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RBCKF-Based Vehicle State Estimation by Adaptive Weighted Fusion Strategy Considering Composite-State Tire Model

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Abstract: The acquisition of vehicle driving status information is a key function of vehicle dynamics systems, and research on high-precision and high-reliability estimation of key vehicle states has significant value. To improve the state observation effect, a vehicle sideslip angle estimation method adopting a robust bias compensation Kalman filter and adaptive weight fusion strategy is proposed. On the basis of the extended Kalman filter algorithm, and with the goals of estimation exactitude and robustness, considering the potential signal deviation, a vehicle state robust deviation compensation Kalman filter estimation algorithm considering bias compensation and residual covariance matrix weighting is proposed. Meanwhile, considering the adaptive and dynamic adjustment capabilities of the observation system in complex state-change scenarios, an estimation strategy based on adaptive weight fusion and a model-based estimator is proposed. The results confirm that the robust bias compensation Kalman filter can ensure estimation exactitude and robustness when the vehicle state fluctuates greatly, and the proposed fusion strategy can ensure that the vehicle maintains optimal estimation performance during operating condition switching.

Keywords: vehicle state; Kalman filter; observer design; adaptive weight



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1. Introduction

Autonomous driving and intelligent control technology of an integrated electrified chassis is currently a core technology pain point for the automotive industry. Many countries have issued multiple policy documents to support the development of vehicle electrification and intelligence. In the process of intelligent driving, information about real-time traffic flow, the driving environment, the vehicle's position and driving status are required to assist the driverless driving system in making optimal control decisions [1–3]. The large information flow means that driverless vehicles need to be equipped with complex sensor systems, which undoubtedly increases the production cost and difficulty of manufacturing the entire vehicle. With the maturity of the automotive industry chain and the improvement of the supply chain, the segmentation functions of the related midstream and downstream automotive parts industry are gradually becoming clear. It helps to greatly reduce the production cost of automobiles while ensuring a large supply. At present, sensor devices for dynamic vehicle control and intelligent driving include speed sensors, acceleration sensors, gyroscopes, onboard cameras and radars, among others. The sensor systems of the vehicle provide strong support for its intelligence, safety, and efficiency. A current practical technical problem is that many key states are difficult to obtain directly through feasible vehicle sensing devices. Therefore, to optimize state acquisition and lower production costs with onboard sensors, precise and reliable vehicle driving state observers are expected to substitute the actual sensing equipment through soft sensing methods [4,5].

At present, model-based Kalman filters are widely used in estimating vehicle sideslip angle [6–8]. Considering that vehicles need to handle completely driverless driving scenarios and process multi-source information, high-precision and highly reliable filtering estimation systems are essential for estimation requirements in complex driving scenarios.

Corresponding research work has significant research value and application prospects. In such a context, the vehicle state observation system needs to have good dynamic response capability and anti-interference ability in order to maintain accurate estimation performance in complex driving scenarios [9,10]. The Kalman filter algorithm and its optimized form are widely used. Its optimized form can cope with the negative impact of unknown disturbances on the estimation results while maintaining the amount of filtering operations, thereby effectively improving the robustness of the filter. The EKF obtains linear measurement equations by applying Taylor's formula to nonlinear observation equations, but the deletion and omission of polynomials can easily cause bias in the filtering results [11,12]. The unscented and volumetric Kalman filter approximates the target state following a Gaussian distribution by selecting Sigma sampling points, which can achieve second-order Taylor approximation accuracy and better stability [13,14]. However, they have high computational complexity and higher complexity in real-time target tracking applications.

On this basis, to achieve better state estimation results, many new concepts and techniques of state observation have also been applied in vehicle state estimation studies, such as AI-based approaches [15–17], tireless model approaches [18,19], tire model-based approaches [20], and multibody model-based approaches [21]. At this point, the introduction of intelligent algorithms, the improvement of filtering algorithms, and the combination of model-based observers and data-driven-based observers are all important aspects of enhancing state estimation technology. The driving environment of vehicles is complex and the road conditions are varied. The information collection and processing system based on electrical signal transmission has a high dependence on the accuracy and reliability of input information values [22,23]. If there are factors such as signal interference and external disturbances of input, there is a high probability that abnormal outliers may appear in the measurements. However, conventional Kalman filtering algorithms do not have robust adjustment capabilities to handle and suppress such outliers, so information fluctuations can easily cause a significant decrease in filter estimation trustworthiness, or even serious deviation of filtering results. In this case, the robustness, anti-interference ability, and adaptability of the vehicle state observer system are crucial [24–26], so the improvement of the robust anti-interference ability of the Kalman filter algorithm is a core point of technological breakthroughs.

At the same time, considering the complex and varied vehicle and road conditions in multi-condition intelligent driving scenarios, the state observation system needs to have the ability to handle large-scale driving condition changes or alternation. At the same time, as road conditions or vehicle states change, tire models are prone to exhibit two typical characteristics of linear and nonlinear switching or dynamic coupling [27–29], which increases the difficulty of accurate state estimation and adaptive tracking for observation systems. The state observation scheme based on a linear tire model can quickly capture and reflect the changing characteristics of the vehicle's dynamic state and its interaction relationships, thus meeting the real-time requirements of state estimation indicators. However, when the tire force relationship exhibits significant nonlinear characteristics, the accuracy of this observer can be expected to decrease [30–32]. A state observer scheme based on a nonlinear tire model can rely on more refined force relationship calculations to reflect the nonlinear changes in forces in the tire model. However, this observation method cannot track the motion relationship and dynamic changes of vehicle status in a timely and effective manner [33,34]. A perfect state estimation method needs to be able to fit or characterize the most accurate vehicle state mapping relationship in various scenarios, and rely on the most accurate model formulas and reliable estimation strategies to improve the state calculation ability during operating conditions and state switching, thereby achieving optimal estimation results [35–37]. To deal with the unique multi-source redundant vehicle information of intelligent vehicles, designing a reliable acquisition scheme for vehicle status with multi-source weight and adaptive adjustment can help improve the estimation accuracy, robustness, and adjustment capability of the observation system [38,39]. Therefore, state observers should be designed based on models using different mechanisms,

and the trustworthiness of different state observers should be determined by combining real-time vehicle state dynamic transitions and coupling relationships. Then, different observers can be matched with different weight coefficients based on their confidence levels, thereby improving the accuracy and reliability of the final estimation results through information fusion.

Based on the above analysis, a vehicle state adaptive estimation strategy with RBCKF is developed to ensure the estimation exactitude, robustness, and adaptability of the vehicle state observation system, in response to the complex driving scenarios that vehicles need to handle during large-scale road driving conditions. The contributions of this study can be summarized as follows: (1). To deal with the uncertainty and interference factors in the measured values, a vehicle state estimator based on RBCKF algorithm is proposed, which improves the robustness of the vehicle state observation system when handling nonlinearity and abnormal measurement results; (2). The influence mechanism of the corresponding vehicle state observation effects of LTM and NLTM is analyzed, and vehicle state observers under different mechanism models are designed; (3). In order to simultaneously improve the estimation performance and adaptive dynamic adjustment capability of the vehicle state observation system in the context of a wide range of operating conditions, a fusion estimation strategy based on adaptive weighting is proposed to enhance the comprehensive state estimation effect.

2. Vehicle Model

2.1. Vehicle Dynamic Model

To facilitate the design of vehicle state observers, vehicle dynamics equations for transversal and yaw directions were established to describe the dynamic mapping relevance of vehicle states. As shown in Figure 1, a dynamic coordination system xoy is established, where the origin of the coordinate system is positioned at the mass center of the vehicle. The x -axis represents the longitudinal motion direction of the vehicle, and the forward direction is positive. The y -axis represents the lateral motion direction of the vehicle, and the forward direction to left is positive. The vehicle suspension system, pitch, roll, and vertical motion are neglected, so the vehicle's dynamic particularities are discussed only in the xoy plane; it is further assumed that all tire particularities are consistent. The dynamic equation of the vehicle monorail model is as follows:

$$m(\dot{u} + v\dot{\varphi}) = T_{yf} + T_{yr}, \quad (1)$$

$$I_z\dot{\varphi} = D_f T_{yf} - D_r T_{yr} + \Delta M_z. \quad (2)$$

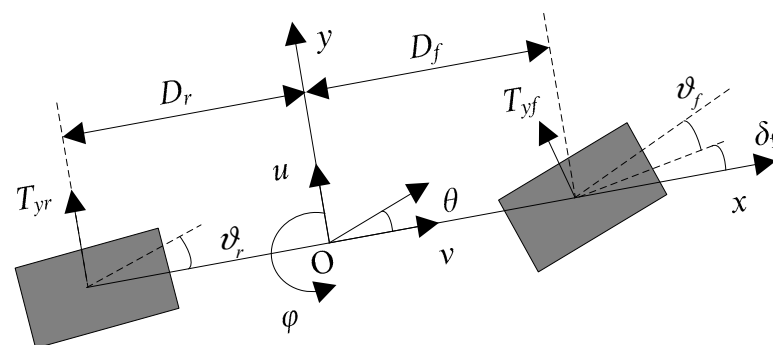


Figure 1. Vehicle dynamics model.

When considering tire force in the linear region, the calculation formulas for T_{yf} and T_{yr} can be expressed as

$$\begin{aligned} T_{yf} &= -2\lambda_f \theta_f \\ T_{yr} &= -2\lambda_r \theta_r \end{aligned} \quad (3)$$

The tire slip angle is

$$\begin{aligned}\vartheta_f &= \delta_f - (D_f \varphi + u)/v, \\ \vartheta_r &= (D_r \varphi - u)/v\end{aligned}\quad (4)$$

The vehicle sideslip angle is

$$\theta = u/v, \quad (5)$$

Combining Equations (1), (2), and (5), the vehicle dynamics model can be calculated as follows:

$$\dot{\theta} = -\frac{\lambda_f + \lambda_r}{mv}\theta + \left(-\frac{\lambda_f D_f - \lambda_r D_r}{mv^2} - 1\right)\varphi + \frac{\lambda_f}{mv}\delta_f, \quad (6)$$

$$\dot{\varphi} = -\frac{\lambda_f D_f - \lambda_r D_r}{I_z}\theta - \frac{\lambda_f D_f^2 + \lambda_r D_r^2}{I_z v}\varphi + \frac{\lambda_f D_f}{I_z}\delta_f, \quad (7)$$

The longitudinal and lateral acceleration of the vehicle is

$$\begin{aligned}\dot{v} &= u\varphi + a_x \\ \dot{u} &= -v\varphi + a_y\end{aligned}\quad (8)$$

Longitudinal and lateral accelerations of the vehicle can be expressed as

$$\begin{aligned}a_x &= \frac{1}{m}\left(T_{xf}\cos\delta_f - T_{yf}\sin\delta_f + T_{xr}\right) \\ a_y &= \frac{1}{m}\left(T_{xf}\sin\delta_f + T_{yf}\cos\delta_f + T_{yr}\right)\end{aligned}\quad (9)$$

2.2. Nonlinear Tire Model

While the tire force is in the nonlinear region, the tire force calculation method in Equation (3) suffers from insufficient representation accuracy. Considering the situation where the tire force locates in a nonlinear region, the Magic Formula is used to calculate the tire force, which can be expressed as longitudinal and transverse tire forces:

$$T_{x,y} = \kappa_D \sin\{\kappa_C \arctan[\kappa_B \vartheta - \kappa_E(\kappa_B \vartheta - \arctan(\kappa_B \vartheta))]\}, \quad (10)$$

The vertical force of each tire is expressed as

$$\begin{aligned}T_{zfl} &= D_f\left(\frac{mg}{2(D_f+D_r)} + \frac{ma_y h}{2b_f(D_f+D_r)}\right) - \frac{ma_x h}{2(D_f+D_r)} \\ T_{zfr} &= D_f\left(\frac{mg}{2(D_f+D_r)} - \frac{ma_y h}{2b_f(D_f+D_r)}\right) - \frac{ma_x h}{2(D_f+D_r)} \\ T_{zrl} &= D_r\left(\frac{mg}{2} + \frac{ma_y h}{2b_r(D_f+D_r)}\right) + \frac{ma_x h}{2(D_f+D_r)} \\ T_{zrr} &= D_r\left(\frac{mg}{2} - \frac{ma_y h}{2b_r(D_f+D_r)}\right) + \frac{ma_x h}{2(D_f+D_r)}\end{aligned}\quad (11)$$

3. Robust Bias Compensation-Based Kalman Filter for Vehicle State Estimation

3.1. Robust Pseudolinear Kalman Filter

The vehicle dynamics model is represented in the form of discrete state space equations, as follows:

$$x_{k+1} = f(x_k, u_k) + w_k, \quad (12)$$

$$y_k = h(x_k) + \zeta_k, \quad (13)$$

where x_{k+1} is the system state, w_k and ζ_k are respectively the process noise and measurement noise, $f(\cdot)$ is the state transition matrix, $h(\cdot)$ is the measurement matrix, and w_k and ζ_k are uncorrelated zero-mean white noise sequences. In this work, the initial variance matrix of prediction error is set as $P_k = \text{eye}(n) \times 10^{-3}$, the initial covariance matrix of process noise is set as $Q_k = \text{eye}(n) \times 10^{-3}$, where n represents the dimensionality of different observers (LTM-based observer or NLTM-based observer). The extended Kalman filtering obtains linear measurement equations by performing first-order Taylor approximation on

nonlinear observation equations, but truncation errors can easily cause filter divergence. To address the issues of nonlinear filtering and measurement result outliers faced by vehicles during complex driving conditions, the observation equation is first pseudolinearized. Then, a robust pseudolinear Kalman filter EKF is derived based on the M-estimation criterion, and the bias caused by pseudolinearization is compensated to improve estimation accuracy. Pseudolinearization is the process of obtaining a new form of linear equation through equivalent algebraic operations. Equation (8) has been converted into a linear equation. When the observed noise is small, the mean of the pseudolinear noise vector ζ_k is approximately 0, and its covariance matrix is $R_k = E(\zeta_k \zeta_k^T)$. By combining the least squares and M-estimation criteria, an error-based objective function is constructed:

$$J_M = \tau_{k|k-1}^T P_{k|k-1}^{-1} \tau_{k|k-1} + \sum_{i=1}^m R_k^{-1}(i, i) \rho(\zeta_k(i)), \quad (14)$$

where $\tau_{k|k-1} = \hat{x}_{k|k-1} - x_k$ is the prediction error, $P_{k|k-1}^{-1}$ is its covariance matrix, $\rho(\cdot)$ is the robust cost function, and m is the dimension of the observation vector. By taking the partial derivative of Equation (14) and making its value 0, it can be designed as

$$P_{k|k-1}^{-1} \tau_{k|k-1} + \sum_{i=1}^m R_k^{-1}(i, i) \varphi(\zeta_k(i)) \frac{\partial \zeta_k(i)}{\partial x_k} = 0, \quad (15)$$

where $\sigma(\cdot)$ is the derivative of $\rho(\cdot)$, and is used to reduce the impact of outliers. $\sigma(\cdot)$ is set as a constrained-limited function so that the outlier error does not exceed a certain limit. Usually, it can be denoted by $\sigma(\zeta_k(i)) = \omega_{k,i} \zeta_k(i)$, and $\omega_{k,i}$ is the weight coefficient. Furthermore, according to the definition of ζ_k , it can be concluded that

$$\frac{\partial \zeta_k(i)}{\partial x_k} = \frac{\partial (\tilde{z}_k(i) - H_k(i, :) x_k)}{\partial x_k} = -H_k^T(:, i), \quad (16)$$

By substituting the expression of $\varphi(\zeta_k(i))$ and Equation (16) into Equation (15), we can obtain

$$P_{k|k-1}^{-1} \tau_{k|k-1} + H_k^T (W_k R_k)^{-1} (H_k x_k - \hat{y}_k) = 0, \quad (17)$$

where $W_k = \text{diag}(\omega_{k,1}^{-1}, \dots, \omega_{k,m}^{-1})$ is the weight matrix. Therefore, the mentioned robust EKF constricts the impact of exception values via the weighted adjustment method, and its filtering steps are as follows.

(1) Time update:

$$\begin{aligned} \hat{x}_{k|k-1} &= f_{k-1} \hat{x}_{k-1|k-1} \\ P_{k|k-1} &= f_{k-1} P_{k-1|k-1} f_{k-1}^T + Q_k \end{aligned} \quad (18)$$

(2) Measurement update:

$$\begin{aligned} K_k &= P_{k|k-1} h_k^T (W_k R_k + h_k P_{k|k-1} h_k^T)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (\hat{y}_k - h_k \hat{x}_{k|k-1}) \\ P_{k|k} &= (I - K_k h_k) P_{k|k-1} \end{aligned} \quad (19)$$

3.2. Overall Estimation Method and Strategy

In the process of model linearization, it is necessary to first consider the correlation of observation matrix H_k and pseudolinear noise matrix ζ_k , leading to estimation bias in the filtering results, especially when the noise is severe and the estimation bias increases rapidly. Therefore, the idea of bias compensation is applied to the robust EKF to improve target tracking accuracy by calculating estimated bias and compensating for it. By using

filtering steps (18) and (19), as well as the matrix inversion formula, the relevant deviation terms can be obtained:

$$E(\sigma_k) = E\left[P_{k|k}H_k^T(W_kR_k)^{-1}\varepsilon_k\right] = P_{k|k}W_{k,1}^{-1}R_k^{-1}E\left(H_k^T\varepsilon_k\middle|x_k\right), \quad (20)$$

Therefore, the estimated state value of RBCKF is

$$\hat{x}_{k|k}^{BC} = \hat{x}_{k|k} - \hat{E}(\zeta_k), \quad (21)$$

Due to the fact that the covariance matrix of the filter estimation result ignores the issue of bias in $\hat{x}_{k|k}$, the actual covariance matrix is

$$E\left[(\hat{x}_{k|k} - x_k)(\hat{x}_{k|k} - x_k)^T\right] = P_{k|k} + \zeta_k\zeta_k^T, \quad (22)$$

Furthermore, the covariance of RBCKF is

$$E\left[(\hat{x}_{k|k}^{BC} - x_k)(\hat{x}_{k|k}^{BC} - x_k)^T\right] = P_{k|k} + \zeta_k\zeta_k^T - \zeta_k\hat{E}(\zeta_k^T) - \hat{E}(\zeta_k)\zeta_k^T + \hat{E}(\zeta_k)\hat{E}(\zeta_k^T), \quad (23)$$

In the process of deriving and calculating the estimated deviation value $\hat{E}(\zeta_k)$, there exists an approximate condition $\sin \zeta_k \approx \zeta_k$. Therefore, when the observation noise is small, the estimated deviation value is approximately equal to the true deviation value, that is $\hat{E}(\zeta_k) \approx \zeta_k$. By substituting this into Equation (18), the covariance matrix of the RBCKF is approximately equal to $P_{k|k}$, so its covariance is updated using Equation (19).

3.3. Weight Function

The RBCKF constricts the impact of exception values by introducing a weight adjustment factor, so the handling of outliers is transformed into the design of weight functions. The weight function needs to determine the interval division of the observed values and the weights of each interval. The commonly used Huber-weight method is a dual-stage function, which constricts the impact of exception values that exceed the threshold while maintaining the weight of normal observed values unchanged. However, the threshold is usually determined based on experience, which is not conducive to its application in actual target tracking systems.

Mahalanobis distance is an important basis for detecting outliers in multivariate data, and its advantage lies in not relying on statistical characteristics of noise, which can avoid determining thresholds through empirical values. Therefore, an outlier discriminant based on Mahalanobis distance can be constructed, and its expression is

$$\Gamma_k = \tau_k^T P_{\tau k}^{-1} \tau_k \sim \chi^2(m), \quad (24)$$

where $e_k = \tilde{z}_k - H_k\hat{x}_{k|k-1}$ is residuals, $P_{\tau k} = H_k P_{k|k-1} H_k^T + R_k$ is the residuals covariance matrix, Γ_k follows a chi-square distribution, and m is the residual vector dimension. According to the assumption testing principle, considering the assigned significance level α , the γ_k satisfies

$$p(\gamma_k < \chi_\alpha^2(m)) = 1 - \alpha, \quad (25)$$

where $p(\cdot)$ is the probability of stochastic information, and $\chi_\alpha^2(m)$ is alpha quantile. While γ_k does not satisfy Equation (20), it indicates that there is an abnormal observation value. The observation vector may only have a few abnormal observations. If a unified weight is used for adjustment, reliable observations will also be reduced in weight. Therefore, it is more reasonable to perform dimension by dimension detection on the new information vector to identify outliers, which can avoid reducing the proportion of conven-

tional observations. For state condition in the residual vector, it can be obtained by the following equation:

$$\Gamma_k(i) = \frac{\tau_k^2(i)}{P_{\tau_k}(i, i)} \sim \chi^2(1).$$

If the chi-square freedom is 1, the proportionality matrix needs to be more finely divided into observation value intervals and determine reasonable weights, not only to suppress the proportion of abnormal values, but also to weaken the influence of significant abnormal values. For this purpose, an optimized triplex-step proportionality matrix is adopted, the expression of which is

$$\omega_{k,i} = \begin{cases} 1, & \Gamma_k(i) \leq \chi_{a_1}^2(1) \\ \exp\left(\frac{\chi_{a_1}^2(1) - \Gamma_k(i)}{\chi_{a_1}^2(1)}\right), & \chi_{a_1}^2(1) < \Gamma_k(i) < \chi_{a_2}^2(1) \\ \tau(i) = 0, & \Gamma_k(i) \geq \chi_{a_2}^2(1) \end{cases} \quad (26)$$

The e -Huber function adjusts the interval of observation values by confidence levels α_1 and α_2 , which is more dependable than the experience judgment of the Huber function and does not depend on the comprehensive quantitative numerical relationship of observation. When the abnormal value is small, the weight is reduced exponentially. When the abnormal value is large, the corresponding dimension's innovation is directly set to 0, which suppresses the negative feedback of the outlier at the practical level. By observing the pseudolinearization of the equation, robust filtering based on M-estimation, the flow of RBCKF can be obtained as shown in Figure 2. The pseudocode of the RBCKF algorithm is shown in Table 1.

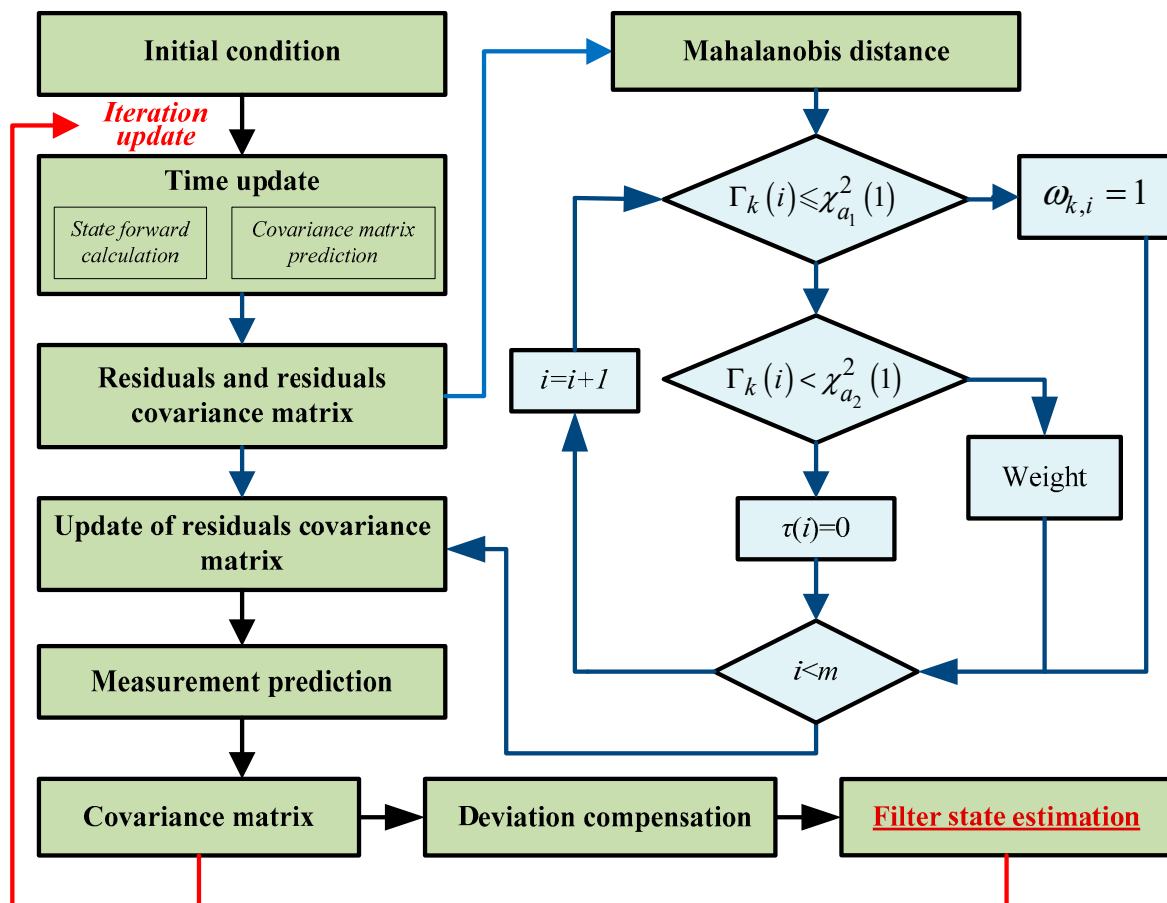


Figure 2. Flow chart of robust bias compensation Kalman filter.

Table 1. Pseudocode of the RBCKF algorithm.

Algorithm of RBCKF
(1) Initialization: Set initial state quantity $\hat{x}_0$ and initial covariance matrix P_0 for filtering. (2) Time update: Calculate predicted state value $\hat{x}_{k k-1}$ and its covariance matrix $P_{k k-1}$. (3) Calculate the innovation vector τ_k and its covariance matrix P_{τ_k} , and then calculate the square of the Mahalanobis distance Γ_k dimension by dimension. (4) Determine the outlier discrimination value of Mahalanobis distance $\Gamma_k(i)$ and the corresponding quantile size, and calculate the corresponding weight coefficients $\omega_{k,i}$. (5) Update the covariance matrix of the new information, and then update the state $\hat{x}_{k k}$ and its covariance matrix $P_{k k}$. (6) Calculate the error of the filter state estimation value and perform state compensation. (7) Repeat steps 2 to 6.

4. Adaptive Weighted Fusion Strategy Considering Composite-State Tire Model

4.1. Vehicle State Observer Considering Linear Tire Model

Firstly, considering the established model and robust bias compensation Kalman filtering algorithm, a vehicle state observer considering a linear tire model was designed. According to Equations (5) and (8), it can be concluded that

$$\dot{v} = v\theta\varphi + a_x, \quad (27)$$

In the process of filter design, the discretized forms of Equations (6), (7), and (27) are represented as

$$\begin{aligned} \theta_{k+1} &= \theta_k + \left(-\frac{\lambda_f + \lambda_r}{mv_k} \theta_k - \left(\frac{\lambda_f D_f - \lambda_r D_r}{mv_k^2} + 1 \right) \varphi_k + \frac{\lambda_f}{mv_k} \delta_{f,k} \right) \cdot T + w_k \\ \varphi_{k+1} &= \varphi_k + \left(-\frac{\lambda_f D_f - \lambda_r D_r}{I_z} \theta_k - \frac{\lambda_f D_f^2 + \lambda_r D_r^2}{I_z v} \varphi_k + \frac{\lambda_f I_f}{I_z} \delta_{f,k} \right) \cdot T + w_k \\ v_{k+1} &= v_k + (v_k \theta_k \varphi_k + a_{x,k}) \cdot T + w_k \end{aligned} \quad (28)$$

where T is the sampling period. By integrating Equations (5), (6), (8), it can be concluded that

$$a_y = \dot{u} + v\varphi = \dot{\theta}v + v\varphi = -\frac{\lambda_f + \lambda_r}{m} \theta - \frac{\lambda_f D_f - \lambda_r D_r}{mv} \varphi + \frac{\lambda_f}{m} \delta_f. \quad (29)$$

Equation (29) is regarded as the measurement update for the vehicle state observer, and then its discrete form can be expressed as

$$a_{y,k} = -\frac{\lambda_f + \lambda_r}{m} \theta_k - \frac{\lambda_f D_f - \lambda_r D_r}{mv} \varphi_k + \frac{\lambda_f}{m} \delta_{f,k} + \zeta_k. \quad (30)$$

By integrating the above models and algorithms, an LTM-based observer is designed using the RBCKF mentioned above, where the observation state is $x_{k+1} = [\theta_{k+1} \quad \varphi_{k+1} \quad v_{k+1}]^T$, the measurement is $y_k = [a_{y,k}]$, and the known input is $u_k = [\delta_{f,k} \quad a_{x,k}]^T$. Thereby, the output observation quantity of the LTM-based observer is denoted as θ_l .

4.2. Vehicle State Observer Considering Nonlinear Tire Model

According to Equations (8) and (9), the model for observer design and considering the nonlinear characteristics of tires can be obtained as

$$\begin{aligned} \dot{v} &= u\varphi + \frac{1}{m} (T_{xf} \cos \delta_f + T_{yf} \sin \delta_f + T_{xr}) \\ \dot{u} &= -v\varphi + \frac{1}{m} (T_{xf} \sin \delta_f + T_{yf} \cos \delta_f + T_{yr}) \end{aligned} \quad (31)$$

Given that a nonlinear tire model is considered in this paper, in order to incorporate the nonlinear tire force relationship into the observation equation, Equation (2) can be extended to

$$I_z \dot{\varphi} = (T_{xfl} + T_{xfr})D_f \sin \delta_f - (T_{xrl} - T_{xrr})b_r - (T_{xrl} + T_{xrr})D_r + (T_{yfl} + T_{yfr})D_f \cos \delta_f + (T_{xrl} - T_{xrr})b_f \sin \delta_f - (T_{xfl} - T_{xfr})b_f \cos \delta_f, \quad (32)$$

In filter design, the discretization equations of Equations (31) and (32) are

$$\begin{aligned} v_{k+1} &= v_k + \left(u_k \varphi_k + \frac{1}{m}(\cdot)_1 \right) \cdot T + w_k \\ u_{k+1} &= u_k + \left(-v_k \varphi_k + \frac{1}{m}(\cdot)_2 \right) \cdot T + w_k \\ \varphi_{k+1} &= \varphi_k + \frac{1}{I_z}(\cdot)_3 \cdot T + w_k \end{aligned} \quad (33)$$

where $(\cdot)_1$ and $(\cdot)_2$ respectively represent the discrete form of states in (23), and $(\cdot)_3$ represents the discrete form of states in (25). Then, using the RBCKF, a vehicle state observer considering the nonlinear characteristics of tires is designed, where the observation state is represented as $x_{k+1} = [v_{k+1} \ u_{k+1} \ \varphi]^T$, the known inputs of the observation system are represented as $u_k = [\delta_{f,k} \ \varphi_k \ T_{xfl} \ \cdots \ T_{xrr} \ T_{yfl} \ \cdots \ T_{yrr}]^T$, and the measured values of the observation system can be represented as $y_k = [\varphi]$. Based on the obtained filtering estimation value, the output observation quantity of the NLTM-based observer is obtained and denoted as θ_n .

4.3. Adaptive-Weight-Based Fusion Strategy Considering Composite-State Tire Model

When the sideslip angle continues to increase and exceeds the linear range, the relationship between tire lateral force and sideslip angle will exhibit nonlinear characteristics. At this point, as the sideslip angle increases, the rate of increase in tire lateral force will gradually slow down, and there may even be a trend of saturation or decrease. This non-linear relationship is crucial for understanding the behavior of vehicles under extreme operating conditions.

The relationship between tire force and tire sideslip angle is influenced by the nonlinearity of the tire model. In nonlinear models, due to the consideration of more physical factors and more complex mathematical expressions, it is possible to more accurately describe the mechanical characteristics of tires at different sideslip angles. This makes the relationship between tire force magnitude and sideslip angle more realistic in nonlinear models. Conversely, the variation in tire force can also serve as an indicator for evaluating the nonlinearity of tire models. If a model can accurately predict the changes in tire slip angle under different forces, it can be considered to have good nonlinear descriptive ability.

In the vehicle state observer considering a linear tire model, the formula for calculating tire lateral force is linear. Based on the multi-parameter coupling mapping relationship of tire force and its empirical state switching law, when the tire sideslip angle is small, the lateral force tends to be linearly positively correlated with the tire sideslip angle. However, when the angle exceeds a certain limit value, the tire force reaches saturation. Thus, when the tire model locates in the linear region, the observer considering the linear tire model has higher estimation accuracy. When the model components contain more connotations of nonlinear states, the accuracy of the observer decreases due to parameter uncertainty and model deviation.

In the vehicle state observer considering the nonlinear tire model, the Magic Formula is used to achieve more accurate tire force input, so the observer will not be directly and quickly affected by the nonlinear state transition changes of tire force. Meanwhile, observation systems based on the nonlinear Magic Formula find it relatively more difficult to dynamically capture rapid fluctuations and transitions in states, resulting in a loss of tracking performance, reducing the ability to quickly provide feedback on the trend of vehicle state changes. Both types of observers have their own advantages and are suitable

for different scenarios. Based on the above analysis, an adaptive fuzzy weighting strategy considering composite-state tire model is proposed to optimize the system's observation capability, as shown in Figure 3.

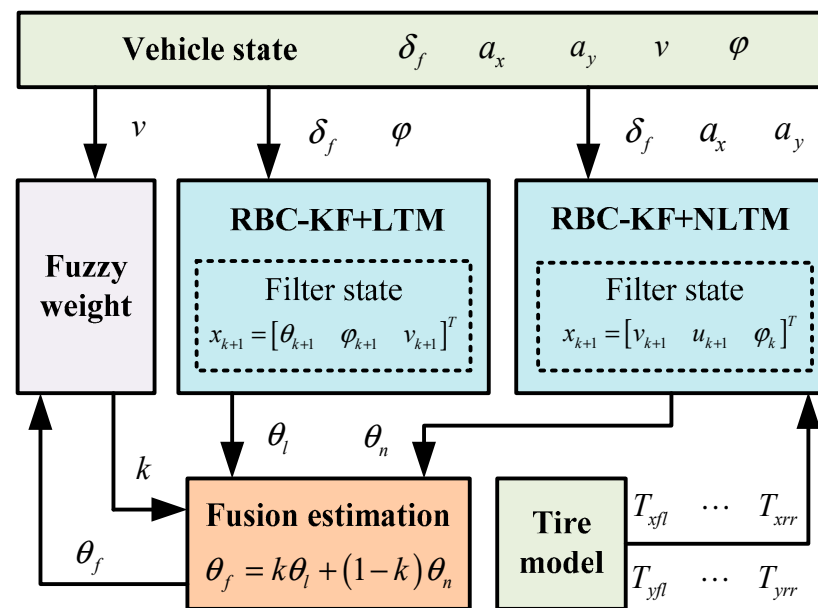


Figure 3. Fusion estimation strategy.

Combining Equation (4), the fuzzy input quantity of nominal tire sideslip angle is

$$\vartheta_n = \vartheta_f + \vartheta_r = \delta_f + \frac{-D_f + D}{v} \varphi - 2\theta_f, \quad (34)$$

where ϑ_n is the nominal tire sideslip angle, which indicates the tire nonlinearity. The vehicle center of mass deflection angle in the formula is computed through the fused results. If we design a fuzzy controller that takes v_x and ϑ_n as inputs and outputs weight coefficients, the system can dynamically adjust the weights of the two observers based on current-time transition change to acquire the optimal fusion estimation result. The membership functions of v_x , ϑ_n , and weight are shown in Figure 4, and the fuzzy rules are shown in Table 2. In Figure 4 and Table 2, the fuzzy rules symbols T, S, M, L, and H respectively represent 'tiny', 'small', 'medium', 'large', and 'huge'. The final result with comprehensive weighting for vehicle sideslip angle estimation θ_f is written as

$$\theta_f = k\theta_l + (1 - k)\theta_n, \quad (35)$$

Table 2. Fuzzy rules.

k		ϑ_n				
		T	S	M	L	H
v	T	T	S	M	M	L
	S	T	S	M	L	L
	M	S	S	M	L	H
	L	H	M	L	H	H

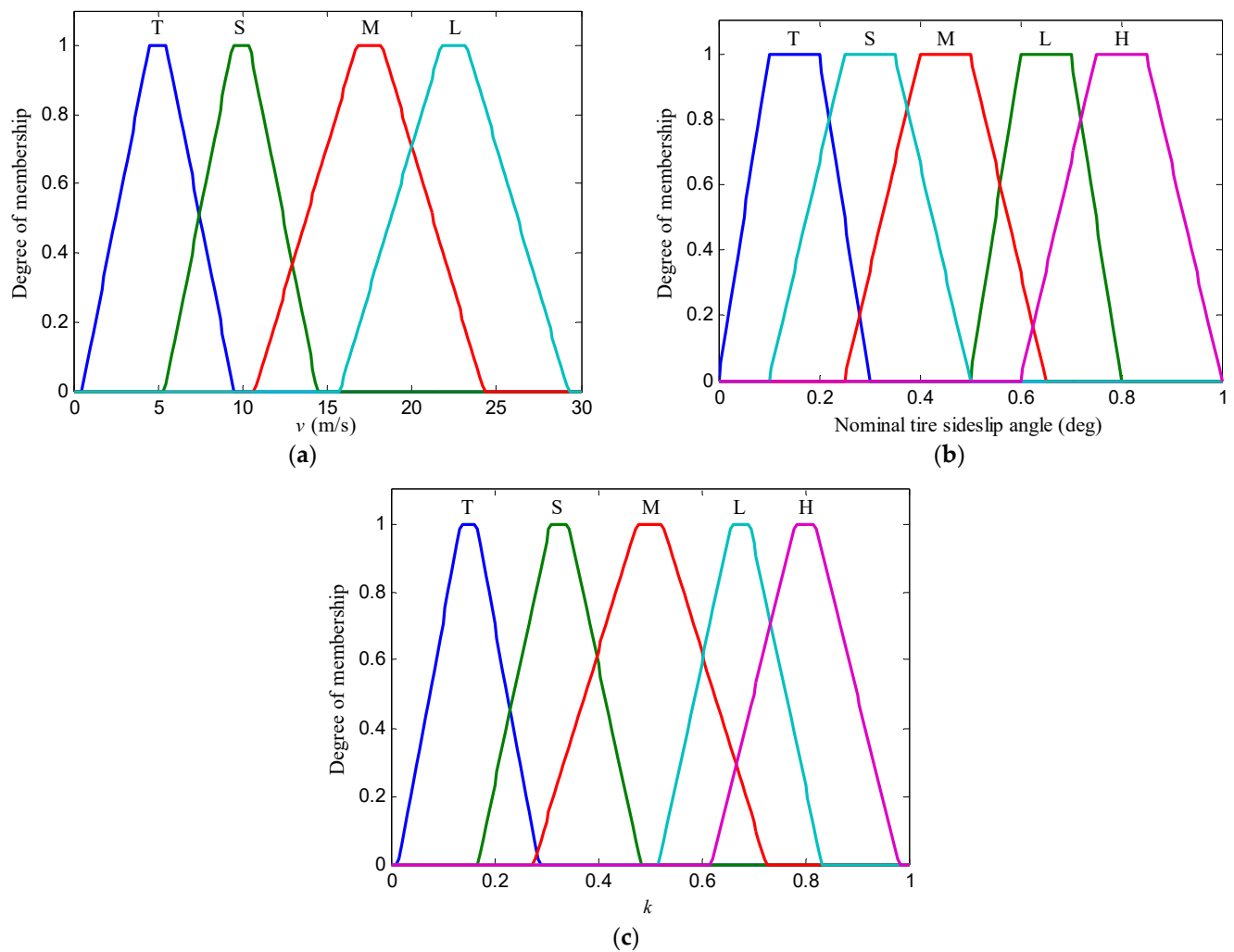


Figure 4. Membership functions of fuzzy controller. (a) Membership function of longitudinal vehicle speed, (b) membership function of nominal tire sideslip angle, (c) membership function of weight.

5. Simulation Verification

5.1. Validation of RBCKF Algorithm

To evaluate the comprehensive application performance in all aspects of robust bias compensation Kalman filter algorithm and fusion estimation strategy in vehicle state estimation, simulation tests and comparative verification were conducted in the Carsim-Simulink co-simulation software environment.

The first step validated the robust bias compensation Kalman filter algorithm. To reflect the estimation effect, a typical and commonly used extended Kalman filter algorithm is selected as the comparative algorithm. Meanwhile, the estimation methods considering linear tire models and nonlinear tire models were also compared. The combination of estimation algorithms used includes 'EKF+LTM', 'RBCKF+LTM', 'EKF+NLTM', and 'RBCKF+NLTM'.

Firstly, simulation tests were conducted under a sinusoidal steering maneuver, with a vehicle speed of 15 m/s and a steering wheel angle as shown in Figure 5. The contrast results of state estimation obtained by different algorithm combinations under sinusoidal steering conditions are shown in Figure 6. As shown in the figure, overall, the estimation results obtained from the combination of the four algorithms can maintain good performance under this operating maneuver. In the local magnified image, comparing the results of 'EKF+LTM' and 'RBCKF+LTM', it can be seen that when using the same LTM-based estimation model, the RBCKF algorithm has relatively better tracking and accuracy in the

estimation results compared to the EKF algorithm. Comparing the results of 'RBCKF+LTM' and 'RBCKF+NLTM', it can be seen that under the same RBCKF algorithm, there are differences in the state results based on different mechanism models due to the dynamic changes in vehicle operating conditions, states, and tire nonlinearity. When the steering wheel angle is large, the lateral angular velocity gradually becomes more intense accordingly, and the NLTM-based state estimation form has better performance. The same trend is also observed in results of the acquired θ in Figure 6. The above cases indicate that the RBCKF algorithm further optimizes the state estimation comportment compared to EKF, and LTM and NLTM have their own advantages in different vehicle states and degrees of nonlinearity.

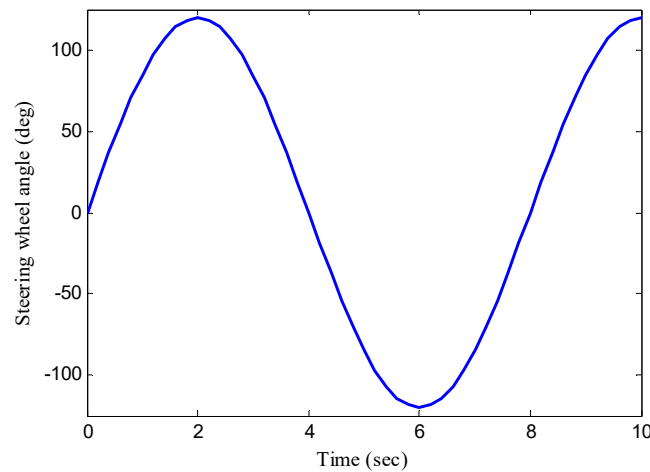


Figure 5. Steering wheel angle in sinusoidal steering maneuver.

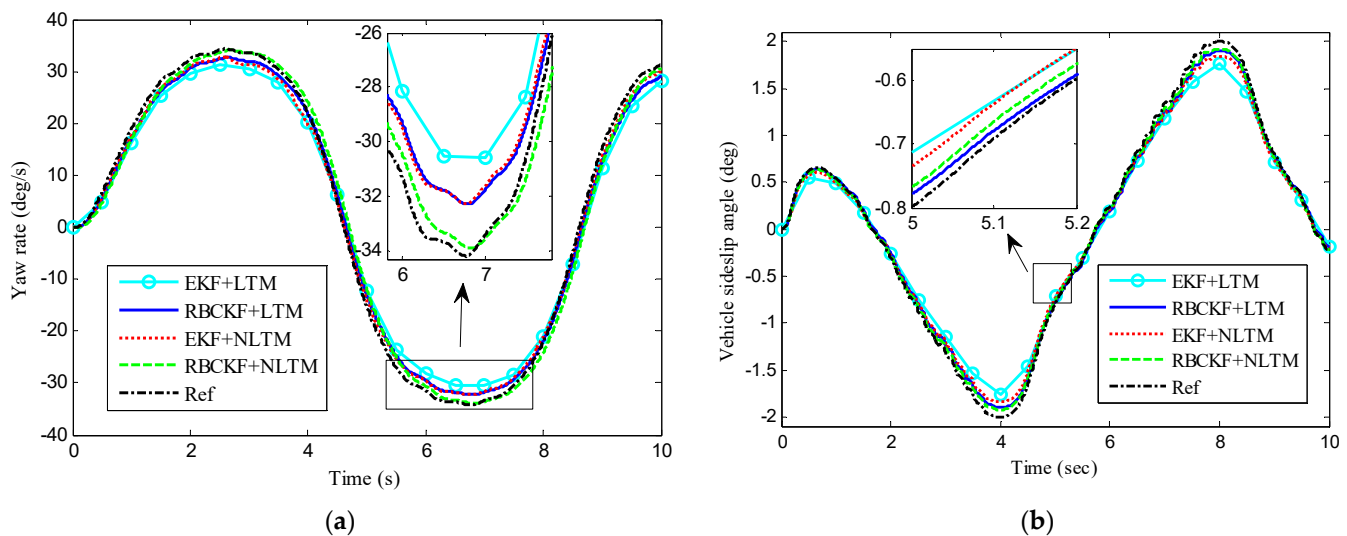


Figure 6. Results in sinusoidal steering maneuver. (a) $\dot{\varphi}$, (b) θ .

On the basis of simulation testing in the sinusoidal steering maneuver, simulation comparison testing was also conducted under a J-turn maneuver. The steering wheel angle and v used to reflect real-time transitions and serve as a reference during the J-turn test are shown in Figure 7. Through testing under variable vehicle speeds and complex steering conditions, we expect to more comprehensively demonstrate the estimation capability of the method. The state estimation results under a J-turn operating maneuver are shown in Figure 8. By observing the estimation results of different algorithm combinations in Figure 8, it is noted that in the distinction of the curves of 'EKF+LTM' and 'RBCKF+LTM', the RBCKF has relatively higher accuracy. Meanwhile, in the comparison between 'RBCKF+LTM' and 'RBCKF+NLTM', the estimation results of NLTM show higher accuracy when the yaw rate

is relatively high, while the estimation accuracy of the two modes is relatively consistent during the initial stages of vehicle starting and steering.

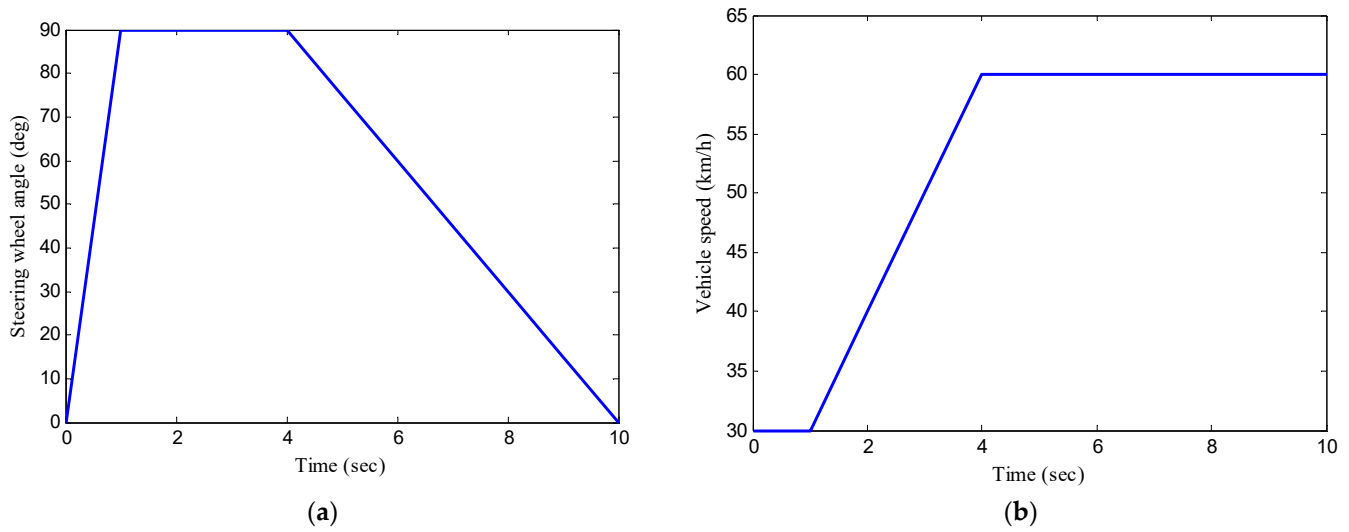


Figure 7. J-turn maneuver. (a) Steering wheel angle, (b) vehicle speed.

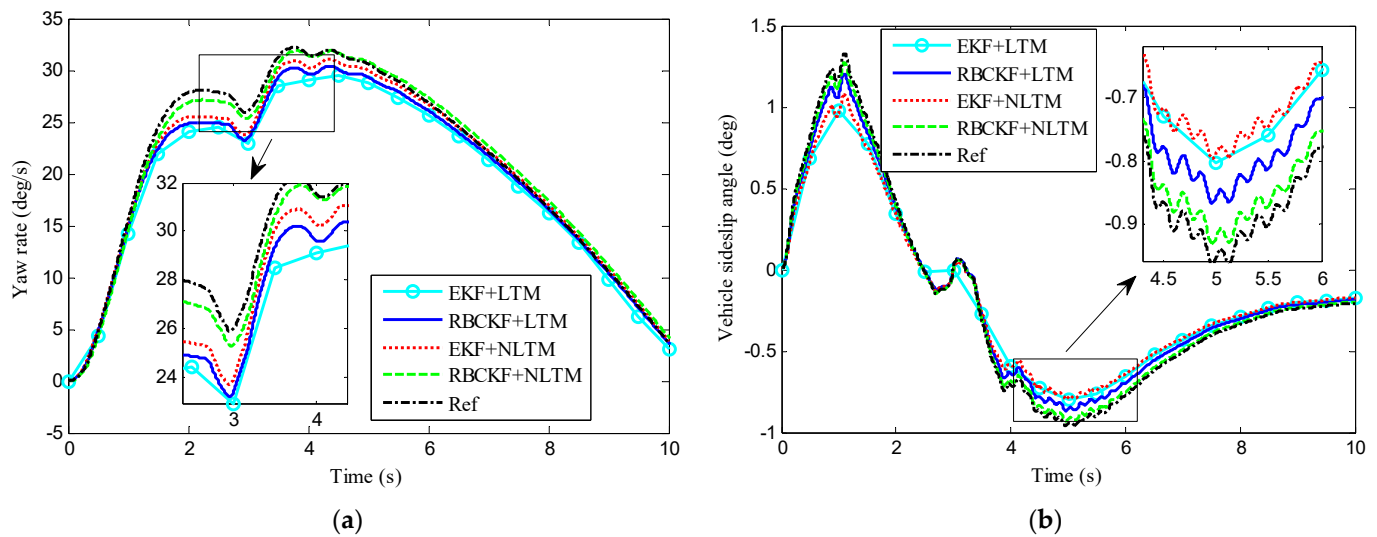


Figure 8. Results in sinusoidal steering maneuver. (a) φ , (b) θ .

In order to verify the application effect of the RBCKF algorithm and observers based on different mechanism models in vehicle state estimation, the observation errors under different estimation modes were further analyzed through data statistics. The averages and root mean square of estimation errors are used for quantitative analysis and are obtained as follows:

$$e_{AVE} = \frac{1}{N} \sum_{i=1}^N (\hat{x}_i - x_i)$$

$$e_{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{x}_i - x_i)^2}$$
(36)

where e_{AVE} is the error average value, e_{RMSE} is the error root mean square, N is the sampling amount, x_i is the referenced value, and $\hat{x}_i$ is the estimated value.

In a sinusoidal steering maneuver, the summative results of e_{AVE} and e_{RMSE} are shown in Table 3. In Table 3, a targeted comparison of the error statistics obtained from the ‘EKF’ and ‘RBCKF’ algorithms of LTM and NLTM shows that when using the same tire model estimation method, the e_{AVE} and e_{RMSE} under the RBCKF algorithm are significantly

smaller. That is to say, under the same set conditions, the RBCKF algorithm can better suppress the overall mean and variance fluctuations of errors, thereby effectively improving the estimation performance. Then, comparing the estimation errors of observers based on LTM and NLTM under the same filtering algorithm, it can be seen that under this operating condition, the overall difference in state estimation errors obtained based on the two mechanism models is not significant. Because in this simulation case of sinusoidal steering, the overall state change of the vehicle is relatively mild, both methods have high estimation accuracy. Among them, the algorithm combination of 'RBCKF+LTM' has a certain advantage in this case.

Table 3. e_{AVE} and e_{RMSE} in sinusoidal steering maneuver.

Vehicle State	Error	EKF+LTM	RBCKF+LTM	EKF+NLTM	RBCKF+NLTM
Yaw rate (deg/s)	eAVE	1.1736	0.7622	1.1771	0.8386
	eRMSE	0.5665	0.2968	0.5927	0.3019
Vehicle sideslip angle (deg)	eAVE	0.1385	0.0634	0.1409	0.0677
	eRMSE	0.3228	0.1693	0.3479	0.1725

Similarly, using the error calculation method in Equation (36), the errors in the J-turn steering manipulator can be obtained as shown in Table 4. In a J-turn steering maneuver, vehicle steering and speed changes become more complex, and tire nonlinearity increases. At this point, NLTM-based observation methods exhibit better observation advantages, resulting in relatively smaller e_{AVE} and e_{RMSE} . At the same time, the RBCKF algorithm can significantly improve estimation accuracy compared to the EKF algorithm. Overall, the combination of 'RBCKF+NLTM' still has the best comprehensive estimation effect, thus verifying the observation advantage of the proposed algorithm.

Table 4. e_{AVE} and e_{RMSE} in J-turn steering maneuver.

Vehicle State	Error	EKF+LTM	RBCKF+LTM	EKF+NLTM	RBCKF+NLTM
Yaw rate (deg/s)	eAVE	1.3975	1.0682	1.1865	0.7931
	eRMSE	0.6465	0.3580	0.4988	0.2997
Vehicle sideslip angle (deg)	eAVE	0.3381	0.2616	0.2903	0.2333
	eRMSE	0.3677	0.2596	0.3018	0.2109

5.2. Validation of Fusion Estimation Strategy

Firstly, simulation verification was conducted under a double lane change maneuver, as shown in Figure 9. In this simulation test, the v was set to 10 m/s, and the simulation results are in Figure 10. By comparing the estimations of θ considering composite-state tire models, when facing the comparatively mild steering condition of low-speed lane changing, the estimated values of θ under the three methods can relatively track the actual value well, and the overall estimation effect is good. By zooming in on the local image, it can be seen that the vehicle state observer considering the linear tire model has relatively good dynamic tracking ability, while the vehicle state observer considering the NLTM exhibits visible hysteresis and lag phenomena in amplitude. The fusion estimation result obtained by weighting is relatively closer to the actual θ , and the accurateness of state acquisition results has been further improved.

In Figure 10b, it can be seen from the comparison chart of weight coefficients and vehicle sideslip angle that, in the initial stage, the fuzzy controller chooses to distribute proportionally more weight to the LTM-based observer. This is due to the fact that at low speeds and small lane changes with a small vehicle sideslip angle, the LTM locates in the linear region, so the observer using LTM has better dynamic response effect. Then, with the dynamic change of θ , the k also changes. When the amplitude of vehicle sideslip angle increases, the weight coefficient shows a decreasing trend, but at this time, the linear tire

model-based observer still occupies a relatively large proportion in the fusion result as a whole.

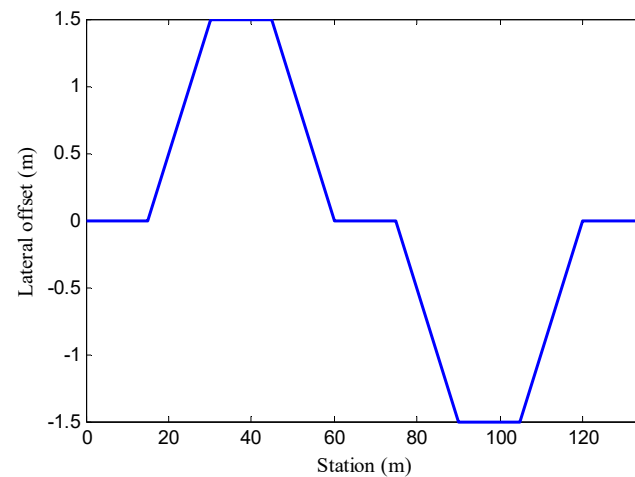


Figure 9. Double lane change maneuver.

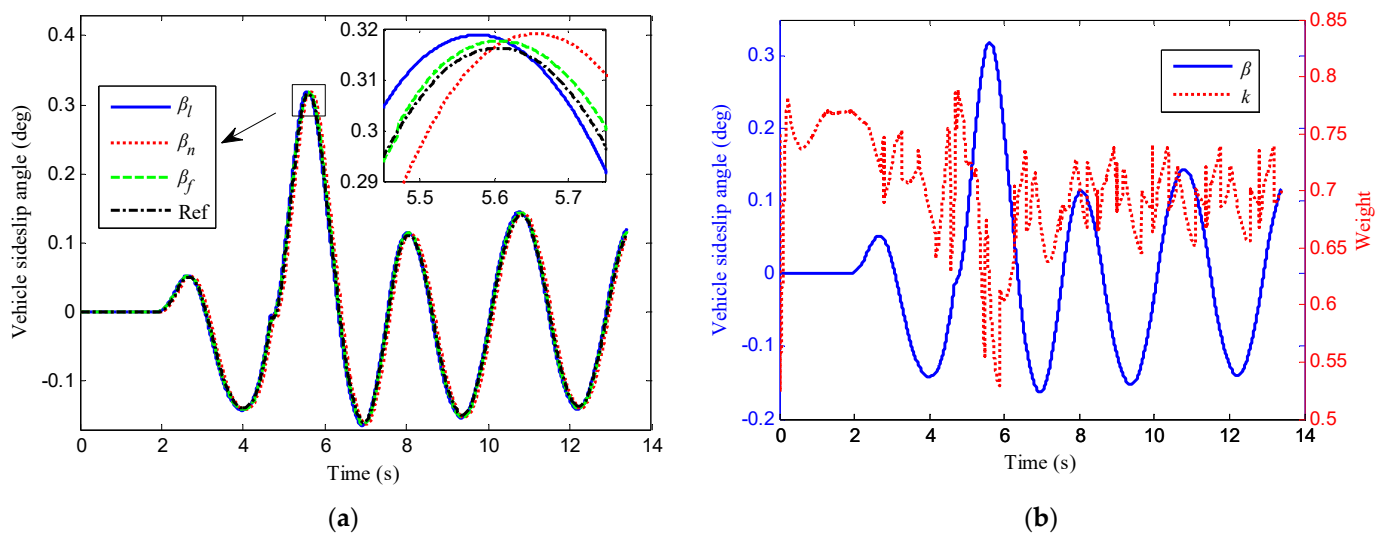


Figure 10. Estimation results in double lane change maneuver. (a) θ , (b) k .

To fully verify the conclusions from different angles and perspectives, the simulation verification was conducted under a fishhook steering maneuver. The vehicle speed under the fishhook steering maneuver is set to 25 m/s, and the steering wheel angle is shown in Figure 11. The estimated contrast curves are shown in Figure 12. Under complex conditions of severe steering, the θ significantly increases. At this time, considering the linear tire model, the estimation accuracy of the vehicle state observer decreases. However, using a fusion estimation method can significantly improve the estimation accuracy. According to the zoomed-in image, it can be seen that during the simulation period of 7.5 s to 9 s, which is the time period when the θ is small, the results obtained by the linear tire model observer, the weighted estimation results, and the actual θ basically coincide. This indicates that the accuracy of different observers varies at different degrees of skewness, further demonstrating the adaptive adjustment effect of the fusion estimation method. Based on k and θ in Figure 12b, it can be seen that under severe steering conditions, the weights matched by the linear tire model-based observer decrease, and the weight coefficients also decrease with the amplitude variation of θ .

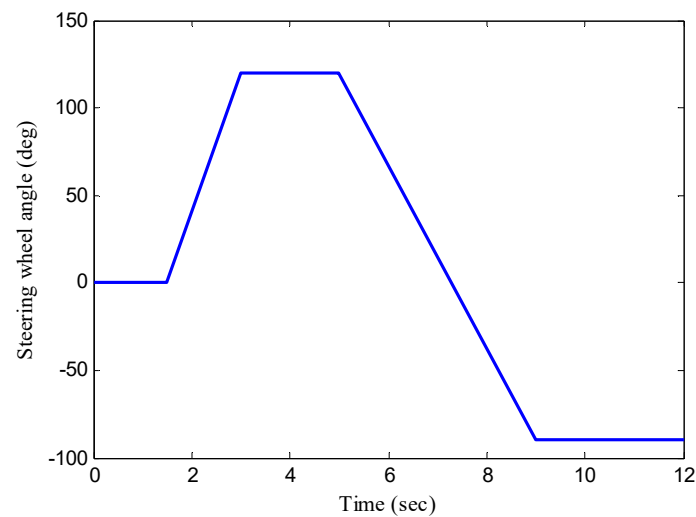


Figure 11. Fishhook steering maneuver.

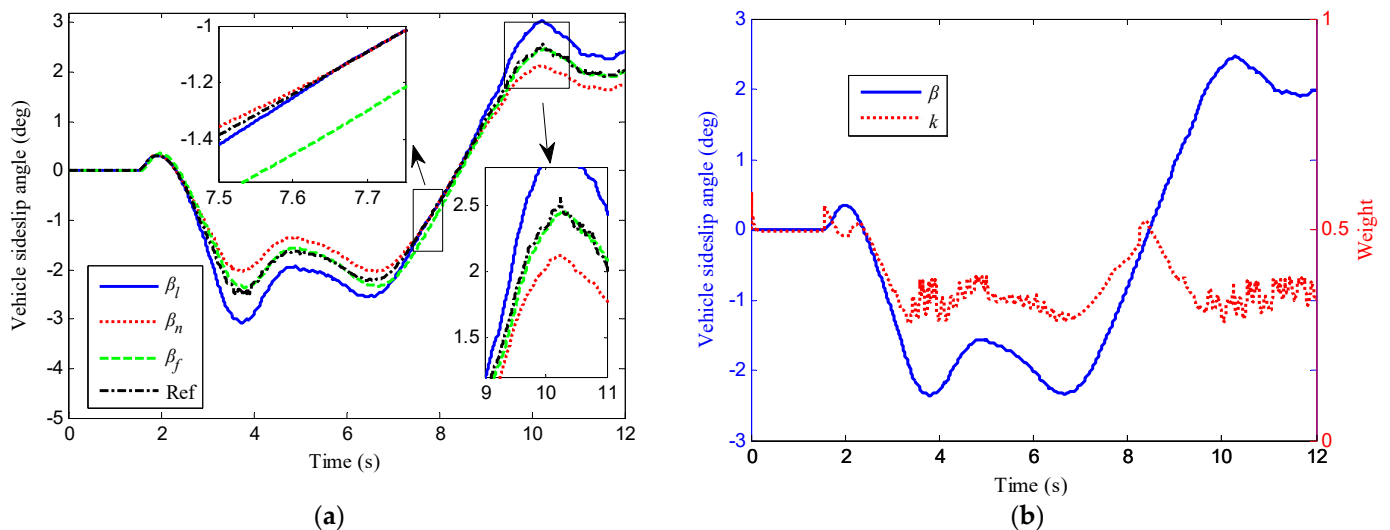


Figure 12. Estimation results in fishhook steering maneuver. (a) θ , (b) k .

Here, the error statistical method in Equation (36) is also used to intuitively reflect the effectiveness of the proposed fusion estimation strategy. The error statistical analyses of fusion estimation results in double lane change and fishhook steering maneuvers are shown in Table 5. According to Table 5, and consistent with the qualitative curve trend analysis results in the comparison chart, the fusion estimation strategy can significantly suppress estimation errors in both different simulation conditions. Compared to observation methods based on a single mechanism model, the e_{AVE} and e_{RMSE} in fusion estimation are significantly smaller.

Table 5. Error statistical analysis of fusion estimation results in double lane change and fishhook steering maneuvers.

Maneuver	Error	θ_n (LTM-Based)	θ_l (NLTM-Based)	θ_f (Fusion Estimation)
Double lane change	e_{AVE}	0.0068	0.0061	0.0034
	e_{RMSE}	0.0182	0.0159	0.0127
Fishhook steering	e_{AVE}	0.1912	0.1753	0.0076
	e_{RMSE}	0.0754	0.0811	0.0535

6. Conclusions

To improve the accuracy and adaptability to different state-transition scenarios and complex disturbances of a vehicle sideslip angle estimation system, a dynamic weight fusion estimation method for vehicle sideslip angle based on RBCKF and composite mechanism model-based observer is proposed. In combination with the RBCKF, vehicle state observers considering LTM and NLTM were designed to achieve dynamic estimation under different operating conditions, vehicle driving states, and degrees of nonlinearity. Then, considering the advantages and applicable scenarios of state observers under different mechanism models, an adaptive fuzzy weight was designed to dynamically adjust the coefficients of the two observers according to vehicle speed and degree of nonlinearity of tires, in order to achieve the optimal estimation effect. The RBCKF algorithm effectively optimized the state estimation accuracy. The results obtained from the fusion estimation strategy have higher estimation accuracy and dynamic tracking ability, and its adaptability to multiple operating conditions is also significantly improved.

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Nomenclature

EKF	extended Kalman filtering
RBCKF	robust bias compensation Kalman filter
LTM	linear tire model
NLTM	nonlinear tire model
v	longitudinal vehicle speed
u	lateral vehicle speed
φ	vehicle yaw rate
m	vehicle mass
I_z	vehicle moment of inertia
D_f/D_r	distances from the vehicle's mass center to the front/rear axles
T_{yf}/T_{yr}	lateral forces of the front/rear axle tires
λ_f/λ_r	lateral stiffness of the front/rear tires
θ_f/θ_r	lateral sideslip angles of front/rear tires
θ_n	nominal tire sideslip angle
δ_f	front-wheel steering angle
θ	vehicle sideslip angle
θ_n	vehicle sideslip angle of NLTM-based observer
θ_l	vehicle sideslip angle of LTM-based observer
θ_f	Fusion estimation result of vehicle sideslip angle
a_x/a_y	longitudinal/lateral accelerations of vehicle
κ_B	stiffness coefficient
κ_C	curve shape coefficient
κ_D	peak coefficient
κ_E	curve curvature coefficient
$F_{zfl}/F_{zfr}/F_{zrl}/F_{zrr}$	vertical tire forces of the left-front/right-front/left-rear/right-rear tires
h	height of vehicle gravity center
g	gravity acceleration

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