


## Article

# Sustainable Value of Investment in Real Estate: Real Options Approach

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**Abstract:** The issue of application of real option valuation approach in the valuation of investment project is presented in the article in a way in which the flexibility of the project could be included in the process of its valuation. The authors apply the valuation approach in case of a specific investment project in the real estate in the capital city of the Czech Republic—Prague, using the option to expand, to contract, and to abandon the project. The main aim of this case study is to present a practical application of the investment valuation and to construct an option pricing model for real estate investment which considers and integrates as many aspects of the investment and market environment as possible to describe the best situation of the real estate market and its development. The valuation of the investment is carried out using a universally applicable numerical method of binomial trees. The results obtained are subjected to the sensitivity analysis with respect to the discount rate, value of the most influential parameter of the volatility and the input option parameters. The results of the valuation of the project obtained using the real option approach are important mainly for the management of the company in the process of quantification of the present value of future investments. Implementation of managerial interventions enables for optimizing the value of the project not only in case of favourable development of the real estate market, but particularly in case of unfavourable development. Therefore, they are important in order to protect an investor from potential high losses. Finally, the valuation of these interventions increases the present value of the project, contributing to the decision of the corporate management regarding its implementation.

**Keywords:** real option; real estate investment; binomial tree; net present value; volatility; discount rate; discounted cash flows; project evaluation; flexibility

## 1. Introduction

The real option valuation, based on the principles of the financial option valuation, has been considered to be one of the most widely applied valuation approaches for some years, not only in the area of real estate investment valuation. By applying this methodology, it is possible to quantify the value of flexibility of managerial interventions during the existence of the project. Moreover, the approach of real option or combination of more real options allows valuating postponement, extension, constriction, abandonment of the project, etc.

The real option valuation follows the dynamic methods of investment evaluation that consider the relationship between the value of initial investments and expected cash flows in their present value

that the project is able to generate during its existence. However, they do not take into account any possibility to make managerial interventions during the existence of the project. It means that, in the process of the evaluation of the investment project, the value of managerial flexibility is not considered at all. In a risky environment, this is a very strong argument because managerial interventions react to the price development and other important information that may fundamentally affect the final value of the project. The managerial interventions are carried out to increase the profitability of the project in case of favourable market conditions or, otherwise, to minimize losses of the project in case of negative development or even early termination of the project because of markedly negative development. The inclusion of the value of real options representing the right to implement intervention in the project greatly increases the value of the project itself.

The purpose of this paper is to construct an option pricing model for real estate investment considering and integrating as many aspects of the investment and market environment as possible. This paper incorporates many practical factors of the real estate investment, which are not investigated in previous studies. It extends the application of the real option theory from the framework borrowed from financial option pricing and considers the case where the investor has an opportunity to intervene in the project during its life. It examines the implications of this additional intervention to the project and its development. This paper therefore contributes to bridge the theoretical models and practical applications. The models constructed in this paper and the results can be directly used in the practical real estate investment evaluation. It also shows the effects of selected factors values on the project value and its development. The evaluation of investment project is realized by the binomial trees technique. This numerical technique is popular in practice despite its comparatively complex mathematical apparatus needed to compute and time-consuming calculation in case of American type options. Currently, the commercial software applications are able to handle these problems. However, they are usually financially demanding and they are not universally applicable due to the uniqueness of the terms of each investment project.

The aim of this paper is also to increase knowledge about real options and understand why and how real options are used, or could be used in the future. The analysis highlights that flexibility in the real estate investment may create additional value enabling investors to react to market trends when new information arrives. This extra value added by managerial flexibility is neglected using the methods of discounted cash flows, *DCF*, and net present value technique. In this way, the real estate management should become more resilient to changes and investments in real estate become more sustainable.

The value added and originality of our case study lie in the suggestion of the application of the real option method for valuation of the real estate investment project in the capital city of the Czech Republic—Prague. Similar studies have not been carried out yet, thereby this study could be considered as a pioneer work in this field. The study used a portfolio approach for valuation of the combined option, consisting of an option to expand a project, to contract the project, and to abandon the project. Thus, the adding value of the study lies in the combination of a portfolio of options applied on the real estate market in Prague and a real option framework for evaluating the flexibility incorporated in a project.

The study is divided into four main chapters. The literature review highlights the current state of the research in the field of the real option application for real estate investments. The second chapter includes description of the intention of the investment project. It offers also a brief methodology of the real option application in practice including the estimation of all input parameters that are needed in the calculation. The third part of the study presents the results of the project evaluation by a real option approach, including the sensitivity analysis of the results to the values of the estimated and input parameters. The discussion includes the analysis of other already existing studies in the field of real option approach application for investments valuation and presents relevant suggestions for its further possible direction.

## Literature Review

In 1977, the US economist Stewart Myers [1] first presented and defined the real options, namely the option to expand, delay, and abandon the strategic project based on new information and circumstances relating to the project under the evaluation. The substance of this new approach to the evaluation of the strategic investment project lies in the analogy with the principles of the financial options. He assumed that the value of each investment project was a derivative of initial capital investments, expected earnings, time and mainly project-related uncertainty. In case some conditions are met, the approaches to the valuation of financial options are applied in the process of the real options valuation, which represents a new method of the investment project evaluation at all. The correctness of his approach to the application of the principle of financial options on the valuation of real options was proven later in the works of Folta and O'Brien [2] and Borison [3].

The methodology of real options is often applied in investment evaluation—for example, in the field of the energetics [4–13], transportation and logistics [14–17], the issue of PPP projects [18,19], evaluating investments in IT technologies [20,21] or evaluation of the projects in other interesting areas [22,23].

In the area of real estate investment evaluation, Titman [24] was the first who used the real option framework. Since then, the real options approach has been used in many studies for setting the prices of various investments in real estates. Bragt et al. [25] in their study created a risk-neutral valuation model for real estate derivatives based on the real estate index process and derived valuation equations for forwards, swaps, and options on this index. They apply their approach to evaluate the Dutch real estate market. The study of Cirjevskis and Tatevosjans [26] illustrates the empirical testing of the real option application on pricing the investment project in Latvia. One of the methods used to evaluate the project is the binomial option pricing model. Authors used a five-step tree with each step for a period of a year. Parthasarathy and Madhumathi [27] provide a case study analysis in India where they applied a real options approach to evaluate the investment in real estate. Wu et al. in their study [28] propose a pricing model based on the real option method to evaluate presale houses and examine some crucial elements that affect the option price. The study of Li et al. [29] proposed a real option-based valuation model for private-owned public rental housing in China. In the paper by Morano [30], real options are used for evaluation of an investment in real estates, namely in urban redevelopment of a former industrial complex. Dortland et al. [31] provide insights about the potential of real option approach for evaluating flexibility of investment in corporate real estate from the owner–user perspective rather than from an investor perspective as most literature on real options. Vimpari and Junnila [32] test option pricing to quantify the option to wait in a residential real estate fund divestment. Baldi [33] used various combinations of options to evaluate an investment for the construction of a new, multi-purpose building in the semi-central zone of the urban area of Rome in Italy. Shen and Pretorius [34] used binomial option pricing framework to construct models for real estate development by considering and incorporating institutional arrangements, direct interactions from outside environment and financial constraints in the model. Bravi and Rossi in their study [35] show the limits of the traditional DCF analysis in capturing flexibility in the real estate investment and present an application of the real option approach in pricing an industrial urban area within a backward risk-neutral valuation process. Kim and Song [36] used a real options framework to measure and manage bubbles in the Korean real estate market taking into account the unique leasing mechanism. Mintah et al. [37] used the real options approach for staging option application to residential development. In their consequent study [38], Mintah et al. involve the exploration of an optimal strategy for residential property investment in Australia development through real option analysis and valuation of a mixed use investment. Lucius in his study [39] focused on the current research in the area of real option application in real estate project valuation. He points out that more practical valuations rather than academic have to be comprehensively carried out and further research concerning the basic prerequisites of real options theory has to be undertaken. Guthrie [40] illustrates

the valuation of the real estate investment by a real options approach using a hypothetical example of a partly completed apartment complex in Las Vegas.

It is obvious that many studies have found the importance of the real options pricing principles in the real estate market investments as the replacement for the traditional *NPV* based valuations [41,42]. In the Czech Republic, Scholleova [43,44] is one of the first authors who started to deal with the real options. At present, this topic is the focus of interest of many Czech authors, namely Culik [45–49] and Zmeskal [50–52]. In case of Slovakia, the real option valuation is mostly discussed in the academic area. We mean mainly the studies of e.g., Guttenova [53], Kramarova and Kicova [54], Fuus and Szolgayova et al. [55–57], Wafi [58], etc. On the other hand, research in the mentioned studies has focused on limited subjects such as energetic industry [46,55–57], construction [49], mining industry [48], business valuation [51] or information technologies [58].

## 2. Methodology and Data

The investment evaluated in this study is the intention of the company to purchase five different older flats in the capital city of the Czech Republic—Prague with a preliminary plan to rent these flats in the long-term for the next eight years, and then to sell them. The flats, situated in blocks of flats, were chosen based on the offers of the real estate offices in May 2018. Their purchase price reached 20.15 million of Czech Koruna, CZK, plus 0.5 million of CZK that represent other expenditures related to purchasing flats and their equipment. The flats had a character of 2 or 3-room apartments and their built-up area ranged from 55 to 65 m<sup>2</sup>. Description of the flats is summarized in Table 1. Flats not located in the basement or aboveground floor, which are significantly cheaper, but are of less interest for both landlords and buyers, were preferred. All flats were required to be located outside of the city centre due to cost considerations. However, all were located no more than 450 m from a metro station, which increased their attractiveness in the real estate market.

**Table 1.** Description of the flats.

Flat	Location	Area in m <sup>2</sup>	Price in Millions CZK	Monthly Rent in CZK
1	Křenická (Praha 10)	65	3.85	16,000
2	Pod Strání (Praha 10)	65	4.63	18,000
3	Sekaninová (Praha 2)	51	3.59	14,000
4	Biskupcova (Praha 3)	54	3.89	15,000
5	Bořislavka (Praha 6)	52	4.19	15,000
Total			20.15	78,000

The monthly rents were determined based on the comparison of monthly rents of similar flats used for renting. In comparison, the authors took into account 124 flats and aspects such as location, built-up area, and apartment equipment. It is considered that the flat rents are paid in advance. The monthly rent paid in advance in total is 78,000 CZK, excluding payments for energy. We considered monthly costs of the basic maintenance of flats and insurance of 7500 CZK. The project is financed by the company itself. We also took into account a 19% corporate income tax. We expected the annual growth of rents, expenditures and property prices of 2%, which corresponded to the target value of the inflation rate of the Czech National Bank. The parameters of the investment under consideration are shown in Table 2.

**Table 2.** Investment parameters.

Description	Parameter Value
Purchased price of the flats and their equipment.	20.65 millions CZK
Monthly flats rents without energy.	78,000 CZK
Monthly costs of the maintenance and insurance.	7500 CZK
Corporate income tax.	19% p.a.
Annual growth coefficient (inflation rate).	2% p.a.

At the same time, the investor admits the possibility of changing the project once during its lifetime according to the situation in the real estate market. This will be done either by expanding the project by purchasing another flat for the purpose of renting, contracting the project by selling one apartment, or abandoning the project by selling all flats in case of a significant decline in flat prices. If the price of the entire set of flats would, due to unfavourable development of the real estate market, approach 75% of the investment, i.e., 15.11 million CZK, the investor would consider selling all of the flats. Also in this case, the total sum is adjusted by an inflation rate of 2% yearly. An additional investment, in case of expansion of the project, is assumed in the amount of 19% of the original value of the investment, i.e., 3.85 million of CZK, while this value is increased every year by expected inflation rate of 2%. Contracting the project, i.e., selling one flat, is assumed to bring in 4.6 million of CZK, i.e., 23% of the original value of the investment. It is also assumed that this amount increases by inflation rate of 2% every year. In evaluating the project, we consider other price alternatives as well. The parameters of this real option are summarized in Table 3.

**Table 3.** Real option parameters.

Option	% of Initial Investment	Amount in Millions CZK	Description
Expansion	19%	$3.85 \times 1.02^t$	Additional investment.
Contraction	23%	$4.6 \times 1.02^t$	Sell-off price of a single flat.
Abandonment	25%	$15.11 \times 1.02^t$	Trigger value for selling off all flats.

The suggested investment project is based on a high interest in renting flats in Prague. In all of the Czech Republic, there is a lack of rental housing, which occurred as a result of gradual privatization of the housing stock. Therefore, more than three quarters of apartments are in personal ownership. The growing interest in living in Prague itself is not only due to its attractiveness, but also due to its safety and the fact that Prague has the lowest unemployment rate across the EU, only 1.7% compared to the EU average of 7.6% [59,60]. Prices of flats in the Czech Republic have increased by over 40% over the last four years. Further growth is expected, although at a more moderate rate [60]. According to the Property Index of the Deloitte consulting firm [61], the Czechs have attained an unenviable leading position—for the first time in the history of the compared countries, an average flat with a size of 70 m<sup>2</sup> costs 11 average gross annual salaries. Real estate prices in the Czech Republic are the fastest growing in the EU in the last two years, which reduces the availability of own housing. Even more significant is the increase in demand in Prague, where three significant factors contributed to the rise of apartment prices—the lack of new development projects, their lengthy approval processes, and low interest rates on mortgages and thus their increased availability. Rental yields in Prague are around 5.5% per year [61], according to other sources [60] ranging from 4 to 6%. In our case study, we choose a discount rate of 5% p.a. At the same time, rentals in Prague are 37% higher than the average in the Czech Republic [62].

Some of the experts are worried about a potential bubble on the real estate market and the Czech National Bank has therefore moved to limit the availability of mortgages due to the risk of a mortgage crisis, which would further aggravate the availability of property housing. For these reasons, the intended investment appears to be meaningful even when the prices of the flats are high [63]. In the case of the expected price increase, the project will be profitable. Therefore, we will consider less favorable alternatives in more detail.

#### *Process of Investment Evaluation*

The value of the analysed project is represented by so-called expanded net present value

$$eNPV = NPV + ROV, \quad (1)$$

where the net present value  $NPV$  is the discounted value of all expected cash flows that the project generates. The real option value  $ROV$  represents the value of the flexibility. Based on the  $NPV$

quantification, the project is considerable undervalued and should be refused by the management of the investor. However, it is clear that the real estate market has undergone development over the years and the investor of the project can respond to these changes by different managerial interventions into the project. The flexibility of managerial decisions represents the value of the real option *ROV*. If it is possible to exercise the option any time during its lifetime, it is an American option. Therefore, the option will be valued using a binomial tree. The binomial tree models the expected development of the underlying asset, e.g., the development of the present value of expected cash flows of the project with the volatility  $\sigma$ . The core component of the *NPV* of the project is the sell price of the flats. Hence, we use the historical flat prices to calculate the volatility. The input data consisted of indices of realized prices of older flats for the period from the beginning of 2008 to the end of the second quarter of 2018, which are reported quarterly by the Czech Statistical Office [64]. The standard deviation thus obtained was annualized and adjusted according to  $\sigma = \sigma_{\text{annually}} \times \sqrt{T}$ , where  $T$  is the time in years. The calculated value of  $\sigma = 12.5\%$  is understood (according to [43]) as the lower limit of the volatility estimate and, for the main proposed calculation, we choose  $\sigma = 15\%$ . However, due to the uniqueness of each purchase and sale of each older flat, it is necessary to translate the *ROV* sensitivity analysis to the value of volatility as part of the flexibility assessment of the project under review.

The flexibility of the project represents three types of the options. The option to expand the project is taken into account in case of favourable development in the real estate market, when the investor will decide to expand the investment by 19%, which means that the value of so-called extension factor is  $f_E = 1.19$ . The option to expand will be exercised if the present value of expected cash flows of the expanded project is higher than the value of the additional investment  $P_t^E$  in the time of decision-making  $t$ . The present value of all expected cash flows of the expanded investment project in time  $t$  is

$$PV_t(CF^E) = f_E \times PV_t(CF) - P_t^E, \quad (2)$$

where the  $PV_t(CF)$  represents the present value of expected cash flows of the original project.

In case of unfavourable development of the value of the investment, the option to contract by 23% can be applied. This means that the value of the factor of contract is  $f_C = 0.77$ . This type of option will be exercised in time  $t$ , if earnings from selling a part of the project are higher than the present value of the terminated cash flows of the original project. The present value of all expected cash flows of the contracted project in time  $t$  is

$$PV_t(CF^C) = f_C \times PV_t(CF) + P_t^C. \quad (3)$$

In case of a very unfavourable situation in the real estate market, the management can make a decision to sell all flats in the investor's portfolio, which represents the option to abandon the project. In this case, the project will be sold for its residual value  $P_t^A$  in time  $t$ . The option will be exercised, if the investor gains higher value than the present value of all expected cash flows until the end of the existence of the project. The present value of all expected cash flows of the expanded investment project in the time  $t$  is

$$PV_t(CF^A) = P_t^A. \quad (4)$$

By combining all of the aforementioned options to expand, to contract and to abandon the project, there is the option to choose. Since particular real options are mutually dependent and are also mutually exclusive, the final value of the option to choose is not given as a sum of the values of individual options. The entire option to choose is therefore valued by one binomial tree. In the calculation, we proceed recurrently back from the expiration date of the option to the time 0. The principle of valuing a real option using a binomial tree is then based on the American-type financial options principle. As stated in the study of Hull [65] for the case of American-type financial options or of [66] for real options, the possibility to exercise the option is valued in each node of the tree. The optimal option is selected (expansion of the project, contracting the project, abandonment of the project, or



continuing in the original project) that maximize the present value of the expected cash flows of the project. The final value in time 0 is then the present value of the expected cash flows including the value of the flexibility that is expressed as the value of the real option, taking into account the value of this possibility of exercising the option at that point of the tree.

In this way, we can calculate the expanded *NPV* of the project *eNPV*, which includes all assumed possibilities of managerial interventions into the project. In an analogous way, it is possible to determine the *eNPV* also in cases where we do not consider all possibilities to intervene in the project, e.g., we consider only the option to abandon the project, to expand the project, or the combination of the option to expand or to contract the project, etc. However, the obtained results are dependent on the selected parameters. The value of the *NPV* can be only influenced by the chosen discount rate *r*, and the value of the real option is most affected by the volatility  $\sigma$ . The impact of the change of these two parameters on the value of the project will be analysed in our case study too.

### 3. Results

#### 3.1. Net Present Value of the Investment Project

The net present value, *NPV*, of the project is the difference between the present values of expected cash flows of the project  $PV_0(CF)$  and initial capital investments *IN*. Thus, we get

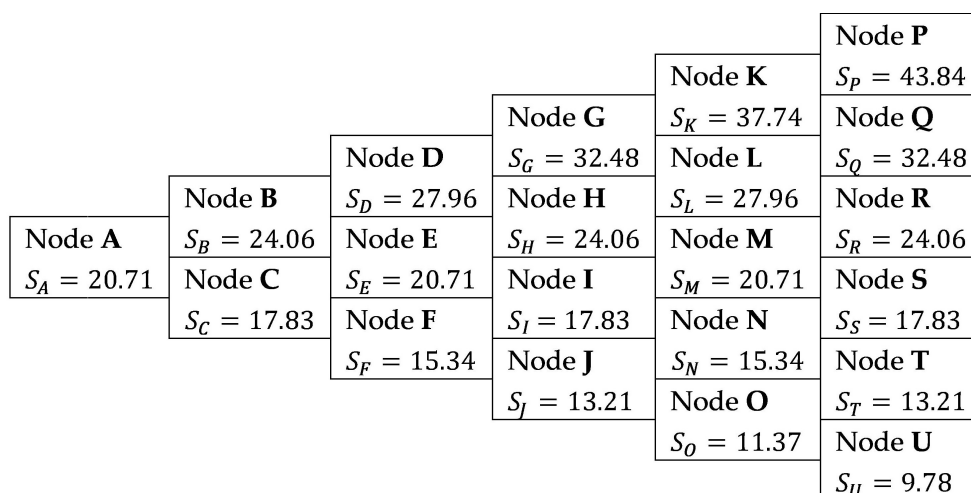
$$NPV = -IN + PV_0(CF) = -20,650,000 + 20,707,176 = 57,176. \quad (5)$$

The present value of the expected cash flows of the project is 57,176 CZK that is a relatively low financial benefit in comparison to the value of the initial investment in the acquisition of property. Thus, taking into account this fact, the management of the project could refuse it.

#### 3.2. The Evaluation of the Project Flexibility

The flexibility of the project can be seen as a complex (real) option to choose American type that includes the option to expand, option to contract, and option to abandon the project. The value of the flexibility is equal to the price of the American option, which is calculated using a five-step binomial tree with the each step for a period of a year. The underlying asset is the present value of cash flows of the project with the spot price  $PV_0(CF) = 20.71$  million of CZK and volatility  $\sigma = 15\%$  p.a. The lifetime of the option is five years, and the risk-free interest rate  $r_f$  is 1.5% p.a. If the option to expand the project is exercised, the project will be expanded to  $f_E = 1.19$  multiple with the exercise price  $P_t^E = 3.85 \times 1.02^t$ . In case of the exercise of the option to contract the project, we calculate with the contract factor  $f_C = 0.77$ , and the strike price  $P_t^C = 4.6 \times 1.02^t$ . The exercise price of the option to abandon the project is  $P_t^A = 15.11 \times 1.02^t$ . All prices and values are indicated in millions of CZK.

Figure 1 illustrates the simulation of evolution of the present value of the expected cash flows (the underlying asset) by applying five-steps binomial tree with each step for a period of a year. The value of  $S_A = 20.71$  million of CZK in the node A is a spot price of the expected cash flows without any managerial interventions in the project. We assume that, during the time interval of a year, the price of the underlying asset will either move up by a coefficient  $u = e^\sigma = 1.1618$  or will move down by a coefficient  $d = 1/u = e^{-\sigma} = 0.8607$ . Therefore, the price of the underlying asset moves up by  $u = 1.1618$  with the probability  $p = ((1 + r_f) - d)/(u - d) = 0.5124$  or moves down by  $d = 0.8607$  with the probability  $q = 1 - p = 0.4876$  [67,68]. Thus, for instance  $S_B = S_A \times u$ ,  $S_I = S_E \times d = S_F \times u$ , etc.



**Figure 1.** Development of the present value of the project cash flows without the managerial interventions.

The present value of the cash flows of the project in the 5th year is expected in the interval  $\langle 9.78, 43.84 \rangle$  millions of CZK. It means that the project could increase its value by more than 23 millions of CZK for five years. However, on the other hand, it also could lose value by almost 11 millions of CZK. In order to eliminate such a large drop in case of unfavourable development and maximize earnings in case of favourable conditions in the real estate market, the flexibility of the project represented by the real option to choose will be taken into account. We will use the values from the binomial tree in Figure 1 together with the parameters of this option in calculation of the present value (in time 0) of the cash flows of the project with inclusion of the optimal managerial decisions in individual nodes in the binomial tree. Similarly, as in the case of evaluation of American options using binomial tree, in each node of the tree, the possibility of early exercising of this option is considered. The value of this possibility has to be taken into account when calculating the current option price. This calculation is illustrated in Figure 2 (the values are in millions of CZK). The present values  $PV(CF)^{RO}$  of the expected cash flows of the project in the nodes are calculated recurrently backwards from the last column towards the first column. For the illustration of the calculations in the last column, we list the calculation of the value  $PV(CF)_P^{RO}$  in the node **P**. This value is the maximum of the present values of the expected cash flows in case of the following alternatives:

1. In case of expansion of the project in the node  $\mathbf{P}$ , following Equation (2), we get:

$$PV_5 \left( CF^E \right)_p = f_E \times S_P - P_5^E = 1.19 \times 43.84 - 3.85 \times 1.02^5 = 47.92 \text{ millions of CZK.}$$

2. In case of contraction of the project in the node **P**, following Equation (3), we get:

$$PV_5(CF^C)_n = f_C \times S_P + P_5^C = 0.77 \times 43.84 + 4.6 \times 1.02^5 = 38.86 \text{ millions of CZK.}$$

3. In case of abandonment of the project in the node **P**, following Equation (4), we get:

$$PV_5(CF^A)_n = P_5^A = 15.11 \times 1.02^5 = 16.68 \text{ millions of CZK.}$$

4. In case the project will continue in the original range in the node **P**, following Figure 1, we get

$$PV_5 (CF)_P = S_P = 43.84 \text{ millions of CZK.}$$

Therefore, in this node, the most advantageous option is to apply the option to expand the project because the present value of the expected cash flows that are generated by this alternative is the highest from all present values. Thus,  $PV(CF)_p^{RO} = 47.92$  millions of CZK.



The calculations of the  $PV(CF)^{RO}$  of the project in other columns are a little bit different. As the example of this procedure, we take the calculation of  $PV(CF)_O^{RO}$  in the node **O**. This value is again the maximum of the present values of the expected cash flows of the project in case of all four considered alternatives:

1. In case of expansion of the project in the node **O**, following Equation (2), we get

$$PV_4(CF^E)_O = f_E \times S_O - P_4^E = 1.19 \times 11.37 - 3.85 \times 1.02^4 = 9.36 \text{ millions of CZK.}$$

2. In case of contraction of the project in the node **O**, following Equation (3), we get

$$PV_4(CF^C)_O = f_C \times S_O + P_4^C = 0.77 \times 11.37 + 4.6 \times 1.02^4 = 13.73 \text{ millions of CZK.}$$

3. In case of abandonment of the project in the node **O**, following Equation (4), we get

$$PV_4(CF^A)_O = P_4^A = 15.11 \times 1.02^4 = 16.36 \text{ millions of CZK.}$$

4. In case the project continues in the original range, we will use the discounted weight average of the present values of the expected cash flows in the nodes **T** and **U** so that

$$\begin{aligned} PV_4(CF)_O &= \frac{1}{1+r_f} \times (p \times PV_5(CF)_T^{RO} + (1-p) \times PV_5(CF)_U^{RO}) \\ &= \frac{1}{1.015} \times (0.5124 \times 16.68 + 0.4876 \times 16.68) \\ &= 16.44 \text{ millions of CZK.} \end{aligned}$$

Based on the calculations, in node **O**, continuing in the original range of the project (open position in the option—OPEN) is the most appropriate alternative from the investor's point of view with the present value of the expected cash flows in amount  $PV(CF)_O^{RO} = 16.44$  millions of CZK.

					47.92
				40.74	EXPAND
			34.57	EXPAND	34.40
		29.43	EXPAND	29.10	EXPAND
	25.33	OPEN	24.94	EXPAND	24.38
22.17	OPEN	21.81	OPEN	21.34	EXPAND
OPEN	19.52	OPEN	19.18	OPEN	18.80
	OPEN	17.72	OPEN	17.51	CONTRACT
		OPEN	16.73	OPEN	16.68
			OPEN	16.44	ABANDON
				OPEN	16.68
					ABANDON

**Figure 2.** Development of the present value of the project cash flows.

By repeating this procedure, we get the present value of the expected cash flows of the project  $PV(CF)_A^{RO} = 22,168,319$  CZK in time 0. After deducting the present value of the cash flows of the project without the option to choose in amount  $S_A = 20.71$  millions of CZK, we get the price of the real option (the value of the flexibility)  $ROV = 1,458,319$  CZK. Based on Formula (1), the expanded value of the investment is then

$$eNPV = NPV + ROV = 57,176 + 1,458,319 = 1.52 \text{ millions of CZK.} \quad (6)$$

By including the value of the flexibility in the form of the combined real option to expand, contract, and abandon the project, the present value of the project has increased more than 25 times and the

project has become more attractive for the investor. In fact, it is possible to interfere in the project in a very complex way and maintain its value. In case of the analysed real option, we included into the value of the project only three exactly specified possibilities of its optimization. Naturally, it is possible to consider just the chosen possibility or chosen possibilities. Table 4 describes the values of the flexibility of the project (in CZK) in the form of the managerial decisions to expand, contract, and abandon the project or their combination. The greatest contribution to the *eNPV* has a combination of all three options that could be expected. Likewise, it can be seen that *ROV* for real options that combine two or three options is not a sum of the values of the individual real options. This fact confirms that individual options are not independent.

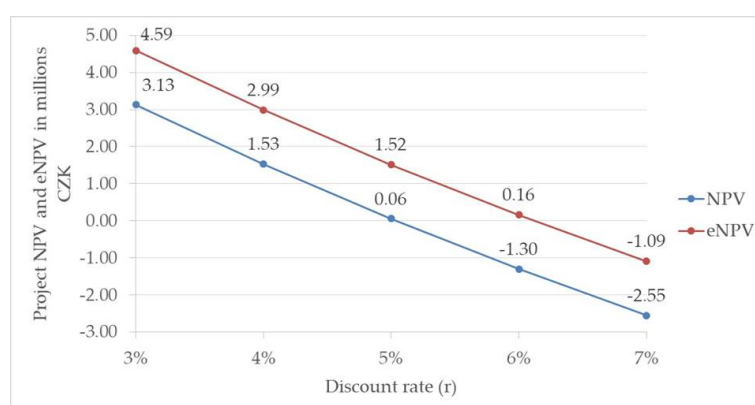
**Table 4.** Values of the real options in case of different combination of individual types of the real options.

Types of the Real Options	ROV	<i>eNPV</i>
without real option	0	57,176
expand	537,606	594,782
contract	623,473	680,649
abandon	644,089	701,265
contract and abandon	920,694	977,870
expand and contract	1,161,088	1,218,264
expand and abandon	1,181,694	1,238,870
expand, contract and abandon	1,458,319	1,515,495

### 3.3. Sensitivity Analysis

Project net present value, *NPV*, and therefore the management's decision to proceed with the project, can be significantly affected by the discount rate. When choosing a discount rate, we will use values ranging from 3 to 6% and we will examine the impacts on *NPV* by means of the sensitivity analysis.

Submitted calculations of both *NPV* and *eNPV* values are performed in a proprietary Microsoft Excel calculation based on parameters provided as inputs (Tables 2 and 3). The values of the *NPV* and *eNPV* in the Figure 3 confirm the generally valid fact that the real option method is of greater importance when the *NPV* is negative or close to zero. In such cases, it can fundamentally reverse the decision whether to implement the project or not. With the increasing *NPV*, the contribution of flexibility on the resulting *eNPV* decreases and has more informative character.



**Figure 3.** Dependence of the project *NPV* and *eNPV* on the discount rate.

Indirect linear dependence of *NPV* on discount rate *r* can be described by the trend line equation  $NPV = -1.419 \times r + 7.271$ . Based on the regression coefficient  $b_1 = -1.419$  it can be assumed that in case the discount rate is increases by 1%, the *NPV* of the project decreases by 1.419 million CZK. In case the discount rate of 5.04% equals to the internal rate of return (*IRR*), the *NPV* of the project is zero. Subsequently, in case of evaluation of the project using the *NPV* method, the management of the company applies the discount rate of 5.04% p.a. or more, the project will be evaluated as a loss and

will be rejected. The lowest discount rates increase the value of  $NPV$ , but, in fact, they are unrealistic in the Czech real estate market. However, the discount rates and  $NPV$  do not reflect real development of the real estate market and managerial interventions in the project related to them.

The result of the real option approach in the flexibility of the investment evaluation needs to be seen in a wider context particularly with respect to the change of individual input parameters since their input values are not determined exactly. The most important parameter that influences the price of the real option  $ROV$  is the volatility of the expected cash flows of the project that was estimated based on the historical data. The sensitivity of  $ROV$  to the changes in the volatility is illustrated in Figure 4. The input values of other parameters are constant and the same as aforementioned. The analysed real option to choose increases with the higher volatility, which confirms the theoretical premises.

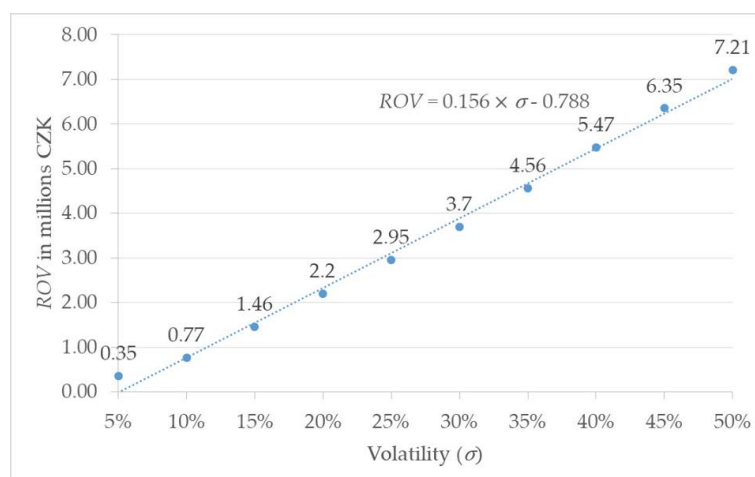


Figure 4. Dependence of the real option value,  $ROV$ , on the volatility.

We can see that more than 99% of the variability of the real option value of real option is explained by the volatility in the form of a linear relationship  $ROV = 0.156 \times \sigma - 0.788$ . The regression coefficient  $b_1 = 0.156$  represents the change of the value of real option in case of the change of the value of volatility by 1%. In case the volatility increases by 1%, the value of the real options increases by 156,000 CZK.

The following figures show the partial dependence of  $ROV$  on the value of factors of expansion, contraction and abandonment, i.e., only one parameter is changed at a time, under the *ceteris paribus* condition. Each considered amount is multiplied by the factor  $1.02^t$ , with  $t$  representing the number of years since the start of the project. Other (constant) parameters include the risk-free interest rate  $r_f = 1.5\%$  and volatility  $\sigma = 15\%$ . Individual partial cases are only theoretical but illustrative of the contributions of individual partial options to the total value of the combined option. The important observation is that, for any range of input parameters,  $eNPV$  remains significantly higher than  $NPV$ .

Figure 5 shows the dependence of  $ROV$  and  $eNPV$  on the trigger value for selling off the entire set of flats. The trigger value is analogous to stop-loss order in stock trading. For example, the trigger value of 25% means that, if the value of the entire set of flats falls below  $15.11 \times 1.02^t$  million CZK (decrease of total value by 25% of the initial investment), the investor sells the flats and abandons the project completely. The tested range of the trigger value for selling off all flats was 12% to 44% of the initial investment. Other input parameters (given in Table 3) are kept constant.

In the binomial tree representing the evaluation of the project cash flows by applying the real option (analogy of Figure 2), the abandonment option is not optimal at any point if the sell-off value for the entire set of flats falls by more than 42% of the initial investment. The abandonment option, thus, has zero value and  $ROV$  is given only by expansion and contraction options.

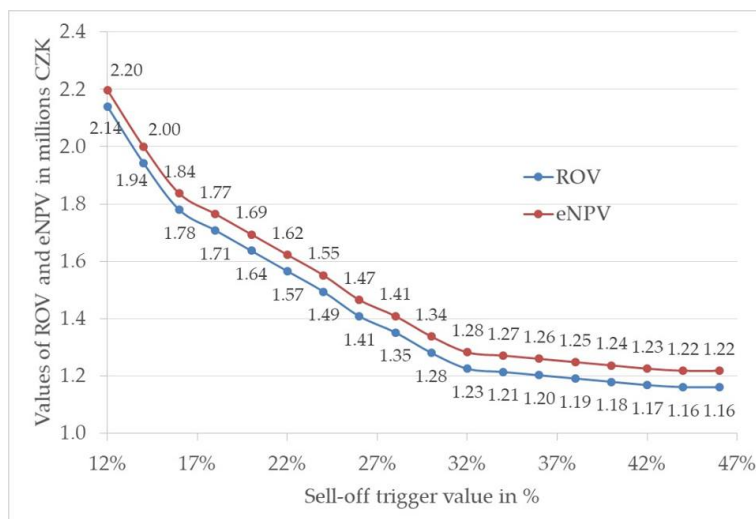


Figure 5. Dependence of ROV and  $eNPV$  on parameter of the project abandonment.

Figure 6 shows the dependence of ROV and  $eNPV$  on the expansion factor. Other input parameters (given in Table 3) are kept constant. We test the impact of an additional investment of  $3.85 \times 1.02^t$  million CZK on the overall increase of the planned cash flows. The tested range of additional investment was 6% to 22% of the initial investment.

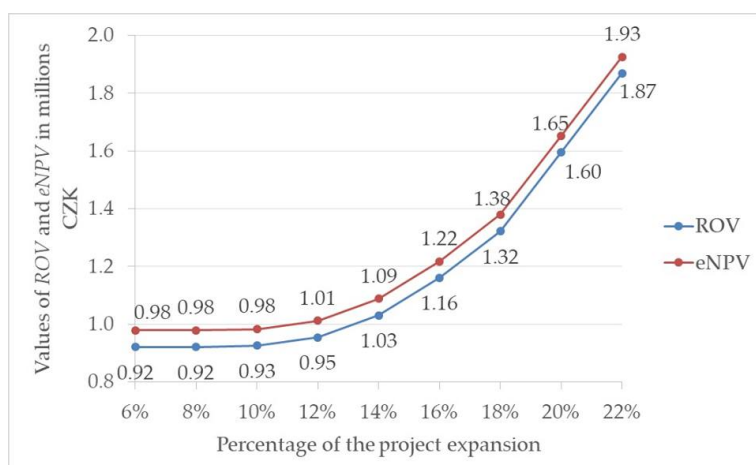


Figure 6. Dependence of ROV and  $eNPV$  on parameter of the project expansion.

By the sensitivity analysis, we found that, if the factor of expansion is below 10%, the expansion option is not optimal at any point of the binomial tree (analogy of Figure 2). The value of the expansion option is, thus, zero and the total value of the combined option consists only of contraction and abandonment options.

Finally, Figure 7 shows the dependence of ROV and  $eNPV$  on the factor of contraction. Other input parameters (given in Table 3) are kept constant. We test the impact of selling off one of the flats for  $4.6 \times 1.02^t$  million CZK, which will decrease the overall cash flows by 16% to 32% of the initial investment.

The sensitivity analysis showed that, if selling a flat for  $4.6 \times 1.02^t$  million CZK means a decrease in planned cash flows by mere 16% to 28%, the option to contract becomes viable in certain nodes of the binomial tree (analogy of Figure 2). This causes significant increase in the total value of the option to choose. On the other hand, if selling a flat decreases planned cash flows by 30% or more, i.e., to less than 70% of initial cash flows, the option to contract is not optimal at any point of the binomial tree. The total value of the option to choose then consists only of options to expand and abandon the project.

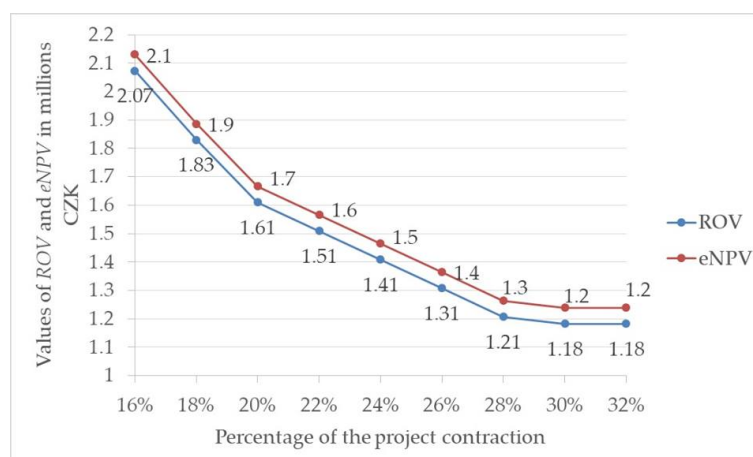


Figure 7. Dependence of ROV and eNPV on parameters of the project contraction.

The results of sensitivity analysis of input parameters from Figures 4–7 are summarized in the Table 5. Other input parameters given in Table 3 are kept constant,  $r_f = 1.5\%$ .

Table 5. Summarization of sensitivity analysis results.

Input	Tested	Change of ROV		Effect on Partial Options	
Parameter	Range (in %)	Dependence	Range (in Millions CZK)	Parameter Value	Type of Partial Option
Volatility	(5; 50)	direct	(0.35; 7.21)	-	Value of all types increases.
Sell-off trigger value	(12; 4)	indirect	(2.14; 1.16)	above 42%	Value of abandonment option is zero.
Factor of expansion	(6; 22)	direct	(0.92; 7.87)	below 10%	Value of expansion option is zero.
Factor of contraction	(16; 32)	indirect	(2.07; 1.18)	above 30%	Value of contraction option is zero.

#### 4. Discussion

Of all the areas that use the concept of the real options, researchers' greatest attention is devoted to the real estate market mainly thanks to the fact that this market is unstable, but huge with the great financial potential. The other important fact is that the availability of empirical data allows researchers to test developed methods empirically.

Titman [24] was one of the first who applied the real options in the area of real estate investment evaluation. The often cited study of Quigg [69] represents the first empirical effort in which the author tried to determine the value of the real option. His methodological framework is a specialization in some sense, on the other hand, the study is a generalization of the methodology presented by Williams [70,71]. This framework was later developed for the re-development of investments [72,73]. The theory has been developed in order to identify the main categories of the real options, such as option to delay, option to abandon, option to contract, and option to expand as well as various compound options that combine aforementioned types of real options [74]. Holland et al. [75] offer the empirical evidence that the models that are based on real options approach are really able to forecast the values of investments on the real estate market. A substantial part of the investment projects on the real estate market, including the investment project presented in this study, is possible to evaluate by applying the option to choose [66], which is the combination of the option to contract, option to expand, and option to abandon the project.

The real option valuation concerns specific features of the real estate market and it takes into account individual possibilities of developers, too. For example, the case study of the land development in Hong Kong analyses the differences between the general real option theory and its practical applicability in the field of development of real estate [34]. The results of the study [76] point to the fact that the values of real options differ in different zone categories for specific land use in the City of Chicago. The application of real options for valuation of the project in the real estate in a similarly unstable environment and under similar legal constraints was applied in Latvia [26]. Rocha et al. [77]

applied the theory of real options to the case of the construction of a two-phase residential housing in the western zone of Rio de Janeiro. Having applied this approach, it is possible to decide whether it is preferable to construct these residences simultaneously or in stages, which can be applied to the situation we have already analysed, in the context of the later purchase of a flat.

Bravi and Rossi [35] state that, if the development is delayed and the land has better alternative use in the future, the value of free land should reflect not only the value based on the best immediate use, but also the value of the option, as it has already been found in our case study. Similar conclusions, which result from the application of options for investment in flats in Prague, were also found in Baldi's study [33] implemented on the project aimed at investing in the construction of a new multi-purpose building in the wider center of Rome. This study points to the fact that the flexibility in the development of real estate prices can bring new value that allows developers to react to new market trends, whereby the use of portfolio of several types of real options is crucial.

In Slovakia and also the Czech Republic, the application of real option methods is the subject of many studies, but, apart from the study [53], the real option valuation has been mainly applied in the energy sector, not for the valuation of the project in the real estates. From this point of view, the study presented in this article can be considered innovative in the region of the Czech Republic and Slovakia.

A common feature of these studies is that the investment valuation methods based on the discounted cash flows are not appropriate tools for evaluating investment projects in the real estate, as we have demonstrated in this case study. Obviously, in a real-life situation, if the value of the investment project developed unfavourably, the management of the investor would definitely implement measures that would prevent high losses from the project and vice versa. If the development were favourable, the intention would be to increase the investment. However, these options cannot be considered in the method of evaluating a project using its *NPV*. It is therefore more appropriate to evaluate projects using the real option method, which uses a number of different options and thus better outlines the investor's situation. Most case studies, including this, actually point to the practical applicability of the real options in this area. The most complicated step in this approach is correct identification of the real options involved in a specific investment project. Subsequently, the flexibility of this project is evaluated by using this portfolio of options. The study elaborated in this article is also limited by this fact.

When applying the proposed evaluation procedure for another investment project, it will be necessary to re-identify real options included in the project as well as their parameters, based on the investor's requirements. One of the most important steps in this process is to estimate correctly the value of the volatility that definitely affects the resulting value of real options best. For the calculation of volatility, real data can be used, but it is also possible to use more complex econometric procedures for modelling volatility. As the sensitivity analysis shows, volatility is the factor that explains more than 99% of the real option price using a linear relationship, determined by the binomial tree method. Obviously, higher volatility means higher price of real option. Greater precision of the obtained numerical estimates of option values can be achieved by using the binomial tree with a greater number of steps of smaller length, for example, half-yearly or quarterly. This study shows the resulting option prices determined using a five-step binomial tree. Likewise, using a twenty-step tree with a one-quarter time step, very similar results were obtained, but with much higher computational demands. Hull [78] suggested using about thirty time steps in a binomial tree to achieve good results in a financial option valuation. On the other hand, for example, Kodukula and Papudesu [79] indicate that, in ROV, from four to six time steps are commonly sufficient for good approximations.

## 5. Conclusions

In this article, we focused on the application of the valuation of projects using real options. By applying the combined real option to choose, we evaluated the project of investment into the real estate situated in the capital city of the Czech Republic, Prague. Considering the net present value method of the investment valuation, we confirmed the theoretical assumption that this approach



significantly undervalued the project. This is due to the fact it did not consider the flexibility of managerial interventions in the project that were dependent on the real estate market development. On the contrary, the value of these interventions is able to quantify and include the real option method in the valuation of the project, which we submitted in this study. Using the binomial tree, we calculated the value of the flexibility of the project in the form of three potential managerial interventions. Proper evaluation of the flexibility of the investment plan is necessary for the investor in case of favourable development. Moreover, it is substantial particularly in case of an unfavourable market situation, or even in case of the catastrophic scenario. We pointed to the fact that the most important factor that influenced the value of flexibility was the volatility of the expected cash flows of the project. We also analysed its impact on the final value of the real option included in the investment. Moreover, we made a sensitivity analysis focused on the impact of other factors, e.g., the impact of the change of the contracting factor, the expanding factor of the project and also the trigger value of the investment for total abandonment of the project.

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