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Optimal Pricing in a Rented 5G Infrastructure Scenario with Sticky Customers

Marta Flamini [1,†] and Maurizio Naldi [2,*,†]

1 Faculty of Engineering, International Telematic University UNINETTUNO, 00186 Rome, Italy
2 Department of Law, Economics, Politics, and Modern Languages, LUMSA University, 00192 Rome, Italy
* Correspondence: m.naldi@lumsa.it
† These authors contributed equally to this work.

Abstract: The ongoing deployment of 5G is accompanied by architecture and pricing decisions. Network sharing is a critical feature, allowing operators to reduce their costs, but introducing a mixed partnering/competition situation, where the infrastructure owner, renting out their infrastructure to virtual operators (who act as customers), also provides services to end customers, competing with virtual operators. Pricing is the leverage through which an optimal balance between the two roles is accomplished. However, pricing may not be the only variable affecting customers' choice, which may prefer (stick to) one operator for several reasons. In this paper, we formulate a game model to analyse the optimal pricing decisions for operators in the presence of such sticky behaviour of customers. After concluding that the game does not allow for a Nash equilibrium, we consider a case when one of the parties (the infrastructure owner, the virtual operators, or the regulator) is responsible for setting prices and analyse how operators' profits are impacted when price-setting powers are shifted among the parties. The scenario where the regulator sets prices leads to the lowest profits for the operators, even lower than when competitors set prices.

Keywords: mobile networks; 5G networks; competition; virtual operators; pricing

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1. Introduction

After its first steps in the previous decade and standardization efforts [1], the diffusion of 5G networks is continuing worldwide. Though research is moving towards the definition of the next standard, i.e., 6G [2], and other architectures are in place for guaranteeing an adequate QoS for specialized services [3,4], the diffusion of 5G networks is still far from reaching a maturity stage. An example of a coverage map providing a detailed view of the availability of 5G networks in the USA is shown in https://www.digitaltrends.com/mobile/5g-availability-map/ (accessed on 19 February 2023). Tools have also been developed to help small and medium operators plan their infrastructure deployment [5].

An important feature in the system architecture of 5G networks is network sharing. Network sharing is the allowance of multiple mobile phone operators to use the same infrastructure, such as cell towers and base stations, to provide mobile network services to their customers. This can be performed through a variety of arrangements, such as leasing capacity on existing networks, sharing the costs of building new networks, or operating a joint venture (similar savings are obtained by merging infrastructures [6]). Though network sharing emerged as a novel concept with 3G networks [7], it has become a fundamental feature in 5G networks [8,9]. The reasons for this include the huge traffic increase faced by customers (due to content-rich applications, an explosion in video traffic, and upcoming vertical market services in sectors such as automotive and e-health) [10] and the difficulty of managing the costs and time required for the growing amount of traffic. A challenge for operators is to accommodate this traffic increase without a significant increase in costs, or at least to share the risks associated with deploying a massive network infrastructure in advance of the actual arrival of customers (it has been shown that some access sites carry a very low portion of traffic that does not justify the full investment [11]). The goal of network sharing is to increase efficiency and reduce costs for the operators while also improving customer coverage and capacity. It has been claimed that network sharing can save up to 20% of the operational costs and even half of the infrastructure costs of passive radio access network (RAN) components [8]. The possibility of partially or fully using the infrastructure of other operators rather than deploying one's own has led to the birth of virtual operators [12], which can carve a profitable business model by bearing null or reduced capital and operational expenses [13].

However, the use of the same infrastructure by different operators poses a competition problem, because the infrastructure owner could serve end customers at the same time, playing the double role of provider/partner and competitor towards the renting (virtual) operators. In order for this business scenario to be sustainable, prices and the subsequent demand of customers for the services of the two groups of operators must be enough to allow for profitable operations for both parties.

A significant body of literature has dealt with the problem of correct/optimal prices offered by a single operator. Only a few works (considered in detail in Section 2) have considered the case generated by network sharing of a double-sided market, where virtual operators are customers of the infrastructure provider and providers of end customers at the same time. Furthermore, very few works have dealt with the more specific scenario where the infrastructure provider also serves end customers, thus creating the competition mentioned above.

In addition, the emphasis has been on prices, which have been considered to be the only leverage (beyond service quality in some cases) for attracting customers. However, despite higher prices, several factors may push a customer towards one of the providers, ranging from brand reputation to service coverage and ancillary services. Unfortunately, the literature has so far avoided the mathematical consideration of factors that act concurrently with and beyond prices.

In this paper, we wish to fill that research gap by introducing a demand function that allows us to model customers' preferences and orientate their decisions in addition to prices. We call this function the stickiness function, because it models how relatively difficult it may be for an operator to attract a customer who is sticking to another operator that he/she prefers. We consider the less-examined competition scenarios between the infrastructure provider and the virtual operators and identify the optimal pricing choices under different price-setting attributions.

Our main contributions are the following:

- We introduce a demand function that takes into account factors that move the balance towards one of the operators other than pricing;
- We formulate a game model for the case where the infrastructure provider and virtual mobile operators compete for the same end customers;
- We show how shifting the power of setting prices from operators to a regulator changes profits and end prices and also benefits end customers.

The paper is organized as follows. We review the literature on two-sided markets in 5G networks in Section 2. The 5G infrastructure and our service model are described in Section 3. We go into the details of the service economics (demand, revenues, costs, and profits) in Section 4. We formulate a game between the infrastructure owner and the virtual operators in Section 5, and show the results of a sample analysis of profits and prices in Section 6.

A glossary of the terms and the pertaining notations used in this paper are shown in Table A1 in Appendix A.

2. Literature Review

Many papers have been devoted to the analysis of economic models for 5G networks. In most cases, the models have been employed to drive resource management, e.g., spectrum allocation or interference and power management. An excellent survey of those models is shown in [14]. Examples of applications in data markets are provided in [15–17]. In this section, we focus instead on pricing models where two-sided markets emerge, which is the specific topic of our paper. For each paper described in this survey, we will outline the major difference between in and our paper.

We first consider papers where there is no direct competition between the two sides of the two-sided market. The roles of the two sides may vary. In some cases, they may be the owner of the infrastructure and the renters. In other cases, they may be the spectrum licensee and the spectrum leasers. In any case, the absence of direct competition means that end users are solely the customer of one of the parties, so the scenarios considered in these papers differ from ours, at least in this basic aspect, i.e., not considering the competition between infrastructure providers and virtual operators.

In [18], a game model was proposed, where wireless resource providers (WRPs), mobile virtual network operators (MVNOs), and users all take part. The set of stakeholders is the same as in our model. WRPs act as providers of RF slices for MVNOs, which in turn sublease their resources to end users. MVNOs act as two-sided players, being customers for WRPs and providers for end users (as in our model). However, WRPs do not directly serve end users and do not compete with MVNOs for those users (contrary to what happens in our model). The game is arranged according to three layers, with WRPs announcing their prices to MVNOs in layer 1 to maximize their own utility, MVNOs announcing their prices to end users in layer 2 to maximize their own utility, and end users optimising their data rate in layer 3. The game model obeys the Stackelberg-like leader–follower scheme.

Another scenario considering a monopolistic infrastructure provider (a small cell provider) and many virtual network service providers was considered in [19]. However, the competition occurs just among virtual providers who compete for end users based on their prices and services and aim to maximize their profits. The authors employed a hierarchical game model. In their model, end users select services, and the operators engage in an upper-level game based on their leasing and pricing strategies to satisfy the service requests of end users. In the lower level, a Stackelberg game takes place between users and virtual service providers, who play the role of Stackelberg leaders.

A further layering of operators was considered in [20], where an infrastructure provider rents its infrastructure to a mobile virtual network operator, which in turn virtualizes the network resources, divides them into slices, and rents them to edge and cloud service providers. They then considered a B2B market made of a single infrastructure provider and a single virtual operator. The problem they dealt with is the maximisation of the social welfare of the ecosystem formed by the MVNO and all the edge and cloud providers.

Another model considering a three-level hierarchy was proposed in [18], where a three-layer game is adopted based on the interactions between wireless resource providers, mobile virtual network operators, and their subscribed wireless users. The wireless resource providers sublease their wireless resources to virtual operators, which in turn try to attract more end-users. The resource provider wishes to maximize its utility. The virtual operators wish to maximize their revenues. Finally, end users wish to maximize their data rates by meeting the imposed quality of service requirements and budget constraints.

A different view of the resources to be shared was considered in [21], where the authors dealt with caching space, which becomes a limited resource due to the exponential growth of mobile data and video consumption. They studied a multi-Stackelberg game between multiple mobile network operators (MNOs) (which act as leaders in the game) and several content providers (which act as followers). In the first step, the multi-leader Stackelberg game leads to definition of the prices that MNOs charge content providers. In the second step, a multi-follower Cournot game between content providers leads to definition of the content cached at the base stations of the mobile network operators. The strategic leverage is the price that mobile network operators set and the quantity of content that each content provider will cache.

Instead, a scenario where two virtual network operators compete, sharing the infrastructure provided by a single provider and offering service to end IoT users, was studied in [22]. The authors proposed a two-stage Stackelberg game, where the two virtual operators first try to maximize their profits by setting the optimal network service prices, and then IoT users make select the network service based on performance and prices.

Though network sharing has been depicted as a sharp move with respect to fully replicated networks, several shades are possible. Different degrees of sharing were explored in [23], and they were compared with the aim of optimizing efficiency.

Several papers have focused on costs only, which is, however, the strongest spur towards adopting a network-sharing approach. An evaluation of architecture costs (capital and operational) was conducted in [24]. A detailed analysis of all the components and an evaluation of the net present values of capital expenses was provided in [25]. In [26], a 5G smart light pole network was considered, and a model was proposed to estimate the total deployment cost, including the capital and operational expenses.

We now turn to consideration of papers that deal with a scenario similar to ours, where MNOs and MVNOs compete for end customers.

A similar approach to ours was employed in [27], where a small cell operator leases the spectrum from the macro cell operator. The scenario is monopolistic at both levels (instead, we consider competition between virtual service providers). The users' utility function is sensitive to both the service price and the expected throughput (i.e., the network performance), whereas we include several aspects—not just performance—in the demand function.

In [28], direct competition between operators was considered in a strategic business model where both the infrastructure owner and the virtual operator compete for end customers by offering different prices. Instead of considering the bandwidth provided to customers, they considered the quality of service. In addition to that, their model also differs from ours in that they did not consider costs and did not include the stickiness of customers in the demand function.

A different kind of B2B service relationship was considered in [29], where a backup reservation scheme to cover for excess-demand needs was considered in place of the fixed provisioning agreement considered in our paper. Although most models consider the demand posed by users, in [30] the demand was represented by slice tenants, which act as agents participating in auctions. Their bids are updated after each auction round to maximize their payoff function. A reversed scenario, where the mobile network operator acts as a customer of local (micro) operators owning the infrastructure, was examined in [31], where the impact of competition on the wholesale prices paid by the MNO to the micro-operator was examined.

Bandwidth, in addition to pricing, was considered as the strategic leverage in [32], where a Stackelberg game framework was adopted for spectrum pricing and spectrum sharing, with bandwidth pricing control for the leader (the spectrum licensee) and bandwidth optimization for the followers (i.e., mobile network operators).

Volume, rather than bandwidth, was considered as the strategic leverage in [33], where a Cournot equilibrium was found for the competition between owner and renter. The optimal price was shown to be a weighted average of the customer's willingness to pay and the operators' costs.

In summary, the literature on the direct competition between MNOs and MVNOs for end customers has considered only the quality of service as a parameter in addition to price to move customers' choices. No studies have considered different factors that may influence these choices. A research gap exists in the inclusion of these factors. Our study aims to include these factors by making the demand function only partially respond to prices, i.e., removing the winner-takes-all approach in the literature where customers all prefer the operator offering the lowest price.

3. The 5G Infrastructure: Network Operators, Pricing Plans, and Customer Preferences

Our analysis of pricing strategies has been conceived in the context of 5G mobile networks, where the phenomenon of virtual operators has grown. In this section, we provide a brief description of 5G networks and the network model we employ in our study.

The 5G network architecture is composed of several key components, including [34,35]:

- The radio access network (RAN), which is responsible for connecting devices to the 5G network and providing wireless communication;
- The core network, which provides the routing and switching functionality needed to connect devices to the internet and other networks;
- The transport network, which provides the physical infrastructure needed to connect the RAN and core network;
- The management and orchestration layer, which is responsible for configuring and controlling the network elements and services;
- The security layer, which provides end-to-end security for the network and its users.

Here we consider a scenario where the infrastructure, namely the radio access portion, is owned by a single operator (which we will refer to as the owner or MNO (mobile network operator) in the following). However, that infrastructure is rented out to other operators which we will refer to as the renters or MVNOs (mobile virtual network operators) in the following.

Mobile network operators own and operate mobile networks. They are responsible for building and maintaining the infrastructure required for mobile communication services, such as cell towers and base stations. They also typically own the spectrum licenses needed to provide these services. Here, we imagine a scenario where we have a single MNO.

Mobile virtual network operators (MVNOs), instead, do not own their own mobile networks but instead purchase network access from MNOs and resell it to their customers. MVNOs typically focus on specific market segments or niches and may offer specialized services or pricing plans.

These operators can offer a variety of pricing plans to their customers. MNOs and MVNOs typically do not differ greatly in their offer portfolio. Some possible pricing plans include:

- Pay-as-you-go plans, which allow customers to pay for only the minutes, texts, and data that they use; this plan can be a good option for customers who do not use their phone frequently or who want to avoid the commitment of a long-term contract;
- Prepaid plans, which require customers to pay in advance for a set amount of minutes, texts, and data and to add more credit to continue using the service when they have used up their prepaid balance;
- Postpaid plans, which allow customers to use the service and pay for it at the end of the month and can be a good option for customers who use their phone frequently and want the convenience of not having to constantly add credit to their account;
- Monthly plans, which typically offer a set amount of minutes, texts, and data for a fixed monthly fee and can be a good option for customers who use their phone frequently and want a predictable monthly bill;
- Family plans, which allow multiple people on the same account to share minutes, texts, and data, and can be a good option for families or groups of friends who want to save money on their mobile service.

MNOs typically charge virtual operators for the use of their network infrastructure and services in one of the following ways:

- Wholesale rate on a per-minute, per-text, or per-data-unit basis; this rate is typically lower than the retail rate that MNOs charge their own customers;
- Flat rate for a set amount of minutes, texts, and data, which can be a good option for MVNOs that want a predictable monthly bill and want to offer fixed plans to their customers;
- Reseller agreements, where the MVNOs resell the MNOs service with a markup;

- Revenue sharing agreements, where the MVNOs pay a percentage of their revenue to the MNOs in exchange for network access.

In our case, we have opted to study a typical situation where we have a single MNO and N_R MVNOs, where MNOs and MVNOS compete for end customers and charge them a flat-rate plan (which encompasses the prepaid, postpaid, and monthly plans described above), where customers pay a fixed amount (respectively, p_{MNO} for the MNO and $p^{(i)}_{\text{MVNO}}$ for the i-th MVNO) for each period of time. On the other hand, the MNO charges a flat rate γB with a fixed amount B of bandwidth allotted to each customer. The revenues coming from end customers are then proportional to the number of customers, whereas the revenues of the MNO coming from MVNOs are proportional to the number of MVNO's customers and the bandwidth allotted to them. The relationship between customers, the MNO, and MVNOs is illustrated in Figure 1.

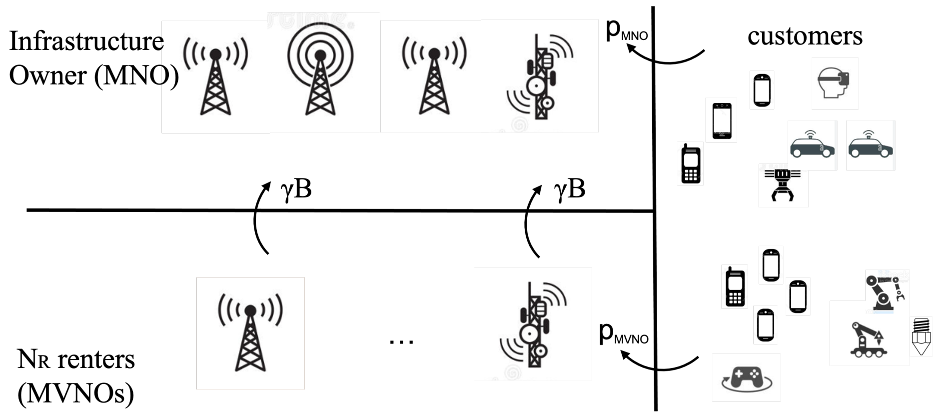

Figure 1. Relationships between customers, the MNO, and MVNOs.

We also assume that customers prefer the MNO, so they may wish to subscribe to its services though its prices are higher. We call this phenomenon stickiness and state the proper demand function in Section 4.2. There are several reasons why mobile customers may prefer an operator even though its prices are higher:

- Network coverage and reliability, because customers may prefer an operator with better network coverage and reliability; a good network coverage allows the customers to have seamless access to services and to be able to connect in more places;
- Quality of service, because customers may prefer an operator that offers a higher quality of service, such as faster internet speeds or better customer service;
- Brand reputation, because customers may prefer an operator that has a good reputation and a long history of providing quality service;
- Offered services, because customers may prefer an operator that offers additional services such as international roaming plans, more flexible plans, more family plans, more roaming plans, and more data plans;
- Device availability, because customers may prefer an operator that offers the latest and most desirable devices for purchase or on contract;
- Bundles and discounts, because customers may prefer an operator that offers bundled services such as TV, internet, and home phone services or offers discounts for combining services;
- Loyalty programs, because customers may prefer an operator that rewards loyal customers with discounts or other benefits.

In the following, we do not provide a detailed analysis of the reasons that may lead a customer to choose the MNO, notwithstanding its (possibly) higher prices. Rather, we consider a demand function that may incorporate any of these factors.

4. Operators' Costs, Revenues, and Profits

Our economic analysis aiming to search for optimal pricing must rely on a detailed description of the economics around 5G service provisioning and the relationship between MNOs and MVNOs. In this section, we provide models for the costs incurred and the revenues obtained by each category of providers. Finally, we combine cost and revenue information to derive their profits.

4.1. Cost Models

The structure of costs is different for the MNO offering its infrastructure for rent and the MVNOs renting it. The expectations for the MVNOs are to have a slimmer set of costs (see the considerations on the strategic advantage of MVNOs in [36]). These lower costs allow their business to be profitable though they cannot expect to set prices as high as those of MNOs. In fact, the latter category is perceived as more reliable or having a stronger brand image, as suggested in [37]. Hence, the structure of costs is essential to set prices for both types of operators. In this section, we describe the cost models separately for the MNO and the MVNOs.

In describing the models, we rely on the distinction between CAPEX (capital expenditures) and OPEX (operational expenditures). CAPEXs include all the expenses made by the company into fixed, long-duration assets. In telecommunications, these include the expenses incurred to purchase, e.g., routers, optical fibres, antennas, and towers. On the other hand, OPEXs include the recurring expenses incurred during day-to-day activities and, more generally, all the expenses that are consumed within a year. OPEXs include consumables and renting fees. The distinction between CAPEX and OPEX is well established in the telco sector, as adopted, e.g., by [38] for LTE (long-term evolution) networks, Ref. [39] for access networks, and by [40–42] for the core network.

However, CAPEX and OPEX refer to different timescales, whereas OPEXs exist during a one-year horizon, CAPEXs extend their effect over several years. In order to arrive at a common time frame reference in the budget equation for the operators, we distribute CAPEXs over the lifetime of the assets. The lifetime can be generally deemed as dictated by the accounting rules of the company for their amortization and recording in ledgers. Here we assume that such distribution over time has been accomplished through a straight-line method (i.e., a uniform distribution of costs over the asset's lifetime) [43] so that we can obtain a yearly CAPEX to be added to the OPEX to arrive at the overall expenses over a year.

Hereafter, we make the following assumptions to define our cost model:

- The CAPEXs of the MNO are linearly related to the bandwidth;
- OPEXs are proportional to the CAPEXs;
- The CAPEXs and OPEXs of MVNOs are a fraction of those pertaining to the MNO;
- Renting is priced linearly in the bandwidth so that the renting fee is expressed per unit bandwidth.

The choice of a linear model for the relationship between CAPEX and bandwidth is based on the data provided by [44].

The assumption of proportionality between CAPEX and OPEX allows us to consider CAPEX through a single proportionality factor. Though in principle OPEX could be determined through a detailed bottom-up approach (see, e.g., the procedure outlined by [45]), OPEXs appear to be proportional to CAPEX in many real cases. For example, Ref. [45] estimate OPEX costs for equipment installation to amount to 30% of the CAPEX costs, and their final estimation for the OPEX/CAPEX ratio is 75%. Though the exact value of the ratio may depend on the operator's business model and the specific network, considering OPEX to be proportional to CAPEX is a common assumption, adopted, e.g., in [46] for virtual networks, where the OPEX/CAPEX ratio appears to depend on the virtual network function utilization rate, or in [47] for data centres, where the OPEX/CAPEX ratio is seen to move in the 31–61% range.

We can now describe the overall cost function for the MNO and the MVNOs.

For the MNO we have

$$C_{\text{MNO}} = (n_{\text{MNO}} + \sum_{i=1}^{N_R} n_{\text{MVNO}}^{(i)})(C_0 + mB)(1 + \alpha), \quad (1)$$

where n_{MNO} and $n_{\text{MVNO}}^{(i)}$ are, respectively, the number of customers of the MNO and the number of customers of the i-th MVNO, C_0 is the zero-consumption fee in the linear cost model, B is the bandwidth, m is the marginal cost of bandwidth, and α is the OPEX/CAPEX ratio.

On the other hand, the MVNOs pay the MNO a renting fee γ for each bandwidth unit, but bear some OPEX and CAPEX anyway, which are, however, only a fraction of those borne by the MNO. The overall cost function of the generic MVNO is

$$\begin{aligned} C_{\text{MVNO}}^{(i)} &= \beta_i n_{\text{MVNO}}^{(i)}(C_0 + mB) + \eta_i \alpha n_{\text{MVNO}}^{(i)}(C_0 + mB) + \gamma n_{\text{MVNO}}^{(i)} B \\ &= n_{\text{MVNO}}^{(i)}[(\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B], \end{aligned} \quad (2)$$

where $\beta_i < 1$ and $\eta_i < 1$ determine how much CAPEX and OPEX the i-th MVNO bears.

4.2. The Demand Function

In this section, we describe the demand function, which provides us with the number of customers joining one of the operators.

We denote the overall number of potential customers (the basin) as n. We need to obtain the number of actual customers of each operator. We obtain that value from the overall number of potential customers through a chain of factors. First, we obtain the number of actual customers through the factor f_{tot} (which can be obviously defined as the ratio of the number of actual customers to the number of potential customers); by actual customers, we mean customers joining either the MNO or one of the MVNOs. Then, we subdivide the actual customers among the operators through an additional factor f_x, where x may take one of the following values: MNO for the infrastructure owner (the MNO) or r_i for the i-th renter (MVNO). The number of customers joining the operator x is finally given by the following composition

$$n_x = n \cdot f_{\text{tot}} \cdot f_x. \quad (3)$$

We can now see how to derive both factors f_{tot} and f_x.

We start with the proportion f_{tot} of actual customers. Here, we postulate that it is an exponential function of the average price $\overline{p}$, which is simply the arithmetic average of the prices imposed by the MNO (p_{MNO}) and the MVNOs ($p_{\text{MVNO}}^{(i)}$ for the i-th MVNO):

$$\overline{p} = \frac{p_{\text{MNO}} + \sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)}}{1 + N_R}. \quad (4)$$

It is to be noted that MVNOs can impose different prices in our model. As to the fraction of actually subscribing customers, we employ an exponential demand function. The exponential function is employed in many contexts [48] and has been adopted for communications services, e.g., in [49–51]. In our case, the price is considered as the average price offered by operators:

$$f_{\text{tot}} = e^{-z\overline{p}} \quad (5)$$

In this equation, the coefficient $z > 0$ is the absolute value of the quasi-elasticity of the overall demand function with respect to the average price. In fact, the quasi-elasticity with respect to the average price $\overline{p}$ (we employ the quasi-elasticity instead of the elasticity because the demand function outputs a value in the $(0, 1]$ range) is

$$\epsilon = \frac{1}{f_{\text{tot}}} \frac{\partial f_{\text{tot}}}{\partial \overline{p}} = e^{z\overline{p}} e^{-z\overline{p}}(-z) = -z. \quad (6)$$

We can now consider the function f_x, which determines how customers are divided between the operators. We call this function the repartition function. As hinted in the Introduction, customers may have reasons for selecting either operator other than a lower price. The reasons may range from the preferred operator holding a more established brand to its having a better reputation. In fact, Turel and Serenko have shown that customer loyalty implies an increased likelihood of repurchasing services from the same provider and lower price sensitivity [52]. In turn, both Choi et al. and Gerpott et al. have shown that customer satisfaction is a key determinant of loyalty [53,54]. Because Calvo-Porral et al. identified service value and corporate image as antecedents of customer satisfaction [55], we can easily conclude that those two factors at least may drive customers' preferences in addition to sheer pricing. The repartition function must then consider price as a relative variable rather than as on that determining customers' preferences, where the latter choice would lead the customer to always choose the operator proposing the lowest price. In this paper, we call a customer who prefers operator A even though operator B is offering lower prices a sticky customer, because they tend to choose an operator despite lower-priced offers by other operators.

Stickiness is a phenomenon different from, yet similar to, lock-in, where customers are bound to an operator for technological or contractual reasons. In the case of lock-in, the customer is currently served by an operator and experiences strong barriers to leaving that operator, whereas we are considering a customer who has to make the initial choice between two (or more) operators and is therefore relatively free to choose. Lock-in has been extensively studied as a general phenomenon of markets and has been reported for telecom services as well. It has been shown to be quite relevant both in broadband services, as shown by [56], and in mobile networks (see the paper by [57]). In some cases, the lock-in effect is incorporated in the contract between the telecom operator and the customer, as considered by [58].

A related phenomenon is churn, where operators suffer from many customer abandonments. Again, in churn, customers leave their present operator rather than choosing one from a set of possible choices, as in our case. Most studies on churn have concentrated on using machine learning techniques, where a large number of features are included at the same time. Typically, those features are mostly related to consumption characteristics as they are extracted from the telecom operator's logs and other information available to the operator (such as the pricing plan). For example, Lu et al. used logistic regression and boosting to predict churn, starting with 700+ features and reducing them to 21 [59]. However, a few studies have highlighted that price is just one of the variables affecting churn. Mahajan et al. classified those variables into two categories: service quality and brand image, with pricing falling into the service quality class [60]. However, brand image includes factors such as the operator's perceived fairness, friendliness, and innovativeness. Furthermore, Dasgupta et al. have highlighted the role of social ties in churn: people whose friends have left their operator tend to do likewise [61].

Going back to stickiness, we can model stickiness through an aptly defined repartition function. In the case of two operators, that function tells us how the aggregate set of customers is divided among operators as a function of their price ratio. In the absence of stickiness, we expect customers to choose either operator with the same probability (50%) when their prices are equal. Of course, the stickiness function becomes progressively unbalanced when the price of either operator tends to zero. The competitor asking for zero price would take all of the market. However, for any case where the price is different from zero, though extremely small, there will always be a non-zero probability that the customer opts for the higher-priced operator. Here, we take the infrastructure owner (the MNO) as the reference, i.e., the operator preferred by customers, that to which customers tend to stick to. For simplicity, we aggregate MVNOs by considering their average price $\overline{p}_{\text{MVNO}}$:

$$\overline{p}_{\text{MVNO}} = \frac{\sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)}}{N_R}. \tag{7}$$

The repartition function f_{MNO} of the MNO provides the average fraction of customers choosing the MNO as a function of the $p_{\text{MNO}}/\overline{p}_{\text{MVNO}}$ ratio. It must exhibit the following properties:

- $f_{\text{MNO}} = 1$ when $p_{\text{MNO}} = 0$;
- $f_{\text{MNO}} = 0$ when all the renters give their service for free, i.e., $p^{(i)}_{\text{MVNO}} = 0$, $i = 1, 2, \ldots, N_{\text{R}}$, so that $\overline{p}_{\text{MVNO}} = 0$.

Due to these properties, we adopt the following repartition function for the owner

$$f_{\text{MNO}} = e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}, \tag{8}$$

where ξ is the stickiness factor. When we have a single renter and there is no stickiness effect, if the MVNO offers the same price as the MNO, the repartition function divides customers equally between the MNO and the MVNO. In that case, solving Equation (8) for the stickiness factor, we obtain $\xi = \ln 2$. In fact, when $p_{\text{MNO}} = \overline{p}_{\text{MVNO}}$, we should obtain $f_{\text{MNO}} = 1/2$. By imposing that condition in Equation (8), we obtain

$$e^{-\xi} = \frac{1}{2} \to -\xi = \ln \frac{1}{2} \to \xi = \ln 2. \tag{9}$$

A stickiness factor deviating from that value signals an imbalance towards one of the two operators. When we have $N_R > 1$ MVNOs, we can similarly find the correct value of ξ when there is no stickiness effect. If all operators set the same price, the repartition function should assign the MNO a fraction $\frac{1}{N_R+1}$ of the customers. In that case, solving Equation (8) provides us with the stickiness factor $\xi = \ln(N_R + 1)$, which is also valid for a single renter. We can call this value ξ_0. When $\xi > \xi_0$, the balance shifts towards the MVNOs, which obtain a larger fraction of customers than the MNO even offering the same price.

The repartition function for the generic i-th renter can be derived by imposing the normalizing condition that

$$f_{\text{MNO}} + \sum_{i=1}^{N_R} f^{(i)}_{\text{MVNO}} = 1 \to \sum_{i=1}^{N_R} f^{(i)}_{\text{MVNO}} = 1 - f_{\text{MNO}}. \tag{10}$$

In order to obtain the share of the generic i-th renter, we can assume that shares distribute linearly with prices so that the operator offering a lower price obtains a higher share. The linear function describing the share of the i-th renter is

$$\begin{aligned} f^{(i)}_{\text{MVNO}} = a_i \sum_{j=1}^{N_R} f^{(j)}_{\text{MVNO}} &= \left[1 - (N_R - 1)\frac{p^{(i)}_{\text{MVNO}}}{\sum_{j=1}^{N_R} p^{(j)}_{\text{MVNO}}}\right] \sum_{j=1}^{N_R} f^{(j)}_{\text{MVNO}} \\ &= \left[1 - (N_R - 1)\frac{p^{(i)}_{\text{MVNO}}}{N_R \overline{p}_{\text{MVNO}}}\right](1 - f_{\text{MNO}}) \\ &= \left(1 - e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}\right)\left[1 - (N_R - 1)\frac{p^{(i)}_{\text{MVNO}}}{N_R \overline{p}_{\text{MVNO}}}\right]. \end{aligned} \tag{11}$$

4.3. Customers

By using the demand functions defined in Section 4.2, we can now compute the number of expected customers for each operator.

After recalling Equations (4) and (5), we first have the overall number of customers subscribing to 5G services

$$n_{\text{tot}} = n \cdot f_{\text{tot}} = ne^{-z\overline{p}} = ne^{-z \frac{p_{\text{MNO}} \sum_{r=1}^{N_{\text{R}}} p^{(i)}_{\text{MVNO}}}{N_{\text{R}}+1}} \tag{12}$$

We can now subdivide those customers among the infrastructure owner and each of the renters. For the MNO, we have the following expression after recalling Equation (8):

$$n_{\text{MNO}} = n \cdot f_{\text{tot}} f_{\text{MNO}} = n e^{-z\overline{p}} e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}} \tag{13}$$

Finally, we can now use the repartition function defined by Equation (11) for each MVNO and obtain the slice of customers for the generic i-th renter:

$$n^{(i)}_{\text{MVNO}} = n \cdot f_{\text{tot}} f^{(i)}_{\text{MVNO}} = n e^{-z\overline{p}} \left(1 - e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}\right) \left[1 - (N_R - 1) \frac{p^{(i)}_{\text{MVNO}}}{N_R \overline{p}_{\text{MVNO}}}\right]. \tag{14}$$

4.4. Profits

We can now compute the profits obtained by each operator.

Because the owner offers a flat-rate price p_{MNO} to end customers and a bandwidth-proportional price to renters, its revenues are represented by those two streams, whereas costs are represented by the sum of CAPEX and OPEX shown in Equation (1). After recalling Equations (13) and (14), the overall profit is then

$$\begin{aligned}
\Pi_{\text{MNO}} &= p_{\text{MNO}} n_{\text{MNO}} + \gamma B \sum_{i=1}^{N_R} n_{\text{MVNO}^{(i)}} - (1+\alpha)(C_0 + mB) n_{\text{tot}} \\
&= p_{\text{MNO}} n e^{-z\overline{p}} e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}} + \gamma B n e^{-z\overline{p}} \left[1 - e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}\right] - (1+\alpha)(C_0 + mB) n e^{-z\overline{p}} \\
&= n e^{-z\overline{p}} \left[p_{\text{MNO}} e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}} + \gamma B \left(1 - e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}\right) - (1+\alpha)(C_0 + mB)\right]
\end{aligned} \tag{15}$$

Similarly, after recalling Equations (2) and (14) for the generic renter, we have the profit

$$\begin{aligned}
\Pi^{(i)}_{\text{MVNO}} &= n^{(i)}_{\text{MVNO}} p^{(i)}_{\text{MVNO}} - n^{(i)}_{\text{MVNO}} [(\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B] \\
&= n^{(i)}_{\text{MVNO}} \left\{ p^{(i)}_{\text{MVNO}} - [(\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B] \right\} \\
&= n e^{-z\overline{p}} \left(1 - e^{-\xi \frac{p_{\text{MNO}}}{\overline{p}_{\text{MVNO}}}}\right) \left[1 - (N_R - 1) \frac{p^{(i)}_{\text{MVNO}}}{N_R \overline{p}_{\text{MVNO}}}\right] \times \\
&\left\{ p^{(i)}_{\text{MVNO}} - [(\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B] \right\}
\end{aligned} \tag{16}$$

5. The Owner–Renter Game

In Section 4, we have set the stage to analyse the strategies of the operators. Now we know the expected number of customers they obtain and the profits they achieve under a demand function that takes into account the preferences of customers that are dictated not just by prices but also by their stickiness to either the infrastructure owner or the renters. In this section, we formulate a game between the operators and look for its Nash equilibrium.

We assume that the aim of all the operators is to maximize their own profit. In order to achieve that goal, they use pricing as their strategic leverage. Precisely, they adjust the prices offered to end customers for which they are competing. So, the infrastructure owner adjusts p_{MNO}, whereas the generic renter will adjust $p_{\text{MVNO}^{(i)}}$, $i = 1, 2, \ldots, N_R$. We assume that end customers may be offered different prices such that $p_{\text{MNO}} \neq p_{\text{MVNO}^{(i)}}$ in general. The price difference will not result in the lowest-price operator taking all of the market due to the sticky demand function.

The maximization of the profits for the operators can be achieved by zeroing the derivative of the profit. We pursue the mathematical details in Appendix B. We show that the equations holding the solution for the maximum profit cannot be solved simultaneously, i.e., there is no Nash equilibrium.

We then consider three different scenarios for price setting that, although they do not envisage an equilibrium, may be employed in a practical setting. The three scenarios differ for the stakeholder in charge of the pricing decision:

- Owner;
- Renters;
- Regulator.

In the first scenario, the owner sets the price for itself and the renters as well. It is the natural extension of a monopolistic setting where the owner is the only operator and is fully in charge of setting prices to maximize its profit. Though it may overestimate the power of the owner and appear impracticable, we can consider it as a reference value (because it is possibly the highest price that can be set). In addition, though it confers great power on the owner, that power would not lead the owner to squeeze the renters out of the market by using the price for the renters as leverage. In fact, setting a high price for the renters would increase their unit profit and would reduce the overall market size due to demand reduction entailed by the demand function described in Section 4.2. On the other hand, setting a low price for the renters would leave them a significant portion of the market, though with reduced unit profit margins.

In the second scenario, the roles are reversed. It is now the renters that set the price both for themselves and the owner to maximize the renters' profits. The same arguments considered for the first scenario apply here as well, with the addition that the owner cannot be ousted from the market, because that would leave the renters themselves without infrastructure.

Finally, in the third scenario, we envisage a regulating authority that sets the prices for both the owner and the renters to maximize the social welfare SW shown in Equation (17), i.e., the sum of the profits of all the operators present in the market. The prices will be different for the owner and the renters, reflecting their different revenue and cost structures.

$$
\begin{aligned}
\mathrm{SW} &= \Pi_{\mathrm{MNO}} + \sum_{i=1}^{N_R} \Pi_{\mathrm{MVNO}}^{(i)} \\
&= p_{\mathrm{MNO}} n_{\mathrm{MNO}} + \sum_{i=1}^{N_R} p_{\mathrm{MVNO}}^{(i)} n_{\mathrm{MVNO}}^{(i)} \\
&\quad - (C_0 + mB)\left[n_{\mathrm{MNO}}(1+\alpha) + \sum_{i=1}^{N_R} n_{\mathrm{MVNO}}^{(i)}(1+\alpha+\eta_i\alpha+\beta_i) \right]
\end{aligned}
\tag{17}
$$

Contrary to what happens for the Nash equilibrium, we find that all those scenarios allow for a solution to the game.

In the following section, we report the results obtained for all the scenarios and a selected choice of values for the parameters.

6. Results

After ruling out the possibility of a Nash equilibrium for prices in Section 5, we can explore the three scenarios set out at the end of the previous section, where prices are set by one of the stakeholders (the MNO, the MVNOs, and the regulatory authority). Here, we report each scenario's prices and profits considering the impact of the bandwidth allotted to customers and the stickiness factor.

We will denote each price-setting scenario by using a specific superscript: "o" when the owner (MNO) acts as the price setter, "r" when the renters (MVNOs) set the price, and "SW" when price setting is left to the regulator. For simplicity, when the renters set prices, we assume that they make the same choice, i.e., they all set the same price. Consequently, we will adopt the notation $p_h^{(k)}$ for prices, which denotes the price offered by operator h

when the price-setting scenario k applies. For example, $p_{\text{MNO}}^{(\text{SW})}$ will be the price offered by the owner and set by the regulator. Similarly, we will adopt the notation $\Pi_h^{(k)}$ for profits.

For the purpose of considering different competition levels, in the following, we consider three cases of competition: monopoly, i.e., no competition ($N_R = 0$); a single MVNO ($N_R = 1$), and three MVNOs ($N_R = 3$). This is largely in line with the real world. Four operators are currently present in the UK (see https://www.rcrwireless.com/20210310/5g/the-latest-on-the-status-of-the-uks-5g-deployments (accessed on 19 February 2023)), whereas 135 operators have launched 5G services in 58 countries according to the latest report of the Global Mobile Suppliers Association (GSA) by [62], which leads to an average of 2.3 operators per country. Hence, our choice lies within those two values.

We have considered the parameter values shown in Table 1. Of course, those are the values for each parameter when that parameter is held constant. In the case of bandwidth, e.g., we may wish to see the effect of changing the bandwidth allowance to each customer on prices and profits. The values shown in Table 1 have been set either on the basis of the literature or postulated as educated guesses. The values considered for C_0 and m have been derived from those reported in [44]. The bandwidth value for B is intermediate among those considered in [44]. The OPEX/CAPEX ratio α lies in the range identified in Section 4.1, i.e., between 30% and 75%. The values for the fraction of CAPEX and OPEX bore by MVNOs (β_i and η_i) are educated guess derived from what is reported in [63], who report CAPEX and OPEX savings due to infrastructure sharing, where savings in the 25–48.6% range are reported for CAPEX and savings in the 14.9–19.9% range are reported for OPEX. Because β_i and η_i may be seen as the complement to 100% of savings, and savings from not deploying the infrastructure (as for an MVNO) are surely larger than those from sharing it, we can expect $\beta_i < 100 - 48.6 = 51.4\%$ and $\eta_i < 100 - 19.9 = 80.1\%$. The values we have employed obey those inequalities. We further remark that we preserve the ordering relationship $\beta_i < \eta_i$ as resulting from the data shown in [63], i.e., that savings in CAPEX are larger than savings in OPEX. The values for the renting fee is an educated guess resulting from the business relationship observed between MNOs and MVNOs. The quasi elasticity of the basin penetration z has been chosen after considering that an average price of 350€ with 80% penetration would lead to $z = 0.00064$. Finally, the stickiness factor ξ has been chosen to tilt stickiness towards the MNO as expected (because it is generally considered more reliable), and having $N_R = 1, 3$ would give a balance value for the stickiness factor, respectively, equal to $\xi_0 = \ln 2 = 0.693$ and $\xi_0 = \ln 4 = 1.386$. The value $\xi = 0.5$ is consistently lower than both.

Table 1. Parameters employed in the analysis.

Parameter	Value
C_0	11.5
m	0.0003
B	60
α	0.5
β_i	0.1
η_i	0.5
γ	0.7
z	0.0005
ξ	0.5

We first compare the different price-setting scenarios by examining the profits generated for each operator. In Table 2, which has been obtained for $B = 60$, the first result is that profits decrease quite heavily as the level of competition grows.

Table 2. Profits of operators under different scenarios.

	$\Pi_o^{(o)}$	$\Pi_r^{(o)}$	$\Pi_o^{(r)}$	$\Pi_r^{(r)}$	$\Pi_o^{(SW)}$	$\Pi_r^{(SW)}$
monopoly	116,439	NA	NA	NA	NA	NA
$N_R = 1$	63,083	51,899	59,276	54,194	57,088	42,208
$N_R = 3$	38,564	23,655	29,548	27,424	20,591	27,350

Even when the MNO sets prices, we observe a real slump as MVNOs enter the picture. Even the introduction of a single competitor slashes down the profits of the infrastructure owner by 45.8%. The decline continues as competition grows. When two further MVNOs add to the group, profits fall by another 38.9%. Competition is bad for MVNOs as well. When we move from one to three MVNOs, the initial renter's profits are halved (precisely, they fall by 54.4%). The renters' profits lag behind those of the owner, which somewhat compensates the owner for the greater burden it suffers from taking care of a much larger infrastructure. When we have just one MVNO, its profits are 82.3% of what the owner obtains, but that fraction goes down to 61.3% when we have three MVNOs.

The situation is improved for renters when they set the prices. Their profits are now close to those of the owner (precisely, 91.4%) when there is a single renter and 92.8% (even better) when there are three MVNOs. This is the combined effect of an increase in absolute terms of the renters' profits and a decrease in the owner's profits. For the owner, releasing the price-setting power to the renter(s) means a loss of 6% of their profits when there is a single renter, but a quite bigger 23.4% when there are three renters.

The intervention of a regulator (who sets prices to maximize social welfare) changes things for the worse for all operators. When we have just one MVNO, the renter receives the most severe negative effects. Its profits fall by 22.1% in comparison to when the renter sets prices and by 18.7% when the owner acts as the price setter. The decline is sharp but less severe for the owner. Its profits fall by 9.5% from it obtains when the owner sets prices and by 3.7% when the renter sets prices.

When the competition grows (here, when we have three renters instead of one), the situation reverses, and the owner is the most penalized. Its profits fall by a whopping 46.6% to its bet case (when the owner sets prices) and yet by 30.3% even when renters set prices. Instead, renters gain from social welfare maximization due to the larger weight in the overall scenario. Their profits increase by 15.6% with respect to what they obtain when the owner sets prices and stay nearly equal to their best case, when they can decide prices for themselves (there is actually a tiny decline of 0.3%).

We can deepen the analysis of profits by considering the impact of bandwidth. In the following, for the purpose of assessing how price setting impacts operators' profits, we consider the worse cases where prices are set by another operator. For example, for the owner, we can consider the cases where prices are set by renters or the regulatory authority. For each case, we define the loss as the percentage of profits that are lost when prices are set by another party:

$$L_h^{((k))} = \frac{\Pi_h^{(h)} - \Pi_h^{(k)}}{\Pi_h^{(h)}}. \tag{18}$$

where h may be either MNO or MVNO, and k may denote one of the three price-setting scenarios (o, r, or SW, for where prices are set by the owner, the renters, and the regulator, respectively).

In Figure 2, we see that the MNO's loss grows with the level of competition (as expected). However, somewhat unexpectedly, the worst case for the MNO is having its prices set by the regulator. Bandwidth has an alleviating effect because the owner is paid by renters proportionally to the bandwidth allotted to each customer, whereas the revenues of the MVNOs grow with the number of customers.

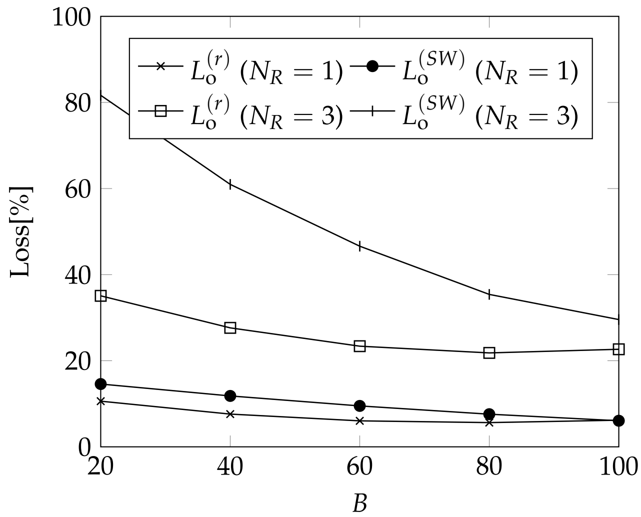

Figure 2. MNO's loss.

Similarly, in Figure 3, we show the loss suffered by each renter. Here, the bandwidth has an adverse effect for the same reason as seen for the owner: renters obtain revenues proportionally to the number of customers but pay the owner proportionally to the allotted bandwidth. Again, we see that the worst cases occur when prices are set by the regulator.

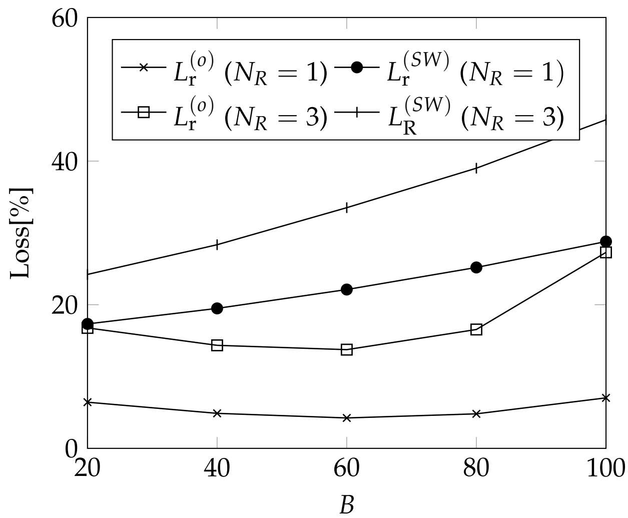

Figure 3. MVNO's loss.

Another factor impacting the strategies of operators is the stickiness of customers. Any deviation of the stickiness factor ξ from its central value ξ_0 moves customers towards either operator. Precisely, if $\xi > \xi_0$, customers move towards the MVNOs. In Figure 4, we see what happens to MNOs' profits when the stickiness factor $\xi = \lambda\xi_0$ moves around its central value. As expected, profit losses grow when the stickiness factor grows and renters set prices. The behaviour is somewhat erratic when the regulator sets prices such that it tends to balance the wishes of both operators.

We observe a similar behaviour for the renter(s) in Figure 5. The stickiness factor does not impress a significant direction to losses when the regulator sets prices. However, the presence of stickiness towards MVNOs does not help, contrasting the strategy by the MNO, which is able to overcome that stickiness and reduce the MVNOs' profits by suitably setting prices.

We now examine the benefits of competition for end users by observing how operators are compelled to change their prices in response to increased competition, which is represented here by a growing number of virtual operators.

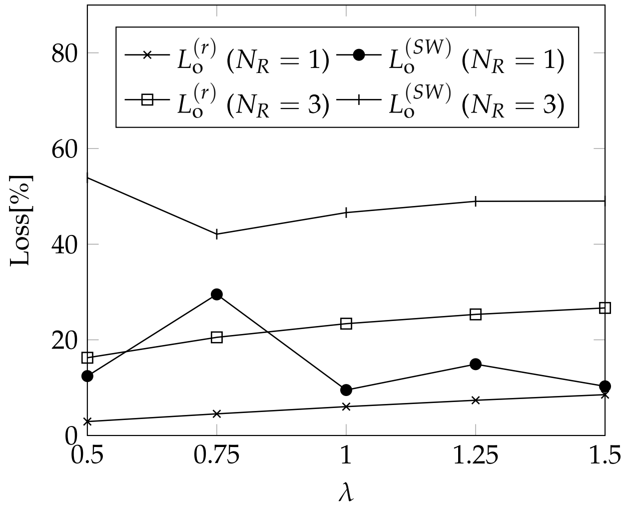

Figure 4. Impact of stickiness on MNOs' profits.

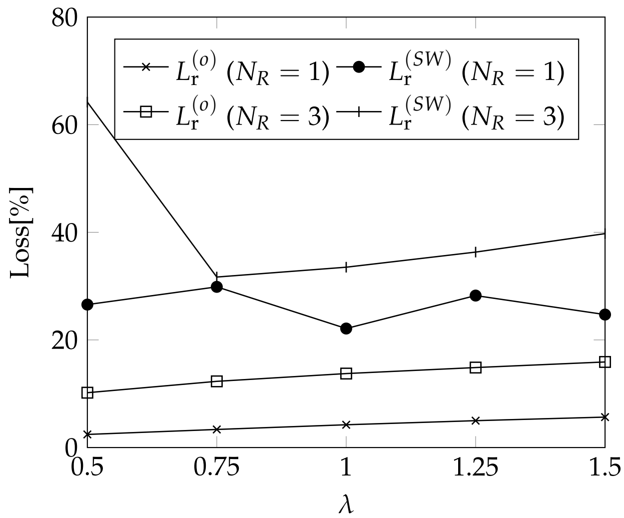

Figure 5. Impact of stickiness on MVNOs' profits.

In Figure 6, we examine the effect of the number or renters on the price set by the owner. We see that the prices decline as expected, though in a less-than-linear fashion. The bandwidth considered here is $B = 60$.

In comparison, the impact of the resources required by users is much smaller. In Figure 7, which is plotted for the case of a single renter, we see that the impact is negligible when prices are set by the regulator to maximize the social welfare. Instead, we observe contrasting effects for prices set by the owner and the renter. The price set by the owner tends to make bandwidth a bit cheaper when the request grows. The renter, instead, makes it significantly more expensive to buy more bandwidth. In the picture, we have omitted the superscript indicating the specific renter because we have assumed for simplicity that all renters set the same prices.

When the competition grows (we consider again the case of three renters), we see in Figure 8 that prices set by the same operator (e.g., the price set by the owner for itself) exhibit the same behaviour as in the case of a lower competition. Renters tend to offset the increase in bandwidth on by charging end customers more, whereas the infrastructure owner allows for a sort of discount on quantity, charging less when bandwidth increases. When setting prices is left to the regulator, the increase in competition does not change the bandwidth-independent behaviour of price for renters, whereas the price for the owner's customers increases.

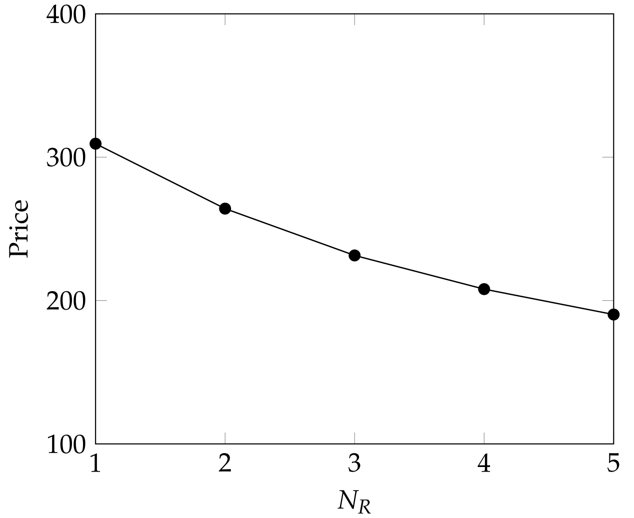

Figure 6. Price set by the owner ($p_{\mathrm{MNO}}^{(o)}$).

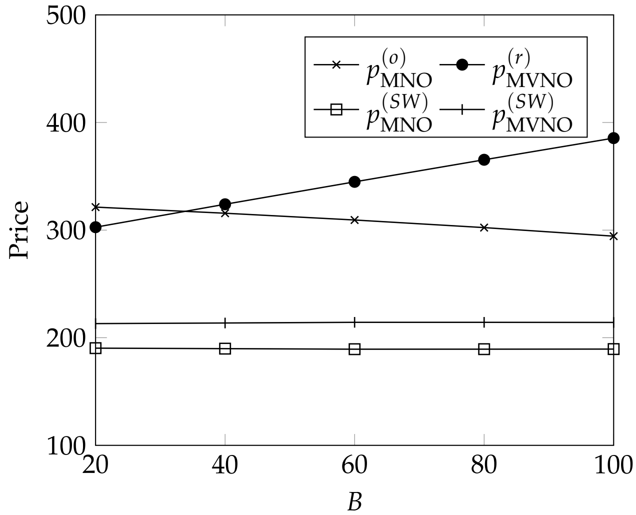

Figure 7. Impact of bandwidth on prices with a single renter.

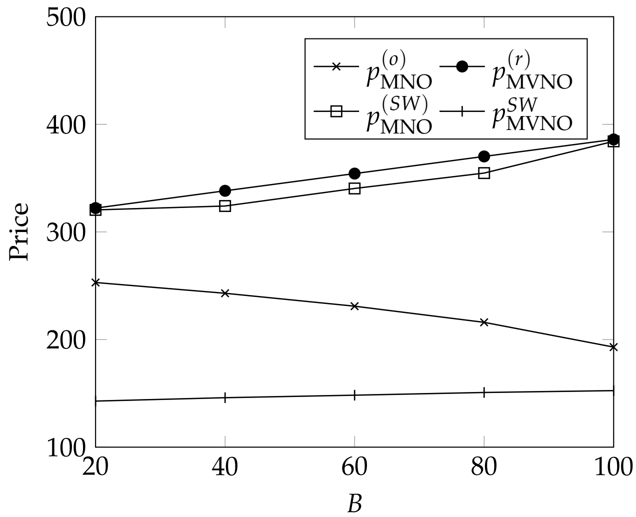

Figure 8. Impact of bandwidth on prices with three renters.

7. Conclusions

We have analysed a scenario where an infrastructure provider and a group of virtual operators compete for end customers, with customers deciding not only based on prices but also based on other factors that lead them to stick with either operator.

The absence of a Nash equilibrium for the game resulting from the interaction of MNO and MVNOs has led us to examine scenarios where one of the stakeholders holds an advantageous position by being able to set prices for all parties.

Aside from the obvious observation that price setters may bend the situation to their advantage and obtain more profits, we have seen that assigning the price-setting power to the regulator (which would maximize the social welfare) may be worse both for the infrastructure owner and the renter. The situation would turn slightly better for the renters when the number of renters grows, because their weight in the overall social welfare would grow so as to push the balance towards them.

An odd finding is that stickiness does not always move profits in the same way as it moves customers, whereas stickiness to MVNOs increases the MNO's losses when MVNOs act as price-setters, we also observe that the same stickiness to MVNOs increases their losses when the MNO sets prices. Furthermore, stickiness does not impress a clear direction to losses when the regulator has price-setting powers and maximizes the social welfare.

A limitation of this study is that is only provides numerical results. Though the results have been obtained for a significant range of values of the parameters involved, finding an analytical solution of the profit-maximization equation in an approximate way,would help us to see the influence of each parameter more clearly. Furthermore, that would help us create a theory or set guidelines for price setting and regulation.

This is also a suggestion for the future work we envisage for the subject. We would also like to explore the range of factors that may lead to stickiness and try their more precise mathematical description.

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Appendix A. Glossary of Terms

We report all the abbreviations employed in the paper in Table A1.

Table A1. Glossary of terms.

Symbol	Definition/Description
n	number of potential customers
n_{tot}	overall number of customers
n_x	number of customers joining the operator x
N_{R}	number of renters
n_{MNO}	number of customers of the MNO
n_{MVNO}	number of customers of the generic MVNO

Table A1. *Cont.*

C_{MNO}	cost function for the MNO
$C^{(i)}_{\text{MVNO}}$	cost function for the i-th MNVO
p_{MNO}	price imposed by the MNO
$p^{(i)}_{\text{MVNO}}$	price imposed by the i-thMVNO
$\bar{p}_{\text{MVNO}}$	average price imposed by the MVNO
$\bar{p}$	average price
Π_{MNO}	profit of MNO
Π_{MVNO}	profit of the generic MVNO
f_{tot}	ratio of the number of actual customers to the number of potential customers
ϵ	quasi-elasticity of the demand function with respect to the average price
$p^{(\text{k})}_{\text{h}}$	price offered by operator h under price-setting scenario k **
$\Pi^{(\text{k})}_{\text{h}}$	profit of operator h under price-setting scenario k **
$L^{(\text{k})}_{\text{h}}$	profit loss of operator h under price-setting scenario k **
SW	social welfare
C_0	zero-consumption fee in the linear cost model
m	marginal cost of bandwidth
B	bandwidth
α	OPEX/CAPEX ratio
β_i	CAPEX fraction borne by the i-th MVNO
η_i	OPEX fraction borne by the i-th MVNO
γ	renting fee for each bandwidth unit
z	value of the quasi-elasticity of the demand function with respect to the average price
ξ	stickiness factor

** $h \in \{MNO, MVNO\}, k \in \{(o), (r), (SW)\}$

Appendix B. Maximization of Profits

In this appendix, we derive the equations whose solution allows us to maximize the profits of operators. We consider first the infrastructure owner (i.e., the MNO) and then the renters (MVNOs). Profit maximization will be sought by zeroing the derivative of profit.

For the infrastructure owner, after recalling Equation (15) and applying the product rule for derivatives, we have

$$\begin{aligned}\frac{\partial \Pi_{\text{MNO}}}{\partial p_{\text{MNO}}} &= ne^{-z\bar{p}}\left\{-\frac{z}{1+N_R}\left[e^{\xi\frac{p_{\text{MNO}}}{\bar{p}_{\text{MVNO}}}}(p_{\text{MNO}}-\gamma B)-(1+\alpha)(C_0+mB)+\gamma B\right]\right\}\\ &+ne^{-z\bar{p}}\left\{e^{\xi\frac{p_{\text{MNO}}}{\bar{p}_{\text{MVNO}}}}\left[1-\xi\frac{p_{\text{MNO}}}{\bar{p}_{\text{MVNO}}}+\gamma B\frac{\xi}{\bar{p}_{\text{MVNO}}}\right]\right\}=0,\end{aligned} \tag{A1}$$

which leads to the following equation

$$e^{\xi\frac{p_{\text{MNO}}}{\bar{p}_{\text{MVNO}}}}\left[1-(p_{\text{MNO}}-\gamma B)\left(\frac{z}{1+N_R}+\frac{\xi}{\bar{p}_{\text{MVNO}}}\right)\right]=\frac{z}{1+N_R}[\gamma B-(1+\alpha)(C_0+mB)] \tag{A2}$$

Unfortunately, Equation (A2) is a transcendental equation that cannot be solved exactly for p_{MNO}, and we have to resort to a numerical solution.

As to the profit of the generic i-th renter, we can use the following expression of the profit:

$$\begin{aligned}\Pi_{\text{MVNO}}^{(i)} &= n_{\text{MVNO}}^{(i)} p_{\text{MVNO}}^{(i)} - n_{\text{MVNO}}^{(i)} \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \\ &= n_{\text{MVNO}}^{(i)} \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) \\ &= n f_{\text{tot}} f_{\text{MVNO}}^{(i)} \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right)\end{aligned} \tag{A3}$$

Because we cannot act on the overall basin dimension n , we can consider the following quantity to be maximized

$$\Pi_{\text{MVNO}}^{(i)} \propto f_{\text{tot}} f_{\text{MVNO}}^{(i)} \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) \tag{A4}$$

After recalling Equation (2), we also note that the following quantity does not depend on the strategic variable $p_{\text{MVNO}}^{(i)}$

$$\frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} = (\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B. \tag{A5}$$

The derivative to be zeroed is then

$$\begin{aligned}\frac{\partial \Pi_{\text{MVNO}}^{(i)}}{\partial p_{\text{MVNO}}^{(i)}} &\propto \frac{\partial (f_{\text{tot}} f_{\text{MVNO}}^{(i)})}{\partial p_{\text{MVNO}}^{(i)}} \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) + f_{\text{tot}} f_{\text{MVNO}}^{(i)} \\ &= \left[-\frac{z}{1+N_R} f_{\text{tot}} f_{\text{MVNO}}^{(i)} + f_{\text{tot}} \frac{\partial f_{\text{MVNO}}^{(i)}}{\partial p_{\text{MVNO}}^{(i)}} \right] \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) + f_{\text{tot}} f_{\text{MVNO}}^{(i)} \\ &= f_{\text{tot}} \left\{ \left[-\frac{z}{1+N_R} f_{\text{MVNO}}^{(i)} + \frac{\partial f_{\text{MVNO}}^{(i)}}{\partial p_{\text{MVNO}}^{(i)}} \right] \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) + f_{\text{MVNO}}^{(i)} \right\}.\end{aligned} \tag{A6}$$

Because the term f_{tot} cannot be brought down to zero, the equation to solve is finally

$$\left[-\frac{z}{1+N_R} f_{\text{MVNO}}^{(i)} + \frac{\partial f_{\text{MVNO}}^{(i)}}{\partial p_{\text{MVNO}}^{(i)}} \right] \left(p_{\text{MVNO}}^{(i)} - \frac{C_{\text{MVNO}}^{(i)}}{n_{\text{MVNO}}^{(i)}} \right) + f_{\text{MVNO}}^{(i)} = 0, \tag{A7}$$

which can be put in the following form after a few algebraic passages:

$$\begin{aligned}&\frac{z}{1+N_R} \left(p_{\text{MVNO}}^{(i)} - (\eta_i \alpha + \beta_i)(C_0 + mB) + \gamma B \right) + (N_R - 1) \frac{\sum_{j \neq i} p_{\text{MVNO}}^{(j)}}{\left(\sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)} \right)^2} = e^{-\xi \frac{p_{\text{PNO}}}{p_{\text{MVNO}}}} \times \\ &\left\{ \xi \frac{N_R p_{\text{MNO}}}{\left(\sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)} \right)^2} \left[1 - (N_R - 1) \frac{p_{\text{MVNO}}^{(i)}}{\sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)}} \right] + (1 - N_R) \frac{\sum_{j \neq i} p_{\text{MVNO}}^{(j)}}{\left(\sum_{i=1}^{N_R} p_{\text{MVNO}}^{(i)} \right)^2} \right\}\end{aligned} \tag{A8}$$

The solutions to Equations (A2) and (A8) can be seen as the best response functions of the owner and the renters, respectively, who are using end prices as their strategic leverages in the game. Unfortunately, the single equations cannot be solved analytically, so the simultaneous solution of the resulting system of equations is not possible in an exact form. That solution would represent a Nash equilibrium. After unsuccessfully trying to solve the system of equations numerically for different values of the parameters at hand, we postulate that they do not have a solution, i.e., there is no Nash equilibrium.

In order to solve the equations emerging in profit-maximization tasks, we have employed the *nleqslv* function in R, whose documentation is available at https://www.rdocumentation.org/packages/nleqslv/versions/3.3.4/topics/nleqslv (accessed on 19 February 2023). That function adopts Broyden and the full Newton method to solve nonlinear Equations [64].

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