

Article

## Resource Allocation for Sustainable Urban Transit from a Transport Diversity Perspective

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**Abstract:** Different transport stakeholders have different needs for transport infrastructure and services. Meeting the needs of all stakeholders implies a trade-off of benefits and costs between supply and demand and creates transport diversity issues. However, the literature has largely ignored these issues. Transport diversity can assess the level to which important needs are satisfied equitably, and monitor whether transportation systems are moving towards sustainability by confirming the targets and basic level of quality of life. Based on the concept of transport diversity, this study utilizes fuzzy multi-objective programming to solve non-linear multi-objective problems involving urban public transit systems to determine the impact of resource allocation on needs satisfaction in relation to stakeholder behaviors. The proposed approach avoids problems of inefficient and inequitable resource allocation. A real-life case is presented to demonstrate the feasibility of applying the proposed methodology. Furthermore, empirical outcomes show that recent investments allocated to public transit systems considered equitable stakeholder satisfaction for both mass rapid transit (MRT) and bus, and also promoted transport diversity in the Taipei metropolitan area.

**Keywords:** sustainability; transport diversity; resource allocation; fuzzy multi-objective programming

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## 1. Introduction

Transportation systems consist of infrastructure, modes, and stakeholders. Different transport stakeholders with diverse demands have different needs for transportation infrastructure and services. In fact, in transportation planning, transport policy-makers must simultaneously consider the trade-offs between differences in the supply of transport infrastructure or modes, as well as the various needs of stakeholders. Feng and Hsieh [1] suggested the concept of transport diversity, defined as different levels of satisfaction within stakeholder needs, and measured this using variations in achievement among needs, to assess the performance of urban transportation systems. Two approaches to improving transport diversity are goal setting (demand side) and resource management (supply side). If the parameters of the demand side, such as the classifications and expected goal values, are given, the critical issue of decision-makers is how to allocate finite available resources to maximize transport diversity and more equitably serve stakeholder needs.

Resource management can improve the performance of transportation system by increasing the quantity, capacity and utilization of resources. Resource utilization is the major tool to influence transport performance while the quantity and capacity of resources are finite and expensive or difficult to increase. Applying inappropriate investments to stakeholder needs causes bias that reduces equity and wastes resources which could otherwise be utilized more efficiently [2]. Consequently, the efficient and effective allocation of limited resources among policies offers a realistic management opportunity for improving transportation performance. Kuhn and Madanat [3] suggested optimization models to deal with asset allocation between of maintenance and rehabilitation. Using the German economy as a model, Conrad [4] provided a comprehensive discussion of transportation resource allocation based on a detailed microeconomic model. Moreover, appropriately designed infrastructure allocation can decrease the costs and improve quality of life, since resource allocation policies directly influence safety, environment and efficiency [5].

However, policy-makers have difficulty prioritizing resource allocation for improving equitable need satisfaction without the appropriate assessment framework. This thus reduces the efficiency of investments and proceeds in opposite direction towards sustainability. Feng and Hsieh [6] determined that the gaps in stakeholder needs are generally inversely related to transport diversity and positively related to private vehicle trips via a hybrid systematic simulation model, and inferred that increasing public transit trips helps the system bridge the gap between satisfaction of stakeholder needs. Accordingly, this study aims to identify aggregate indicators representing stakeholder needs derived from mass rapid transit (MRT) and bus riding behavior to optimizing public resource allocation based on transport diversity. The definition of transport diversity and indicators referring to transit stakeholder needs are illustrated in the next section, followed by a Pareto-based multi-objective approach. Section 4 discusses the constructions of the resource allocation model, followed by the results and discussion. Finally, Section 6 describes the conclusion and future research directions.

## 2. Transport Diversity

Transport diversity is defined as the level of satisfaction, which is the gap between expected goal and present values, of stakeholder needs in the form of the Entropy to tackle the issue of how to more equitably satisfy diverse stakeholder needs [1]. Once the stakeholders' needs are determined, minimizing the need gaps, the remainder of the needs achievement, between the expected goals and present values (as shown in Equation 1) is a key objective:

$$m_y = \frac{O_y^{goal} - V_y}{O_y^{goal} - O_y^{threshold}} \quad (1)$$

where  $m_y$  denotes the normalized gap of the indicator referring to stakeholder need  $y$ ,  $O_y^{goal}$  and  $O_y^{threshold}$  represent the expected goal and minimum threshold of need  $y$ , respectively, and  $V_y$  is the present value of need  $y$ . The normalized value prevents need gaps resulting from differences in unit scale. Meanwhile,  $n_y$  denotes the positive remainder of the gap of needs, namely the achievement indicated by Equation 2. Moreover, transport diversity deals with the equal satisfaction of stakeholder needs, the other critical objective of transportation planning, in the form of Entropy presented in Equation 3. Transport diversity comprises two components: richness, measured by the number of stakeholder groups, which determines the number of terms in the summation, and equability, measured by the evenness of needs distribution across groups:

$$n_y = \text{Max}(0, 1 - m_y) \quad (2)$$

$$H = -\sum_y \frac{n_y}{\sum_y n_y} \times \ln \frac{n_y}{\sum_y n_y} \quad (3)$$

Greater diversity indicates that as the distribution between compartments becomes more equitable, the gradients between compartments reduce, and larger numbers of compartments come to be involved in the system [7]. Moreover, Feng and Hsieh [1] have proposed that the expected goals and minimum thresholds of needs refer to the level of sustainability and basic life quality, respectively. The contents of quality of life could be tailored to fit different sustainable development targets. Improving sustainability and quality of life with regard to transportation requires the support of transport diversity. Transport diversity is thus the necessary condition for improving quality of life in a sustainable manner. However, resources may be allocated inefficiently when transport diversity is improved excessively. To prevent inefficient and inequitable resource allocation policies, this study employs a Pareto-based multi-objective approach to simultaneously maximize transport diversity and minimize the gaps in stakeholder needs.

## 3. Pareto-Based Multi-Objective Approach

Most real-world optimization problems are multi-objective, meaning they require the simultaneous consideration of several objectives. However, such problems are complex because they lack a single optimal solution, with there instead being a set of trade-offs called efficient solutions or Pareto-optimal solutions subject to resource constraints. The most crucial challenge is to find efficient feasible solutions

for dealing with numerous multi-objective planning issues. Consequently, methods for solving resource allocation problems, one of the most widely discussed issues in combinatorial optimization theory, should address the best allocation for performing a given task according to distinct targets given limited resources. The optimal or potential efficient solutions should be determined based on consideration of a set of diverse and conflicting criteria [8]. Some researchers thus utilized heuristic algorithms to solve multi-objective optimization problems [9,10]. However, most studies have used an aggregative approach to reduce the multi-objective problem to a single objective optimization problem. A convex solution set is the necessary condition for aggregative approaches to generate proper Pareto optimal solutions [11].

During recent decades, fuzzy multi-objective programming, an effect method of identifying compromised solutions for optimization problem, has been applied to solve multi-objective linear, as well as nonlinear, programming problems [12-14]. Moreover, fuzzy multi-objective programming has been used in many fields. Li and Lee [15] proposed a two-phase approach for getting a non-dominated solution and adapting it to *de novo* programming with fuzzy parameters. Additionally, Bhattacharya *et al.* [16] utilized fuzzy multi-objective programming to solve a multi-objective facility location problem. The genetic algorithm approach has been proved capable of solving fuzzy multi-objective programming with fuzzy nonlinear function goals and nonlinear constraints [17]. Liang [18] utilized fuzzy linear programming to assist in interactive multi-objective transportation planning decisions. Furthermore, some studies have concluded that a compromise solution can easily be found by applying fuzzy multi-objective programming to large problems, and is applicable to all types of multi-objective transportation problem [19,20]. The achievements of previous studies have increased the practicability of fuzzy multi-objective programming.

Accordingly, fuzzy multi-objective programming is utilized in this allocation model. In compromise programming, the weights indicate the importance of the relative deviation of the objectives from the ideal, but in fuzzy multi-objective programming they express the importance of the deviations from the anti-ideal [21]. Following the procedures of the fuzzy multi-objective programming algorithm, the ideal solution set  $I^* = \{W_s^*\}$  and anti-ideal solution set  $I^\# = \{W_s^\#\}$  should first be determined for the basic model, where  $W_s^*$  denotes the independently optimal performance for each indicator  $s$  while  $W_s^\#$  represents the worst performance for each indicator  $s$  due to the optimization of the objective indicators non- $s$ . For example, the model considers two objectives, including transport diversity and the gap between sustainable goal and present value, e.g.,  $s = 1, 2$ .  $W_1^*$  shows the optimal solution when transport diversity is identified as the objective function. Conversely,  $W_1^\#$  illustrates the worst performance value for transport diversity in the optimization for minimizing the gap between the sustainable target and the present situation.

Furthermore, both the ideal and anti-ideal solution sets are employed as a reference to define the membership function  $DS_s(W_s)$ , indicating the satisfaction degree of each objective  $W_s$ . The membership functions are represented as Equation 4 for solving minimization problems:

$$DS_s(W_s) = \begin{cases} 1, & W_s^* > W_s \\ \frac{(W_s^\# - W_s)}{(W_s^\# - W_s^*)}, & W_s^\# > W_s > W_s^* \\ 0, & W_s > W_s^\# \end{cases} \quad (4)$$

Moreover, a compromise-grade  $\lambda$ , referring to overall satisfaction of the optimization model, is expressed as Equation 5. Through maximizing  $\lambda$ , the multi-objective problem can be transformed into the following problem and the compromised solutions, including the values of decision variables  $x_i$ , compromise-grade  $\lambda$  and compromised objectives  $W_s$  with each degree of satisfaction  $DS_s$ , are thus obtained in Equation 6. According to the optimization using fuzzy multi-objective programming, there are two main assumptions in this study. First, the solution set for both maximal diversity as well as minimal gaps in stakeholder needs are convex. Second, the weights of stakeholder needs are dealt by the settings of goal and threshold values during transport diversity assessment rather than by fuzzy multi-objective programming:

$$\lambda = \text{Min}_s \{DS_s(W_s)\} \quad (5)$$

$$\begin{aligned} & \text{Max } \lambda \\ \text{s.t. } & \lambda \leq \frac{(W_s^\# - W_s)}{(W_s^\# - W_s^*)} \\ & \sum A_i x_i \leq B \\ & 0 \leq \lambda \leq 1 \end{aligned} \quad (6)$$

#### 4. Resource Allocation Model

Based on indicators used in previous works [6,22], this study selects ten indicators representing urban public transit stakeholder needs, such as accessibility, affordability and operator profit in both MRT and bus systems, reliability and mobility for bus operation, as well as emission and energy over-consumption for non-users. In fact, these ten indicators are classified into three groups based on the stakeholder characteristics. Transit authority focuses on what non-users concern including emission and energy over-consumption, while transit operators express interest in profits. Relatively, transit passengers attach importance to transit performance, such as accessibility, affordability, reliability and mobility. Since the aggregate data are employed in metropolitan area scales, this study assumed that individuals of each stakeholder cluster including MRT users, bus users, the MRT operator, bus operators and non-users are homogeneous. Each of the following indicator is plugged into Equations 1–3 to determine the gaps in stakeholder needs and transport diversity becomes a part of objective functions in programming model. The existing value of transportation infrastructure and service are marked with the suffix 0 for each variable and calculated as a constant in the following analyses. Appendix A presents the notation table.

#### 4.1. Accessibility

The literature review indicates that accessibility can be used to assess the equitable distribution of transport infrastructure and services. In public transit sub-systems, the accessibility indicated in Equations 7 and 8 is defined as the ratio of the resident population served by public transit, including mass rapid transit and bus, to the total resident population. The population served by public transit is identified as the population residing in the service area, namely with 500 meters of MRT, bus or feeder bus stations. In fact, the service population for MRT is related to the length of MRT lines and feeder buses routes (Equation 7), while the service population for bus is related to the length of bus routes (Equation 8). Appendix B describes the regression formulations in detail:

$$V_{Ac^{MRT}} = \frac{g_1(L_0^{MRT} + x_1) + g_2(L_0^{f-bus} + x_2)}{P_0} \quad (7)$$

$$V_{Ac^{bus}} = \frac{g_3(L_0^{bus} + x_3)}{P_0} \quad (8)$$

where  $P_0$  denotes the total resident population in the Taipei metropolitan area,  $L_0^m$  represents the existent length of operation routes for mode  $m$  including MRT, feeder bus and bus in kilometers. Meanwhile,  $x_1$ ,  $x_2$  and  $x_3$  refer to decision variables namely investment in construction of MRT lines, feeder bus routes as well as bus routes, respectively.

#### 4.2. Affordability

Affordability denotes the ability of particular consumer groups to bear the cost of a minimum level of a certain service [23]. Discussion of the relationship among social diversity, as represented by household income levels, mobility and transportation expenditure in Brazil revealed that those with the lowest household monthly income had a very low mobility but spent about 30% of their income on transportation [24]. Affordability thus becomes a key social equity issue. Moreover, a common acceptable measurement of affordability is utility payment relative to monthly disposable income. Therefore, the indicators for MRT and bus affordability are calculated as Equations 9 and 10, in which total transportation expenditure is the product of average cost per trip and monthly number of trips. The decision variables  $x_4$  and  $x_5$  denote the subsidies to MRT users and bus users, respectively:

$$V_{Af^{MRT}} = \frac{\hat{T}_0^{MRT} \times (F_0^{MRT} - x_4)}{Inc_0} \quad (9)$$

$$V_{Af^{bus}} = \frac{\hat{T}_0^{bus} \times (F_0^{bus} - x_5)}{Inc_0} \quad (10)$$

### 4.3. Operator Profit

From an operator perspective, economic health implies profitability, assisting an enterprise in achieving financial sustainability. Equations 11 and 12 reveal the operator profit represented by the product of monthly trips and the difference between average fare box revenue and average operational cost per trip for the MRT and bus:

$$V_{R^{MRT}} = (F_0^{MRT} - C_0^{MRT}) \times T^{MRT} \quad (11)$$

$$V_{R^{bus}} = (F_0^{bus} - C^{bus}) \times T^{bus} \quad (12)$$

### 4.4. Mobility

The investigation of MRT mobility is excluded since the mobility of MRT users is relatively higher and more predictable than that of other transport users. The mobility for bus users is measured using the ratio of the travel time by private vehicles to that by bus. The indicator of level of bus service is based on the network performance [25] and indicated in Equation 13. Bus user travel time is calculated as the sum of actual bus travel time (in-vehicle time) and average waiting time. Average waiting time is calculated by halving average bus headway and reliability. The policy parameter reducing headway of bus operation in minute is represented by  $x_6$  in Equation 13:

$$V_{M^{bus}} = \frac{TT_0^{private}}{TT^{bus} + \frac{(h_0^{bus} - x_6)}{2 \times V_{rel^{bus}}}} \quad (13)$$

### 4.5. Reliability

Reliability is defined as the probability of failure-free operation for a specified time and space. In transportation systems, reliability refers to the durability of facilities in terms of engineering and punctuality in terms of management. In fact, the discussion of private vehicle reliability is deficient due to the difficulty of identifying and determining the failure. Besides, engineering reliability is closely related to safety concerns. This study thus adopts a management perspective on public transit reliability. This study defines public-transit sub-systems as comprising MRT and bus. Notably, MRT has higher and more stable punctuality than bus owing to the reduced influence of external factors. The bus reliability indicated in Equation 14 is the probability of the punctuality in which buses does not fail, i.e., the average waiting time for bus users are less than half of bus headway:

$$V_{rel^{bus}} = \Pr(\overline{TT}^{bus} \leq \frac{1}{2}(h_0^{bus} - x_6)) \quad (14)$$

#### 4.6. Resource Over-Utilization

Indicators that focus on non-renewable resources are an important environmental issue in relation to sustainability. For example, Canadian sustainable development indicators emphasize the use of non-renewable resources, renewable resources, land and soil, and air and water qualities [26]. Tong *et al.* [27] argued that the efficiency of non-renewable resource utilization is a critical indicator for assessing sustainable development performance. Additionally, fossil energy is the most important non-renewable resource used in transportation systems. The indicator shown as Equation 15 expresses the consumption of fossil energy as the product of monthly trips and average energy consumption per trip for each mode, including MRT, bus and private vehicle. Oil equivalent is used as the unit of measurement for the average:

$$V_{EnCs} = \alpha_1 \times T^{MRT} + \alpha_2 \times T^{bus} + \alpha_3 \times T^{private} \quad (15)$$

#### 4.7. Externality

Along with the waste, hydro-resource pollution and negative habitat impacts associated with transportation infrastructure construction, externalities caused by transportation consist of air pollution, noise and vibration in operation periods. Regarding quality of life and sustainability, the greenhouse effect and global warming have attracted attention during recent decades. How to mitigate emissions has become a more crucial challenge than other negative influences. Air pollution, measured in terms of emissions per trip for each transportation mode, is thus considered a substitute for externality:

$$V_{Emi} = \alpha_4 \times T^{private} + \alpha_5 \times T^{bus} \quad (16)$$

Additionally, Equation 17 reveals that bus travel time is affected by the length of exclusive bus lane when average system travel speed is considered as an external factor, in which  $x_7$  indicates the incremental length in bus exclusive lanes. The operational cost, as shown in Equation 18, relates to the policy variable of headway. Monthly MRT and bus trips, represented in Equations 19 and 20, respectively, are influenced through the needs satisfaction in terms of urban public transit stakeholders. The functions of adjustment factors are established by meetings to establish expert consensus. Moreover, this study explores the connections among the metadata from GIS through the regression model. The analytical results reveal that a non-linear regression model, particularly a logarithm regression model, has better goodness of fit, moreover the all coefficients are statistically significant at the 0.05 level:

$$TT^{bus} = g_4(L_0^{BEL} + x_7) \quad (17)$$

$$C^{bus} = g_5(h_0^{bus} - x_6) \quad (18)$$

$$T^{MRT} = f_1(V_{Ac^{MRT}}) \times f_2(V_{Af^{MRT}}) \times T_0^{MRT} \quad (19)$$

$$T^{bus} = f_3(V_{Ac^{bus}}) \times f_4(V_{Af^{bus}}) \times f_5(V_{M^{bus}}) \times f_6(V_{rel^{bus}}) \times T_0^{bus} \quad (20)$$

The Taipei metropolitan area, the largest in Taiwan, provides the empirical data used to discuss the managerial implications of the model. Except for unavailable data, including driver behavior and conflict patterns, the average speed is assumed to be constant, and minor variations in trips are not taken as indicators of the accident rate. Besides, it is difficult to quantify the impact of level of universal design on system behavior. This study thus excludes accident rate and level of universal design. Furthermore, this study assumes that the impacts of subsystems, such as pedestrians, bicycles, private vehicles, as well as land use patterns, are given. Diverse transport stakeholders have different needs for urban transport infrastructure and services. The main issue in transport diversity thus is how to more equitably satisfy diverse stakeholder needs. Transport diversity is defined as different levels of satisfaction of stakeholder needs, expressed via appropriate indicators and measured using the variations in achievement among indicators.

Additionally, minimizing the indicator gaps (as shown in Equation 1) between the expected goals and present values is a key objective in urban transportation planning and thus the first objective of the proposed model. Moreover, the second objective is to maximize transport diversity in the form of Entropy (as shown in Equation 3) to equitably meet the various conflicting needs of urban public transit stakeholders.

Besides, budget constraints, indicated in Equation 21, express the limited available resources that should be allocated efficiently and equitably. Furthermore, Equation 22 denotes public transit system capacity. Notably, total trips in the Taipei metropolitan area are constant (as shown in Eqn. 23) due to the deficient consideration of trip generation. Because MRT operator makes a fixed positive profit, Equation 24 prevents bus operators from losing money. The domain of each policy variable  $x$  is identified from Equations 25 to 27, respectively. The upper boundaries of the policy variables,  $x_3$ ,  $x_5$  and  $x_6$ , are employed to minimize unreasonable travel costs and headway:

$$\sum_r c_r x_r \leq B_0, \quad r = 1, 2, \dots, 7 \quad (21)$$

$$T^m \leq b_0^m, \quad m = MRT, bus \quad (22)$$

$$T^{MRT} + T^{bus} + T^{private} = T_0 \quad (23)$$

$$V_{R^{bus}} \geq 0 \quad (24)$$

$$0 \leq x_r \leq F_0^m, \quad r = 3 \quad \text{if } m = MRT, r = 5 \quad \text{if } m = bus \quad (25)$$

$$0 \leq x_6 \leq h_0^{bus} \quad (26)$$

$$0 \leq x_r, \quad r = 1, 2, 4, 7 \quad (27)$$

## 5. Results and Discussion

This study develops a non-linear programming approach for assessing achievement level in satisfying diverse needs of urban public transit stakeholders. If maximizing total achievement level of satisfaction,

i.e., minimizing the sum of the normalized gaps between the target and present values for stakeholder needs, is considered the sole objective, the finite resource may be allocated inequitably. Inequitable allocation leads to the neglect of some needs and deficiencies in the transportation service provided to stakeholders. On the other hand, equitable but inefficient resource allocation can reduce total urban public transit system quality if the achievements of stakeholder needs sink further. Therefore, the proposed multi-objective model helps decision-makers allocate resources equitably and efficiently. The optimization problems are solved using Lingo 9.0.

To further demonstrate the applicability of the constructed model, an experimental analysis of public transit systems in the Taipei metropolitan area is conducted. Along with the consentaneous influence functions provided by expert discussion (as shown in appendix B), the actual data, such as population, length of operation lines, average income, headway, and so on, used in this analysis are obtained from the annual reports published by Ministry of Transportation and Communications. Table 1 lists the results of the baseline alternative considering actual modern data without resource allocation policies. The goal values denoting the expected target of sustainable development and the threshold value referring to the basic level of needs to maintain quality of life are set by the government for system monitoring.

**Table 1.** Results of the baseline (no-action) alternative.

| Indicator           | Present Value | Goal Value | Threshold Value | $m_i$                                                 | $n_i$ | $P_i = \frac{n_i}{\sum_i n_i}$ |
|---------------------|---------------|------------|-----------------|-------------------------------------------------------|-------|--------------------------------|
| MRT Accessibility   | 0.7565        | 0.85       | 0.60            | 0.37                                                  | 0.63  | 0.13                           |
| MRT Affordability   | 0.0811        | 0.05       | 0.15            | 0.31                                                  | 0.69  | 0.14                           |
| MRT Operator Profit | 65.3247       | 20.00      | 150.00          | 0.35                                                  | 0.65  | 0.13                           |
| Bus Accessibility   | 0.6867        | 0.85       | 0.60            | 0.65                                                  | 0.35  | 0.07                           |
| Bus Affordability   | 0.0508        | 0.03       | 0.10            | 0.30                                                  | 0.70  | 0.14                           |
| Bus Operator Profit | 140.4401      | 200.00     | 50.00           | 0.40                                                  | 0.60  | 0.12                           |
| Bus Mobility        | 0.5794        | 0.80       | 0.40            | 0.55                                                  | 0.45  | 0.09                           |
| Bus Reliability     | 0.7300        | 0.85       | 0.65            | 0.60                                                  | 0.40  | 0.08                           |
| Energy Consumption  | 35.8534       | 20.00      | 45.00           | 0.63                                                  | 0.37  | 0.07                           |
| Emission            | 28.1338       | 15.00      | 30.00           | 0.88                                                  | 0.12  | 0.03                           |
|                     |               |            |                 | $\sum m_i = 5.04 \quad H = -\sum P_i \ln P_i = 2.227$ |       |                                |

### 5.1. Single Objective Problem-Gap Minimization

The optimal allocation corresponding to seven policies is determined by the proposed constraints with the sole objective of minimizing the sum of the normalized gaps. Table 2 lists the analytical results. The sum of the normalized gap declines from 5.04 to 3.89, a 22.96% improvement, due to the significant improvements in MRT affordability, bus affordability and bus mobility. The investment policies include subsidizing public transit fares and constructing exclusive bus lanes. However, the affordability of each public transit system scores relatively highly in terms of satisfaction in the baseline alternative. The variation in the achievement of different stakeholder needs is enlarged from 0.04 (in the baseline alternative) to 0.09 owing to the inequitable allocation. Accordingly, the transport diversity

reduces 47.98% to 2.190 because the energy consumption and emission values are related to the mode share. Therefore, increasing the mode share of public transit is an effective strategy for mitigating the environmental impacts. To transfer trips from private vehicles to public transit, resources allocation should focus on needs with low satisfaction, such as accessibility and reliability, ahead of those with high satisfaction, such as affordability.

**Table 2.** Solution to the allocation model under a minimum gap objective.

[illegible]

### 5.2. Single Objective Problem—Transport Diversity Maximization

The analytical results of the maximizing transport diversity problem are expressed in Table 3. Investments, such as constructing exclusive bus lanes, extending bus operation routes, and reducing bus headway, decrease the total normalized gap by 5.96% whereas the transport diversity increases to 2.2429.

**Table 3.** Solution to the allocation model under a maximum diversity objective.

[illegible]

Satisfaction with bus accessibility, mobility and reliability thus clearly improves because reducing average bus headway simultaneously and significantly improves bus mobility and reliability. Since the achievements of stakeholder needs are distributed uniformly in the baseline alternative, budgetary restrictions become less effective as a means of equity improvement. Particularly, bus operator profit performs poorly in comparison to the baseline alternative. Moreover, the need satisfactions derived from the most equitable resource allocation are inferior to those from the most efficient allocation, with exceptions in the cases of bus accessibility and bus reliability.

### 5.3. Multi-Objective Problem

Traditionally, goal programming is employed to solve problems involving conflicting objectives, such as those no optimized solution exists. The main idea is to convert the original multi-objective into a single combined goal, and then seek a compromise solution based on the relative importance of each objective. In fact, both proposed objectives in this study, normalized gap and transport diversity, simultaneously consider the setting and weighting of each goal, and thus fuzzy multi-objective programming is utilized. According to the solution of each single objective, listed in Tables 2 and 3, the ideal and anti-ideal solution set comprises  $I^* = \{3.89, 2.243\}$  and  $I^\# = \{4.75, 2.190\}$ , respectively. The multi-objective problem can be transformed into a single objective problem of maximizing  $\lambda$  using the ideal and anti-ideal solution set. Along with the equations mentioned in Section 4, the following two more constraints (Equation 28) are added into the model according to Equation 6:

$$\begin{aligned}\lambda &\leq (4.75 - W_1) / (4.75 - 3.89) \\ \lambda &\leq (W_2 - 2.190) / (2.243 - 2.190)\end{aligned}\quad (28)$$

Table 4 lists the analytical results of fuzzy multi-objective programming in which the maximized compromise-grade generated within the membership function equals 0.4810. The optimal allocation indicates that all policy variables are invested, except extending MRT lines and reducing average bus headway. Since manufacturing new MRT infrastructures is costly, MRT accessibility is improved via extending feeder buses routes rather than lengthening MRT lines. Along with slightly raising bus mobility and reliability, building exclusive bus lanes avoids the severe negative impacts on government finance and bus operator profit from reducing average bus headways.

**Table 4.** Compromise solution to the fuzzy allocation model.

| Indicator           | Present Value | Goal Value | Threshold Value | $m_i$ | $n_i$ | $P_i = \frac{n_i}{\sum_i n_i}$ |
|---------------------|---------------|------------|-----------------|-------|-------|--------------------------------|
| MRT Accessibility   | 0.7716        | 0.85       | 0.60            | 0.31  | 0.69  | 0.12                           |
| MRT Affordability   | 0.0745        | 0.05       | 0.15            | 0.24  | 0.76  | 0.13                           |
| MRT Operator Profit | 65.8027       | 20.00      | 150.00          | 0.35  | 0.65  | 0.11                           |
| Bus Accessibility   | 0.6969        | 0.85       | 0.60            | 0.61  | 0.39  | 0.07                           |
| Bus Affordability   | 0.0474        | 0.03       | 0.10            | 0.25  | 0.75  | 0.13                           |
| Bus Operator Profit | 148.2331      | 200.00     | 50.00           | 0.35  | 0.65  | 0.12                           |
| Bus Mobility        | 0.7111        | 0.80       | 0.40            | 0.22  | 0.78  | 0.14                           |

**Table 4.** *Cont.*

|                                                       |         |       |       |      |      |      |
|-------------------------------------------------------|---------|-------|-------|------|------|------|
| Bus Reliability                                       | 0.7462  | 0.85  | 0.65  | 0.52 | 0.48 | 0.08 |
|                                                       | 35.4217 | 20.00 | 45.00 | 0.62 | 0.38 | 0.07 |
| Emission                                              | 27.8308 | 15.00 | 30.00 | 0.86 | 0.14 | 0.03 |
| $\sum m_i = 4.33 \quad H = -\sum P_i \ln P_i = 2.230$ |         |       |       |      |      |      |

The ideal transport diversity for the ten given indicators has a value of 2.30 if the achievement of stakeholder needs follows a uniform distribution. The fact that the transport diversity reaches 2.227 in the present situation confirms that the Taipei metropolitan area performs well in terms of equitably satisfying the needs of public transit stakeholder. Accordingly, resources utilized to bridge 75% gaps in stakeholder needs achieve a mere 21.23% improvement in transport diversity. The allocation based on compromise solution significantly increases the satisfaction of stakeholder needs, including MRT accessibility, MRT affordability, bus affordability, bus mobility and bus reliability. Furthermore, energy consumption and emissions are mitigated due to the increment of public transit trips.

After comparing the proposed Pareto-based model with single objective strategies, most need achievements of the compromised model locates between those using single objective models with the exception of MRT accessibility. This phenomenon demonstrates that investments in improving MRT accessibility benefits the consideration of tradeoffs between efficiency and equity but can be harm individual targets. The Pareto based allocation contributes to a 52.24% improvement in transport diversity compared with the single objective within minimizing gaps in stakeholder needs. Meanwhile, the proposed model bridges 48.10% stakeholder need gaps in the single objective within maximizing transport diversity. Consequently, the Pareto based approach prevents inefficient and inequitable resource allocation.

Moreover, analytical outcomes show that recent investment in public transit systems considered equitable stakeholder satisfaction for both MRT and bus users, and promoted transport diversity in the Taipei metropolitan area. Although bus accessibility, mobility and reliability performed relatively poorly in public transit system, variation in satisfaction of considered stakeholder needs was only slight, and thus most public transit stakeholders were able to achieve adequate transportation quality for meeting their daily needs. The outcomes suggest that future investments should be allocated to improve levels of bus services to prevent resource constraints arising from biased and inefficient MRT-oriented allocation. Besides, the empirical results suggest that the Pareto based approach is superior to single objective strategies, since the multi-objective model generates a compromise solution with higher cardinality and better diversity along the Pareto frontier.

Consequently, the proposed model considering transit stakeholder regarding to MRT and bus systems simultaneously in a meso-scopic urban transportation system as well as the invested transport infrastructures and improvements of transit services describes more specific relationships in urban transit system than macroscopic models. Comparatively, the developed model dealing with not only quantitative indicators but also qualitative needs can generate a better performance than the models involving either pure quantitative or pure qualitative parameters. Moreover, the proposed model combining systematic thinking based on system simulation and optimization resource allocation assists decision-makers in understanding the system behaviors.

## 6. Conclusions

The previous literature has neglected resource allocation in urban transportation systems, meaning decision-makers lack specific and operational methods for allocating resources in urban transportation systems. This study constructs a model for optimizing resource allocation in terms of transport diversity considering appropriate indicators related to critical stakeholder needs. Because a goal programming evaluation is considered in the setting of individual goals including maximizing transport diversity and minimizing normalized gaps, this study employs fuzzy multi-objective programming, which combines fuzzy set theory with multi-criteria decision-making methods to solve multi-objective problems. The objective functions are represented via a fuzzy set, and the decision rule is used to select the solution with the highest membership of the decision sets.

The proposed model seeks to determine the resource allocation for public transportation infrastructures and services to simultaneously maximize transport diversity and minimize gaps in stakeholder needs. This model evaluates investments in both transportation infrastructure and services to equitably serve transit stakeholder needs, as well as gaps in sustainable targets related to needs to make recommendations regarding efficient resource allocation. The developed multi-objective model based on Pareto optimization leads to acceptable compromise solutions. The inequitable supplies, which ignore the needs of certain disadvantaged minorities, aggravate the disparity in demand satisfaction. This contradicts the common perception that the best means of increasing need achievements is an effective policy of achieving sustainability because some improved needs are more sustainable than others. Resources can be allocated to improve needs satisfaction in areas where unsatisfied needs are impacting daily travel behaviors, while assigning a lower priority to needs that are already satisfied. Decision-makers thus should identify appropriate targets, and seek to achieve a basic level of satisfaction of each stakeholder need.

To illustrate the proposed approach, this study presents an empirical example in which the resource allocation model is applied to an urban public transit system in the Taipei metropolitan area. Specifically, the statistical framework and expert consensus are utilized to estimate quantitative and qualitative relationships, respectively, among variables in urban public transit systems. Since individual parameters and behaviors are unavailable, the experiments are carried out on average valued instances. Moreover, the interactions between public transit and private vehicle systems are assumed to be constant, and pedestrians, cyclists and land use patterns are ignored. The conclusions are limited by the deliberate simplification of the model and incomplete validation.

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## Appendix A. Notation Table.

| Variable              | Description                                                                                                                                                                                                              |
|-----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $H$                   | the value of diversity                                                                                                                                                                                                   |
| $m_y$                 | the normalized gap of the indicator referring to stakeholder need $y$                                                                                                                                                    |
| $O_y^{goal}$          | the expected goal of need $y$                                                                                                                                                                                            |
| $O_y^{threshold}$     | the minimum threshold of need $y$                                                                                                                                                                                        |
| $V_y$                 | the present value of need $y$                                                                                                                                                                                            |
| $n_y$                 | the positive remainder of the gap namely the achievement of need $y$                                                                                                                                                     |
| $L^m$                 | the length of operation routes for mode $m$ in kilometers                                                                                                                                                                |
| $P_0$                 | the total resident population in the Taipei metropolitan area                                                                                                                                                            |
| $\alpha_y^m$          | the average coefficient for need $y$                                                                                                                                                                                     |
| $TT^m$                | the actual travel time for mode $m$ in minutes                                                                                                                                                                           |
| $\overline{TT}^{bus}$ | the average bus waiting time                                                                                                                                                                                             |
| $h^{bus}$             | the average headway of buses                                                                                                                                                                                             |
| $T_0$                 | the constant total trips in the Taipei metropolitan area                                                                                                                                                                 |
| $\hat{T}^m$           | the average monthly trip amount for each user taking mode $m$                                                                                                                                                            |
| $F^m$                 | the average travel cost paid by user and the fare box revenue of operator per trip for mode $m$                                                                                                                          |
| $Inc$                 | the average monthly disposable income                                                                                                                                                                                    |
| $C^m$                 | the average operational cost for mode $m$                                                                                                                                                                                |
| $b^m$                 | the limited capacity for mode $m$ in trips                                                                                                                                                                               |
| $g(x_r)$              | the regression formulation to determine the impact of decision variable $r$ on concerned needs                                                                                                                           |
| $f_y^m(V_y^m)$        | the adjustment function of need $y$ on mode $m$ trips                                                                                                                                                                    |
| $T^m$                 | the monthly trip amount of mode $m$                                                                                                                                                                                      |
| $W_s$                 | the solutions under the criterion $s$ in the fuzzy multi-objective programming                                                                                                                                           |
| $DS_s(W_s)$           | the membership function related to the solutions in the fuzzy multi-objective programming                                                                                                                                |
| $\lambda$             | the compromise-grade of the optimization in the fuzzy multi-objective programming                                                                                                                                        |
| $x_r$                 | the decision variables in the fuzzy multi-objective programming                                                                                                                                                          |
| <b>Suffix</b>         |                                                                                                                                                                                                                          |
| 0                     | the existent value of transportation infrastructure and service for each variable as a constant                                                                                                                          |
| $r$                   | the strategy for resource allocation in the fuzzy multi-objective programming, $r = 1, 2, \dots, 7$                                                                                                                      |
| $s$                   | the evaluation criteria used in the fuzzy multi-objective programming, $s = 1, 2$                                                                                                                                        |
| $y$                   | the identified stakeholder need in urban transportation system including emission, safety, accessibility, mobility, reliability, affordability, resource over-utilization, operator profit and level of universal design |
| $Emi$                 | the analysis for the need externality in the substitute emission                                                                                                                                                         |
| $Ac$                  | the analysis for the need accessibility                                                                                                                                                                                  |
| $M$                   | the analysis for the need mobility                                                                                                                                                                                       |
| $rel$                 | the analysis for the need reliability                                                                                                                                                                                    |
| $Af$                  | the analysis for the need affordability                                                                                                                                                                                  |
| $EnCs$                | the analysis for the need resource over-utilization in the substitute energy over-consumption                                                                                                                            |
| $R$                   | the analysis for the need operator profit                                                                                                                                                                                |
| <b>Superscript</b>    |                                                                                                                                                                                                                          |
| $m$                   | the modes, such as MRT, bus, feeder bus, passenger car and motorcycle                                                                                                                                                    |
| *                     | the ideal situation in the fuzzy multi-objective programming                                                                                                                                                             |
| #                     | the anti-ideal situation in the fuzzy multi-objective programming                                                                                                                                                        |

**Appendix B.** Coefficients of Resource Allocation Model.**Table B-1.** Constant coefficients.

| Item       | Applied in  | Estimation | Unit                            | Source                                   |
|------------|-------------|------------|---------------------------------|------------------------------------------|
| $\alpha_1$ | Equation 15 | 7.32       | liter oil-equivalent/vehicle-km | Taipei Rapid Transit Corporation         |
| $\alpha_2$ | Equation 15 | 0.45       | liter oil-equivalent/vehicle-km | Taipei City Department of Transportation |
| $\alpha_3$ | Equation 15 | 0.16       | liter oil-equivalent/vehicle-km | Bureau of Energy, MOEA                   |
| $\alpha_4$ | Equation 16 | 0.1248     | kg/vehicle-km                   | Environmental Protection Administration  |
| $\alpha_5$ | Equation 16 | 0.5956     | kg/vehicle-km                   | Environmental Protection Administration  |

**Table B-2.** Coefficient formulation sourced by regression.

| Coefficient | Applied in  | Formulation                              | Unit of $x_i$ | R-squared |
|-------------|-------------|------------------------------------------|---------------|-----------|
| $g_1(x)$    | Equation 7  | $-3375.2 + 1140.7 \ln(L_0^{MRT} + x_1)$  | km            | 0.92      |
| $g_2(x)$    | Equation 7  | $-22193 + 3901.4 \ln(L_0^{f-bus} + x_2)$ | km            | 0.95      |
| $g_3(x)$    | Equation 8  | $-12716 + 2263.7 \ln(L_0^{bus} + x_3)$   | km            | 0.94      |
| $g_4(x)$    | Equation 17 | $33.869 - 6.2806 \ln(L_0^{BEL} + x_7)$   | km            | 0.88      |
| $g_5(x)$    | Equation 18 | $33.048 - 7.155 \ln(h_0^{bus} - x_6)$    | minute        | 0.89      |

**Table B-3.** Consentaneous impact function via expert discussion meeting.

| Coefficient | Applied in  | Formulation                                      |
|-------------|-------------|--------------------------------------------------|
| $f_1(V_y)$  | Equation 19 | $\frac{1.05}{1 + e^{-9(V_{Ac}^{MRT} - 0.35)}}$   |
| $f_2(V_y)$  | Equation 19 | $1 - \frac{1}{1 + e^{-20(V_{Af}^{MRT} - 0.25)}}$ |
| $f_3(V_y)$  | Equation 20 | $1.05 \times (1 - e^{-5V_{Ac}^{bus}})$           |
| $f_4(V_y)$  | Equation 20 | $1 - \frac{1}{1 + e^{-20(V_{Af}^{bus} - 0.25)}}$ |
| $f_5(V_y)$  | Equation 20 | $\frac{1.05}{1 + e^{-4(V_{M}^{bus})}}$           |
| $f_6(V_y)$  | Equation 20 | $\frac{1.07}{1 + e^{-10(V_{rel}^{bus} - 0.5)}}$  |

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