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Article

An Improved Weise's Rule for Efficient Estimation of Stand Quadratic Mean Diameter

Róbert Sedmák [1,2,*], Ľubomír Scheer [1], Róbert Marušák [2], Michal Bošeľa [2,3], Denisa Sedmáková [4] and Marek Fabrika [1]

[1] Faculty of Forestry, Technical University in Zvolen, Zvolen 96053, Slovakia; E-Mails: scheer@tuzvo.sk (L.S.); fabrika@tuzvo.sk (M.F.)

[2] Faculty of Forestry and Wood Sciences, Czech University of Life Sciences Prague, Prague 6 165 21, Czech Republic; E-Mail: marusak@fld.czu.cz

[3] National Forest Centre—Forest Research Institute in Zvolen, Zvolen 96053, Slovakia; E-Mail: bosela@nlcsk.org

[4] Institute of Forest Ecology, Slovak Academy of Sciences, Zvolen 96053, Slovakia; E-Mail: sedmakova@savzv.sk

* Author to whom correspondence should be addressed; E-Mail: robert.sedmak@tuzvo.sk; Tel.: +421-455-206-305, Fax: +421-455-332-654.

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Abstract: The main objective of this study was to explore the accuracy of Weise's rule of thumb applied to an estimation of the quadratic mean diameter of a forest stand. Virtual stands of European beech (*Fagus sylvatica* L.) across a range of structure types were stochastically generated and random sampling was simulated. We compared the bias and accuracy of stand quadratic mean diameter estimates, employing different ranks of measured stems from a set of the 10 trees nearest to the sampling point. We proposed several modifications of the original Weise's rule based on the measurement and averaging of two different ranks centered to a target rank. In accordance with the original formulation of the empirical rule, we recommend the application of the measurement of the 6th stem in rank corresponding to the 55% sample percentile of diameter distribution, irrespective of mean diameter size and degree of diameter dispersion. The study also revealed that the application of appropriate two-measurement modifications of Weise's method, the 4th and 8th ranks or 3rd and 9th ranks averaged to the 6th central rank, should be preferred over the classic one-measurement estimation. The modified versions are characterised by an improved accuracy (about 25%) without statistically significant bias and measurement costs comparable to the classic Weise method.

Keywords: quadratic mean diameter; diameter dispersion, sample quantile; rule of thumb; simulation; European beech; forest inventory

1. Introduction

Exact mathematical descriptions of stand diameter distributions are one of the important tasks of forest growth modelling [1]. Diameter structure is a basic modelling component of many complex forest growth and yield models linking individual tree characteristics with stand variables [2,3]; therefore, modelling stand diameter distribution is a rapidly evolving research field [4–7]. Several probability density functions based on statistical probability theory are used as a mathematical model of diameter distributions. Normal distribution modified by Gramm-Charlier expansion [8], Weibull distribution [9,10], Beta distribution [11], Johnson's SB distribution [12,13], and logit-logistic distribution [14] are popular probability density functions.

Principally, four main approaches are used for parameter estimation of diameter distributions: (i) the parameter prediction method [15]; (ii) the parameter recovery and percentile-based parameter recovery method [16]; (iii) the non-parametric percentile-based distribution-free method [17]; and (iv) the quantile regression method [18]. In particular, the quantile regression method has gained increased attention in the last few years [19].

Effective, unbiased, and accurate determination of stand mean diameter in the field is an important task for forest inventory and empirical modeling. Arithmetic and quadratic mean diameters (QMD) are the most important descriptive characteristics of diameter distributions, as derived from the 1st and 2nd non-central moments. They are often used for the estimation of parameters of the selected distribution model for the application of parameter recovery or percentile-based parameter recovery approaches [18]. Stand QMD is a basic input variable for the calculation of growing stock at a particular stand age. However, the errors associated with mean diameter determination have four times greater impact on the accuracy of growing volume calculations than errors of mean height determination [20].

An efficient method to quickly estimate the QMD in the field is an empirical rule known as Weise's rule [21]. It is a rule of thumb for estimating QMD using the 60th percentile from the visually ordered set of diameters selected at a given sampling point. A practical procedure of estimation is based on the visual ranking of 10 trees nearest to the sampling point according to their size (smallest to largest) and measurement of diameter of the 6th stem in rank order. Stand QMD is then calculated as an arithmetic mean from the appropriate number of sampling point estimates (approximately 1 sampling point ha^{-1} is empirically suggested). Weise's rule of thumb is widely used within Slovak and Czech forest state surveys and precedes the elaborate forest management plans obligatory for all forest owners in both countries [22]. From a broader European perspective, the method has been almost forgotten in spite of its rational approach and practical applicability.

Due to widespread utilization of QMD in forest planning and its empirical character, Weise's rule has become a subject of extensive validation in Slovak natural and forest management conditions [23,24].

The percentiles corresponding to real QMD were empirically determined from a large sample of single species forest stands of different tree species covering almost all of Slovakia. Two separate surveys revealed two different sets of recommended percentiles according to different diameter distributions (Table 1).

Table 1. Empirical percentiles and ranks of measured stems for QMD estimation.

Shape of Diameter Distribution	Percentiles and Rank of Weise's Stem According to	
	[23]	[24]
Negatively (right) skewed	57% (6th)	52% (5th)
Symmetrical	61% (6th)	55% (5th–6th)
Positively (left) skewed	66% (7th)	60% (6th)
Reverse J-shaped	74% (7th)	68% (7th)

Consequently, two practical problems arose: (i) determining which set is correct; and (ii) how to evaluate the shape of the tree diameter distribution before the stand is measured. Both problems likely encouraged wider implementation of Weise's rule refinements in forestry practice. Regardless, the original simple rule of 6th stem in rank is typically applied irrespective of the actual distribution shape. Another issue is that simple estimation of sample percentiles as rank/sample size is not valid for small samples [25,26]. The correct percentiles for a sample size of 10 are theoretically derived from a cumulative density function of binomial distribution and are displayed in Table 2.

Table 2. Rank and theoretical sample quantiles at a sample size of 10 [25].

Rank	1	2	3	4	5	6	7	8	9	10
Quantile	6.7	16.2	25.9	35.5	45.2	54.8	64.5	74.1	83.8	93.3

Table 2 implies that the measurement of the 6th diameter in rank corresponds to approximately the 55th percentile and not to the desired 60th percentile; for a correct estimation of the 60th percentile, the measurement and averaging of the dimater of the 6th and 7th ranked trees should be used instead. According to Table 1, the 60th percentile can be considered an optimal estimate of QMD only for left-skewed diameter distributions according to [24], or symmetrical distributions according to [23]. Obviously, the rules for practical applications of Weise's method are not clear and the current practice of the measurement of the 6th diameter in rank may lead to serious biases in estimated stand QMD.

Another important issue related the implementation of Weise's method is a purely empirical question regarding the determination of adequate sample size. No information exists regarding the variability of sample point estimates in different forest stand types; thus, the determination of correct sample size prior to measurement is impossible. Current practice supposes that the estimation accuracy meets practical needs, but exact evaluations have not been conducted.

The objective of this study was to develop clear recommendations and rules for the practical application of Weise's method by employing a simulation approach. Different types of stand structures for European beech (*Fagus sylvatica* L.), the most widespread species of Slovak forests (31.3% or approx. 600,000 hectares) across the Western Carpathians, were stochastically generated and randomly sampled. The aim was to obtain exact information about the variability, bias, and accuracy of stand QMD estimates employing different ranks of measured stems in single species stands of shade-tolerant species growing in Central Europe. In addition, we proposed several modifications of the original Weise's rule based on the measurement and averaging of two measured diameters with different ranks.

2. Materials and Methods

2.1. Data Generation and Simulation

Stochastic generation of adequate numbers, spatial distribution and dimensions of individual trees growing at predefined site conditions was modelled for 27 virtual stands, each 9 ha in size. The virtual list of tree values and their coordinates in each stand were stochastically generated by an original modelling approach utilizing several existing submodels and equations constructed for single species beech stands in Slovakia.

The modelling approach requires some basic predefined values, including site quality, defined by a site index of 30 m (mean stand height at reference age of 100 years) commonly found in Slovak beech stands, and the arithmetic mean and its degree of dispersion. Stands were differentiated by mean diameter size intervals of 5 cm, from 10 to 50 cm, and three categories (low, medium, and high) of degree of diameter dispersion (DoD). The DoD description is a qualitative measure of tree diameter variability that can be easily estimated in the field by visual inspection of a stand. A low degree of variability is typical for even-aged, single-species stands growing on homogenous sites managed by silviculture approaches to encourage tree uniformity (e.g., understorey thinning). Conversely, high DoD is characteristic of uneven-aged, highly-structured forest stands of variable site conditions tended by close-to-nature approaches that preserve high variability of individual tree characteristics [20]. In total, 27 combinations of 9 mean diameters and 3 DoD yielded 27 virtual beech stands covering almost all stand structure types of European beech that can be found in Slovakia on a single predefined site quality. An effort was made to adequately capture the whole variety of stand types in order to secure a wider generalization of the study results.

The virtual stands were generated through mathematical modelling using the following approach:

(1) Estimation of the stand age, number of trees per hectare, and mean height corresponding to the selected mean diameter for a given height site index of 30 m from valid growth and yield tables [27]; estimation of the diameters coefficients of variation and standard deviations from the mean diameters according to different DoD categories through regression equations [28];
(2) Estimation of the diameter distribution skewness and curtosis corresponding to selected mean diameters and DoD categories from results published by Halaj [23];
(3) Mathematical modelling of the diameter distributions based on predefined mean diameter, DoD categories, and estimated coefficients of skewness and excess by means of normal distribution modified by the second order Gram-Charlier expansion; and
(4) Stochastic generation of individual tree dimensions (tree diameter, height, and crown width) and tree coordinates within the stand area.

An overview of input variables required for a simulation of virtual stands (results of steps (i)–(iii)) is given in Table 3.

Table 3. Input variables for simulation of individual trees in virtual stands.

Stand Variables	Degree of Dispersion	Mean Diameter (cm)								
		10	15	20	25	30	35	40	45	50
Age (year)		35	45	60	75	90	110	135	155	180
Number of trees (ha^{-1})		3788	2160	1227	828	615	455	343	287	241
Mean height (m)		13.5	17.2	21.7	25.3	28.3	31.5	34.7	36.8	39
Coefficient of variation (%)	Low	27.5	26.1	25.1	24.5	24.2	24.4	24.9	25.8	27.0
	Medium	37.0	34.6	33.0	32.1	32.0	32.6	34.0	36.2	39.1
	High	46.3	42.9	40.5	39.4	39.3	40.4	42.6	45.9	50.4
Skewness A	Low	0.45	0.50	0.60	0.65	0.68	0.65	0.60	0.48	0.30
	Medium	0.40	0.48	0.52	0.55	0.60	0.55	0.50	0.30	0.10
	High	0.30	0.40	0.48	0.52	0.60	0.58	0.50	0.35	0.10
Kurtosis E	Low	–0.70	0.00	0.20	0.50	0.70	0.65	0.50	0.15	–0.10
	Medium	–0.70	–0.20	0.10	0.35	0.40	0.35	0.10	–0.30	–0.80
	High	–1.20	–0.50	–0.30	0.00	0.12	–0.05	–0.30	–1.00	–1.20

The diameter distributions were mathematically expressed by a probability density function of normal distribution modified by the Gram-Charlier expansion [29]:

$$f(d_j) = \frac{1}{\sqrt{2\pi}s_d} e^{-\frac{z_j^2}{2}} \left[1 + \frac{\kappa_3}{6s_d^2}(z_j^2 - 3z_j) + \frac{\kappa_4}{24s_d^4}(z_j^4 - 6z_j^2 + 3) \right] \quad (1)$$

where $f(d_j)$ is relative frequency for j-th diameter class with 1 cm width. Variable z_j is a normalized variable for j-th diameter class calculated as $z_j = (d_j - \bar{d}_g)/s_d$, where d_j is the central value of j-th diameter class, and $\bar{d}_g$ is QMD. κ_3 and κ_4 are the 3rd and 4th cumulated diameter distribution modified symmetry and excess of normal distribution, respectively. The cumulants are functions of central moments, m_2, m_3, m_4; when $\kappa_3 = m_3$ and $\kappa_4 = m_4 - 3\,m_2$. The 2nd–4th central moments, m_x are calculated from estimated values of s_d; A and E are listed in Table 3. An overview of all input variables in Equation (1) is given in Table 4. The use of the modified normal distribution is justified by its flexibility and by the possibility to obtain reliable estimates of its parameters for all considered types of beech stands.

Table 4. Input variables for diameter distribution modelling.

Variable	Degree of Dispersion	Mean Diameter (cm)								
		10	15	20	25	30	35	40	45	50
s_d	Low	2.8	3.9	5.0	6.1	7.3	8.5	10.0	11.6	13.5
	Medium	3.7	5.2	6.6	8.0	9.6	11.4	13.6	16.3	19.5
	High	4.6	6.4	8.1	9.8	11.8	14.1	17.0	20.7	25.2
κ_3	Low	36	42	114	176	269	425	660	982	1111
	Medium	50	108	215	336	627	952	1687	1982	3055
	High	104	231	485	771	1474	2762	4836	9514	6227
κ_4	Low	−134	−94	−51	491	2123	3439	4904	1085	20024
	Medium	−934	−587	−510	1326	2796	5266	−3769	−47673	−244450
	High	−2530	−2785	−5271	−4588	−5789	−28267	−109209	−545833	−1407101

After obtaining the mathematical representation of diameter distributions, the list of individual trees and their dimensions for each modelled stand was generated (step iv). Initially, a list of individual diameters was stochastically generated from the diameter distribution described by Equation (1). Two numbers, r_1 and r_2 from uniform distribution (0.1), and the random diameter, d_r from range 0.1–120 cm, calculated using the formula $d_r = 0.1 + r_1 120$, were determined. The random diameter, d_r was assigned corresponding to the 1 cm diameter class calculated by Equation (1) and the relative frequency of diameters $f(d_j)$ in that particular class. The random diameter d_r was stochastically accepted if $r_2 < f(d_j)$; otherwise, the stochastic generation was repeated with a new pair of random numbers [r_1, r_2] The process was repeated until the number of accepted tree diameters equaled the expected number of trees per hectare (Table 3).

Individual tree heights were estimated using a generalized height-diameter model [30] and the generated tree diameter, d defined mean diameter, $\bar{d}_g$ and corresponding mean height, $\bar{h}_g$ variables (Table 3); regression estimates of heights were stochastically modified. Normal distribution of height residuals in a particular diameter class with 0 mean and variance were provided by Halaj [31]. Tree crown widths were estimated by non-linear regression from known tree height and diameter [28], and the distribution of trees was modelled across the stand area.

Tree distribution was simulated based on a model parameterised from long-term research plot data from Slovak single-species beech stands [32]. This approach was based on the stochastic generation of individual tree coordinates (x, y) from a uniform distribution of possible coordinates that was stochastically verified by a probability, p (l_r) determined by a generalized logistic equation using relativised distance, l_r. Relativised distance can be considered a special competition index calculated from the crown widths and spatial distances between randomly selected pairs of trees. Tree coordinates are accepted as long as the random number r generated from uniform distribution with range 0–1 is $r < p$ (l_r); otherwise, the procedure is repeated with new random coordinates, and the procedure is repeated until all generated trees are positioned within the stand area.

2.2. Methods

The average of simulated point samples of QMD estimates according to Weise's rule determined the estimated QMD in each virtual stand. For each virtual stand (Table 5), simple systematic sampling was used with a sample size calculated to achieve an accuracy of Δ% = 5% at the confidence level $P = 95\%$, using the formula:

$$n = \left(\frac{t_{\alpha/2,f} s_d \%}{\Delta\%} \right)^2 \quad (2)$$

where $t_{\alpha/2,f}$ is a critical value of Student's t distribution and $s_d\%$ is the tree diameter coefficient of variation (Table 3).

Table 5. Number of sampling points for accuracy of Δ% = 5% at the 95% confidence level.

Degree of Dispersion	Mean Diameter (cm)								
	10	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
Low	121	100	81	81	81	81	81	100	100
Medium	196	169	144	144	144	144	144	169	196
High	289	256	225	196	196	225	225	256	289

Several variants and modifications of Weise rule applied to QMD estimation were examined:

- Estimations based on the diameter measurement of the 5th, 6th, 7th, and 8th ranked largest trees of the 10 individuals nearest to the sampling point
- Estimations based on the average of the two diameter measurements with appropriately selected ranks:
    - Five rank combinations, [5. and 6.]—[4. and 7.]—[3. and 8.]—[2. and 9.]—[1. and 10.], centered between the 5th and 6th rank (50% percentile according; Table 2)
    - Four rank combinations, [5. and 7.]—[4. and 8.]—[3. and 9.]—[2. and 10.], centered on the 6th rank (55% percentile)
    - Four rank combinations, [6. and 7.]—[5. and 8.]—[4. and 9.]—[3. and 10.], centered on between the 6th and 7th rank (60% percentile)
    - Three rank combinations, [6. and 8.]—[5. and 9.]—[4. and 10.], centered on the 7th rank (65% percentile)
    - Three rank combinations, [7. and 8.]—[6. and 9.]—[5. and 10.], centered between the 7th and 8th ranks (70% percentile)
    - Two rank combinations, [7. and 9.]—[6. and 10.], centered on the 8th rank (75% percentile)

In principle, all ranks or combinations of ranks that have the potential to produce good estimates of the QMD were explored. The two-measurement variants were applied at only half of the sampling points to maintain the same number of diameter measurements and to approximate equal field sampling time and measurement costs necessary for stand QMD estimation for comparisons between one- and two-measurement variants.

The individual estimates of the QMD at each sampling point, $\bar{d}_j$ were obtained using Weise's variants. The arithmetic mean of sampling point estimates, $\hat{d}_g$ provided the final estimate of real stand QMD, $\bar{d}_g$ and the relative standard deviation of sampling point estimates, $s_{\bar{d}_j}\%$, provided a measure of variability of sampling point estimates within the stand. The standard error of estimation, $s_e\%$ describing the precision was determined using the equation:

$$s_e\% = s_{\bar{d}_j}\% \big/ \sqrt{n-1} \tag{3}$$

where n is the number of sampling points. Relative deviation of the final estimate $\hat{d}_g$ from known mean stand diameter $\bar{d}_g$ or the relative estimation bias was determined to be:

$$\bar{e}\% = (\hat{d}_g - \bar{d}_g) \big/ \bar{d}_g \, 100 \tag{4}$$

Statistical significance of biases were tested by a *t*-test at the level of significance α = 5%. Final measurements of estimated accuracy were calculated as:

$$\mathrm{RMSE} = \sqrt{\bar{\mathrm{e}}\%^2 + s_e^2\%} \quad (5)$$

where RMSE denotes the percent root mean square error.

The generation of virtual stands, the simulations of tree samples, and the application of different QMD estimation methods associated with calculations of biases, precisions, and accuracies of applied methods were done in the Borland Pascal programming environment [33].

3. Results

A trend of significant under-estimation to significant over-estimation of QMD was evident with increasing rank for one-measurement variants of Weise's rule (Figure 1A). The smallest, but still the most significant negative bias (calculated as average of all examined stands differentiated by mean diameter size and DoD), was achieved for the 6th rank. Because biases composed a substantial part of the accuracy, the smallest average RMSE of 2.8% at 68% confidence level (*i.e.*, approx. 5.6% at 95%) was also achieved at the 6th rank. Still, the 6th rank was the most accurate and was higher than the originally intended 5.0% at the 95% confidence level. On the other hand, the average precision, $s_e\%$, was rather high for all ranks (1.10%–1.26% at the 68% level) with only small and random variations among them (Figure 1B).

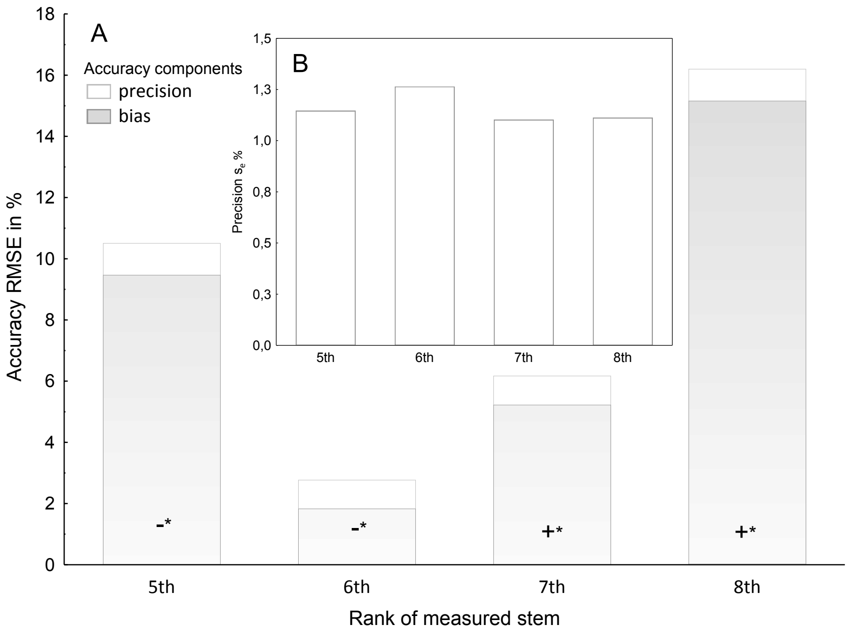

Figure 1. Comparison of accuracy (**A**); and precision (**B**) for one-measurement variants of Weise's estimation averaged across all examined stands. The stacked accuracy bars representing averaged proportions of bias (**grey**) and precision (**white**) in terms of accuracy were calculated as ratios of squared bias/precision on squared RMSE. Signs in the bias proportions shows the prevalent direction of original biases (under- or overestimation). The signs denoted by an asterisk indicate the prevalence of statistically significant biases at a *p*-value of 5%.

More detailed analysis of the best accuracy results according to different mean diameter size and degree of diameter dispersion confirms that the measurement of the 6th stem in rank is the best variant in 24 out of 27 cases (Table 6). More than 90% (22 out of 24) of 6th rank RMSE% contained negative biases of which approximately 60% (13 out of 22) are significant at a *p*-value of 5%. The influence of different mean diameter size on RMSE was not unambiguous, but a weak tendency of RMSE to decrease at larger mean diameters was observed.

Table 6. The optimal rank, accuracy, and bias of QMD estimation for the best one-measurement variants of Weise's rule (the values in column for each combination of QMD and DoD are optimal rank followed by RMSE and bias in %).

Degree of	**Real QMD in cm**									**Mode**
Dispersion	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**Mean/Mean**
Low	6 1.38 −0.81	6 3.01 −2.82 *	6 1.21 −0.32	6 2.08 −1.83	6 2.36 −2.16 *	6 1.94 −1.70	6 3.00 −2.76 *	6 2.61 −2.40 *	6 2.02 −1.69	6 2.18 −1.83
Average	6 3.10 −2.91 *	6 1.74 −1.34	6 2.40 −2.17 *	6 3.16 −2.99 *	6 2.09 −1.87	6 3.46 −3.29 *	6 1.61 −1.26	6 1.77 −1.42	6 1.44 −0.90	6 2.31 −2.02
High	6 3.48 −3.31 *	7 4.99 4.89 *	6 4.08 3.95 *	7 4.87 4.77 *	6 3.25 −3.07 *	7 4.26 4.16 *	6 3.52 −3.35 *	6 3.09 −2.88 *	6 1.19 0.14	6 3.04 [1] −2.24
Mode	6	6	6	6	6	6	6	6	6	
Mean/Mean	2.65 −2.34	3.25 0.25	2.56 −2.15	3.37 −0.02	2.56 −2.36	3.22 −0.28	2.71 −2.46	2.49 −2.23	1.55 −0.82	

[1] the averages for this DoD category are calculated only for optimal 6th rank values, * the statistically significant biases at *p*-level 5%

The influence of DoD was more pronounced. The measurement of the 6th stem in rank was the most accurate variant for stands with low and medium DoD with mean RMSE% of 2.2% and 2.3%, respectively, at the 68% confidence level. In spite of the most significant negative biases, mean RMSEs of these categories were smaller than the proposed accuracy of 2.5%. For the high DoD category, the results were more complicated, although the 6th rank was generally still optimal. However, measurement of the 7th stem in rank was the best option in some cases (particularly with smaller mean diameters) because they had smaller RMSE% than the 6th stem in rank. Application of 7th stem in rank was accompanied by a significant overestimation of QMD in contrast to underestimations characteristic of the 6th rank. The mean RMSE% (3.04%) was about 20% higher than the proposed accuracy of 2.5%, which was not surprising considering the higher variability of diameters in the generated stands of the high DoD category.

Modification of the classic Weise's method based on averaging of two diameter measurements in selected ranks on half the number of sampling points clearly improved the estimation accuracy (Figure 2A). The most successful variants of the two-measurement modifications, the 6th rank centroid obtained by averaging the measurements of the 4th and 8th stems in rank and the 6th rank centroid obtained by averaging the measurements of the 3rd and 9th stems in rank, achieved average RMSE of

2.17% and 2.12%, respectively, for all stands. They were approximately 15% lower than the planned accuracy of 2.5% and approximately 25% lower than the accuracy at 2.8% of the best 6th rank's variant of the classic one-measurement Weise's method. The 6th variant of 3–9 had a lower positive bias of 0.86% (on average) in comparison to the 6th variant of 4–8 with a larger negative bias of 1.25% (*i.e.*, 33%). Both biases were statistically non-significant at a *p*-level of 5% in most stands. Precisions of the best two-measurement variants varied between 1.25 and 1.55% on average, which was slightly worse than the precisions of one-measurement variants of Weise estimation (Figure 2B). More detailed analysis of the best two-measurement variants according to different mean diameter sizes and DoD revealed no significant differences (Table 7).

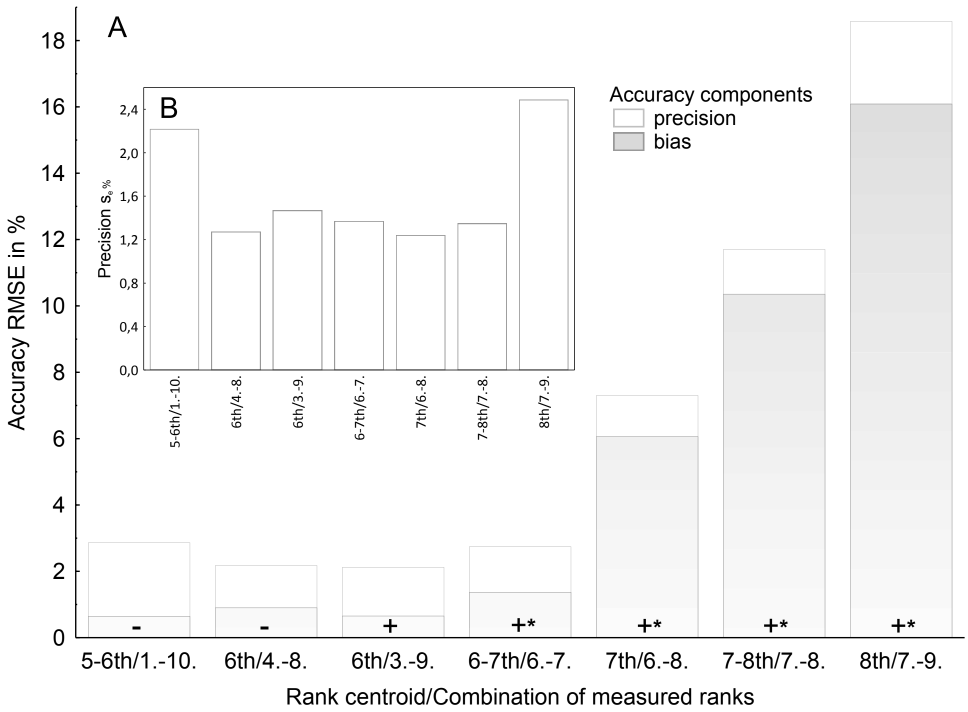

Figure 2. Comparison of accuracy (**A**); and precision (**B**) for two-measurement modifications of Weise's rule averaged across all stands. The stacked accuracy bars representing averaged proportions of bias (**grey**) and precision (**white**) on accuracy were calculated as ratios of squared bias/precision on squared RMSE. Signs in bias proportion parts show the prevalent direction of original biases (under- or overestimation), and the signs denoted by an asterisk indicate the prevalence of statistically significant biases at a *p*-value of 5%).

Almost all biases of the best variants were not statistically significant and two variants identified as optimal varied according to different mean diameters and DoD. The 6th variant of 3–9 was the best in 13 of 27 cases, while the 6th variant of 4–8 was best in 14 of 27 cases. In general, negative bias was prevalent amongst most estimations. The 6th variant of 4–8 had negative biases in 8 of 14 cases and the 6th variant of 3–9 had negative biases in 9 of 13 cases. The slight statistically non-significant tendency to underestimate existed if the two most accurate two-measurement variants were used for a QMD estimation.

No clear pattern of optimal two-measurement variants was visible according to the mean diameter size of the generated stands, but a weak pattern was detected according to DoD. The 6th variant of 4–8 occurred with a higher frequency in low DoD (7 of 9 cases), which was opposite to the high DoD stands whereby the 6th variant of 3–9 prevailed (6 of 9 cases). Medium DoD was also consistent with average frequency analysis, e.g., the 6th variant of 3–9 was identified and recommended due to its slight prevalence (in 5 of 9 cases).

Table 7. The optimal centroid rank/combination of measured ranks, accuracy, and bias of QMD estimation for the best two-measurement variants of Weise's rule (the values in columns for each combination of QMD and DoD are optimal rank centroid/combinations of measured ranks followed by RMSE and bias in %).

Degree of	**Real QMD (cm)**									**Mode**
Dispersion	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**Mean/Mean**
	6/4–8	6/3–9	6/4–8	6/4–8	6/4–8	6/4–8	6/4–8	6/3–9	6/4–8	6/4–8
Low	1.56	1.37	1.71	1.62	1.38	1.57	1.65	1.53	1.76	1.57
	–0.77	–0.21	–0.15	–0.97 *	–0.51	0.3	0.03	–0.53	1.21	–0.18
	6/3–9	6/4–8	6/4–8	6/3–9	6/3–9	6/3–9	6/4–8	6/3–9	6/4–8	6/3–9
Medium	1.29	1.38	1.22	1.5	1.14	2.19	1.50	1.81	1.42	1.49
	–0.06	–0.05	–0.32	–0.60	0.16	–1.68 *	0.01	1.18	0.38	–0.11
	6/3–9	6/3–9	6/4–8	6/3–9	6/4–8	6/3–9	6/3–9	6/4–8	6/3–9	6/3–9
High	1.24	1.37	1.62	2.33	1.48	1.49	1.53	1.97	1.34	1.60
	0.26	–0.21	–0.96 *	–1.92 *	0.39	–0.32	0.77	–1.46 *	–0.36	–0.42
Mode	6/3–9	6/3–9	6/4–8	6/3–9	6/4–8	6/3–9	6/4–8	6/3–9	6/4–8	
Mean/Mean	1.36	1.38	1.52	1.82	1.34	1.75	1.56	1.77	1.51	
	–0.19	–0.16	–0.48	–1.16	0.01	–0.57	0.27	–0.27	0.41	

* The statistically significant biases were at a *p*-level of 5%.

4. Discussion and Conclusions

A simple QMD estimation using an appropriate sample quantile is an efficient, practical solution to quickly estimate the QMD of forest stands in the field. Although QMD estimation using Weise's rule is widely used in Czech and Slovak forest inventory practice, an evaluation of the accuracy of QMD quantile estimations and exploration of different estimation alternatives has been missing until now.

Our study clearly demonstrates the usefulness of Weise's method for a range of forest structure types in the Western Carphatians, particularly for single-species beech forests. The advantage of Weise's method is the estimation of QMD at the sampling point using one diameter measurement; systematic group sampling and measurement of ten trees are transformed into a single systematic sampling of one specific tree. The inclusion of information about specific stem ranks into the mathematical measurements provides variable sample point QMD estimates, similar to the variability of sampling point estimates calculated as an average of ten measured diameters. Therefore, the number of sampling points required for an arbitrarily selected accuracy and confidence level could remain equal, but the number of diameter measurements at the sampling point is ten times smaller. Thus, it seems reasonable to expect ten times lower time consumption and measurement costs in comparison to full group sampling.

This simulation study confirmed the recommendation to apply the 6th rank corresponding to the 55% percentile of diameter distribution in accordance with the original empirical rule. However, our study also revealed that the application of the appropriate two-measurement variants of the modified Weise's method (measurement and averaging of 4th and 8th or 3rd and 9th diameters in rank centered to the 6th rank) over a single measurement of the 6th diameter in rank should be preferred in beech stands growing in the Western Carphatians. The accuracy of the best variants of the modified Weise's method was about 15% higher compared to the original accuracy of 5% at the 95% confidence interval, and it was about 25% higher compared to the classic Weise method at comparable measurement costs. One advantage of the two-measurement variants was the absence of significant bias, which was notably decreased in the final accuracy of the single 6th rank estimation.

Recommendations for the two-measurement variant application differ according to DoD category. The variant of the 6th diameter in rank of the 4th–8th rank measurements is recommended in stands with low diameter dispersion, irrespective of the mean diameter size (e.g., artificially regenerated even-aged beech stands tended by thinning from below growing on homogeneous sites). Averaging the 3rd and 9th ranks is suggested for stands with medium and high diameter dispersion, irrespective of the mean diameter size (e.g., stands growing on less homogenous sites with more differentiated age composition, or more variable spatial and vertical stand structure resulting from the application of modern silviculture approaches, such as natural regeneration, intensive crown or target tree thinnings, more close-to-nature management, *etc.*).

It should be noted that the explicit recommendation of the 6th rank is closely related to the characteristics of the empirical material. According to [23], the tendency toward slightly left-skewed diameter distributions persists in beech stands in general in the Western Carpathians. This is clearly reflected in Table 3, where values of skewness (A) and kurtosis (E) varied between 0.1–0.68 and −1.2–0.7, respectively. This indicates that most of the diameter distributions included in the study had a slight left asymmetry and a nearly normal kurtosis distribution. In such cases, simple arithmetic and quadratic mean diameters are always greater than the median and utilization of percentiles over 50% is to be expected as *a priori*. Because of the slight deviation from normal symmetry, the 60% percentile and the 6th rank of measured stem is a natural choice. These stand types were characteristic of managed even-aged, normal age-classed stands favored in the past.

Changing environmental conditions and public demands have encouraged more close-to-nature silviculture and management strategies supporting the diversification of tree age composition and stand structure. Close-to-nature management of beech stands with natural regeneration under shelterwood management supports the desired variability in shapes of diameter distribution curves, where highly asymmetric or even reverse J-shaped distributions are becoming more frequent in forest practice. Therefore, if it has been found that management approaches have favored practices that encourage a divergence of stand structure from normal, even-aged forest structures, we suggest utilizing the two-stage approach. In the first preliminary stage, measurements of a small preliminary set of diameters, e.g., 10 trees, distributed across the stand area in a systematic fashion are recommended. Therefore, sample ordering, estimation of sample quantiles corresponding to measured diameters (for example according to Table 2), and the calculation of QMD from the preliminary diameter set are suggested. Subsequently, regression estimation of parameters of a cumulative density function for a suitable diameter distribution model (e.g., Weibull or SB distribution) according to the principles of percentile-based parameter recovery approaches, and the calculation of exact sample quantile corresponding to preliminary QMD are suggested. Alternatively, the exact sample quantile corresponding to preliminary QMD can be determined using simple interpolation between sample quantiles reported in Table 2.

The calculation of QMD from a preliminary set of diameters and determination of corresponding sample percentile (exact or aproximative) will determine the correct rank (or combination of ranks) of measured stem(s) at sampling points in the second phase. The second phase has the character of common presently applied approaches, which means: (i) the formation of representative systematic sample of sampling points; (ii) visual ordering of ten nearest stems at each sampling point; (iii) measurement of the specific proper rank(s) at each sampling point; and (iv) average QMD point estimates over the whole stand.

A key part of all the above-described approaches is a simple non-parametric rank estimation of sample quantiles. Different definitions of sample quantiles used in several statistical packages are reviewed by [26]. Further research is needed in order to find the optimal definition of sample quantiles from QMD estimation and/or growing stock calculation.

Overall, this study confirmed the usefulness of Weise's rule of thumb. Original and modified versions of Weise's method attained almost invariant bias and accuracy according to different mean diameter sizes, *i.e.*, they achieved similar accuracy in stands with completely different age and site quality. Only the degree of diameter dispersion significantly affected QMD accuracy; diameter dispersion is easily determined from forest management information records, or on the basis of a simple visual inspection of the stand. Two-stage sampling application of Weise's method is recommended in beech stands with very diversified structure, unmanaged or managed by more close-to-nature approaches, to ascertain correct rank(s) of stems measured at a sampling point.

Empirical estimation of QMD by Weise's method is a simple yet highly effective way to determine mean diameter with reasonable accuracy. Moreover, two-measurement modifications of the original rule show a tendency to remain unbiased and achieve a high accuracy of QMD estimation compared to the original method. Therefore, they could be recommended for application in even- and uneven-aged beech forests growing in Central Europe.

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Author Contributions

Róbert Sedmák and Ľubomír Scheer conceived and designed the study, processed and analysed the data, and wrote the paper. Róbert Marušák and Michal Bošeľa contributed to the data analysis and results interpretation. Denisa Sedmáková contributed to the discussion and language correction of the paper. Marek Fabrika supervised and reviewed the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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