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Quantifying the Decision-Making of PPPs in China by the Entropy-Weighted Pareto Front: A URT Case from Guizhou

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Abstract: In recent years, value for money (VFM) evaluations using the Public Sector Comparator (PSC) method have gradually been adopted by governments worldwide in the field of public investment, and used as decision-making tools for public–private partnership (PPP) projects. However, there has been little research concerned with such emerging decision-making questions with VFM. This paper proposed a quantitative decision-making method of entropy-weighted Pareto front applied in the specific context of Chinese PPPs, with a case study of an Urban Transit Railway (URT) PPP project from Guizhou province. Factor analysis was used to extract the qualitative indicators of VFM evaluation, associate them with the quantitative characteristics of the project, and thus help public sector decision-makers choose the proper quantitative decision variables.

1. Introduction

Governments have used public–private partnerships (PPP) contracts worldwide as a tool to develop and manage public infrastructure and services, with the underlying goal of providing the best service at the lowest cost [1]. The World Bank 'PPP Reference Guide 2.0' provides a broad definition of PPP [2]: a long-term contract between a private party and a government entity, for providing a public asset or service, in which the private party bears significant risk and management responsibility, and remuneration is linked to performance. In China, whether or not a PPP project provides value for money (VFM) is measured by a VFM evaluation. The approach and techniques of VFM evaluation contain both qualitative and quantitative aspects to determine the reduction of project life cycle cost, optimize risk allocation, and improve the operational efficiency of a PPP project [3]. How to quantitatively determine the appropriate share of the private sector involvement, the proportion of risk transfer, charge to users, and the project duration becomes an important issue, since the result of decision-making should ensure not only that the public sector pay the lowest PPP project cost, also that the private sector could obtain an expected return.

Since the last decade, research studies have mainly focused on the problems of quantitative decision-making in PPPs. Studies on decision-making objects have included the concession period duration, the capital proportion of the private sector, extra profit allocation, the charge to users, project capacity, operation patterns, and risk transfer. The decision-making methods varied according to these decision objects.

Jin et al. [4] developed a neuro-fuzzy decision support system (NFDSS) for the risk-allocation decision-making process in PPP projects by combining fuzzy synthesis decision-making and neural network techniques. Tang [5] built a risk allocation model on the framework of cumulative prospect theory to help the public and private sector reach consensus with each other. Yin et al. [6] designed a risk-sharing scheme for the PPP projects based on the cloud model, which was used to solve the problem of existing decision-making method that the opinions of a few experts who hold the right views are ignored, by grading on thickness of the 'cloud'.

Song and Fei [7] provided one type of group-decision technique to moderate expectations of PPP participators by an iterative algorithm to provide a solution for the group's satisfaction. Xie and Ng [8] established a decision-making model using a Bayesian network (BN) to connect the decision items, evaluation criteria, and objectives with a weighted score approach applied to combine each objective of the stakeholders into a single value. Alireza et al. [9] identified that one of the key issues in a multi-objective problem of a PPP project was the most feasible and satisfactory solution of the allocation of excess costs among the public sector, private sector, and the users; then, a decision-making model was proposed and solved via a max–min composition algorithm.

Xu et al. [10] developed a pricing model using the system dynamics (SD) technique based on a concession project pricing parameters, and then verified the effectiveness of the proposed model by a real toll tunnel project located in China. Xue et al. [11,12] proposed a Bi-level Programming (BLP) decision model of the public bus system to determine the capacity and charge of a bus PPP project where the upper problem was to maximize the total surplus, including the value-of-time (VOT) of passengers as constrained by ticket fare; the lower problem was the passenger's surplus, which was constrained by service capability and the lowest return rate of the private sector.

Liu et al. [13] analyzed the government guarantee of restrictive competition in PPP projects, and constructed an evaluation model with real option theory.

Hu et al. [14] constructed a model of the operational pattern selection of a PPP project on an Support Vector Machine (SVM) classifier, taking investment scale, project VFM value, and the types of income as the three main factors.

These research studies substantially developed the contemporary decision-making methodology with the introduction of the machine learning techniques and intelligent algorithms. However, only a few of these decision-making models referred to the specific requirement of a VFM evaluation of the PPP projects, which is accepted and promoted by more countries worldwide, especially in Asia. Besides, the decision-making of the PPPs is a typical multi-objective optimization problem providing many optimal solutions that are equally good from the perspective of the given objectives, which are known as the Pareto front and as non-dominated solutions (Pareto solutions) [15].

This paper proposed a quantitative decision-making method of entropy-weighted Pareto front for PPP projects in China, then illustrated how to use it via a real Urban Transit Railway (URT) PPP project case. Firstly, the factor analysis method was applied to extract the main factors associated with the quantitative characteristics of a URT PPP project from 15 qualitative indicators, which were used in 19 collected cases [16]. Secondly, entropy weights that were distributed to the net cost of the private sector, the public sector, and the users were calculated by the predicted operation income, general public budget, and passenger volume from the selected case. Thirdly, two types of multi-objective optimization models were built with the constraint that both the VFM and the returns of the private sector are positive; then, each of models was solved by a genetic algorithm solver to generate the Pareto front (the Pareto solution matrix). Finally, the entropy weights were applied to all of the elements in each of the Pareto solution sets. After that, the set with the minimum-weighted sum was picked out, and the best decision was determined. Furthermore, a sensitivity analysis of the net cost of the private sector and the public sector on the parameters in each optimization model were carried out.

2. Materials and Methods

2.1. The Modeling Process

2.1.1. Phase 1—Implement Factor Analysis

All of the steps in Phase 1 will be implemented with SPSS (version 21.0.0.0) software.

Step 1. Identify whether a certain linear relationship exists between the qualitative evaluation indicators. If yes, it will be suitable to extract factors by factor analysis. The common test methods include correlation coefficient matrix, the Bartlett Sphericity (BS) test, and the Kaiser–Meyer–Olkin (KMO) test. The correlation coefficient matrix will be generated at this step.

Step 2. Make a tentative extraction by principal component analysis (PCA) method. According to the correlation coefficient matrix of the original indicators, the initial component analysis will be applied to extract the factors with eigenvalues greater than 1. The initial solution of the factor analysis (the factor-loading matrix) will be obtained at this step.

Step 3. Determine the number of factors to be extracted by observing the scree plot and the result of the total variance interpreted.

Step 4. Implement the orthogonal rotation to the factor-loading matrix by the varimax method, and then output the result by the descending order of first factor loading. The rotated factor-loading matrix will be obtained at this step.

2.1.2. Phase 2—Calculate Entropy Weights

Step 1. Determine the indicators that will be used to calculate the entropy weights of the net cost of the private sector, the public sector, or the users. Suppose that there are n indicators, and that each has m evaluation objects. A $m \times n$ matrix will be obtained at this step. a_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots n$) is supposed to be an element of the matrix as the value of the i-th item in the j-th indicator.

Step 2. Standardize the values of all of the items. a'_{ij} is supposed to be the standardized value of an item. Use:

$$a'_{ij} = \frac{a_{ij} - \min\limits_{i}\{a_{ij}\}}{\max\limits_{i}\{a_{ij}\} - \min\limits_{i}\{a_{ij}\}}, (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots n) \tag{1}$$

for the indicator on which project has better performance when the value of a_{ij} is larger; or use:

$$a'_{ij} = \frac{\max\limits_{i}\{a_{ij}\} - a_{ij}}{\max\limits_{i}\{a_{ij}\} - \min\limits_{i}\{a_{ij}\}}, (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots n) \tag{2}$$

for the indicator on which project has the better performance when the value of a_{ij} is smaller.

Step 3. Calculate the proportion y_{ij} of a'_{ij} in the matrix.

$$y_{ij} = \frac{a'_{ij}}{\sum\limits_{i=1}^{m} a'_{ij}}, (j = 1, 2, \ldots, n) \tag{3}$$

Step 4. Calculate the entropy value e_j of every indicator.

$$e_j = -\frac{1}{\ln m} \sum_{i=1}^{m} y_{ij} \ln y_{ij}, (j = 1, 2, \ldots, n) \tag{4}$$

when $y_{ij} = 0$, $y_{ij} \ln y_{ij} = 0$.

Step 5. Calculate the difference coefficient d_j of every indicator.

$$d_j = 1 - e_j, (j = 1, 2, \ldots, n) \tag{5}$$

Step 6. Calculate the entropy weight of every indicator

$$w_j = \frac{d_j}{\sum\limits_{j=1}^{n} d_j} \tag{6}$$

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2.1.3. Phase 3—Build the Optimization Model

Step 1. Determine the decision variables of the optimization model based on the factor analysis results in Phase 1.

Step 2. Determine the functions of the optimization model objectives and constraints by the PSC method.

Step 3. Identify whether the optimization model has available solutions when it is required to satisfy the constraints that both the VFM and returns of the private sector are positive.

2.1.4. Phase 4—Get the Entropy-Weighted Results

Step 1. Solve the optimization model by 'gamultiobj' solver on MATLAB 9.1.0.441655 (R2016b), and then get the Pareto solutions sets.

Step 2. Calculate the entropy-weighted sum of each set in the Pareto front solution sets.

Step 3. Find the minimum weighted sum, and the value set of its corresponding decision variables will be determined as the best quantitative decision of the project.

2.2. The Two Types of Optimization Models

There are usually two typical scenarios in the specific context of Chinese PPPs, so two types of optimization models need to be built.

For a newly-built PPP project, if there is no similar project built or under operation within the same city that could provide some guidance in project decision-making, i.e., there is no reference information of the project charge or concession period duration, etc., these pieces of information need to be treated as decision variables in the optimization model. However, the concrete discount rate to calculate the net present value (NPV) of the cost of the public sector should be determined as a parameter with a given value.

Else, if there are similar projects located within the same city, the project charge and concession period duration of the newly-built PPP project probably need to be consistent with the existing projects, i.e., these two parameters, charge and duration, should be treated as parameters with given values in the optimization model. In that scenario, the decision-makers of the public sector need to decide the discount rate within a reasonable range, i.e., the discount rate of the public sector cost could be treated as a decision variable.

In this paper, the optimization model of the first scenario is noted as 'Model 1', while that of the second scenario is noted as 'Model 2'.

2.3. A URT PPP Project Case

Guizhou is a province located in southwest China that is well-known for its enriched tourism resources. The capital city is Guiyang city, where two URT lines are to be opened to the public. The case study in this paper is that of Line 2, which is planned to have eight stations: six underground and two on-the-ground. The total length of Line 2 will be 13 km: 9.83 km of underground sections, and 3.17 km

of on-the-ground sections. The concession period duration is 29 years: four years of construction and 25 years of operation. The basic public investment benefit rate is 3–8%, the reported discount rate for the public sector is 6.22%, and the expected return rate for the private sector is 6.37%.

Table 1 listed the source data extracted from the project report [17], where C_{ci} is the construction cost spent in the *i*-th year, yuan $\times 10^4$; C_{oi} is the operation cost spent in the *i*-th year, yuan $\times 10^4$; C_{riski} is the risk cost occurred in the *i*-th year, yuan $\times 10^4$; Q_i is the passenger turnover volume in the *i*-th year, km $\times 10^4$; D_i is the total planning operation kilometers of the *i*-th, km $\times 10^4$, E_i is the project earning of the *i*-th year, yuan $\times 10^4$; and B_i is the general public budget of Guiyang city of the *i*-th year, yuan $\times 10^8$.

Table 1. Source data of Guiyang Urban Transit Railway (URT) Line 2 public–private partnerships (PPP) project.

Year	i	C_{ci}	C_{oi}	C_{riski}	Q_i	D_i	E_i	B_i
2017	1	158,658.8	0	10,744.6	0	0	0	518.7
2018	2	237,988.2	0	16,116.89	0	0	0	534.26
2019	3	237,988.2	0	16,116.89	0	0	0	550.29
2020	4	158,658.8	0	10,744.6	0	0	0	566.79
2021	5	0	20,946.09	8629.92	9256.4	1933.07	3725.7	612.14
2022	6	0	20,946.09	8592.37	10,285.71	1933.07	4139.99	661.11
2023	7	0	20,946.09	8550.68	11,428.14	1933.07	4599.83	714
2024	8	0	20,624.54	8455.49	13,099.86	1933.07	5272.69	771.12
2025	9	0	20,624.54	8385.71	15,012.46	1933.07	6042.51	832.81
2026	10	0	20,624.54	8305.68	17,206.11	1933.07	6925.46	899.43
2027	11	0	20,624.54	8213.92	19,720.94	1933.07	7937.68	971.39
2028	12	0	20,624.54	8108.85	22,600.8	1933.07	9096.82	1049.1
2029	13	0	20,624.54	7988.34	25,904.06	1933.07	10,426.38	1133.03
2030	14	0	20,624.54	8224.58	19,428.94	1933.07	7820.15	1223.67
2031	15	0	22,433.06	8375.35	20,491.11	2188.19	8247.67	1260.38
2032	16	0	22,433.06	8334.6	21,608	2188.19	8697.22	1298.19
2033	17	0	22,433.06	8291.59	22,786.94	2188.19	9171.75	1337.13
2034	18	0	22,701.02	8277.53	24,031.6	2188.19	9672.72	1377.25
2035	19	0	22,701.02	8229.59	25,345.6	2188.19	10,201.6	1418.57
2036	20	0	22,701.02	8179.12	26,728.94	2188.19	10,758.4	1461.12
2037	21	0	22,701.02	8125.85	28,188.94	2188.19	11,346.05	1504.96
2038	22	0	22,701.02	8069.79	29,725.6	2188.19	11,964.55	1550.11
2039	23	0	22,701.02	8010.53	31,349.86	2188.19	12,618.31	1596.61
2040	24	0	22,701.02	8044.35	30,422.74	2188.19	12,245.16	1644.51
2041	25	0	22,701.02	8017.05	31,171	2188.19	12,546.33	1693.84
2042	26	0	22,701.02	7989.09	31,937.51	2188.19	12,854.84	1744.66
2043	27	0	22,701.02	7960.46	32,722.26	2188.19	13,170.71	1797
2044	28	0	22,701.02	7931.16	33,525.26	2188.19	13,493.91	1850.91
2045	29	0	22,701.02	7901.06	34,350.14	2188.19	13,825.94	1906.43

## 3. Results

3.1. The Factor Analysis

There were 15 different indicators used in the VFM qualitative evaluation of 19 URT PPP projects [17]: 'Integration' (I1), 'Risk Allocation' (I2), 'Performance' (I3), 'Competitiveness' (I4), 'Government Competency' (I5), 'Bankability' (I6), 'Procurement Potential' (I7), 'Scale' (I8), 'Life span' (I9), 'Capital Diversity' (I10), 'Cost Accuracy' (I11), 'Legal Support' (I12), 'Income Potential' (I13), 'Exemplary' (I14), and 'Acceptance' (15). In the current Chinese PPPs' VFM evaluation system, the first six indicators are stipulated as basic ones that should be used in every PPP case, while the last 10 are specified as optional indicators that could be weighted as 0. In each PPP case, the sum of the weights is 1. Tables A1 and A2 listed the original weights of the 15 indicators among 19 URT PPP projects, in which project 7 did not use the basic indicator 'Bankability'.

Table 2 listed the correlation matrix of 15 indicators. Due to the existence of optional indicators, this matrix is not a positive definite, so that the BS test and KMO test could not proceed. Linear correlation could be seen between most indexes, which means these indicators could be used for factor analysis.

Table 2. Correlation matrix of 15 indicators.

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15
I1	1.000	−0.164	0.286	−0.678	−0.015	0.057	0.067	0.236	0.370	0.336	−0.258	0.053	−0.783	0.117	0.027
I2	−0.164	1.000	−0.841	−0.255	0.300	0.112	−0.238	−0.637	0.414	−0.146	0.667	−0.187	−0.179	−0.162	−0.095
I3	0.286	−0.841	1.000	0.219	−0.591	−0.279	0.429	0.484	−0.296	0.388	−0.739	0.337	0.055	0.114	0.171
I4	−0.678	−0.255	0.219	1.000	−0.522	−0.489	0.383	0.072	−0.395	0.066	−0.061	0.302	0.558	−0.119	−0.230
I5	−0.015	0.300	−0.591	−0.522	1.000	0.514	−0.701	0.008	−0.053	−0.522	0.419	−0.552	−0.142	0.166	0.080
I6	0.057	0.112	−0.279	−0.489	0.514	1.000	−0.821	−0.256	0.096	−0.662	0.180	−0.694	0.063	0.336	0.224
I7	0.067	−0.238	0.429	0.383	−0.701	−0.821	1.000	0.099	−0.069	0.817	−0.318	0.543	−0.026	−0.420	−0.139
I8	0.236	−0.637	0.484	0.072	0.008	−0.256	0.099	1.000	−0.073	0.096	−0.393	0.217	−0.143	−0.031	−0.356
I9	0.370	0.414	−0.296	−0.395	−0.053	0.096	−0.069	−0.073	1.000	0.035	0.264	−0.190	−0.502	−0.274	−0.197
I10	0.336	−0.146	0.388	0.066	−0.522	−0.662	0.817	0.096	0.035	1.000	−0.096	0.517	−0.456	−0.545	−0.207
I11	−0.258	0.667	−0.739	−0.061	0.419	0.180	−0.318	−0.393	0.264	−0.096	1.000	−0.301	−0.249	−0.511	−0.282
I12	0.053	−0.187	0.337	0.302	−0.552	−0.694	0.543	0.217	−0.190	0.517	−0.301	1.000	−0.021	−0.160	−0.109
I13	−0.783	−0.179	0.055	0.558	−0.142	0.063	−0.026	−0.143	−0.502	−0.456	−0.249	−0.021	1.000	0.212	0.289
I14	0.117	−0.162	0.114	−0.119	0.166	0.336	−0.420	−0.031	−0.274	−0.545	−0.511	−0.160	0.212	1.000	0.226
I15	0.027	−0.095	0.171	−0.230	0.080	0.224	−0.139	−0.356	−0.197	−0.207	−0.282	−0.109	0.289	0.226	1.000

In Figure 1, the horizontal axis is the number of the factors that were supposed to be extracted, and the vertical axis is the eigenvalue of the factor. The eigenvalues of the first four factors are high, in which these factors have strong interpretation to 15 indicators. From the fifth factor to the last factor, their eigenvalues are low, i.e., these factors are weak in interpretation of all 15 indicators.
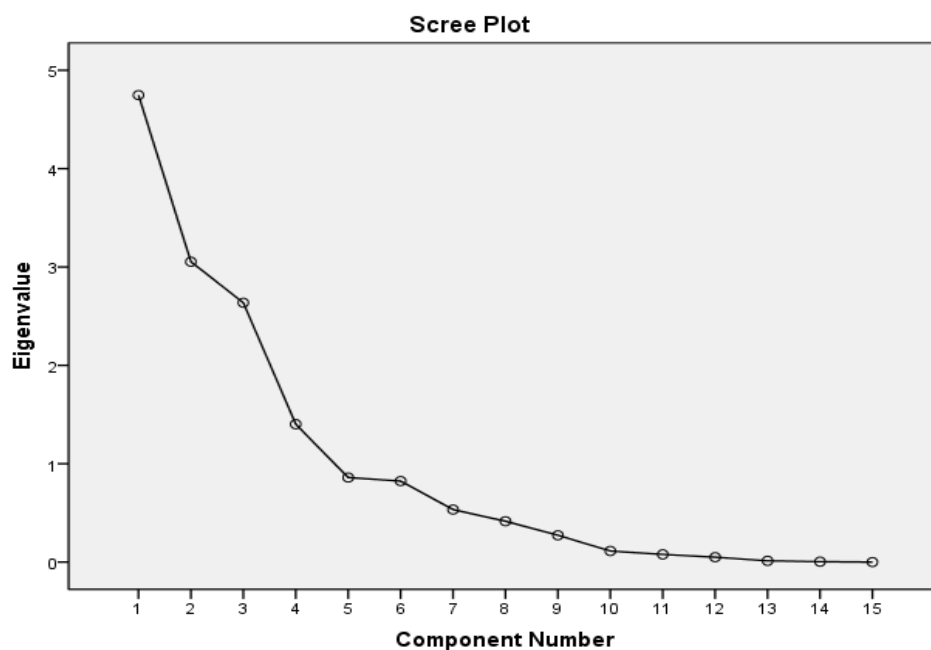
Figure 1. Scree plot chart of factor analysis.

Table 3 is the component matrix of the four extracted factors. The loading of each indicator on the first factor is relatively higher than those of the other three factors. However, the meaning of these four factors is still obscure.

Table 3. Component matrix.

	Component			
	1	**2**	**3**	**4**
Procurement Potential	0.841	0.292	−0.203	0.207
Government Competency	−0.777	−0.040	0.159	−0.294
Bankability	−0.773	−0.289	0.308	0.018
Performance	0.762	−0.315	0.434	0.033
Legal Support	0.712	0.136	−0.126	0.087
Capital Diversity	0.700	0.588	0.015	0.165
Risk Allocation	−0.604	0.485	−0.423	0.265
Cost Accuracy	−0.572	0.508	−0.492	−0.173
Income Potential	0.051	−0.789	−0.511	0.083
Life span	−0.258	0.647	0.177	0.062
Exemplary	−0.205	−0.628	0.397	0.105
Integration	0.097	0.434	0.853	0.135
Competitiveness	0.500	−0.338	−0.679	−0.218
Scale	0.426	−0.078	0.417	−0.732
Acceptance	−0.126	−0.423	0.190	0.716

Extraction method: principal component analysis.

Table 4 displayed the rotated component matrix outputted in the first factor loading descending order. After orthogonal rotation of the original component matrix, the meanings of the four factors become clearer. As could be seen from Table 4, the loadings of 'Procurement Potential', 'Capital Diversity', 'Government Competency' and 'Legal Support' are high on the first factor, in which these indicators are the main components that are interpreted by the first factor. Similarly, the second factor provide interpretation mainly of 'Performance', 'Competitiveness', and 'Scale'. The third factor mainly interpreted 'Integration' and 'Life span'. The fourth factor mainly interpreted 'Acceptance'.

Table 4. The rotated composition matrix.

	Component			
	1	**2**	**3**	**4**
Procurement Potential	0.928	0.120	−0.026	−0.017
Capital Diversity	0.858	0.021	0.340	−0.110
Bankability	−0.841	−0.052	0.127	0.226
Government Competency	−0.785	−0.244	0.142	−0.149
Legal Support	0.706	0.205	−0.074	−0.062
Exemplary	−0.505	0.480	−0.088	0.336
Risk Allocation	−0.122	−0.890	0.120	0.164
Cost Accuracy	−0.182	−0.868	0.022	−0.266
Performance	0.396	0.842	0.022	0.048
Integration	0.053	0.319	0.916	0.044
Income Potential	−0.110	0.141	−0.888	0.268
Competitiveness	0.412	0.060	−0.806	−0.220
Lifespan	0.012	−0.398	0.596	−0.084
Acceptance	−0.139	0.190	−0.029	0.829
Scale	0.015	0.630	0.102	−0.700

Extraction method: principal component analysis. Rotation method: Varimax with Kaiser normalization. Rotation converged in five iterations.

From the above paragraph, it seems that the indicators that were mainly interpreted by the first factor on related to government commitments and guarantees; the second factor was associated with the output and performance of the private sector involvement; the third factor was directly related to concession period duration; and the fourth factor reflects the degree of the public service accepted by users who are supposed to be sensitive with price level. Thus, the decision variables could be

determined as the benchmark benefit ratio of public investment (r_g), which will be used as the discount rate of the public sector cost, the proportion of the public sector's share in the project company (a), the unit payment to the private sector (b), and the public transport services charge to the users (p).

The degree of total variance interpreted by the extracted factors is shown in Table 5. There are two groups of datasets; each has three columns of data, in turns: the eigenvalue of the factor, the contribution rate of variance interpretation, and the accumulative contribution rate of variance interpretation. In the first group dataset, 78.923% of the total variance of the 15 indicators was interpreted by the four factors before rotation. The second group dataset is the ultimate factor solution after rotation. The accumulative contribution rate of variance interpreted after rotation is not changed. However, after rotation, the factor contributions to the 15 indicators are reallocated, and the meanings of the four factors are less obscure.

Table 5. The total variance interpreted by four extracted factors.

Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
4.747	31.647	31.647	4.082	27.211	27.211
3.053	20.354	52.001	3.323	22.150	49.361
2.636	17.577	69.578	2.825	18.834	68.195
1.402	9.345	78.923	1.609	10.728	78.923

3.2. Two Nonlinear Multi-Objective Models

3.2.1. Model 1

Model 1 has three objectives: the minimum cost of the private sector, the public sector, or a single user, expressed as equations (7)–(9):

$$\min_{a,b,p,n} Z1 = C_{pri} = (1 - ar_{cap} - E_{other})C_c\lambda_p/Q + C_{oy}D/Q + (1-a)C_{risky}/Q - bD/Q - p(1+r_t) \tag{7}$$

$$\min_{a,b,p,n} Z2 = C_{pub} = aC_c r_{cap}\lambda_p/Q + bD/Q + aC_{risky}/Q \tag{8}$$

$$\min_{a,b,p,n} Z3 = C_{user} = p \tag{9}$$

s.t.

$$C_{PRI} \leq 0 \tag{10}$$

and

$$C_{VFM} \geq 0. \tag{11}$$

In the above:

$$C_{PRI} = (1 - ar_{cap})C_c + C_{oy}D/\lambda_p + (1-a)C_{risky}/\lambda_p - bD/\lambda_p - pQ(1+r_t)/\lambda_p - C_cE_{other} \tag{12}$$

$$C_{PUB} = ar_{cap}C_c + bD/\lambda_g + aC_{risky}/\lambda_g \tag{13}$$

$$C_{PSC} = C_c + C_{oy}D/\lambda_g + C_{risky}/\lambda_g - pQ(1+r_t)/\lambda_g \tag{14}$$

$$C_{VFM} = C_{PSC} - C_{PUB} \tag{15}$$

$$\lambda_p = (1+r_p)^n r_p/((1+r_p)^n - 1) \tag{16}$$

$$\lambda_g = (1+r_g)^n r_g/((1+r_g)^n - 1) \tag{17}$$

where [17],

a is the project share of the public sector, which is capped at 49% (according to current policy in China);

b is the unit payment for the private sector at every year, yuan/km;

p is the unit charge to users of the public transportation service provide by the PPP project, yuan/person/km;

n is the concession period duration, in years (in the 19 cases collected, the shortest duration is 10 years, and the longest one is 35 years.);

λ_g and λ_p are the coefficients that convert the present value into the unit time (e.g., every year) value;

C_{PRI} is the net cost present value of the private sector, including the fund of equity share, the operating and maintenance cost, and risk expenditure (the payments from the public sector and users should be deducted);

C_{PUB} is the net cost present value of the public sector, including the share funds, payment for the private sector, and risk expenditure;

C_{PSC} is the Public Sector Comparator (PSC), which is the net cost present value of the public sector when implement the URT project in a traditional pattern other than the PPP pattern;

C_{VFM} is the difference between C_{PSC} and C_{PPP} that reflects the savings of the public investment for using PPP instead of a traditional pattern;

C_{pri} is the unit cost undertaken by the private sector every year, yuan/person/km;

C_{pub} is the unit cost undertaken by the public sector every year, yuan/person/km;

C_{user} is the unit cost of user, yuan/person/km, which is exactly the p;

C_c is the total investment of the project, which is also the sum of C_{ci} in Table 1, 793293.91 yuan $\times 10^4$;

r_{cap} is the capital fund proportion of total investment, 30%;

D is the mean of D_i in Table 1, 2086.14 km/year;

C_{oy} is the unit operating cost per year, 10.49, yuan/km/year, $Coy = \sum\limits_{i=5}^{29} C_{oi}/25/D$;

Q is the mean of Q_i in Table 1, 23533.16 km $\times$ person/year;

C_{risky} is the mean of C_{riski} in Table 1, 8928.13 yuan $\times 10^4$/year;

r_g is the minimum basic public investment benefit rate, 3%, which is also the discount rate for calculating the net cost present value of public sector;

r_p is the minimum rate of return required by the private sector, 6.37%, which is also the discount rate for calculating the net cost present value of the private sector;

r_t is the ratio of non-ticket revenue taken up to the total ticket revenue, 15%;

E_{other} is the ratio of the external commercial development revenue taken up to the total project investment, which is set at 20% to ensure that the optimization model does have available solutions.

3.2.2. Model 2

Model 2 has two objectives: the minimum cost of the private sector and the public sector, as expressed in equations (18) and (19). The cost of single user p is fixed in this scenario. Constraints are the same as with Model 1.

$$\min_{a,b,r_g} Z1 = C_{PRI} = \sum_{i=1}^{N} \left((1 - ar_{cap})C_{ci} + C_{oi} + (1-a)C_{riski} - bD_i - pQ_i(1+r_t) \right)/(1+r_p)^i \quad (18)$$

$$\min_{a,b,r_g} Z2 = C_{PUB} = \sum_{i=1}^{N} \left(ar_{cap}C_{ci} + bD_i + aC_{riski} \right)/(1+r_g)^i \quad (19)$$

In Model 2, b is capped by 89.49. This value is calculated by:

$$b_{\max} = (1+r_p)(1+r_g)^{25}C_c/25 + (1+r_p)\max\{C_{oi}\} - \min\{E_i\}, \quad (20)$$

which is stipulated by current PPP policy in China. C_{oi}, C_{riski}, D_i, and Q_i are listed in Table 1.

3.3. The Entropy Weights

The entropy weights of C_{pri}, C_{pub}, and C_{user} in Model 1 are calculated on the variation degrees of E_i, B_i, and Q_i in Table 1. The entropy weights of C_{PRI} and C_{PPP} in Model 2 are calculated by E_i, and B_i only.

Table 6 presents the weights of C_{pri}, C_{pub}, and C_{user} of Model 1, where the weights of C_{pri} and C_{user} are equal, and the weight of C_{pub} is the smallest.

Table 6. The entropy weights of C_{pri}, C_{pub}, and C_{user} in optimization Model 1.

| of	C_{pri}	C_{pub}	C_{user}
entropy value	0.909389	0.934844	0.909388
entropy weight	0.367772	0.264454	0.367774

Table 7 presents the weight of C_{PRI} and C_{PPP} of Model 2, where the weight of C_{PRI} is larger than that of C_{PPP}.

**Table 7.** The entropy weights of C_{PRI} and C_{PUB} in optimization Model 2.

of	C_{PRI}	C_{PUB}
entropy value	0.909389	0.934844
entropy weight	0.58171	0.41829

3.4. Pareto Front

Figure 2 depicts the Pareto front of Model 1. Figure 2a–c presents the correlation between each pair of C_{pri}, C_{pub}, and C_{user}, and Figure 2d presents the distribution of the Pareto solutions of the three objectives in a three-dimensional (3D) space. It seems that C_{pub} and C_{user} are linear-correlated. However, it is not easy to determine the relation between C_{pri} and C_{pub} through the displayed complexion. Model 1 and Model 2 used the same configuration of the 'gamultiobj' solver to obtain optimized solutions. The maximum generation is set to 100,000, the initial population size is set to 1000, and the ratio of Pareto front to population size ratio is set to 0.3.
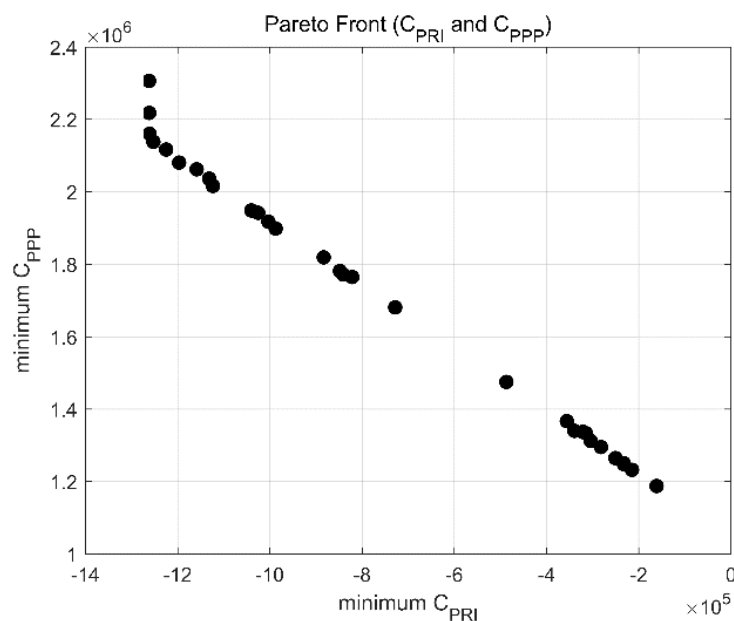

(a)   (b)

Figure 2. *Cont.*

(**c**) (**d**)
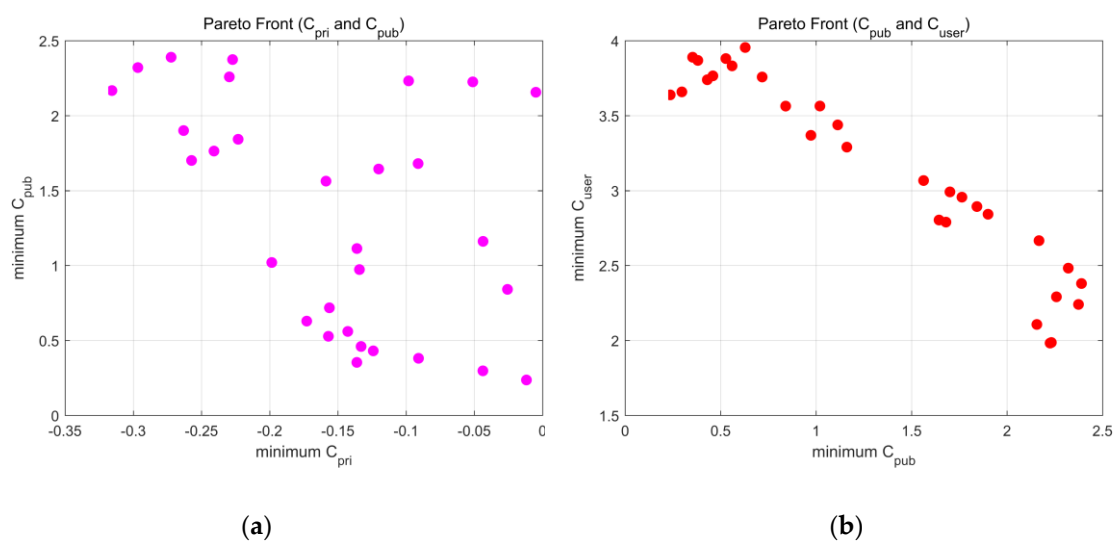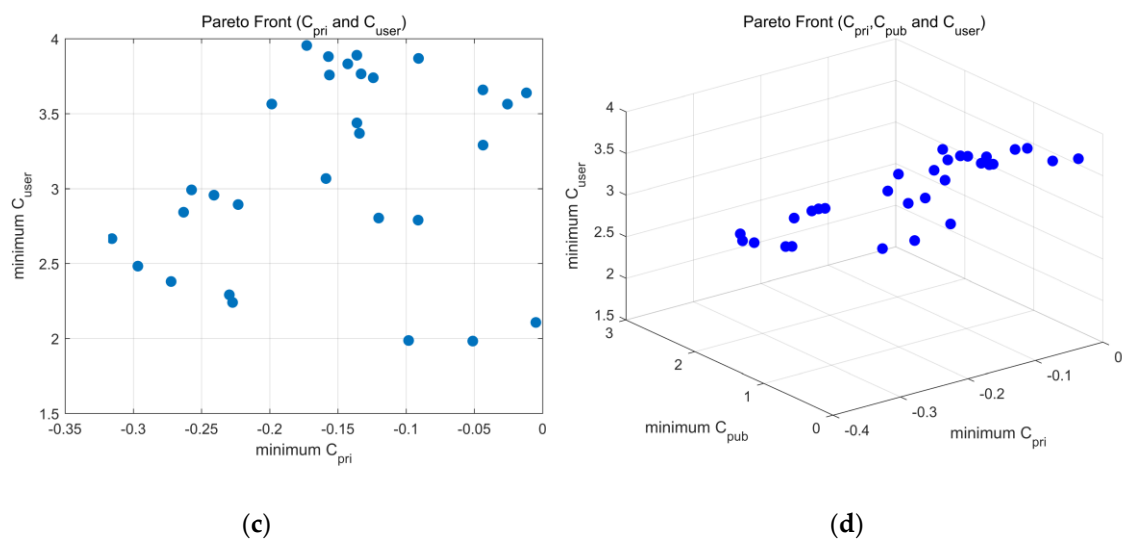
Figure 2. Pareto front solutions distribution chart of Model 1. (**a**) C_{pri} and C_{pub}; (**b**) C_{pub} and C_{user}; (**c**) C_{pri} and C_{user}; (**d**) C_{pri}, C_{pub} and C_{user}.

Figure 3 presents the Pareto front of Model 2. It seems that there is a kind of strict linear correlation between C_{PRI} and C_{PPP}. The full version of the Pareto front generated by Model 1 and Model 2 are listed in Table A2.

Figure 3. Pareto front solutions distribution chart of Model 2.

3.5. The Minimum Weighted Result

Table 8 is the minimum weighted sum and the value of the related decision variables and the value of the three objectives. C_{pub} is slightly more than C_{user}, which means that the user paid less than the public sector to the project. The share ratio of the public sector was close to its upper limit, and the whole concession period duration was much shorter than the longest one of 19 cases (35 years).

Table 8. The minimum entropy-weighted sum of Pareto solutions of Model 1.

a	b	p	n	C_{pri}	C_{pub}	C_{user}	Weighted Sum
0.487381	17.64895	1.98632	12.38446	-0.0982	2.231788	1.98632	1.284605

As could be seen from Table 9, each decision variable is close to its own upper limit, and the net return obtained by the private sector through 25 years of operation exceeds the total investment of the project.

Table 9. The minimum entropy-weighted sum of Pareto solutions of Model 2.

| a	b	r_g	C_{PRI}	C_{PUB}	Weighted Sum
0.487917	89.15039	0.079737	−1,252,278	2,137,844	165,775.7

4. Discussion

From the collected 19 URT PPP project cases, it is understandable that to certain extent, the weight distribution among the 15 indicators in VFM qualitative evaluation reflected some of the quantifiable characteristics of the project that were valued by the experts who graded on the mark sheet. However, it may be difficult to directly obtain the correlation between the weight of the qualitative indicators and quantifiable characteristics. Factor analysis, as a common method of dimensionality reduction, could be used to explore such correlations by analyzing the data weights, and to find out the most important aspects and which qualitative evaluation indicators contributed to them substantially. Due to the application of the optional indicators in Chinese PPP practice, not all 15 indicators could be given non-zero weights, which then means that the correlation matrix is not positive definite, and the application of factor analysis tests are restricted. The unifying indicators in each evaluation may help in determining the quantitative decision variables with a more explanatory result factor analysis.

The entropy weight method is one kind of objective weighting that could avoid the bias caused by the expert preference that may occur in the subjective weighting methods. Under the same evaluation indicator, the greater the difference of the observed values, the smaller the entropy value of the corresponding indicator, and the greater the entropy weight. Initially, for Model 1, the author used the Pareto solutions directly as evaluation items under all three objectives—C_{pri}, C_{pub}, and C_{user}—and found out that no matter how many times the experiment was conducted, the weights were equally distributed among the three evaluation indicators. The possible reason for that is the variance of observed values under each indicator produced by the solver equally. Thus, in order to reflect the difference degree of C_{pri}, C_{pub}, and C_{user}, substituted indicators are needed.

This paper used E_i, Bi, and Q_i as the entropy-weighting indicators for Model 1, because the operation cost in every year is fixed by C_{oy} and Q. To be consistent with Model 1 in entropy-weighting, Model 2 selected E_i and Bi for calculating the weights of C_{PRI} and C_{PUB}. As seen from the results, in Model 1, the private sector and the users are equally weighted, and the public sector is slightly lower. In Model 2, the private sector has a slightly higher weight than the public sector. These results are basically consistent with the reality of Chinese PPPs practice, in which the public sector tent gives more benefits to the private sector and the users to enhance their willingness of participation, with non-negative VFM as the prerequisite of the project.

As seen in the Pareto front results of both Model 1 and Model 2, the private sector achieved profits based on the payment from the public sector and the users. In Model 1, b and p had the upper limit as positive infinity, while in Model 2, b had the upper limit and p was a constant parameter. However, b was much lower in Model 1 than in Model 2; since the users were burdened with a higher unit cost in Model 1, that unit price may be too expensive for the local citizen. In Model 2, in which the private sector yielded at a high level, the possible reason was that the decision variables a, b, and r_g could not keep the private sector from over-profiting, reflecting that the current policies may not be reasonable.

Figure 4 displays the variance (%) of C_{PRI} and C_{PUB} calculated by the decision picked out according to the minimum weighted sum in both models when some parameter added its own −20%, −10%, 0, +10%, +20%, respectively. Figure 4a shows that C_{PRI} is very sensitive to most of the six parameters, r_g and C_{riksy} not included. Table 10 shows the actual values of C_{PRI} when these parameters changed; the positive values are underlined, revealing that the private sector had a net loss, which failed to satisfy the prerequisites of a PPP project. Figure 4c presented that the C_{PRI} value in Model 2 was insensitive to the parameters, except for r_p. Figure 4b–d showed that C_{PUB} was insensitive to the parameter changes in both models.
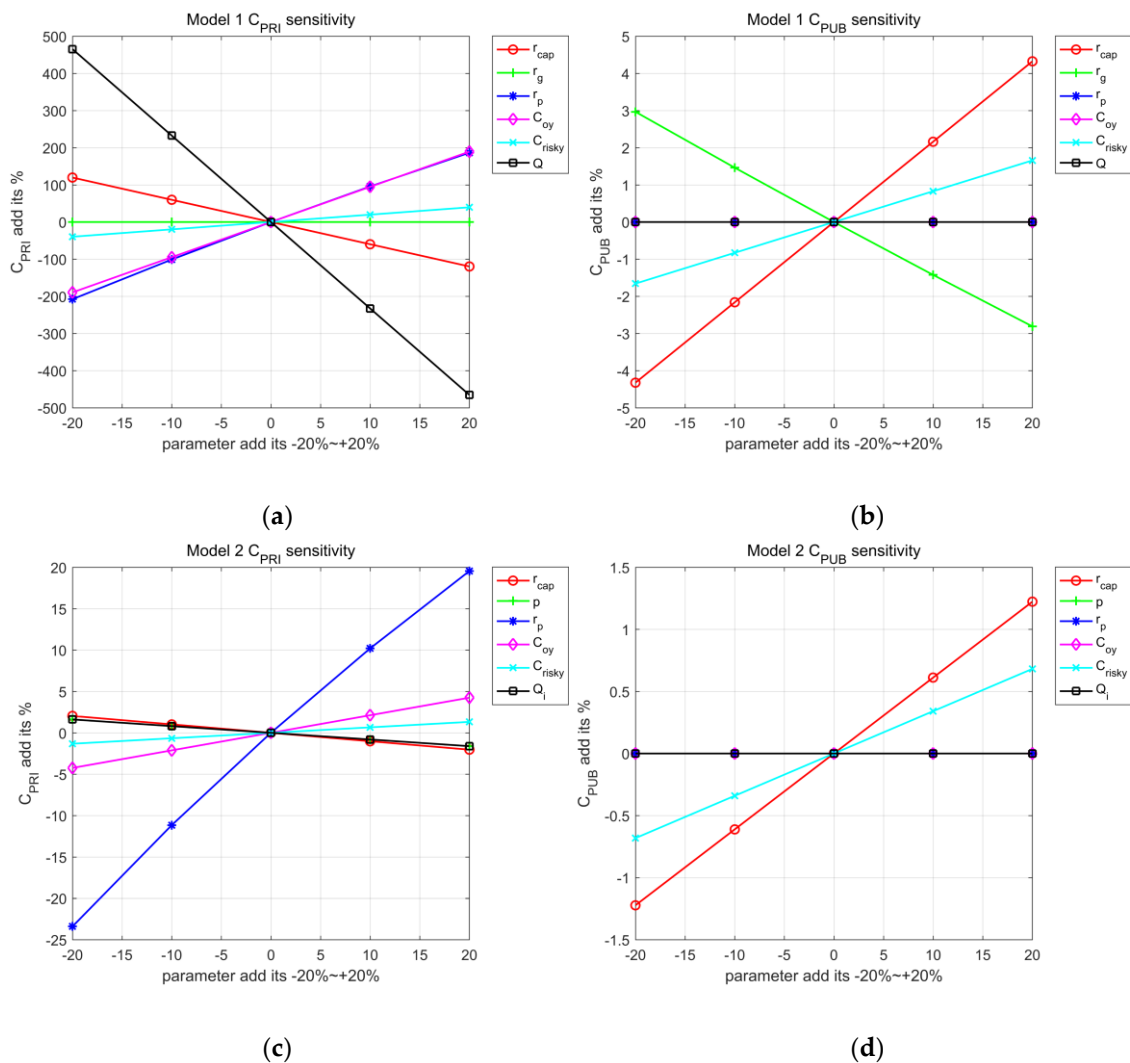
Figure 4. Sensitivity results of Model 1 and Model 2. (**a**) C_{PRI} in Model 1, (**b**) C_{PUB} in Model 1, (**c**) C_{PRI} in Model 2, (**d**) C_{PUB} in Model 2.

Table 10. The minimum entropy-weighted sum of the Pareto solutions of the optimization of Model 1.

	−20%	−10%	0	10%	20%
when r_{cap} changed	3804.055	−7795.05	−19,394.1	−30,993.3	−42,592.4
when r_g changed	−19,394.1	−19,394.1	−19,394.1	−19,394.1	−19,394.1
when r_p changed	−59,664.9	−38,980.1	−19394.1	−835.989	16,760.19
when C_{oy} changed	−56,123.2	−37,758.7	−19,394.1	−1029.62	17,334.92
when C_{risky} changed	−27,075.6	−23,234.9	−19,394.1	−15,553.4	−11,712.7
when Q_i changed	70,829.07	25,717.46	−19,394.1	−64,505.8	−109,617

5. Conclusions

In this paper, factor analysis was carried out on the qualitative evaluation weights of the 19 URT PPP cases, and the results were used to determine the quantitative decision variables of the optimization models. Two types of nonlinear multi-objective models were established to solve the Pareto front solution sets, with constraints expressed as non-negative VFM values, and the return of the private sector upon the PSC method. The entropy-weights method was applied to evaluate the Pareto solutions, and the one with the minimum weighted sum was picked up. The calculated entropy weights showed consistency with the reality of Chinese PPPs practice. Finally, the sensitivity analysis of the present value of the public sector and private sector cost calculated by the determined decision variable value is obtained.

This paper provided a basic model under the scenario of static decision-making. The following two were not discussed throughout this article, but deserve further study:

One is the financing decision model of a cost-recovery URT PPP project based on add–value–capture (LVC) theory as the extension of the basic model in this paper. The other is the multi-stage dynamic and flexible decision-making problems over a whole life cycle of the URT PPP project, which can be divided into different stages of financing, bidding, construction, operation, maintaining, retrofitting, capital transferring, etc.

Author Contributions: F.L. contributed to the research design, data collection, implementation of model, and manuscript writing. J.L. and X.Y. acted as supervisory roles and contributed equally to problem analysis and the application of methodology.

Appendix A

The VFM assessment reports of 19 URT PPP projects containing original manuscripts of grates and weights that listed in Tables A1 and A2 are available online at http://www.cpppc.org:8086/pppcentral/map/toPPPList.do.

Table A1. Part 1 of the original weights of 15 indicators from 19 URT PPP cases (Case 1 to 10).

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10
Integration	0.1	0.15	0.15	0.15	0.17	0.17	0.15	0.15	0.15	0.15
Risk Allocation	0.2	0.15	0.17	0.15	0.18	0.18	0.15	0.15	0.15	0.1
Performance	0.1	0.15	0.12	0.15	0.12	0.12	0.15	0.15	0.15	0.15
Competitiveness	0.2	0.1	0.13	0.15	0.08	0.08	0.15	0.15	0.15	0.15
Government Competency	0.1	0.1	0.13	0.05	0.13	0.13	0.05	0.05	0.1	0.1
Bankability	0.1	0.15	0.1	0.08	0.12	0.12	0	0.03	0.1	0.15
Procurement Potential	0	0	0	0.1	0	0	0.1	0.05	0	0
Scale	0	0	0.04	0	0.04	0.04	0.06	0.05	0.05	0.07
Life span	0	0	0	0	0.04	0.04	0	0.05	0	0
Capital Diversity	0	0	0	0.07	0.04	0.04	0.07	0.03	0	0
Cost Accuracy	0.1	0	0.05	0.05	0.08	0.08	0	0.03	0	0
Legal Support	0	0	0	0	0	0	0.07	0.03	0	0
Income Potential	0.1	0.08	0.04	0.05	0	0	0.05	0.05	0.05	0.07
Exemplary	0	0.06	0.07	0	0	0	0	0.03	0.1	0.06
Acceptance	0	0.06	0	0	0	0	0	0	0	0

Table A2. Part 2 of the original weights of 15 indicators from 19 URT PPP cases (Case 11 to 19).

	Case 11	Case 12	Case 13	Case 14	Case 15	Case 16	Case 17	Case 18	Case 19
Integration	0.15	0.13	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Risk Allocation	0.15	0.15	0.2	0.2	0.15	0.15	0.15	0.15	0.15
Performance	0.15	0.12	0.1	0.1	0.15	0.15	0.15	0.15	0.15
Competitiveness	0.1	0.13	0.1	0.1	0.1	0.15	0.15	0.15	0.15
Government Competency	0.1	0.15	0.1	0.1	0.1	0.05	0.05	0.05	0.05
Bankability	0.15	0.12	0.15	0.15	0.15	0.15	0.1	0.04	0.1
Procurement Potential	0	0	0	0	0	0	0.03	0.1	0.03
Scale	0.06	0.06	0	0	0.04	0.05	0.03	0.06	0.03
Life span	0	0	0.05	0.05	0.03	0.05	0	0.04	0
Capital Diversity	0	0	0	0	0.04	0	0.04	0.06	0.04
Cost Accuracy	0	0.07	0.05	0.05	0.03	0.05	0.03	0	0.03
Legal Support	0	0	0	0	0	0	0.03	0	0.03
Income Potential	0.08	0.07	0.05	0.05	0.03	0.05	0.05	0.05	0.05
Exemplary	0.06	0	0.05	0.05	0.03	0	0.04	0	0.04
Acceptance	0	0	0	0	0	0	0	0	0

Table A3. The full version of Pareto Front generated by Model and Model 2.

	Model 1			Model 2	
No.	C_{pri}	C_{pub}	C_{user}	C_{PRI}	C_{PUB}
1	−0.01182	0.236268	3.638375	−161,293	1,187,172
2	−0.13615	0.353558	3.889556	−1,261,708	2,305,979
3	−0.05116	2.225221	1.982355	−1,196,545	2,080,527
4	−0.31566	2.167159	2.665843	−987,204	1,898,145
5	−0.04375	0.297243	3.658421	−304,068	1,311,654
6	−0.13424	0.972897	3.368459	−1,260,641	2,159,870
7	−0.0982	2.231788	1.98632	−847,846	1,781,122
8	−0.14278	0.560115	3.831897	−821,201	1,764,332
9	−0.13303	0.45965	3.765208	−1,261,407	2,217,593
10	−0.15699	0.527611	3.880541	−1,224,720	2,116,505
11	−0.12411	0.4304	3.73927	−339,572	1,339,597
12	−0.00492	2.1559	2.106714	−281,836	1,294,707
13	−0.29665	2.320473	2.482393	−1,002,835	1,917,117
14	−0.02571	0.840748	3.563561	−321,014	1,336,837
15	−0.09123	1.67997	2.789534	−232,086	1,247,415
16	−0.2296	2.258401	2.291057	−728,035	1,680,591
17	−0.13609	1.113198	3.438026	−314,521	1,332,921
18	−0.24086	1.763577	2.955981	−356,058	1,366,419
19	−0.19851	1.020047	3.563972	−1,025,183	1,941,797
20	−0.12015	1.643778	2.803375	−486,996	1,474,725
21	−0.26312	1.9003	2.842046	−883,049	1,818,761
22	−0.22314	1.842073	2.893234	−1,131,454	2,036,124
23	−0.22722	2.373951	2.240217	−214,515	1,231,844

Table A3. *Cont.*

	Model 1			**Model 2**	
No.	C_{pri}	C_{pub}	C_{user}	C_{PRI}	C_{PUB}
24	−0.15876	1.562877	3.066917	−1,039,812	1,948,281
25	−0.15625	0.717709	3.757613	−1,158,419	2,061,794
26	−0.09092	0.380833	3.868423	−1,123,306	2,016,009
27	−0.27229	2.38963	2.379703	−232,574	1,250,608
28	−0.17288	0.628949	3.954517	−840,417	1,771,252
29	−0.2573	1.700806	2.991098	−1,252,278	2,137,844
30	−0.04364	1.161033	3.289835	−250,612	1,264,479

References

1. Cruz, C.O.; Marques, R.C. Theoretical considerations on quantitative PPP viability analysis. *J. Manag. Eng.* **2013**, *30*, 122–126. [CrossRef]

2. Asian Development Bank (ADB); Inter-American Development Bank (IDB); World Bank Group; Public-Private Infrastructure Advisory Facility (PPIAF). *Public-Private Partnerships: Reference Guide, Version 2.0*; World Bank Group: Washington, DC, USA, 2014.

3. Jiang, D.M.; Zhao, Z.; Management, S.O.; University, Q.T. Research on Investment Risk Management of Expressway PPP Project. *Value Eng.* **2017**, *11*, 35–38. [CrossRef]

4. Jin, X.H. Neurofuzzy Decision Support System for Efficient Risk Allocation in Public-Private Partnership Infrastructure Projects. *J. Comput. Civ. Eng.* **2010**, *24*, 525–538. [CrossRef]

5. Tang, Q. Research on the Risk of the PPP Project Based on the Cumulative Prospect Theory. *Value Eng.* **2017**, *12*, 69–71. [CrossRef]

6. Yin, H.; Li, Y.; Zhao, D. Method of Decision Making of PPP Project Risk Allocation Scheme Based on Cloud Model. *Scie. Technol. Manag. Res.* **2016**, *36*, 201–206. [CrossRef]

7. Song, B.; Fei, X.U. Partner-Selection in Public-Private Partnership Project Based on an Iterative Algorithm for the Multi-Objective Group Decision Problem. *J. Syst. Manag.* **2011**, *20*, 690–695.

8. Xie, J.; Ng, S.T. Multiobjective Bayesian Network Model for Public-Private Partnership Decision Support. *J. Constr. Eng. Manag.* **2013**, *139*, 1069–1081. [CrossRef]

9. Sharafi, A.; Iranmanesh, H.; Amalnick, M.S.; Abdollahzade, M. Financial management of Public Private Partnership projects using artificial intelligence and fuzzy model. *Int. J. Energy Stat.* **2016**, *4*, 1650007. [CrossRef]

10. Xu, Y.; Sun, C.; Skibniewski, M.J.; Chan, A.P.C.; Yeung, J.F.Y.; Cheng, H. System Dynamics (SD)-based concession pricing model for PPP highway projects. *Int. J. Proj. Manag.* **2012**, *30*, 240–251. [CrossRef]

11. Xue, Y.; Guan, H.; Corey, J.; Qin, H.; Han, Y.; Ma, J. Bilevel Programming Model of Private Capital Investment in Urban Public Transportation: Case Study of Jinan City. *Math. Probl. Eng.* **2015**, *2015*, 1–12. [CrossRef]

12. Xue, Y.; Guan, H.; Corey, J.; Wei, H.; Yan, H. Quantifying a financially sustainable strategy of public transport: Private capital investment considering passenger value. *Sustainability* **2017**, *9*, 269. [CrossRef]

13. Liu, J.; Yu, X.; Cheah, C.Y.J. Evaluation of restrictive competition in PPP projects using real option approach. *Int. J. Proj. Manag.* **2014**, *32*, 473–481. [CrossRef]

14. Hu, Z.; Fan, X.F.; Dong, Q. Decision-making model of public-private partnerships projects' paradigm choice—Based upon the SVM classified theory. *J. Xi'an Univ. Archit. Technol.* **2012**, *4*, 568–571. [CrossRef]

15. Wang, Z.; Rangaiah, G.P. Application and Analysis of Methods for Selecting an Optimal Solution from the Pareto-Optimal Front obtained by Multi-Objective Optimization. *Ind. Eng. Chem. Res.* **2016**, *56*. [CrossRef]

16. CPPPC China Public Private Partnerships Center—Project Database. Available online: http://www.cpppc.org:8086/pppcentral/map/toPPPList.do (accessed on 10 May 2018).

17. Committee, G.M.T. Guiyang Rail Transit Line 2 Public Financing Feasiblity Report. Available online: http://www.cpppc.org:8083/efmisweb/ppp/projectLibrary/getProjInfoNational.do?projId=d4f66527c80b408f8a7729af7590fe1f (accessed on 10 May 2018).