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## Vehicle Dynamic Simulation for Efficiency Optimization of Four-wheel Independent Driven Electric Vehicle

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#### Abstract

Electric vehicle is considered to be the solution for energy crisis and environment pollution in individual transportation. However, electric vehicles are still not competitive with conventional vehicles limited by the driving range with the state-of-art power battery technology. Therefore, energy efficiency is of crucial importance for battery electric vehicles due to the limited stored energy. In order to optimize the energy efficiency of a four-wheel independently driven electric vehicle, the model consisting of the vehicle dynamics model, wheel motor model and PNGV model of LiFePO<sub>4</sub> power battery was developed in Matlab/Simulink. As the motor efficiency is low in small torque range, motors should work as much as possible in high efficiency range in order to extend the driving range. A new control algorithm was developed to distribute the total torque requirement dynamically to four wheel motors. When the total torque requirement is low, only two motors drive the vehicle and when the total torque requirement is high, four motors should work together to deliver the required torque. The vehicle switches between 2-wheel drive mode and 4-wheel drive mode according to the driving condition using the new control algorithm. Simulations of the new control algorithm was achieved with the new control algorithm.

Keywords: Electric vehicle, four-wheel independently driven, energy efficiency optimization

#### 1. Introduction

With the challenges of energy and environment crisis, new-energy vehicles have been the focus of automotive industry. Pure electric vehicle has the advantages of high efficiency and zero emission, and has been developed intensively by automobile manufactures. However, the power battery technology has always been the bottleneck of electric vehicles. As a result, short driving range is one of the most drawbacks which prevent electric vehicles from being competitive enough with the conventional vehicles.

In order to extend the driving range, many studies have been carried out to improve the efficiency performance of the motors and motor controllers<sup>[1][2][3][4]</sup>. As for electric vehicles driven by four in-wheel motors, more DOFs can be taken advantage of to exploit the maximal potential of energy efficiency improvement.

"Micro-Harry" is an experimental EV developed by Tsinghua University, shown in Figure 1 and Figure 2. The EV is driven by four PMSM wheel motors and equipped with a LiFePO<sub>4</sub> power battery. Specifications of the vehicle are listed in Table 1.



Figure 1: "Micro-Harry" EV developed by Tsinghua University



Figure 2: System configuration of "Micro-Harry" EV

Table 1: Specifications of "Micro-Harry"

Name (Unit)	Value
Length/Width/Height (mm)	2500/1200/1530
Wheelbase (mm)	1645
Vehicle curb mass (kg)	500
One motor power (kW)	2
Battery capacity (Ah)	110
Battery voltage (V)	48

This paper firstly described the details of the vehicle model which consists of vehicle dynamics model, driver model, wheel motor models and Liion power battery model. Then the new control algorithm was explained after the motor efficiency analysis. Simulation on a modified urban drive cycle in Matlab/Simulink was carried out to show the performance of the new control algorithm. In the end a conclusion was drawn.

### 2. Vehicle Model

The block diagram of the vehicle model is shown in Figure 3.



Figure 3: Block diagram of the vehicle model

#### 2.1 Vehicle Dynamics

Vehicle dynamics model only concerns the longitudinal dynamics of the vehicle. The inputs are motor driving torque and braking torque, and the output is the vehicle speed.

The driving force of the vehicle can be expressed as

$$F_{\rm t} = \frac{T_{\rm tq} - T_{\rm br}}{r} \tag{1}$$

 $T_{\rm tq}$  is the motor torque,  $T_{\rm br}$  is the braking torque and *r* is the effective rolling radius.

The overall resistance  $F_r$  of the vehicle is

$$F_{\rm r} = (m_{\rm v} + m_{\rm cap})gf\cos\alpha + (m_{\rm v} + m_{\rm cap})g\sin\alpha + \frac{1}{2}\rho C_{\rm D}Au^2 + m_{\rm v}e\frac{du}{dt}$$
(2)

 $m_{\rm v}$  is the curb vehicle mass,  $m_{\rm cap}$  is the capacity mass, f is the rolling resistance coefficient,  $\alpha$  is the angle of gradient,  $\rho$  is the air density,  $C_{\rm D}$  is the drag coefficient, A is the cross sectional area exposed to air flow, u is the vehicle velocity, e is the mass factor.

The driving force and the overall resistance force should be in equilibrium.

$$F_{\rm t} = F_{\rm r} \tag{3}$$

Vehicle speed could be calculated from the equations above.

#### 2.2 Wheel motor model

The wheel motor might work in drive mode or regeneration mode according to the command from the vehicle controller. The wheel motor model calculates the actual torque and power of the motor. Electric motors are characterized with fast and accurate response, so the dynamics of the motor could be neglected.

The wheel motor model is built based on the efficiency map of motor, which is shown in Figure 4. As can be seen, motor efficiency is quite

low when the motor is running in the region of low speed and low torque.



Figure 4: Efficiency map of wheel map

The power of motor is calculated according to the motor efficiency.

$$P_{i} = \begin{cases} \frac{2\pi T_{i}n_{i}}{60\eta_{i}}, & \text{when driving} \\ \frac{2\pi T_{i}n_{i}\eta_{i}}{60}, & \text{when braking} \end{cases}$$
(4)

 $n_i$ ,  $T_i$  and  $\eta_i$  denote the rotation speed, torque and efficiency of the four motors respectively.

#### 2.3 Power battery model

The power battery is simulated using an equivalent circuit model<sup>[5]</sup>, as shown in Figure 5 and expressed in state space equation (5).



Figure 5: Equivalent circuit model of the power battery

$$\begin{cases} \begin{bmatrix} U_{\rm L} \end{bmatrix} = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} U_{\rm pb} \\ U_{\rm pp} \end{bmatrix} + \begin{bmatrix} -R_{\rm po} \end{bmatrix} \begin{bmatrix} I_{\rm L} \end{bmatrix} + \begin{bmatrix} U_{\rm OCV} \end{bmatrix} \\ \begin{bmatrix} \dot{U}_{\rm pb} \\ \dot{U}_{\rm pp} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{C_{\rm pp} R_{\rm pp}} \end{bmatrix} \begin{bmatrix} U_{\rm pb} \\ U_{\rm pp} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{\rm pb}} \\ \frac{1}{C_{\rm pp}} \end{bmatrix} \begin{bmatrix} I_{\rm L} \end{bmatrix}$$
(5)

 $U_{\rm OCV}$  is the open circuit voltage of the battery,  $R_{\rm po}$  is the ohm resistance,  $U_{\rm po}$  is the voltage over  $R_{\rm po}$ ,  $R_{\rm pp}$  is the polarization resistance of the battery which consists of chemical and concentration polarization resistance,  $U_{\rm pp}$  is the voltage over  $R_{\rm pp}$ ,  $C_{\rm pp}$  is the polarization capacitor of the battery,  $C_{\rm pb}$  is the capacitor representing the open circuit voltage variation by the charge and discharge accumulation,  $U_{\rm pb}$  is the voltage over  $C_{\rm pb}$ ,  $I_{\rm pp}$  is the load current,  $U_{\rm L}$  is the end voltage of the battery.

The variables of the model are identified using experiment data at different SOC. This model shows a quite good accuracy<sup>[5]</sup>.

#### 2.4 Driver model

Driver model simulates the behavior of the driver when driving. The driver manipulates the acceleration or brake pedal based on the inputs of target speed and actual speed. It is basically a PI controller.

#### 2.5 Vehicle controller

The vehicle controller gives orders to the wheel motors with the inputs of driver manipulation and states of vehicle and battery. It interprets the driver manipulation into the total torque requirement. Figure 6 shows the torque requirement when accelerating. The required torque would be negative when regenerative braking.



Figure 6: Torque requirement when accelerating

The vehicle controller is also responsible to distribute the total torque requirement to four wheel motors, and also the mechanical brake system when the vehicle brakes and the mechanical brake is required to assist the regenerative braking.

# **3.** Control algorithm for energy efficiency optimization

#### 3.1 Formulation of the control problem

As the efficiency map of the wheel motor shows, the efficiency is very low at low speed and low torque range. In order to improve the energy efficiency, the motor should work as much as possible in the high efficiency range.

At a given operation point, the speed and the total torque requirement are determined by the driver, therefore are fixed. But how to distribute the total torque requirement to four wheel motors is what can be done to improve the system overall efficiency. This control problem can be formulated as in (6).

$$J = \begin{cases} \min\left(\sum_{i=1\sim4} \frac{n_i T_i}{\eta_i}\right), & \text{when driving} \\ \max\left(\sum_{i=1\sim4} n_i T_i \eta_i\right), & \text{when braking} \end{cases}$$
(6)

As only longitudinal dynamics is considered, no differential torque between left and right wheels is necessary. And the speeds of the four wheels should be the same as required.

$$\begin{cases} T_{1} = T_{2}, T_{3} = T_{4} \\ n_{i} = n_{req} \\ \sum_{i=1\sim4} T_{i} = T_{req} \end{cases}$$
(7)

So the cost function (6) could be rewritten as

$$J = \begin{cases} \min\left(\frac{T_{\rm f}}{\eta_{\rm f}} + \frac{T_{\rm r}}{\eta_{\rm r}}\right), & \text{when driving} \\ \max\left(T_{\rm r}n_{\rm r} + T_{\rm r}n_{\rm r}\right), & \text{when braking} \end{cases}$$
(8)

With

$$T_{\rm f} + T_{\rm r} = T_{\rm req} \tag{9}$$

 $T_{\rm f}$  and  $T_{\rm r}$  are the torques of the front and rear wheel motors.  $T_{\rm req}$  is the total torque requirement.

#### 3.2 Motor efficiency model

The efficiency of the electrical motor in driving condition  $\eta_d$  can be expressed as

$$\eta_{\rm d} = \frac{P_{\rm o}}{P_{\rm i}} = \frac{P_{\rm o}}{P_{\rm o} + P_{\rm Cu} + P_{\rm Fe} + P_{\rm m} + P_{\rm str}}$$
(10)

 $P_{\rm o}$  and  $P_{\rm i}$  are the output and input power of the motor,  $P_{\rm Cu}$  is the copper loss, which is proportional to the square of the motor torque.  $P_{\rm Fe}$  is the iron loss, which is mainly decided by the motor speed.  $P_{\rm str}$  is the stray loss, which could be

neglected. At a fixed motor speed, the electrical losses could be approximated as in  $(11)^{[2][6]}$ .

$$P_{\rm Cu} + P_{\rm Fe} + P_{\rm str} = C_1 T^2 + C_2 \tag{11}$$

 $P_{\rm m}$  is mainly decided by the motor speed and could be treated as constant at a fixed speed. Therefore, at a given speed the motor efficiency could be expressed as in (12).

$$\eta_{\rm d} = \frac{T}{C_1 T^2 + T + C_3} \tag{12}$$

 $C_1$  and  $C_3$  are two parameters to be derived by fitting the motor efficiency map. Figure 7 shows the curve fitting result at the motor speed of 240r/min. The motor efficiency increases with the motor torque at low torque region and decreases at high torque region, which means that the copper loss takes a larger part with increasing torque, while the mechanical loss is predominant at low torque region.

When the motor is working in regenerative braking mode, motor efficiency can be expressed as in (13).

$$\eta_{\rm b} = \frac{P_{\rm o}}{P_{\rm i}} = \frac{P_{\rm i} - \left(P_{\rm Cu} + P_{\rm Fe} + P_{\rm m} + P_{\rm str}\right)}{P_{\rm i}}$$
(13)

And based on the analysis above, (13) could be rewritten as (14).

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$$\eta_{\rm b} = \frac{P_{\rm o}}{P_{\rm i}} = \frac{T - C_1 T^2 - C_3}{T} \tag{14}$$



Figure 7: Fitting of motor efficiency at motor speed of 240r/min

When the torque of regenerative braking is very low, the efficiency could be minus, because energy is consumed instead of being regenerated at this condition.

## **3.3** Control algorithm for efficiency optimization

A new control algorithm is developed to optimize the overall energy efficiency. Take the driving condition for example.

Apparently, if the total torque requirement can be met by front motors or rear motors alone, the total torque should be distributed to front motors or motors wheels only, because in this condition the total output power is working at a higher overall efficiency.

If the total torque requirement exceeds the limits of four motors working together, the total torque requirement could not be met and all the motors should work at its maximum output torque.

If the total torque requirement could not be met by front motors or rear motors alone, but stills remains in the limits of four motors working together, a strategy to distribute the total torque requirement to four motors needs to be studied.

$$f(x) = \frac{T_{\rm f}}{\eta_{\rm f}} + \frac{T_{\rm r}}{\eta_{\rm r}} = \frac{T_{\rm f}}{\eta(T_{\rm f})} + \frac{T_{\rm req} - T_{\rm f}}{\eta(T_{\rm req} - T_{\rm f})}$$
(15)

With

$$0 < T_{\rm f} < T_{\rm max}$$
  
$$0 < T_{\rm req} - T_{\rm f} < T_{\rm max}$$
 (16)

According to Equation (12),

$$f(x) = C_1 T_f^2 + T_f + C_2 + C_1 (T_{req} - T_f)^2 + T_{req} - T_f + C_2$$
(17)  
$$= C_1 (T_f^2 + (T_{req} - T_f)^2) + T_{req} + 2C_2$$

Obviously, f(x) get its minimum value when  $T_f = T_{req}/2$ . That is to say, the total torque requirement should be distributed evenly to four motors to achieve the maximum overall energy efficiency.

The vehicle will switch between 2-wheel drive mode and four-wheel drive mode according to the driving condition using the new control algorithm. The energy distribution strategy for regenerative braking condition is similar to driving condition. Therefore, we are not going to discuss it in details here.

#### 4. Simulation Results

The new control algorithm is simulated in Matlab/Simulink. The simulation is run on an urban drive cycle given in the form of "speed - time" as the input to driver model. Because of the limit of maximal vehicle speed, the drive cycle is a shrunk version by the ratio of 50/74, shown in Figure 8. The maximum speed of this drive cycle is 50km/h, and the average speed is 15 km/h.

## 4.1 Simulation of the new control algorithm

The new control algorithm is compared with the conventional control algorithm which distributes the required torque evenly all the time. The range per charge is used to compare the energy efficiency.



Figure 8: Urban drive cycle

The simulation shows that, the vehicle can reach the range of 74.4km with a full charge using the original control algorithm, while the new control algorithm can achieve 83.9km, which means an improvement by 12.84%. Figure 9 shows the time percentage of each efficiency zone in a full drive cycle. As can be seen from the figure, the time when the vehicle is operating in low efficiency zone decreases, while high efficiency operating time increases.



Figure 9: Comparison of the new control algorithm

Some study still needs to be conducted before the new control algorithm is applied in real vehicle control. A transition strategy is necessary to provide a smooth switch between 2-wheel driving mode and 4-wheel driving mode. Besides, more factors should be taken into consideration when making driving mode switch. For example, temperature of the driving motor affects not only on the motor efficiency but also operation safety. Suppose that the motor has just experienced a tough driving and its temperature is quite high. In this condition, the efficiency of the motor is definitely lower than normal. These motors should be relieved from heavy load to maintain a high efficiency and safe operation.

### 5. Conclusion

In order to extend the driving range of a fourwheel independently driven electric vehicle, a new control algorithm trying to optimize the overall energy efficiency is developed.

Firstly, the model of the vehicle is built, which includes the vehicle dynamics model, wheel motor model and power battery model. The efficiency model of permanent magnet synchronous motor is analyzed. Based on the analysis, the new control algorithm is designed to distribute the total torque requirement to four motors. The vehicle switches between 2-wheel driving mode and 4-wheel driving mode according to the driving condition using the new control algorithm. Simulations on a modified urban driving cycle in Matlab/Simulink are carried out, which show that an improvement of the driving range by 12.84% is achieved with the new control algorithm.

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