A Brief Introduction to Compressed Sensing

In the light of compressed sensing theorem, if a signal is sparse, or sparse in a specific transform domain, it can be projected into a low-dimensional space by an observation matrix which is not related to the sparse basis. Then, the original signal can be reconstructed from few projections by solving an optimization problem.

The mathematical explanation of compressed sensing is as follows. Given an original signal $x \in R^{N\times 1}$, which is non-sparse, real-valued, finite-length, one-dimensional and discrete-time, it can obtain a sparse representation on $N \times N$ orthogonal basis matrix $\Psi = [\psi_1, \psi_2 \dots \psi_N]$:

$$x = \sum_{i=1}^{N} s_i \psi_i, \tag{1}$$

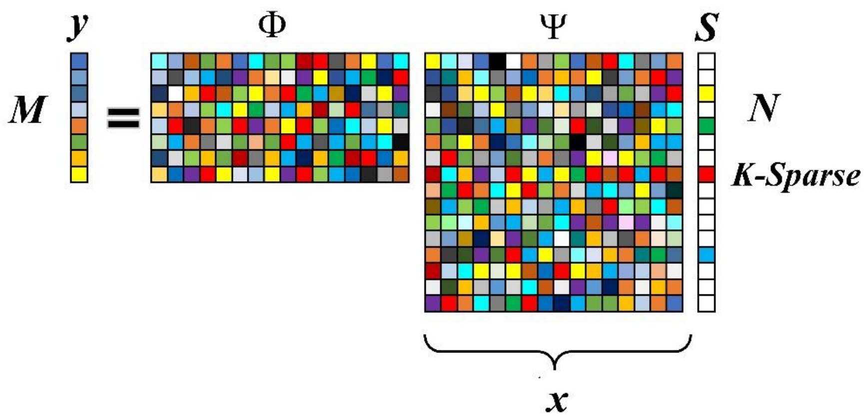
where weighting coefficients $s_i = \langle x, \psi_i \rangle = \psi_i^T x$ and superscript T denotes transposition. Thus, an equivalent sparse representation of original signal x is obtained:

$$x = \Psi S, \tag{2}$$

where $S = [s_1, s_2 \dots s_N]$. Supposed that $K \ll N$ and N is extremely large but the rest N-K elements are very small, S is K-sparse and compressive. Then, an observation matrix $\Phi \in R^{M\times N}$ uncorrelated with the sparse basis is applied to measuring the signal x:

$$y = \Phi x, \tag{3}$$

where measurement result $y \in R^{M\times 1}$. Note that the length of y is M, and $M \ll N$. Hence, a compressed signal y is obtained when sampling the original signal x, and y contains enough information to reconstruct the original signal x.



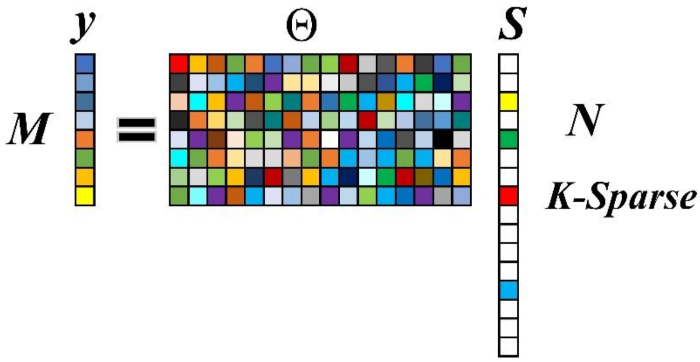
(**a**)

By merging Equation (2) and Equation (3), we get the following equation (Figure S1a):

$$Y = \Phi \Psi s, \tag{4}$$

For simplicity, let $\Theta = \Phi\Psi$, Eq. (4) can be rewritten as (Figure S1b):

$$y = \Theta s. \tag{5}$$



(**b**)

Figure S1. Compressed sensing schematic. (**a**) Schematic diagram of compressed sensing with observation matrix and sparse matrix. (**b**) The sensing matrix is obtained through merging the observation matrix and sparse matrix for simplicity.

In practice, due to $M \ll N$, that is, the number of equations is far less than the number of unknowns. The Equation (5) is an undetermined equation and there is no definite solution. Nevertheless, since S is K-sparse and observation matrix Φ satisfies RIP, S can be accurately reconstructed from M measurements. Finally, the original signal x can be obtained by Equation (2).