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Effect of Particle Form and Surface Friction on Macroscopic Shear Flow Friction in Particle Flow System

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Abstract: The damage caused by landslide disasters is very significant. Among them, landslides after forest fires have been widely concerned by scholars in recent years due to their particular physical and chemical properties. This large-scale shear flow of particulate matter has similarities to fluid systems. However, due to the discontinuity of the particle system, its flow process has significant random characteristics. To investigate the random properties of particle systems, this study conducted a series of ring shear tests on four particle systems. The effects of the particle shape, normal stress, and shear velocity on the systems' shear rheological features were investigated using experimental data. The particle form has an important effect on the macroscopic properties of the system. In a spherical particle system, the macroscopic friction fluctuation is determined by the friction of the particle surface and the system's normal stress. The shear velocity has a minor effect on this characteristic. Three elements simultaneously influence the macroscopic friction fluctuation of a breccia particle system: the particle surface friction, system normal stress, and shear velocity. The origins of macroscopic frictional fluctuations in particle systems with various shapes are fundamentally distinct. This study contributes to a better understanding of the causes of particle system fluctuations, and establishes the theoretical foundation for the future development of disaster prevention technology.

Keywords: granular flow; ring-shear test; fluctuation characteristics

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1. Introduction

Avalanches and landslides are examples of the aggregate movement of an enormous number of solid materials exhibiting considerable fluid-like qualities in nature. These ubiquitous particle flow systems can potentially be disastrous. A series of rain-induced landslides occurred in the Mocoa region on 31 March 2017, and resulted in 333 deaths, 398 injuries, and 76 missing people [1]. On 1 July 2017, a landslide occurred in Ningxiang County, Hunan, China, and nine people were killed in the process of searching for survivors [2]. A statistical study considering the Chittagong area in Bangladesh reported that 730 landslides occurred over 17 years, causing enormous economic losses [3].

Abundant vegetation conditions have been shown in several studies to be of great significance for slowing/suppressing landslides [4,5]. However, forest areas face a very specific and dangerous risk: post-fire mudslides. Researchers used alluvial stratigraphy to reconstruct 32 fire-related alluvial events, indirectly demonstrating the prevalence and hazard of debris flow disasters after major fires [6]. A study on the debris flow after the fire in Australia pointed out that for the same area, the regional peak flow before and after burning can differ by more than 30 times, and the sediment concentration increases by three orders of magnitude [7]. After the fire, the physical and chemical properties of the soil will change greatly [8,9]. How this change in particle properties affects landslides is unclear.

Particle flow occurs naturally in various situations, including landslides and other natural calamities [10,11]. Studies have discovered that particle flow exhibits complicated

random properties in practical engineering [12], and is a classic example of an unstable system in a flow process. In practical engineering, the unstable properties exacerbate the difficulty of predicting landslides. Ceccato et al. calculated the impact force of particle flow on stiff retaining walls and discovered that the impact force fluctuation of a discrete system is substantially larger than that of a continuous system. Zhang et al. [13] investigated the flow impact characteristics of several particle systems by conducting an indoor model experiment and discrete element simulation. They discovered that the temporal history of the impact force is significantly unstable. The peak difference in the impact force between two particle systems operating under the same boundary condition may be more than three-fold. Owing to the uneven nature of the granular system's flow mechanism, more precise landslide control is required.

Most recent studies on the flow characteristics of granular systems have focused on the average macroscopic parameters [14,15]. Numerous particle flow constitutive models can accurately describe the velocity distribution [16], friction coefficients [17], blocking circumstances [18], and flow state transitions [19,20] occurring during the system's flow process. With the advance of research on particle flow systems in recent years, the unstable properties of the particle flow systems' macroscopic motion have been increasingly receiving attention. Artoni et al. [21] investigated the velocity fluctuation and self-diffusion impact of particles in dense particle flow through the discrete element simulation of the ring shear model. They concluded that the fluctuation of a single particle is strongly connected to the system's overall velocity. Similar investigations have been conducted by Meng et al. [22], who established that the boundary effect influences the fluctuation of local particles. Lu et al. [23] conducted a discrete element simulation of overland flow and demonstrated that there exists a considerable correlation between the dynamic characteristic fluctuation of the particle flow system and the system particle size. Huang et al. [24] conducted a series of ring shear experiments to investigate the relationship between particle fragmentation and the fluctuations in the macroscopic shear behavior of particle flow systems. However, the source of the variation in the flow properties of a particle system has not been elucidated to date, and experimental data are lacking. Therefore, additional experimental studies are required to fully clarify the variation mechanism.

Most previous studies have established particle flow shear and rheological models by considering simple spherical particle systems. However, research has revealed that the breccia system used in actual engineering has significantly different compressibility, shear friction, and other properties compared with a simple spherical system. Sun et al. [25] reported that, in non-spherical particle systems, the particle motion is mainly caused by sliding friction and energy dissipation resulting from the great increase of particle rotation/dislocation. Jiang et al. [26] conducted ring-shear experiments on glass beads/quartz sand and demonstrated that the volume compressibility, peak strength, shear residual strength, and other properties of particle systems with varying shapes change significantly during the shear rheological process. To better guide engineering practice, it is very important to investigate complex particle systems. A detailed explanation of the effect of the particle characteristics on the macroscopic features of the system can help in understanding the sources of the random motion characteristics of particle flows. This study investigated the effect of particle features on the shear wave behavior of macroscopic systems by conducting 24 groups of annular shear experiments on various materials' particles with different shapes.

2. Experimental Design of Ring Shear Test

This study used a GCTS ring shear instrument, which can provide significant shear distance without affecting the shear area, and is, therefore, ideal for investigating massive deformation systems. The sample had an exterior diameter of 150 mm, inner diameter of 100 mm, and height of 20 mm to satisfy the H > 10D requirement. Four types of glass beads, quartz sand, spherical corundum, and brecciated corundum were used to investigate the effect of the particle form and surface friction on the system's shear characteristics. Corun-

dum is a widely used granular industrial abrasive with consistent chemical properties, high surface roughness, and low cost. Therefore, this unique material was selected to experimentally investigate the high friction of particles. Each sample was initially screened to a particle size of 2–2.5 mm. In the ring shear experiment, the normal stress gradient was adjusted to 100–200–300 kPa, the shear speed was set to 5°/min–30°/min–90°/min, and the shear distance was set to 360°. There are few velocity groupings and normal stress groupings set in this paper. This allows subsequent data analysis to only achieve trend analysis. In future research, we will use experimental or simulated methods to further supplement the data set to give more detailed conclusions. Table 1 lists the experimental groups and their associated numbers. Experimental groups with repeated trials are highlighted in the table. The sampling rate was set to 10–100 Hz, depending on the duration of the experiment. Pre-experiments were conducted to determine the filling quality of each material and ensure that each sample had the same initial height under a positive pressure of 100 kPa. Finally, the weight of each group of glass beads and quartz sand samples was determined as 330 g, whereas the weight of each group of the corundum samples was determined as 212 g. To ensure consistent filling, the zonal filling approach was adopted. Figure 1 shows the experimental equipment, materials and zonal filling method. This figure is modified from a previously published article by the author [24]. After completing the zonal filling, normal stress was applied to achieve system pre-consolidation and retain strong particle contact. Shear stress was added after the system volume had stabilized. Only the stationary shear part of the system was evaluated during data processing.

Table 1. Experimental group summary.

Number	Material	Normal Force	Shear Velocity
G1-3 *	Glass bead	100 kPa	5°/min–30°/min–90°/min
G4-6		200 kPa	5°/min–30°/min–90°/min
G7-9		300 kPa	5°/min–30°/min–90°/min
Q1-3*	Quartz sand	100 kPa	5°/min–30°/min–90°/min
Q4-6		200 kPa	5°/min–30°/min–90°/min
Q7-9		300 kPa	5°/min–30°/min–90°/min
CS1-3	Spherical corundum	100 kPa	5°/min–30°/min–90°/min
CS4-6		200 kPa	5°/min–30°/min–90°/min
CS7-9		300 kPa	5°/min–30°/min–90°/min
CB1-3	Brecciated corundum	100 kPa	5°/min–30°/min–90°/min
CB4-6		200 kPa	5°/min–30°/min–90°/min
CB7-9		300 kPa	5°/min–30°/min–90°/min

* indicates experimental groups with repeated tests.

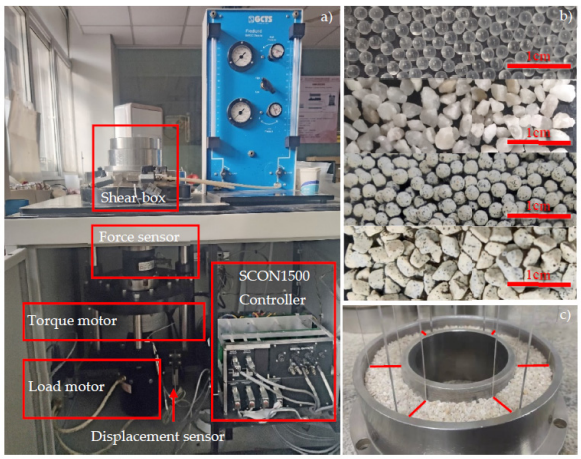

Figure 1. The basic condition of the ring-shear test. (**a**) Structure of GCTS ring-shear apparatus; (**b**) Initial photos of experimental materials; (**c**) Partition filling method. This figure was modified from Huang et al. [24].

3. Analysis of Shear Flow Characteristics of Different Particle Systems

3.1. Compression Characteristics of the Particle System

Figure 2 illustrates the typical shear compression curves of spherical particle systems with different friction coefficients. The shearing trend of the spherical particle system is not substantial, and the overall volume change rate is less than 2%. The compression of the samples mainly occurred in the pre-shear period. The change rate of the sample volume gradually decreased as the shear displacement increased. In the later shear period of the sample with low friction, the volume change rate tends to increase. The particle surface friction is proportional to the particle system's volume change rate and steady-state volume. As the particle surface friction increased, the initial volume changes became slower and the steady-state volume eventually became smaller.

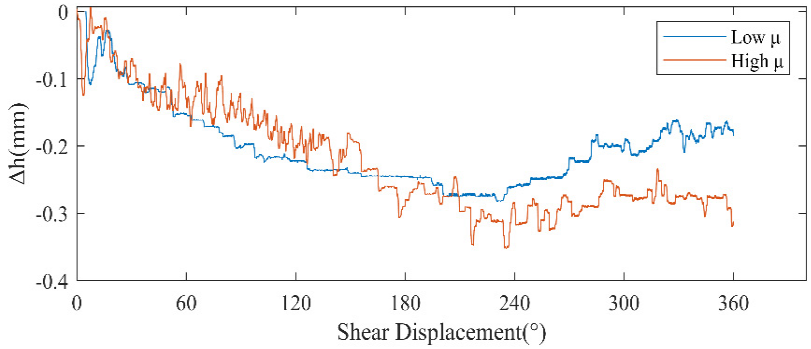

Figure 2. Vertical displacement versus shear displacement of glass beads and spherical corundum at the same shear velocity.

Figure 3 shows the typical shear compression curve of the breccia system with varying friction coefficients. The sample volume changed significantly during the shear process of the angular particle system, and the total volume change rate is approximately 4%–6%. Owing to the pre-treatment of samples, significant particle rearrangement did not occur, and therefore, "early dilatancy—late shear shrinkage" phenomena did not occur. The system's continuous shear shrinkage during shearing is linearly proportional to the shear displacement. For the breccia system, the particle surface friction is proportional to the system's volume change rate. The volume change rate of a high-friction system is much slower than that of a low-friction system.

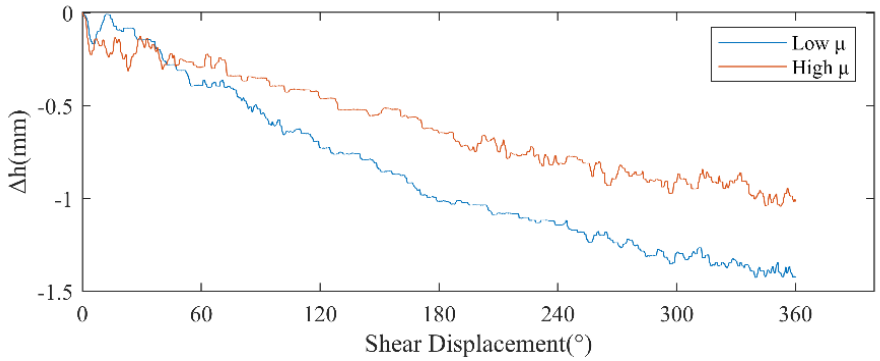

Figure 3. Vertical displacement versus shear displacement of quartz sand and brecciated corundum at the same shear velocity.

According to the particle form, the reasons of shear shrinkage vary, resulting in a variety of shear shrinkage curves for various particle systems. Owing to the highly symmetrical shape of a spherical particle, the volume change of the spherical particle system is governed solely by the particle position distribution. In a complex breccia system, the particle form is complex and the symmetry is inadequate. The volume change

of the system is also affected by distribution parameters such as the long axis angle of the particles. In shear motion, the interaction of particles is more intricate. Hence, it is considered that the spherical arrangement can more easily achieve extreme compression. Notably, breccia systems are more prone to particle shear breakage, which results in system volume loss. In practice, however, it is impossible to avoid the inaccuracy induced by particle breakage, and additional research into novel granular materials is required to supplement the experimental results.

3.2. Macroscopic Friction Analysis of Particle System

Ideally, the system boundary conditions should be stable during the investigation of the shear rheological properties of granular systems. However, owing to the feedback-adjustment mechanism of the ring shear instrument's normal stress loading system, the normal stress cannot be kept constant and exhibits highly random variation. The shear stress of a system is directly proportional to the normal stress, and the normal stress fluctuation has a significant effect on the shear stress. In order to avoid such systemic inaccuracies, the subsequent analysis did not directly address shear stress, but instead statistically evaluated the system's macroscopic friction coefficient. Figure 4 shows the correlation analysis of the stress ratio and shear velocity in the spherical particle system. Figure 3a shows the experimental result for the glass bead system. The macroscopic friction coefficient is 0.33–0.39, and has little association with the shear velocity, normal tension, and other boundary conditions. Moreover, there existed substantial divergence between the macroscopic friction and the rest of the data in the low-speed shear experiment. Figure 3b shows the experimental results for the corundum particle system. The macroscopic friction is 0.385–0.41, which is much larger than that of the glass bead system. In a spherical particle system, the particle surface friction dominates the macroscopic friction coefficient because the relationship between macroscopic friction and boundary conditions is weak. The macroscopic shear friction was not affected by boundary conditions such as the shear velocity and normal tension.

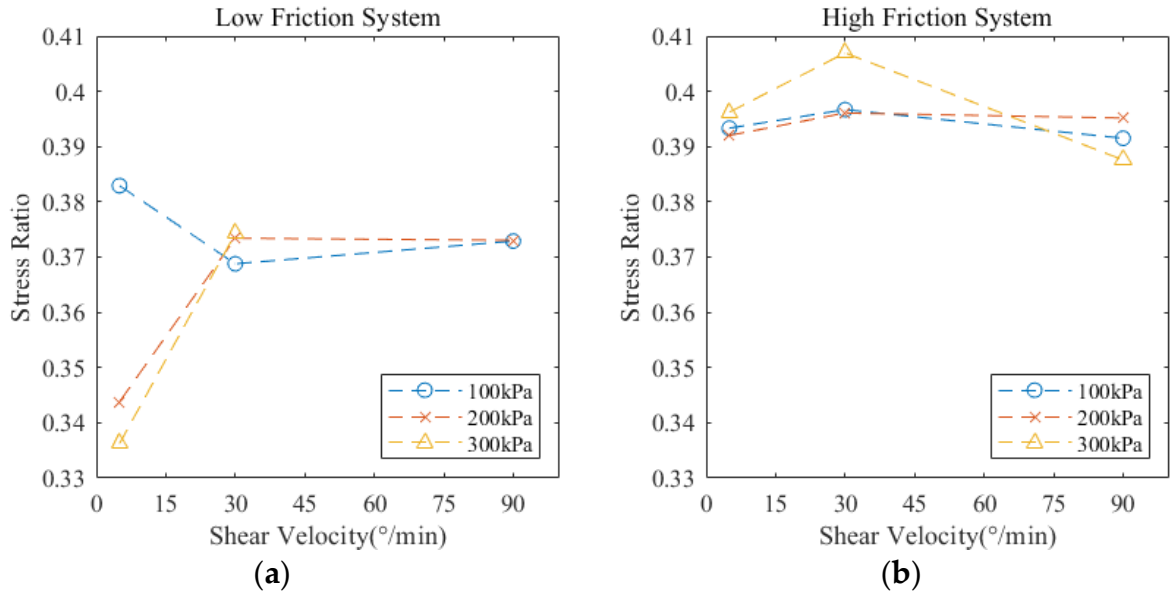

Figure 4. Relationship between shear velocity and stress ratio under different normal stresses. (**a**) Glass bead; (**b**) Spherical corundum.

Figure 5 shows the correlation analysis of the stress ratio and shear velocity in the brecciated particle system, where Figure 5a,b correspond to the quartz sand and brown corundum systems, respectively. There is a modest negative association between the macroscopic friction and shear velocity in breccia particle systems. The normal stress did not obviously influence the macroscopic friction. The macroscopic friction coefficients of

the two distinct granular materials are 0.5–0.55. Therefore, the particle surface friction did not affect the macroscopic friction in the diagonal gravel system.

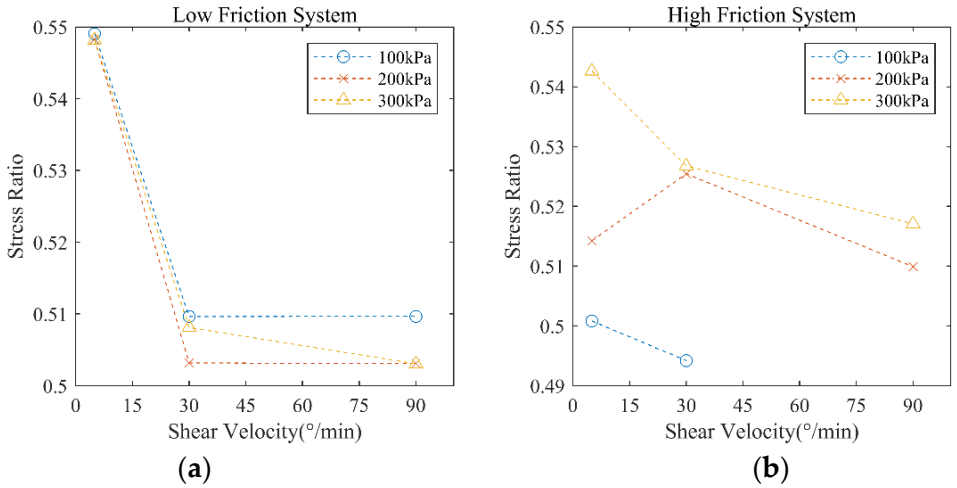

Figure 5. Relationship between shear velocity and stress ratio under different normal stresses. (**a**) Quartz sand; (**b**) Brecciated corundum.

The macroscopic friction of a particle system is mainly determined by the particle form and surface friction, but not by the boundary conditions. The particle form is complicated, and the poor sphericity of the particles increases the system's macroscopic friction. In turn, higher particle surface friction increases the macroscopic friction of a spherical particle system.

3.3. Analysis of Macroscopic Frictional Fluctuation Characteristics of Particle System

Figure 6 shows the investigation of the correlation of the macroscopic frictional fluctuation and shear velocity in the spherical particle system, where Figure 6a shows the glass microbead system and Figure 6b shows the brown corundum spherical system. The link between the shear velocity and the macroscopic fluctuation is low in the spherical system. The system fluctuation was significantly dampened by the normal stress. The particle surface friction significantly affects the system's macroscopic friction fluctuation, which is substantially smaller in a low-friction particle system compared with a high-friction particle system.

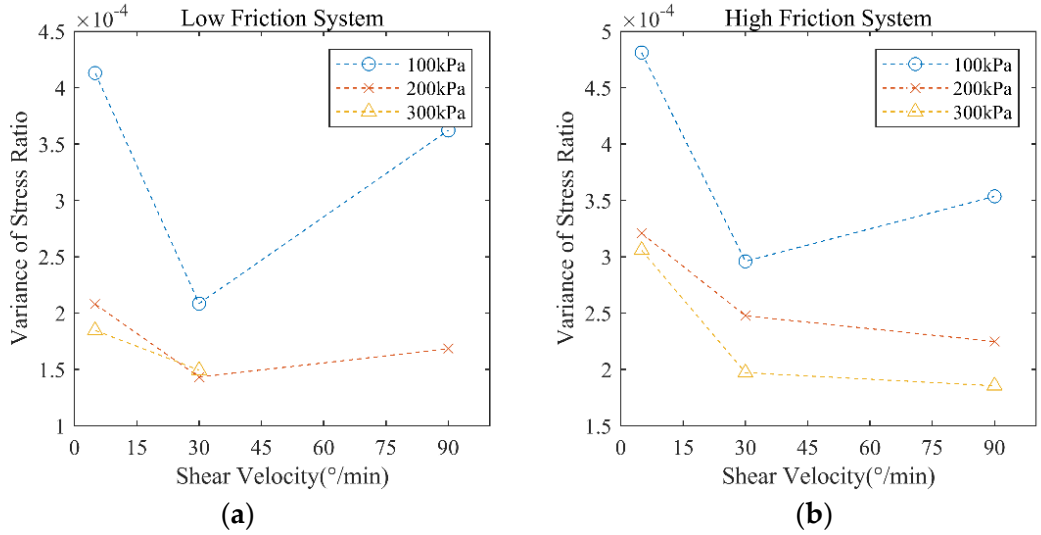

Figure 6. Relationship between shear velocity and the variance of stress ratio under different normal stresses. (**a**) Glass bead; (**b**) Spherical corundum.

Figure 7 shows the correlation analysis of the macroscopic friction fluctuation and shear velocity in the breccia particle system, where Figure 7a shows a quartz sand particle system and Figure 7b shows an angular brown corundum particle system. Higher shear velocity decreases the macroscopic friction fluctuation in breccia systems. Normal stress has a complex effect on the system's macroscopic friction. In a low-friction particle system, the normal stress is inversely proportional to the system's macroscopic friction fluctuation. Higher normal stress suppresses the system fluctuation. However, at V = 30°/min, aberrant behavior was observed, but the reason behind this phenomenon is unknown. In high-friction systems, the normal stress is positively correlated with macroscopic friction fluctuation. Specifically, the normal stress levels increase to the point where they can no longer restrain the system's volatility. Comparative investigation revealed that there exists positive correlation between particle surface friction and the system's macroscopic fluctuation. Particles with a rougher surface exhibit greater system shear fluctuation.

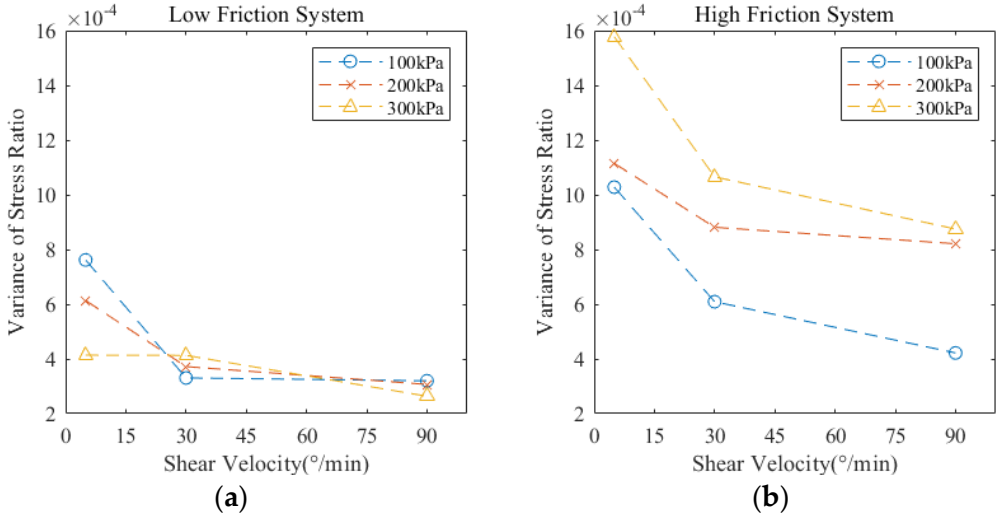

Figure 7. Relationship between shear velocity and the variance of stress ratio under different normal stresses. (**a**) Quartz sand; (**b**) Brecciated corundum.

An additional quantitative investigation revealed that the logarithmic function may be employed to fit the relationship between the macroscopic variation the and shear velocity with remarkable precision.

$$\sigma^2 = Aln(v) + B \quad (1)$$

where A and B are state parameters related to the particle form, surface friction, and stress conditions, respectively. When $A = 0$, the spherical particle system can be considered as a particular solution. The precise expression form of A/B can be further investigated using discrete element simulation and other techniques.

4. Analysis of Causes of Fluctuant Features in Particle Systems

The factors influencing the mean value of macroscopic friction and the fluctuation features of the particle systems were investigated based on the experimental data. The relevant laws are more complicated owing to the numerous factors involved. To simplify the discussion in this section, the pertinent laws obtained as described in the previous section are provided in tabular form. Table 2 summarizes the conclusions drawn for the spherical particle system.

Table 2. Conclusions in the spherical particle system.

	Particle Surface Friction	Normal Stress	Shear Velocity
Macroscopic friction	+	x	x
Macroscopic frictional fluctuation	+	-	x

+ indicates a positive correlation, - indicates a negative connection, and x indicates an irrelevant association.

According to previous studies, macroscopic variation is generated by intergranular split-layer occlusion. The primary cause is thought to be the "particle bite–slip–over–rebite" process. If this factor is significant in a spherical particle system, the macroscopic fluctuation will be positive relative to the shear velocity. However, such positive correlation does not exist in the experimental data obtained by this study. Hence, split-layer occlusion is not responsible for the macroscopic variation in a spherical particle system.

The mesoscopic mechanism of macro fluctuations may be related to the shift between rolling and sliding friction. Owing to the high degree of symmetry of spherical particles, their relative motion is mainly rolling motion. However, sliding friction between particles may exist under specific boundary conditions. This type of local dislocation increases the local stress ratio, which in turn results in the fluctuation of the system's macroscopic friction coefficient. To verify this hypothesis, discrete element modeling must be carried out to capture the system's mesoscopic motion characteristics.

Table 3 summarizes the conclusions drawn for the breccia system.

Table 3. Conclusions in the breccia system.

	Particle Surface Friction	Normal Stress	Shear Velocity
Macroscopic friction	x	x	(-)
Macroscopic frictional fluctuation	+	(+)	-

+ indicates a positive correlation, - indicates a negative connection, x indicates an irrelevant association, and ( ) indicates a weak correlation.

The biting force between particles may be the main factor controlling the macroscopic friction fluctuation in breccia systems. The system fluctuation increases when the inter-particle occlusion is tighter. The roughness of the particle surface, which increases in normal stress and decreases in shear velocity, contribute to the increase in the biting force between the particles and system fluctuation. This conclusion is strongly supported by the experimental results. Hence, for the breccia system, the non-stationary "intergranular occlusal–over–reocclusal" process is the primary cause of the system's macroscopic mechanical behavior fluctuation.

This study ignored the influence of particle breakage, which is a limitation of the investigation presented herein. The experimental conditions considered in this study cannot ensure that particles are not shattered in a breccia system. Therefore, numerical tests or the development of new high-strength materials are required to further refine the experimental conclusions.

5. Conclusions

Particle systems have fluid-like properties and exhibit significant instability during the flow process. The physical and chemical properties of the soil will change significantly after forest fires, and the changes in the properties of the particles themselves have an impact on the macroscopic flow characteristics of the system, such as particle shape, particle surface friction, shear boundary conditions, and other factors. This study investigated the causes of fluctuation in four particle systems by conducting a series of ring shear experiments. The consideration of the random properties of particle flow systems in practical engineering can assist in the prevention of landslides and other common geological disasters and provides

a theoretical foundation for the development of relevant engineering technology. The following main conclusions were drawn from this study:

(1) The shape of the particles has a significant effect on the trend of volume variation during the system's shear process. Compression limits exist for spherical particle systems. The sample volume of a breccia particle system diminishes as the shear distance increases.

(2) For a spherical particle system, the mean value of the macroscopic friction is independent from the boundary conditions and proportional to the particle surface friction. The macroscopic friction fluctuation of the system is determined by the particle surface friction and the system's normal stress and is completely independent of the shear velocity.

(3) In a breccia system, the mean value of the macroscopic friction is only weakly connected to the shear velocity. Additionally, three elements affect the macroscopic friction fluctuation: particle surface friction, system normal stress, and shear velocity.

(4) The fluctuating macroscopic friction of the spherical particle system is a result of the changing friction mode between particles. The "biting-over-rebiting" mechanism of the interlayer particles is responsible for the macroscopic friction fluctuation of the system in a breccia particle system.

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