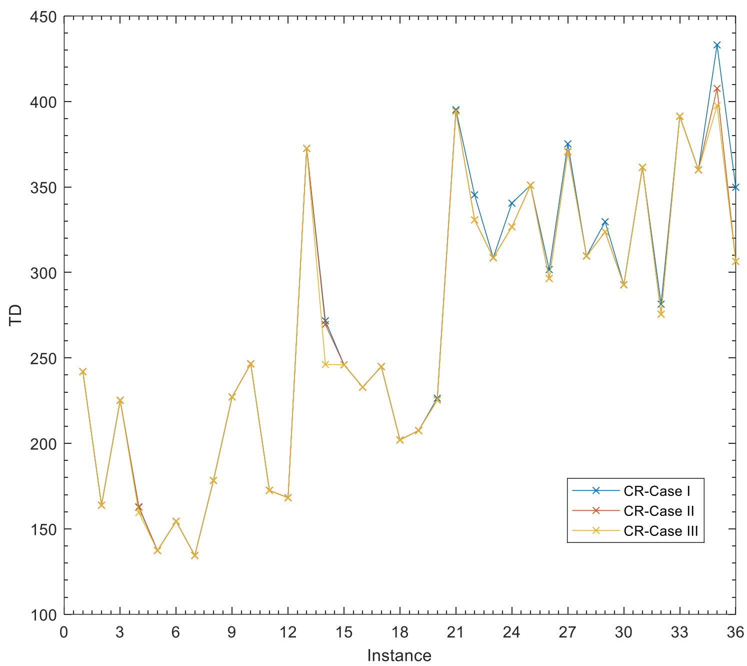

Figure 4. Numerical results for comparison with different recharging rates.

4.3.3. The Sensitivity Analysis of Budget

The results of Section 4.3.2 show that faster charging rates can improve service efficiency, reduce the total traveled distance, and even reduce the number of vehicles used. However, fast charging equipment requires high costs, so in this section we will set the budget as two values of 8 and 15 (Corresponding data is included in the Supplementary Materials, folder Section 4.3.3, which is contained in folder Section 4.3), and then the results are compared with the results when the budget is 10. The comparison of the three different budgets is shown in Table 5.

From Table 5, we find that ten results become worse when the budget is reduced and ten results become better when the budget is increased, but the examples are different by comparing the $\#v$, TD, and R_A columns for the three cases. More budgets can allow for more devices with fast charging rates to be built to increase the charging rate, and further reduce the total traveled distance, conversely, the total traveled distance increasing. Therefore, the results seem to be reasonable. Similarly, because the objective function is to minimize the running distance of all of the MCVs, we further demonstrate the distance of the results of the three budgets in Figure 5. We even find that for the first 12 examples, including five customers, the distance results completely coincide. When the number of customers is 15, the running distances of all of the vehicles are separated in three cases, which is also a realistic result.

Table 5. The sensitive analysis on budget.

No.	*Instance*	*Eight*			*Ten*			*Fifteen*		
		#v	*TD*	*R_A*	*#v*	*TD*	*R_A*	*#v*	*TD*	*R_A*
1	C101–5	3	241.97	267.98	3	241.97	267.98	3	241.97	267.98
2	C103–5	2	163.77	34.83	2	163.77	34.83	2	163.77	34.83
3	C206–5	2	225.21	231.58	2	225.21	231.58	2	225.21	231.58
4	C208–5	1	162.89	282.06	1	162.89	282.06	1	162.89	282.06
5	104–5	2	137.32	156.88	2	137.32	156.88	2	137.32	156.88
6	R105–5	2	154.4	105.74	2	154.4	105.74	2	154.4	105.74
7	R202–5	1	134.41	194.35	1	134.41	194.35	1	134.41	194.35
8	R203–5	1	178.31	153.76	1	178.31	153.76	1	178.31	153.76
9	RC105–5	2	227.19	0	2	227.19	0	2	227.19	0
10	RC108–5	2	246.54	62.94	2	246.54	62.94	2	246.54	62.94
11	RC204–5	1	172.49	169.06	1	172.49	169.06	1	172.49	169.06
12	RC208–5	1	168.21	153.85	1	168.21	153.85	1	168.21	153.85
13	C101–10	3	372.55	283.8	3	372.55	283.8	3	372.55	283.8
14	C104–10	**2**	**273.80**	**266.84**	2	271.72	249.76	2	271.72	249.76
15	C202–10	2	245.98	274.51	2	245.98	274.51	2	245.98	274.51
16	C205–10	2	232.92	410.85	2	232.92	410.85	2	232.92	410.85
17	R102–10	**3**	**248.38**	**213.45**	3	244.9	98.28	3	244.9	98.28
18	R103–10	3	202.01	116.7	3	202.01	116.7	3	202.01	116.7
19	R201–10	3	207.53	294.24	3	207.53	294.24	3	207.53	294.24
20	R203–10	1	226.4	289.04	1	226.4	289.04	**1**	**225.35**	**287.99**
21	RC102–10	3	395.18	141.88	3	395.18	141.88	3	395.18	141.88
22	RC108–10	3	345.35	189.61	3	345.35	189.61	**3**	**344.98**	**271.91**
23	RC201–10	**2**	**308.90**	**253.10**	2	308.59	271.78	2	308.59	271.78
24	RC205–10	3	340.51	301.86	3	340.51	301.86	**2**	**331.74**	**360.18**
25	C103–15	3	350.99	414.78	3	350.99	414.78	3	350.99	414.78
26	C106–15	4	301.64	229.71	4	301.64	229.71	**3**	**296.45**	**324.68**
27	C202–15	**3**	**376.16**	**384.22**	3	375.19	575.1	**2**	**370.58**	**415.04**
28	C208–15	2	309.62	380.09	2	309.62	380.09	2	309.62	380.09
29	R102–15	**4**	**329.84**	**176.28**	4	329.64	107.43	4	329.64	107.43
30	R105–15	**3**	**298.93**	**114.18**	3	292.81	216.15	**3**	**292.75**	**219.09**
31	R202–15	3	361.47	477.85	3	361.47	477.85	3	361.47	477.85
32	R209–15	**3**	**307.07**	**245.63**	2	281.41	353.83	**2**	**275.56**	**397.65**
33	RC103–15	4	391.34	74.01	4	391.34	74.01	4	391.34	74.01
34	RC108–15	**3**	**377.38**	**161.32**	3	359.98	269.52	**3**	**355.73**	**324.76**
35	RC202–15	**4**	**437.77**	**326.54**	4	433.09	228.75	**3**	**407.65**	**372.54**
36	RC204–15	**3**	**351.35**	**315.94**	2	349.82	410.29	**2**	**308.99**	**367.56**
Average		2.47	272.38	226.37	2.42	270.65	235.36	2.31	267.97	243.07

Note: "$\#v$" denotes the vehicle number, "TD" the traveled distance, and "R_A" the recharging amount.

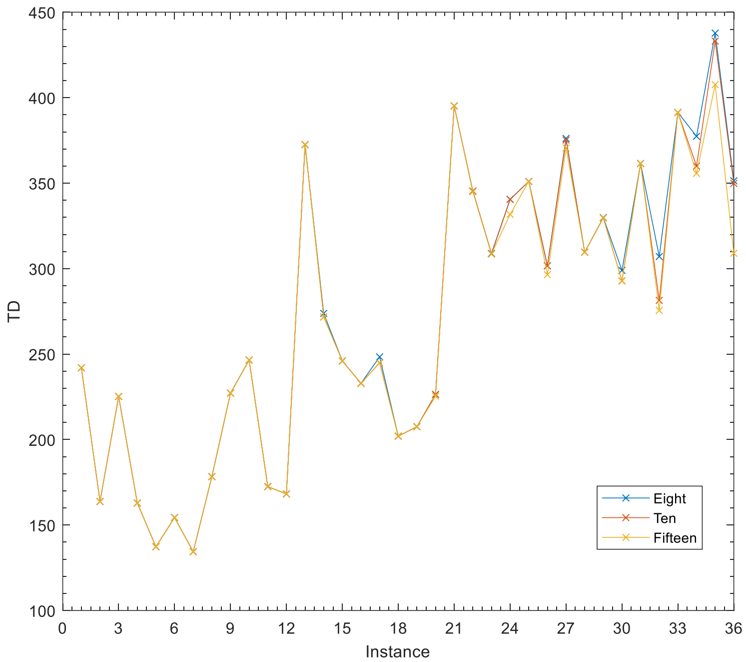

Figure 5. Numerical results for comparison with different budgets.

4.3.4. The Sensitivity Analysis of the Number of Corresponding Dummy Nodes of One Candidate Charging Station

Through the studies of Sections 4.3.2 and 4.3.3, we can find that the improvement of the recharge rate can improve the solution, where Section 4.3.2 improves the recharge rate directly, while Section 4.3.3 increases the number of high recharge rates of a charging station. If a charging station can be used multiple times, the frequency of the used charging station with a fast charging rate will be increased, so the solution can also be improved. Therefore, in this part, the number of corresponding dummy nodes of one candidate charging station α, is improved and set as 2, which means a charging station can be used up to two times at most. We test the battery capacity to be seven and nine times the average value of the total power demand at the customer nodes, respectively. (Corresponding data is included in the Supplementary Materials, folder Section 4.3.4, which is contained in folder Section 4.3).

The results are shown in Table 6. We compare the results in Table 6 with the results in Table 3, and find that six results are improved, which are bolded, regardless of the battery capacity. Although a charging station can be used multiple times, its position has been fixed, so the solutions are not as good as Sections 4.3.2 and 4.3.3. We show the distance results of Tables 3 and 6 in Figure 6. From the figure, we can find that when there are fewer customer nodes, only two color lines can be observed, indicating that the visit frequency of each charging station has little effect on the results. When the number of customers increases, we can observe the lines of three colors. Combined with the data in Table 6, it can be seen that when the battery capacity is seven times the average value of the total power demand at the customer nodes, different frequencies can impact the distance results, while when the battery capacity is nine times, different frequencies have no effect on the results.

Table 6. The sensitive analysis on the number of corresponding dummy nodes of one candidate charging station.

No.	Instance	Seven			Nine			Δv(%)	ΔTD(%)
		#v	TD	R_A	#v	TD	R_A		
1	C101–5	3	241.97	267.98	3	234.72	127.25	0	3
2	C103–5	2	163.77	34.83	2	161.26	0	0	1.53
3	C206–5	2	225.21	231.58	2	219.54	47.46	0	2.52
4	C208–5	**1**	**162.26**	**138.38**	**1**	**157.72**	**87. 76**	0	2.8
5	R104–5	2	137.32	156.88	2	136.45	22.72	0	0.63
6	R105–5	2	154.4	105.74	2	151.15	0	0	2.1
7	R202–5	1	134.41	194.35	1	126.83	80.17	0	5.64
8	R203–5	1	178.31	153.76	1	178.17	118.25	0	0.08
9	RC105–5	2	227.19	0	2	227.19	0	0	0
10	RC108–5	2	246.54	62.94	2	245.87	0	0	0.27
11	RC204–5	1	172.49	169.06	1	172.49	69.37	0	0
12	RC208–5	1	168.21	153.85	1	163.33	108.21	0	2.9
13	C101–10	3	372.55	283.8	3	366.5	218.68	0	1.62
14	C104–10	2	271.72	249.76	2	261.21	151	0	3.87
15	C202–10	2	245.98	274.51	2	243.19	266.12	0	1.13
16	C205–10	2	232.92	410.85	2	228.32	172.51	0	1.97
17	R102–10	3	244.9	98.28	3	242.42	47.44	0	1.01
18	R103–10	3	202.01	116.7	3	193.69	0	0	4.12
19	R201–10	3	207.53	294.24	3	206.05	59.69	0	0.71
20	R203–10	**1**	**224.71**	**287.35**	**1**	**214.62**	**261.43**	0	4.49
21	RC102–10	3	395.18	141.88	3	385.79	0	0	2.38
22	RC108–10	3	345.35	189.61	3	331.14	80.33	0	4.11
23	RC201–10	2	308.59	271.78	2	304	0	0	1.49
24	RC205–10	3	340.51	301.86	2	325.27	195.29	33.33	4.48
25	C103–15	3	350.99	414.78	3	343.47	399.38	0	2.14
26	C106–15	**3**	**283.24**	**293.87**	**3**	**267.82**	**103.17**	0	5.44
27	C202–15	3	375.19	575.1	2	362.59	359.57	33.33	3.36
28	C208–15	**2**	**309.48**	**519.20**	**2**	**305.90**	**286.04**	0	1.16
29	R102–15	4	329.64	107.43	4	326.01	98.5	0	1.1
30	R105–15	3	292.81	216.15	3	290.12	98.8	0	0.92
31	R202–15	3	361.47	477.85	3	358.28	340.91	0	0.88
32	R209–15	2	281.41	353.83	2	272.14	247.6	0	3.29
33	RC103–15	4	391.34	74.01	4	389.89	32.89	0	0.37
34	RC108–15	3	359.98	269.52	3	358.86	56.47	0	0.31
35	RC202–15	**3**	**419.07**	**386.99**	**3**	**398.76**	**208.77**	0	4.85
36	RC204–15	**2**	**309.69**	**359.98**	**2**	**306.37**	**256.65**	0	1.07
Average		2.36	268.57	239.96	2.31	262.7	127.85	1.85	2.16

Note: "#v" denotes the vehicle number, "TD" the traveled distance, and "R_A" the recharging amount.

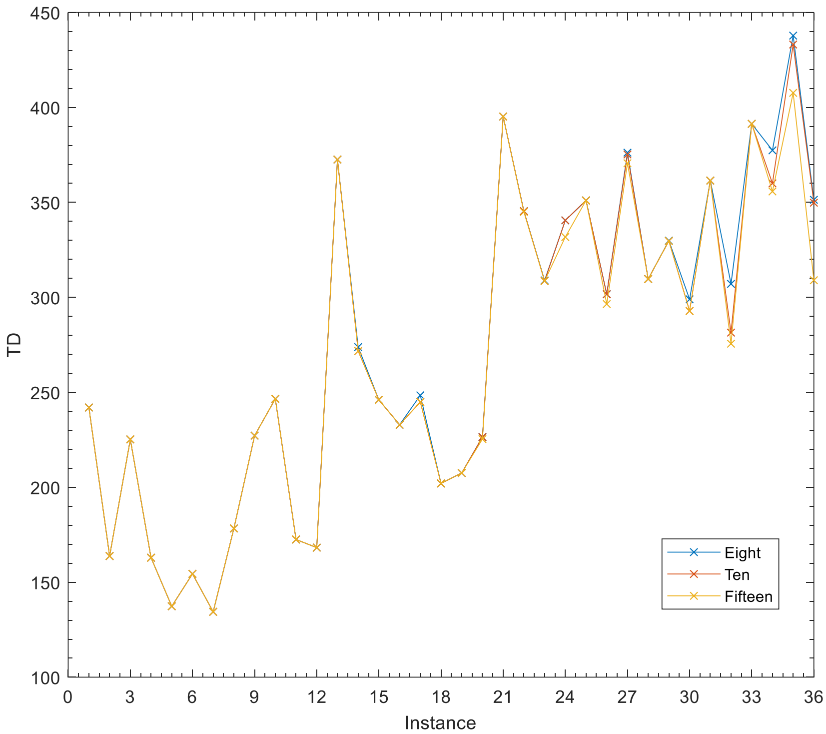

Figure 6. Numerical results for comparison with different visit frequencies.

5. Conclusions and Discussions

In this paper, a novel approach to providing a service for the battery charge replenishment of BEVs is presented. Facility location and vehicle routing are two of the most crucial decisions in reducing the cost of many companies. Because the strategy of plug-in charging station location directly determines the route of each vehicle, and meanwhile, the station location strongly depends on the vehicle routing plan, when these two interrelated components are tackled separately, the solution could be suboptimal, which has been proven by many studies. Therefore, we have built our problem on LRP, and present the multiple types of plug-in charging facilities' location-routing problem with time windows for mobile charging vehicles (MTPCF-LRPwTW-MCVs) in this paper. MTPCF-LRPwTW-MCVs can determine the station location and vehicle routing plan simultaneously.

Multiple types of charging stations with different charging speeds have been discussed by Wang and Lin [36]. Hence, in this paper, we not only determine the location of the charging station, but also the type of chargers at each charging station. However, when the multiple types of charging stations are considered in this problem, a common bilinear term will be generated, which is nonlinear and leads to the nonconvex property of the whole problem. To simplify the solution, the nonconvex problem here is transformed into an equivalent MILP by the reformulation-linearization technique (RLT) [53].

From RLT, we formulate the MTPCF-LRPwTW-MCVs as an MILP, which can be solved directly using the GAMS commercial solver. To demonstrate the model, 36 small instances are designed based on the benchmark instances for VRPTW, proposed by Schneider et al. [43]. All of the instances can be solved exactly, and the sensitivity analyses, such as the battery capacity, the recharging rate, and so on, are also conducted. It could be concluded that the larger battery capacity or quicker charging rate could decrease the number of mobile charging vehicles and the total traveled distances, respectively. With more budgets, the number of fast charging stations can increase, so the number of mobile charging vehicles and total traveled distances will decrease, respectively. In the latter, we improve the number of corresponding dummy nodes of one candidate charging station, α, to allow a charging station to be used up to two times at most. This method improves the using frequency of the fast charging station as well as the efficiency, so the result is the same as raising the budget. In the above-mentioned four sensitivity analysis cases, we will find that the difference in the total distance of the optimization target increases when the number of customers increases. These are reasonable results due to more candidate customer nodes and more vehicle routing options.

Furthermore, the model can be extended in several ways. Firstly, it is interesting to investigate how problems with real-life road networks will affect the performance of the commercial solvers. The solution rate of the solver GAMS has a certain limit. When the total number of nodes is 100, the model solution time will reach more than 20 hours. Therefore, more efficient algorithms can be expected in future work. Moreover, the heterogeneity of both the mobile charging vehicles and the requests should be further discussed. It can be seen from the test results that when the number of customer nodes is small, as the vehicle battery capacity increases or decreases, the total running distance of all of the vehicles does not change much, indicating that the power of all of the vehicles used to provide services and operate by themselves is greater than the total demand. There is a certain waste of resources, so the combination of heterogeneous vehicles will be a worthy research direction, which can reduce the waste of resources. Then, it should be pointed out that the mobile charging vehicle routing will be more complex when the charging requests are dynamically considered. Because, on the way to providing services for the next customer, the drivers of the MCVs may receive new service requests. Therefore, under the previous customer's service time window, how to re-plan the path, leading to maximum profit, has a certain research significance. Furthermore, the maximum travel distance can vary according to environmental factors, including weather, temperature, and so on. Therefore, these various factors should be reflected in future studies.

Supplementary Materials: The following are available online at www.mdpi.com/2071-1050/10/8/2855/s1. The 36 small benchmark instances with 5, 10, 15 customers and 21 candidate recharging stations per instance are presented in folder Section 4.2. Each text file is named using the corresponding example name, and the file contains the data of this example, such as: type, location, time windows, service time, the number of requests, the power demand of the battery electric vehicle (BEV), battery capacity of the mobile charging vehicle (MCV), the recharging rate of the MCV, and the average velocity of the MCV. Folder Section 4.3 contains folder Section 4.3.1, Section 4.3.2 and Section 4.3.3. The files contained in each folder are the data requirements of the corresponding part of the paper.

Author Contributions: Conceptualization, H.Z.; Methodology, S.C.; Software, S.C.; Writing-Original Draft Preparation, S.C.; Writing-Review & Editing, H.Z.; Funding Acquisition, H.Z. and C.Z.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

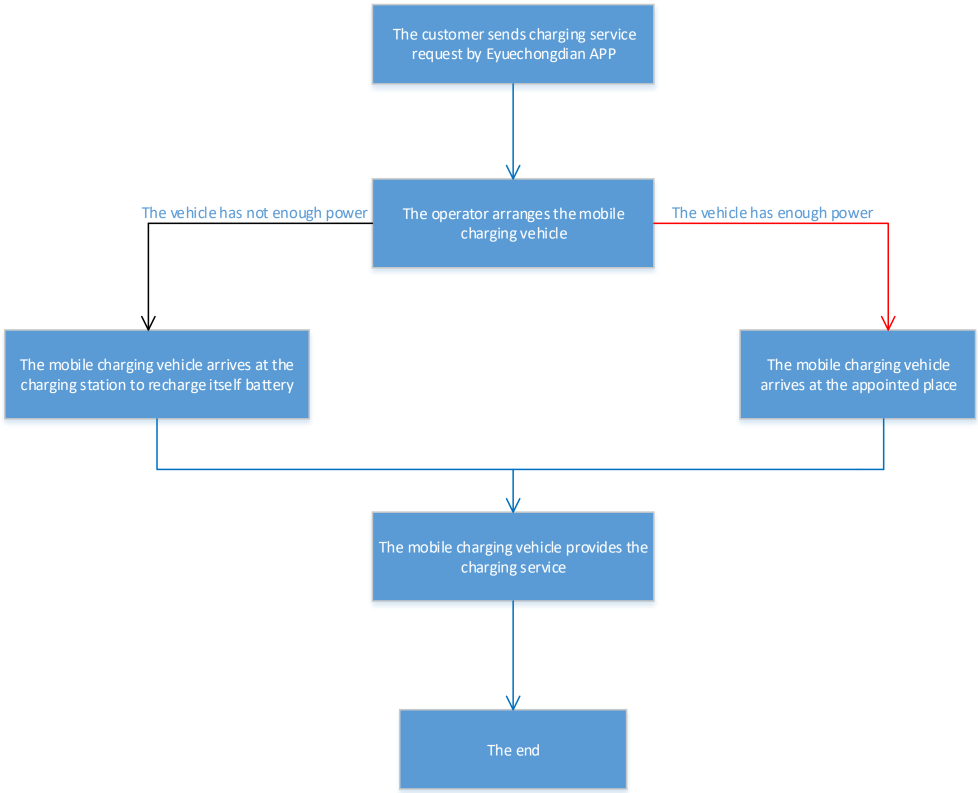

Figure A1. The mobile charging service flow chart.

Firstly, the customer sends a charging service request from the app to an operator. Then, the operator will arrange the mobile charging vehicle to provide service. If the car has enough power, it will provide service directly (along red line); otherwise, it first goes to the charging station to recharge its battery, and will then provide service (along black line).

(**a**) (**b**)

Figure A2. The mobile charging vehicle and charging equipment. (**a**) The mobile charging vehicle; (**b**) The charging equipment of the car.

Figure A3. The mobile charging car is providing charging service for two vehicles at the same time.

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