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# A Stochastic Model Predictive Control Strategy for Energy Management of Series PHEV

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## Abstract

Splitting power is a tricky problem for series plug-in hybrid electric vehicles (SPHEVs) for the multi-working modes of powertrain and the hard prediction of future power request of the vehicle. In this work, we present a methodology for splitting power for a battery pack and an auxiliary power unit (APU) in SPHEVs. The key steps in this methodology are (a) developing a hybrid automaton (HA) model to capture the power flows among the battery pack, the APU and a drive motor (b) forecasting a power request sequence through a Markov prediction model and the maximum likelihood estimation approach (c) formulating a constraint stochastic optimal control problem to minimize fuel consumption and at the same time guarantee the dynamic performance of the vehicle (d) solving the optimal control problem using the model predictive control technique and the YALMIP toolbox. Our simulation experimental results show that with our stochastic model predictive control strategy a series plug-in hybrid electric vehicle can save 1.544 L gasoline per 100 kilometers compared to another existing power splitting strategy.

*Keywords: Hybrid Systems, Model Predictive Control, Markov Prediction, Energy Management, Hybrid Electric Vehicle*

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## 1. INTRODUCTION

Series plug-in hybrid electric vehicles (SPHEVs) are emerging as an attractive alternative for fuel-efficient vehicles. They have a relatively longer driving range and lower cost compared to battery electric vehicles as an auxiliary power unit (APU) is included in the powertrain to supplement the power output [1]. As shown in Figure 1(a), this series architecture only allows the motor to provide propulsion power to meet the power demand at wheels, but the two energy sources---battery pack and APU---allow a flexibility for the manipulation of splitting the power demand of the vehicle.

As shown in Figure 1(b), when driver accelerates the vehicle, an AC/DC couples the electricity from a battery pack and an APU, or from one of them [2], and with the

electricity, an electric motor outputs a power to drive the vehicle; when driver brakes the vehicle, part braking energy is recovered through the electric motor (working in generation mode) to the battery pack for storing; whenever the APU outputs more electricity other than the necessary for driving the vehicle, the redundant part is transferred to the battery pack for storing; and when the vehicle is parked and plugged into power grid, the battery pack is charged [3]. In addition to the multiple working modes of the powertrain mentioned above, other hybrid dynamics also create the hybrid nature of the powertrain of SPHEVs, such as the variations in engine state (start/stop), and the limited availability of the battery pack due to the upper and lower boundaries on its state of charge (SOC) [4]. Furthermore, for higher fuel efficiency, the APU is designed with a small size and always outputs limited electricity, which is unable to

satisfy the power request of the vehicle independently. In order to guarantee the dynamic performance of the vehicle, the SOC of the battery pack is usually required to be higher than 25% during the whole driving range [5]. Nevertheless, for lower cost of the electricity than the fuel oil per unit power, it is preferred to discharge the battery pack to provide electricity for driving the vehicle. Thus, modeling the powertrain with hybrid dynamics, predicting a power request sequence, and then splitting the power for the battery pack and the APU in real time are the necessary technologies for SPHEVs. However, for different driving habits and changing driving conditions, it is hard to predict an accurate power request sequence for the vehicle. Therefore, the hybrid essence of the powertrain and the hard prediction of the power request make the power splitting tricky.

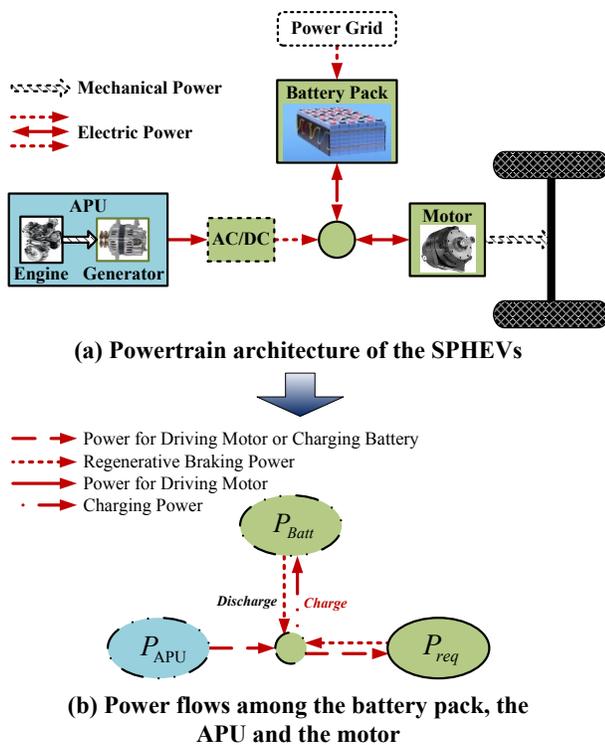


Figure 1 : Powertrain architecture and power flows of the SPHEVs

Previous works on the power splitting of the SPHEVs mainly focus on fuel cost minimization and emission reduction. The first methodology is based on the deterministic optimal technique [6, 7, 8, 9, 10]. It formulates the optimal control problem with a certain driving cycle by discretizing the continuous state space and control space into finite grids, and then applies the deterministic dynamic programming (DDP) to solve the optimal problem numerically [8]. Although (almost) global optimization solution is obtained with the DDP technique, it strongly depends on a specific driving cycle, and it is

impractical to apply the DDP algorithm in the vehicle-mounted embedded controller for its high complexity of computation. The second methodology is based on the non-deterministic optimal control technique [6, 7, 11, 18, 19]. Lin C C et al. [11] first model the power request as a Markov chain, and then use a Markov prediction model to estimate the probabilistic distribution of the future power request based on the previous power requests and vehicle speeds, and finally, formulate a stochastic optimization problem to minimize the fuel cost over an infinite horizon and solve the problem with stochastic dynamic programming (SDP) technique. The prediction of the power request in [11] is particularly appealing in this work, since it makes the optimal control independent of a specific driving cycle for the optimization is based on a probabilistic distribution, rather than a single cycle [2]. However, SDP also has a drawback of high complexity of computation. Recently, the model predictive control (MPC) is emerging as an attractive technique to solve a constraint optimal control problem with a finite horizon, which reduces the computation complexity greatly if the objective function can be built as a quadric form [12, 13]. For applications, the MPC has been used to split power for hybrid electric vehicles (HEVs) [14, 15, 16] and plug-in hybrid electric vehicles (PHEVs) [17], where the researchers are mainly focus on tracking a certain driving cycle. For an uncertain driving cycle, Bernardini, D. et al. [18] propose a methodology to transform the stochastic model predictive control (SMPC) problem to a standard MPC problem through an optimization tree with the maximum likelihood estimation and a cost function with the probability factors. With a low computation complexity, the SMPC approach has been applied to split the power for HEVs [19]. Compared to HEVs, the power splitting of SPHEVs need to guarantee the dynamic performance of the vehicle while minimizing the fuel cost. To this end, we add a time-varying constraint to the state of charge of the battery pack while splitting the power for SPHEVs.

For modeling the powertrain of SPHEVs, previous works treat this as a linear system [19]. In practice, the components in the powertrain have multiple working modes during the vehicle driving as discussed before. The state-of-the-art techniques from hybrid system modeling and control provide an approach to model the powertrain with strong soundness and split the power with the stochastic model predictive control.

In this work, we present a methodology for splitting the power for SPHEVs. Firstly, we develop a hybrid automation (HA) model to capture the power flows among the battery pack, the APU and the drive motor. Secondly, we construct a constraint optimal control problem with a

transformation model from the HA form. [21] Thirdly, we model the power request as a homogeneous Markov chain, and then estimate its probabilistic distribution with reference to the current states. Fourthly, we propose a novel method with a SOC penalty function to guarantee the vehicle dynamic performance while minimizing the fuel consumption. Finally, we solve this optimal control problem with stochastic model predictive control technique. This work is the first instance of applying hybrid system modeling and SMPC techniques to optimize the power splitting for SPHEVs. The four key steps in our methodology are:

- **Modeling.** We design an optimal operation curve with best fuel economy for the APU to reduce the complexity of the control problem, and then based on the curve and the experimental data, we build the steady-state and dynamic fuel consumption models for the APU. We define a piecewise linear model to describe the changes of the state of charge (SOC) of the battery pack based on the charging and discharging modes, and then we decouple the system. Then, we build a quasi-static vehicle simulation model, and present the driving distance model and the energy consumption model. Finally, we develop a HA model to capture the evolution of the power flow of the powertrain.
- **Power Request Prediction.** We divide the feasible region into  $S$  intervals based on the distribution of the values of the power request, and use the average value of the power request fall on the interval  $i$  to represent the power level of state  $i$ . Subsequently, we model the power request as a homogeneous Markov chain, and propose an algorithm to estimate the transition probability matrix of the power request based on the history driving cycle.
- **SMPC Design and Solution.** We design a power splitting scheme for the powertrain, and then transform the HA model to a piece wise affine (PWA) model with two disturbances including the power request and the vehicle speed. Furthermore, we design a time-varying SOC reference, and then define a SOC penalty function for battery energy consumption control. We formulate a constraint stochastic control problem to minimize the fuel consumption while guarantee the vehicle dynamic performance, and apply SMPC technique to transform the stochastic control problem to a standard MPC problem. Finally we formulate and solve the problem in YALMIP toolbox [22].
- **Simulation and Results.** We use the china typical driving cycle for city bus to estimate the transition

probabilistic matrix. And then, we test the SMPC approach on the diving cycle, and compare the performance of SMPC approach with a deterministic MPC technique mentioned in [19].

## 2. MODELING

As shown in Figure 1, the battery pack, the APU and the drive motor are the main components in the powertrain. We begin with modeling the state of charge (SOC) of the battery pack, the steady-state and dynamic fuel consumption of the APU, and the dynamics of vehicle, and then we develop a hybrid automaton model to capture the power flows among the components.

### 2.1 Battery Pack

The state of charge (SOC) is a normalized physical variable used to indicate the remaining electric energy of the battery pack (SOC=0 indicates that the battery pack is discharged completely, SOC=1 indicates that the battery pack is fully charged). Since the existence of internal resistance of the battery pack, we compute SOC with the energy losses, which are represented by an efficiency coefficient ( $\eta_{Batt}$ ). For the discharging and charging modes of the battery pack, we use a piecewise linear function to approximate the evolution of SOC as

$$\dot{SOC}(t) = \begin{cases} -K_1 P_{Batt}, & \text{if } P_{Batt} < 0, \\ -K_2 P_{Batt}, & \text{if } P_{Batt} \geq 0, \end{cases} \quad (1)$$

where

$$K_1 = \frac{1}{E_{Batt}} \cdot \eta_{Batt}, \quad K_2 = \frac{1}{E_{Batt}} \cdot \frac{1}{\eta_{Batt}} \quad (2)$$

$E_{Batt}$  represents the electric energy storage of the battery pack when it is fully charged.

### 2.2 Auxiliary Power Unit

The APU consists of a gasoline engine and a generator, and the output shaft of the engine is directly connected to the input shaft of the generator (see Figure 1a). As a matter of fact, the APU has two degrees of freedom (DOF) which are engine speed ( $\omega_e$ ) and generator torque ( $T_m$ ), or engine torque ( $T_e$ ) and generator speed ( $\omega_m$ ). However, to reduce the complexity of the prediction model and solution algorithm, we take the output power ( $P_{APU}$ ) as the only input of the APU, rather than use of  $\omega_e$  and  $T_m$  as the inputs. To realize this, we design an optimal operation curve for the APU based on the comprehensive consideration of the efficiency maps

of the engine and generator. In the curve, the corresponding speed and torque can make the APU obtain the highest efficiency for a given output power. Once the target output power of APU is optimized by the controller, a low level controller will adjust the APU to the target power in terms of optimal engine speed and generator torque in the curve. Therefore, we take the target power ( $P_{APU}^*$ ) as the input of the APU, and take the output power ( $P_{APU}$ ) as the state, and we build a quasi-static model based on the assumption as following.

$$P_{APU}(t) = P_{APU}^*(t) \quad (3)$$

To compute the fuel consumption of the APU, we build a steady-state fuel consumption model based on the efficiency maps of the engine and the generator, and we also consider the dynamic fuel consumption. Owing to the low efficiency of the APU for a small output power, we restrict the minimum generation power to 5 kW when the APU starts. In other words, when the APU input is less than 5 kW, we set it to 0 kW and then shut down the engine. Therefore, we define the steady-state fuel consumption as a piecewise model,

$$fuel_{steady} = \begin{cases} 0, & P_{APU} = 0 \\ f_2(P_{APU}), & 5 \leq P_{APU} \leq 75 \end{cases} \quad (4)$$

where  $fuel_{steady}$  is the specific fuel consumption of APU, and its unit is g/(kW·h). In order to formulate a quadratic problem, we define  $f_2$  as a quadratic function.

$$f_2 = a(P_{APU} - P_{opt})^2 + b, 5 \leq P_{APU} \leq 75 \quad (5)$$

Based on the fuel consumption experimental data of the APU as shown in Figure 2, we use a second-degree polynomial to fit the points of  $(P_{APU,i}, fuel_i)$  but except  $(0, 0)$  based on the principle of least square, and we obtain  $P_{opt} = 39.3617$  kW.

To simplify the problem, we extend the domain of  $P_{APU}(k)$  in  $f_2$  to zero, and redefine  $fuel_{steady}$

$$fuel_{steady} = a(P_{APU} - P_{opt})^2 + b, 0 \leq P_{APU} \leq 75 \quad (6)$$

and then, when the input of the APU is optimized, we use the equation (7) to approximate the optimization solution to guarantee the equation (4).

$$P_{APU}^* = \begin{cases} 0, & P_{APU} < 5 \\ P_{APU}^*, & P_{APU} \geq 5 \end{cases} \quad (7)$$

Furthermore, the fuel consumption experiments show that reducing the frequencies of start-stop and transients from one power point to another can improve the fuel

economy of the APU, and if we limit the output power variations, the dynamic regulating process will be short and smooth, and the APU can almost operate along the optimal curve. Thus, the dynamic fuel consumption is considered in this work, and we model it as a function of  $\Delta P_{APU}$ .

$$f_{dynamic} = c(\Delta P_{APU})^2 \quad (8)$$

where  $f_{dynamic}$  is the fuel consumption of APU, its unit is g/h, and  $c$  is a constant. Obviously, by applying equation (3), the fuel consumption models of (6) and (8) can be defined as a function of  $P_{APU}^*$  and  $\Delta P_{APU}^*$  respectively.

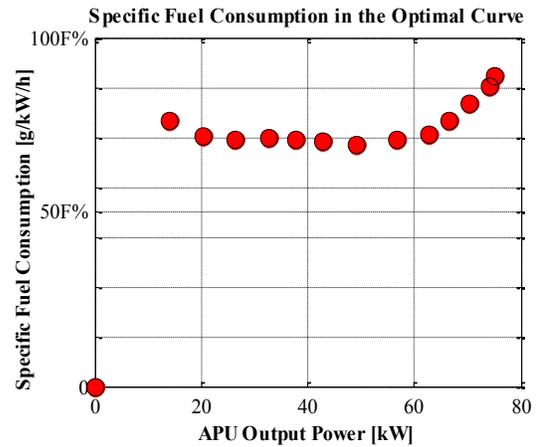


Figure 2 : APU optimal specific fuel consumption curve

### 2.3 Vehicle

The vehicle model used in this work is also quasi-static, which is wrote as a program code in MATLAB based on the maps and equations of different components of the vehicle.

As previously mentioned, to guarantee the vehicle dynamic performance, we must reasonably distribute the remaining electric energy of the battery to the remaining trip. To realize this, we estimate the future energy demand of the remaining trip based on the energy consumption level in the past. Thus, we present the models of the driving distance and energy consumption of the vehicle as follows.

$$\dot{s}(t) = u_a \quad (9)$$

$$\dot{W}(t) = P_{req} \quad (10)$$

where  $s$  (km) is the driving distance for the past, and  $W$  (kW·h) is the energy consumption for the past.

## 2.4 System Decoupling

In order to satisfy the power request from the motor, the powertrain need to satisfy the following constraint during the driving process.

$$P_{Batt}(t) + P_{APU}(t) = P_{req}(t) \quad (11)$$

On the basis of previous analysis for the coupling system as shown is Figure 1, we find the strategy based on the input  $P_{APU}^*$  is equal to  $P_{Batt}^*$ . In order to directly control the APU to keep it almost operating along the optimal curve, we choose  $P_{APU}^*$  as the input. By integrating the formulas of (3) and (11), we can obtain

$$P_{Batt}(t) + P_{APU}^*(t) = P_{req}(t) \quad (12)$$

Through combing equation (12) and (1), we displace  $P_{Batt}$  by  $P_{APU}^*$  to decouple the system. And then the SOC model (1) is redefined as a function of  $P_{APU}^*$  and  $P_{req}$ .

$$\dot{SOC}(t) = \begin{cases} -K_1(P_{req}(t) - P_{APU}^*(t)), & \text{if } P_{req}(t) - P_{APU}^*(t) < 0 \\ -K_2(P_{req}(t) - P_{APU}^*(t)), & \text{if } P_{req}(t) - P_{APU}^*(t) \geq 0 \end{cases} \quad (13)$$

## 2.5 Hybrid Automaton Model

With the charging and discharging modes of the battery pack, we develop a HA model to capture the power flow in the powertrain. We treat the vehicle speed  $u_a$  and the power request  $P_{req}$  as two disturbances (see Figure 3).

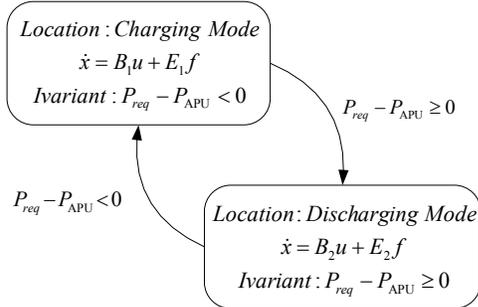


Figure 3 : The HA model for the powertrain of SPHEVs

where  $x = [SOC, s, W]'$ ,  $u = P_{APU}^*$ ,  $f = [P_{req}, u_a]'$ , and

$$B_1 = \begin{bmatrix} K_1 \\ 0 \\ 0 \end{bmatrix}, E_1 = \begin{bmatrix} -K_1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} K_2 \\ 0 \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} -K_2 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

## 3. POWER REQUEST PREDICTION

### 3.1 Stochastic Prediction Model

The vehicle power request is affected by the combination of various complex factors, such as driving conditions and driving habits. And the participation of the humans led to the future power request changes randomly. As a matter of fact, the future power request sequence  $\{P_{req}(k+1), P_{req}(k+1), \dots, P_{req}(k+N)\}$  is difficult to exactly estimate during the driving process. However, building a reasonable and scientific mathematic prediction model to forecast the future power request is the premise of realizing the optimal control for power splitting of SPHEVs.

In this work, we apply the theory of stochastic process to analyze the probabilistic characteristics of power request from the history driving cycle. Firstly, we divide the feasible region into  $S$  intervals (see Figure 4). Each interval constitute a state represented by an index  $j$  respectively, and the average value  $(P_j, j=1,2,\dots,S)$  of all power request in interval  $j$  is used to represent the size of power level.

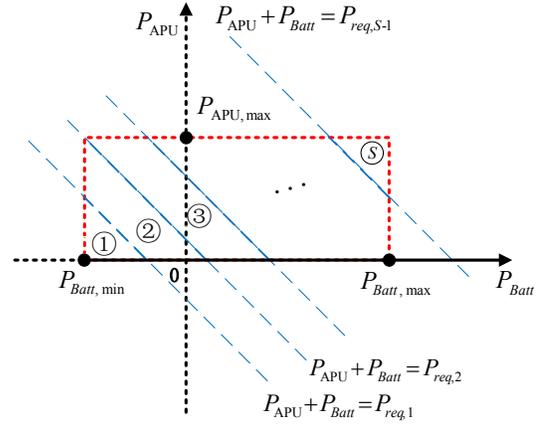


Figure 4 : Interval division of the feasible region

Secondly a homogeneous Markov prediction model is built to describe the probabilistic distributions of future power request. And the model is defined by a transition probability matrix  $\Pi \in R^{S \times S}$

$$\mathbf{P}\{P_{req}(k+1) = P_j | P_{req}(k) = P_i\} = \pi_{i,j}, j = 1, 2, \dots, S, \sum_{j=1}^S \pi_{i,j} = 1 \quad (14)$$

Thirdly, we design an algorithm to calculate the transition probability matrix  $\Pi$  from the driving cycle.

Finally, we predict the probabilistic distributions of power request in each step of the future during a prediction horizon  $N$  is by using model (14).

$$\begin{matrix}
P_{req}(k+1) & P_{req}(k+2) & \dots & P_{req}(k+N) \\
P_1 & \begin{bmatrix} \pi_{i,1}^{(1)} & \pi_{i,1}^{(2)} & \dots & \pi_{i,1}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ P_j & \pi_{i,j}^{(1)} & \pi_{i,j}^{(2)} & \dots & \pi_{i,j}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ P_S & \pi_{i,S}^{(1)} & \pi_{i,S}^{(2)} & \dots & \pi_{i,S}^{(N)} \end{bmatrix} & & & 
\end{matrix} \quad (15)$$

### 3.2 Transition Probability Matrix Estimation

In this work, we use the following procedure to calculate transition probability matrix  $\Pi$  from the driving cycle.

- (i) calculating the power request sequence with respect to the history driving cycle,
- (ii) defining the classification intervals, and defining state index to represent each interval,
- (iii) classifying each power request of the sequence based on the classification intervals, and calculating the mean value of power request belonging to the same state,

Table 1 : Power Classification Rules

Intervals	State Index	Average Power
$(-\infty, P_{req,1}]$	1	$P_1$
$(P_{req,1}, P_{req,2}]$	2	$P_2$
$\vdots$	$\vdots$	$\vdots$
$(P_{req,S-1}, +\infty)$	$S$	$P_S$

- (iv) counting the frequency  $\{f_{i,j} \mid i, j=1,2,\dots,S\}$  which is the number of the occurrences of the transition from state  $i$  to  $j$  in the sequence, and calculating the transition probability  $\pi_{i,j} = f_{i,j} / \sum_{j=1}^S f_{i,j}$ ,
- (v) setting a threshold  $\pi_{min}$ , and normalizing each row of the transition probability matrix  $\Pi$  again after deleting the probability less than  $\pi_{min}$ .

## 4. STOCHASTIC MODEL PREDICTIVE CONTROL DESIGN

### 4.1 SMPC Approach

Predicting the sequence of future power request or giving a reference sequence is the premise of using optimization method to solve the problem of power splitting for SPHEVs. In this work, we use a Markov prediction model to forecast the probabilistic distribution of the future power request, and then we apply the SMPC approach designed by Daniele Bernardini and Alberto

Bemporad [18] to solve the stochastic control problem. The main idea of SMPC technique is designing an optimization tree with maximum-likelihood estimation method to provide a reference sequence for future power demand, and then building a cost function with the probabilistic factors to transform the SMPC problem to a standard deterministic MPC problem.

### 4.2 Model Transformation

In order to use SMPC technique to solve the control problem, we transform the HA model to a piece wise affine (PWA) model, and we begin with designing a power splitting scheme as show in Figure 5. Since the dynamic fuel consumption is modeled as of function of gradual variations of target power of the APU, we take the  $\Delta P_{APU}^*$  as the new input of the system to reduce the complexity of the optimization problem.

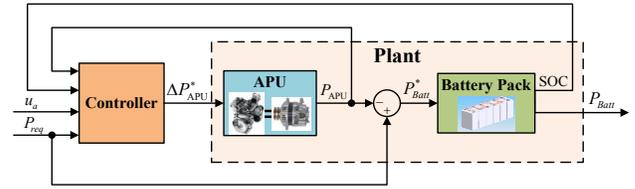


Figure 5 : Closed-loop system structure

where  $\Delta P_{APU}^*(k)$  is defined as

$$\Delta P_{APU}^*(k) = P_{APU}^*(k) - P_{APU}^*(k-1) \quad (16)$$

according to equation (3), we have

$$\Delta P_{APU}^*(k) = P_{APU}(k) - P_{APU}(k-1) \quad (17)$$

Additionally, we add the output power  $P_{APU}(k-1)$  to the state vector. And with a sampling time  $T_s = 2s$ , we discretize the HA model to a PWA model as follows.

$$x(k+1) = \begin{cases} A_{1d}x(k) + B_{1d}u(k) + E_{1d}f(k), & \text{if } [1, 0]f(k) - [1, 0, 0, 0]x(k) - u(k) < 0 \\ A_{2d}x(k) + B_{2d}u(k) + E_{2d}f(k), & \text{if } [1, 0]f(k) - [1, 0, 0, 0]x(k) - u(k) \geq 0 \end{cases} \quad (18)$$

where  $x(k) = [P_{APU}(k-1), SOC(k), s(k), W(k)]'$  is the system state vector,  $u(k) = \Delta P_{APU}^*(k)$  is the input,  $f(k) = [P_{req}(k), u_d(k)]'$  is the disturbance vector, and

$$A_{1d} = \begin{bmatrix} 1 & & & \\ K_1 T_s & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}, \quad B_{1d} = \begin{bmatrix} 1 \\ K_1 T_s \\ 0 \\ 0 \end{bmatrix}, \quad E_{1d} = \begin{bmatrix} 0 & 0 \\ -K_1 T_s & 0 \\ 0 & T_s \\ T_s & 0 \end{bmatrix},$$

$$A_{2d} = \begin{bmatrix} 1 & & & \\ K_2 T_s & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}, \quad B_{2d} = \begin{bmatrix} 1 \\ K_2 T_s \\ 0 \\ 0 \end{bmatrix}, \quad E_{2d} = \begin{bmatrix} 0 & 0 \\ -K_2 T_s & 0 \\ 0 & T_s \\ T_s & 0 \end{bmatrix}$$

### 4.3 Controller Synthesis

Based on the power prediction model, PWA prediction model of the system and the SMPC technique, we design a controller to minimize the fuel economy while guarantee the dynamic performance of the vehicle. And the objective function is composed of three parts: (a) the first part denotes steady fuel consumption used to keep the APU almost operate along the optimal curve during the generation process; (b) the second part is a penalty function used to limit the frequency and amplitude of the regulation of APU to reduce the dynamic fuel consumption; (c) and the last part is also a penalty function used for battery SOC control to guarantee dynamic performance of the vehicle.

#### 4.3.1 Battery SOC Control

As previously mentioned, management of the electric energy consumption of the battery pack is necessary for guaranteeing the dynamic performance of the vehicle. Thus, we design a SOC reference line for the prediction horizon in each control step (see Figure 6), and then we use the quadric difference of the SOC and SOC reference values to define a penalty function for battery SOC control.

The main idea of SOC reference design is to equally distribute the remaining electric energy of the battery pack into the rest trip based on the future energy demand estimation. First of all, we calculate the average energy consumption per kilometer ( $\bar{w}$ ) for the past driving distance, and limit its minimum value as the average energy consumption of china typical driving cycle for city bus ( $\bar{w}_0 = 1.2866 \text{ kW} \cdot \text{h/km}$ ).

$$\bar{w}(k) = \begin{cases} \bar{w}_0, & s(k) = 0 \\ \max\left(\frac{W(k)}{s(k)}, \bar{w}_0\right), & s(k) > 0 \end{cases} \quad (19)$$

Simultaneously, we estimate the future energy demand for the rest trip based on the energy consumption level in the past.

$$W_{rest}(k) = \bar{w}(k)(s_0 - s(k)) \quad (20)$$

Afterwards, for each control step  $k$ , we equally assign the remaining SOC according to the future energy estimation value  $W_{rest}(k)$  and the power demand prediction sequence. Moreover, to ensure the SOC reference can

reach the desired value ( $\text{SOC}_{ref,end}$ ) at the end of the trip, we assign the remaining SOC reference instead of the real SOC of the battery. So we define the desired variations of the SOC reference as

$$\Delta \text{SOC}_{ref}^k(i) = \frac{\text{SOC}_{ref}^k(1) - \text{SOC}_{ref,end}}{W_{rest}(k)} \sum_{j=1}^{i-1} \left( P_{req}(j) \cdot \frac{T_s}{3600} \right) \quad (21)$$

$$i = 2, 3, \dots, N$$

Finally, we define the SOC reference as following.

$$\text{SOC}_{ref}^k(i) = \begin{cases} \text{SOC}_{ref}^k(1) - \Delta \text{SOC}_{ref}^k(i), & \Delta \text{SOC}_{ref}^k(i) > 0 \\ \text{SOC}_{ref}^k(1), & \Delta \text{SOC}_{ref}^k(i) \leq 0 \end{cases} \quad (22)$$

$$i = 2, 3, \dots, N$$

where

$$\text{SOC}_{ref}^k(1) = \begin{cases} \text{SOC}(1), & k = 1 \\ \text{SOC}_{ref}^{k-1}(2), & k \geq 1 \end{cases} \quad (23)$$

Therefore, when the prediction value of future energy demand of  $i$  step is negative, the SOC reference value keeps the value of the former.

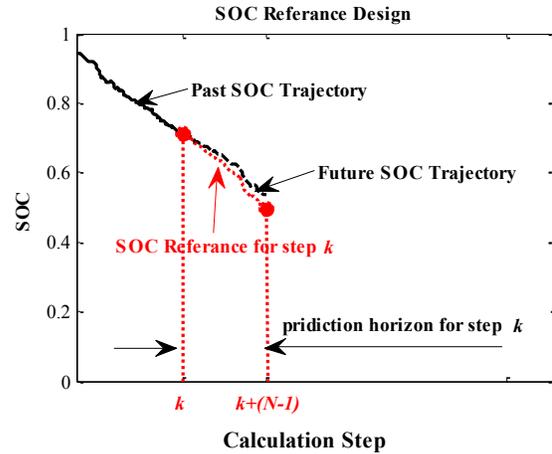


Figure 6 : Tracking the SOC Reference

#### 4.3.2 Optimization Problem Formulation

As we use the algorithm of optimization tree design in [18], here, we repeat the definition of the relevant symbols as follows.

- $T$  : the set of the optimization tree nodes, defined as  
 $T = \{T_1, T_2, \dots, T_N\}$ ,
- $succ(i)$  : the successor of node  $i$  in the optimization tree,
- $pre(i)$  : the predecessor of node  $i$  in the optimization tree,
- $S \subset T$  : the set of leaf nodes, defined as

$$S = \{T_i : succ(T_i, j) \notin T, i=1,2,\dots,N, j=1,2,\dots,S\},$$

$\pi_i$  : the probability of reaching  $T_i$  from  $T_1$ .

Based on the optimization tree nodes  $T = \{T_1, T_2, \dots, T_N\}$ , we obtain the sequence of the future power request  $\{P_{req}(k), P_{T_2}, \dots, P_{T_N}\}$ , where  $P_{T_i}, i=2, \dots, N$  is the average power of state  $T_i$  (see Table 1). To simplify the notation, in the following formulation, the symbols  $x_i, f_i, u_i, y_i, x_{ref,i}, \pi_i, pre(i)$  are used to denote  $x_{T_i}, f_{T_i}, u_{T_i}, y_{T_i}, x_{ref,T_i}, \pi_{T_i}, pre(T_i)$  respectively. Thus, we model the SMPC problem as

$$\min J = \sum_{i \in T \setminus T_1} \pi_i (x_i - x_{ref,i})' Q (x_i - x_{ref,i}) + \sum_{j \in T \setminus S} \pi_j (u_j' R u_j) \quad (24a)$$

subject to,

$$x_i = x(k) \quad (24b)$$

$$f_i = f(k) \quad (24c)$$

$$x_i = \begin{cases} A_{1d} x_{pre(i)} + B_{1d} u_{pre(i)} + E_{1d} f_{pre(i)} \\ A_{2d} x_{pre(i)} + B_{2d} u_{pre(i)} + E_{2d} f_{pre(i)} \end{cases}, \forall i \in T \setminus \{T_1\} \quad (24d)$$

$$x_{ref,i} = [P_{opt}, SOC_{ref}^k(i), 0, 0]' \quad (24e)$$

$$y_i = C x_{pre(i)} + D u_{pre(i)} + F f_{pre(i)}, \forall i \in T \setminus S \quad (24f)$$

$$x_i \in \mathbf{X}, \forall i \in T \setminus \{T_1\} \quad (24g)$$

$$u_i \in \mathbf{U}, \forall i \in T \setminus S \quad (24h)$$

$$y_i \in \mathbf{Y}, \forall i \in T \setminus S \quad (24i)$$

and

$$\mathbf{X} = \{x : SOC_{min} \leq [0, 1, 0, 0]x \leq SOC_{max} \\ 0 \leq [1, 0, 0, 0]x \leq P_{APU, max}\} \quad (24j)$$

$$\mathbf{U} = \{\Delta P_{APU, min} \leq u \leq \Delta P_{APU, max}\} \quad (24k)$$

$$\mathbf{Y} = \{P_{Batt, min} \leq y \leq \Delta P_{Batt, max}\} \quad (24l)$$

where  $Q = diag(Q_{11}, Q_{22}, 0, 0)$  is a diagonal matrix,  $Q_{11}$ ,  $Q_{22}$  and  $R$  are nonnegative value scalar weights,  $C = [1, 0, 0, 0]$ ,  $D = 1$ ,  $F = [1, 0]$ . Note that the objective function (24a) is modeled with two functions: one is to minimize the fuel consumption of the APU. We keep the APU operate around the optimal point  $P_{opt}$  by imposing  $P_{APU, ref} = P_{opt}$  to maximize the fuel economy of the APU. And then we limit the variations of the output power to make the APU almost operate along the optimal curve and shorten the dynamic regulation process through a penalty function of the input. The other function is to make the trajectory of SOC evolve along the reference line to ensure the dynamic performance of the vehicle for the whole trip  $s_0$  by imposing  $SOC_{ref} = SOC_{ref}^k(i)$ . In addition, the objective function

is constrained by (24b)-(24l), where (24b) and (24c) define the initial states and disturbances of the system respectively. For a given prediction horizon, the second element ( $u_a$ ) of the disturbance  $f$  is only used to calculate the state variable  $s(k)$ , and we only use the initial value of  $s(k)$  (except  $s(k+1)$   $s(k+2)$  ...) to estimate the reference line of SOC. Since the vehicle speed is an external input of the closed loop system, here we don't need to care its future value. But the future value of the first element ( $P_{req,i}, i \geq 2$ ) of the disturbance  $f$  is obtained by optimal tree design algorithm based on the Markov model. Other constraints are related to the input and output characteristics of the APU and battery pack.

## 5. SIMULATION and RESULTS

We test the SMPC approach on the china typical driving cycle for city bus (see Figure 7) based on the vehicle simulation model designed by us. The cycle is a sequence consists of vehicle speed to be tracked, and the driving range of the cycle is 5.904km. Thus, we repeat this cycle several times to form an 80km driving cycle.

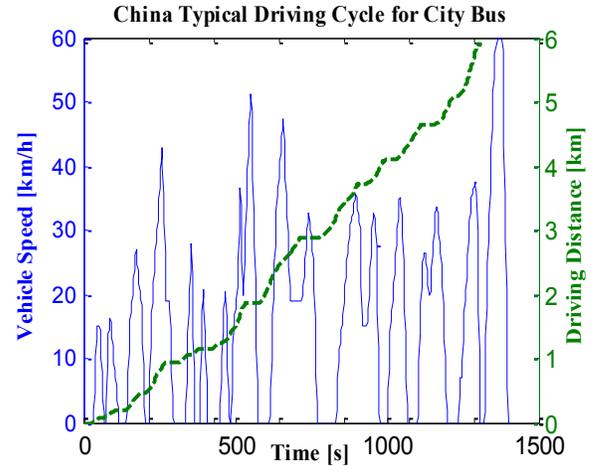


Figure 7 : China typical driving cycle for city bus

Even though the driving cycle is specific, we use it to estimate the transition probabilistic matrix of the power request for the Markov prediction model (14). First of all, we use the formula given by vehicle dynamics to calculate the power demand at wheels.

$$P_{dmd} = f_2(u_a, \dot{u}) = \frac{u_a}{3600 \eta_T \cdot \eta_M} \left( Gf + Gi + \frac{C_D A u_a^2}{21.15} + \delta m \dot{u} \right) \quad (25)$$

where  $u_a$  (km/h) is the vehicle speed,  $\dot{u}$  is the acceleration ( $m/s^2$ ), and other variables are vehicle parameters. Simultaneously, we consider the braking energy recovery to improve the fuel economy, and we define the recovery proportion as a function of the deceleration

tion ( $\dot{u} < 0$ ), and then the power request of the motor is defined as

$$P_{req} = \begin{cases} P_{dmd}, & P_{dmd} \geq 0 \\ f_3(\dot{u})P_{dmd}, & P_{dmd} < 0 \end{cases} \quad (26)$$

We calculate the power request sequence for the driving cycle in Figure 7 by using the formula (26). As shown in Figure 8, the minimum and maximum values of the power request are -71.79 kW and 174.55 kW respectively.

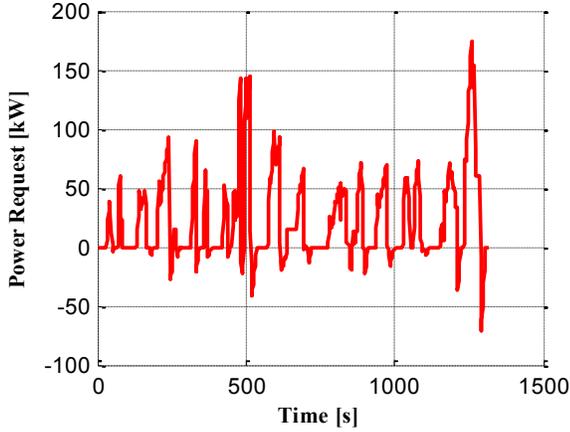


Figure 8 : Power request sequence for the driving cycle

Afterwards, we estimate the transition probability matrix using the procedure introduced in chapter 3.2 (see Figure 9), and then we built the Markov prediction model for prediction of probabilistic distribution of the future power request.

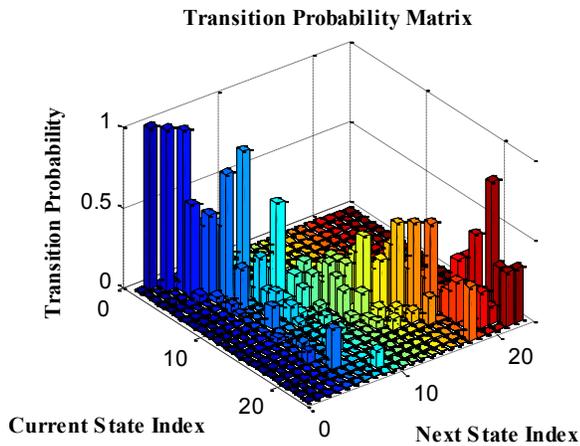


Figure 9 : Transition probability matrix

Based on the previous work, we test the SMPC approach in MATLAB software for the 18 tons city bus, and use the YALMIP tool box to solve the control problem for each step. For simulation, the system's initial conditions

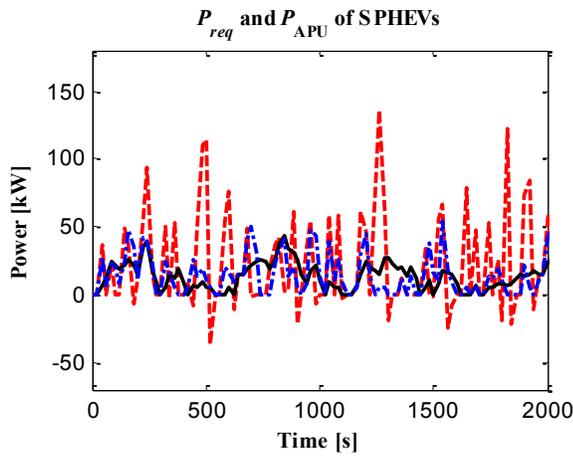
are  $f(1) = [P_{req}(1), u_a(1)]' = [0, 0]'$ ,  $x(1) = [P_{APU}(0), SOC(1), s(1), W(1)]' = [0, 0.95, 0, 0]'$ ,  $s_0 = 80$  km,  $SOC_{ref, end} = 0.26$ ,  $SOC_{min} = 0.25$ ,  $SOC_{max} = 1.0$ ,  $P_{APU, max} = 70$  kW,  $\Delta P_{APU, min} = -10$  kW,  $\Delta P_{APU, max} = 10$  kW,  $P_{Batt, min} = -120$  kW,  $P_{Batt, max} = 240$  kW,  $E_{Batt} = 60$  kW·h,  $\eta_{Batt} = 0.92$ , and we choose  $Q_{11} = 1 \times 10^{-5}$ ,  $Q_{22} = 10000$ ,  $R = 0.0267$  as a set of weights in the objective function of (28a). The prediction horizon is  $N = 10$ , and the nodes of the optimization tree are built with the same length as  $N$ .

Here, we compare the performance of SMPC to a deterministic MPC approach presented in [19], namely the frozen-time MPC (FTMPC). For a given prediction horizon  $N$ , the FTMPC also has no information about the driving cycle, but assumes the future power request as a constant equals the current value. And the simulation results for SMPC and FTMPC are list in Table 2.

Table 2 : Fuel consumption comparison

		FTMPC	SMPC
$\ \Delta P_{APU}^*\ _2$	[kW]	481.9896	223.2843
steady fuel cons.	[L/100km]	30.3041	32.2123
dynamic fuel cons.	[L/100km]	4.3075	0.9244
equivalent fuel cons.	[L/100km]	39.9176	38.3636
economy improve	[%]	—	3.89

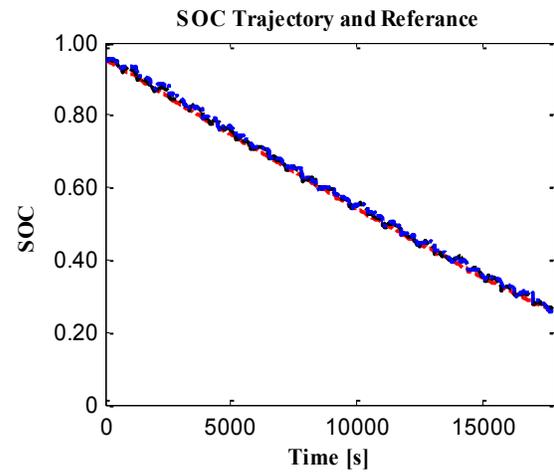
Where  $\|\Delta P_{APU}^*\|_2$  is the Euclidean Norm of the variation of the APU output power for the whole simulation interval  $N_{sim} = 8929$ , and its value indirectly represents the frequencies and the amplitudes of the variations of the APU output power. As shown in Table 2, the APU operates more smoothly for SMPC, which can be directly observed in Figure 10. And if we limit the variations of the output power of the APU, the fuel economy will be improved. The equivalent fuel consumption consists of steady-state fuel consumption, dynamic fuel consumption and the equivalent conversion of the electricity consumption to fuel in terms of the cost. According to the results in Table 2, the city bus tested in this work can save 1.554 L gasoline by applying SMPC approach compared with FTMPC (economy improve 3.89%).



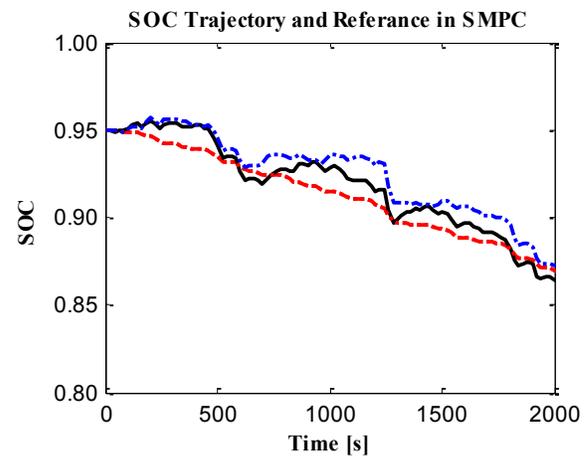
Partial enlarged figure for the first 2000 seconds: power request of the vehicle (dashed line), output power of the APU for SMPC (solid line), output power of the APU for FTMPC (dashed-dotted line)

Figure 10 : Comparison of the output power of APU based on SMPC and FTMPC approaches

Since the two SOC reference lines almost overlap together, we only plot the SOC reference for SMPC, and the reference consists of the second value of SOC reference of each calculation step  $k$ , namely,  $\{SOC_{ref}^1(2), SOC_{ref}^2(2), \dots, SOC_{ref}^{N_{sm}}(2)\}$ . In Figure 11(a), we find the reference can keep the SOC trajectory to track itself from the initial value to a low level close to the minimum SOC, while never permit the SOC trajectory overpass the lower boundary. Obviously, during the whole trip, the approach realize that keeping the battery pack release energy slowly and equally for the whole trip through tracking the reference. That is to say, the feasible conditions of the optimization problem for each calculation step are guaranteed. In Figure 11(b), it is clear the SOC trajectory for SMPC tracks the reference better than the trajectory for FTMPC, because the power request prediction helps to adjust the output power of the APU.



(a) SOC reference line for SMPC(dashed line), SOC trajectory for SMPC (solid line), SOC trajectory for SMPC (dashed-dotted line)



(b) Partial enlarged figure for the first 2000 seconds: SOC reference line for SMPC(dashed line), SOC trajectory for SMPC (solid line), SOC trajectory for FTMPC (dashed-dotted line)

Figure 11 : The trajectory of SOC of the battery pack for SMPC and FTMPC

The results of the fuel consumption of the APU and the equivalent fuel consumption of the vehicle are show in Figure 12. We find their increasing tendency is linear. The main reason is we make the battery release electric power equally for the whole trip.

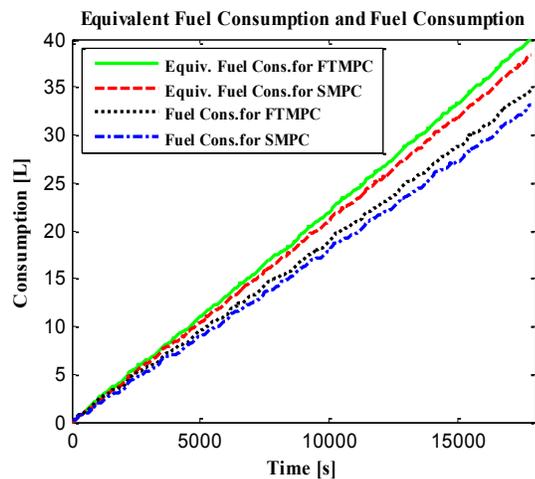


Figure 12 : Equivalent fuel consumption of the vehicle and fuel consumption of the APU

## 6. CONCLUDING REMARKS

In this work, we propose a methodology for online optimal splitting power between the APU and battery pack based on hybrid system modeling, the theory of stochastic process and the SMPC technique, and our approach makes an 18 tons city bus save 1.544 L gasoline per 100 kilometers compared with a deterministic MPC approach. By modeling the power demand as a homogeneous Markov model where the transition probabilistic matrix can be estimated from the history data, we make the optimal control independent of a specific driving cycle. We build a HA model to capture the power flow of the powertrain, and we first synthesize the hybrid system modeling and SMPC approach to solve the power splitting problem for SPHEVs. In addition, we set a time-varying SOC reference to guarantee vehicle dynamic performance. However, the verification of the approach is not studied in this work, and the component dynamics are ignored. Thus, our future work will focus on verification, improving the algorithm of optimization tree design, and system dynamics modeling.

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## Appendix

Table 3 : Symbol description

Symbols	Description	Units
$P_{dmd}$	: power demand at the wheels	kW
$P_{req}$	: power request of the drive motor	kW
$P_i$	: average value of power request belong to the state $i$	kW
$P_{APU}$	: output power of the APU	kW
$\Delta P_{APU}$	: variation of the APU output power	kW

Symbols	Description	Units
$P_{Batt}$	: output power of the battery pack	kW
SOC	: state of charge of the battery pack	-
$s$	: driving distance	km
$W$	: energy consumption for the past	kW·h
$SOC_{ref}^k$	: SOC reference for step $k$	-
$fuel_{steady}$	: fuel consumption of steady-state	g/kW/h
$f_{dynamic}$	: dynamic fuel consumption	g/h

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