

**Supplementary Materials.** Detailed results from the piecewise linear regression analysis of the various sap flow methods against gravimetric measurements of sap flow. SRFM: slow rates of flow method (also known as the heat ratio method, HRM), see equation 4; Tmax\_Coh and Cohen's Tmax Method: see equation 2; Tmax\_Klu and Kluitenberg's Tmax method: see equation 3;  $k_{Vand}$  and  $k_{Hogg}$ : sapwood thermal diffusivity, see equation 5; Slope was determined from the linear regression curve and Error is based on the deviation of this slope from 1; RMSE is the root mean square error of the linear regression (smaller values indicate a more accurate model); n is sample size;  $V_h$  is heat velocity;  $V_{m\_critical}$  *a priori* Method was determined on heat velocity (cm/hr) data: see equation 10;  $V_{m\_critical}$  *posteriori* Method was determined via a breakpoint analysis on sap flux density (cm<sup>3</sup>/cm<sup>2</sup>/hr) data.

| Method                                 | Tmax Method   | k Method | Slope | R <sup>2</sup> | RMSE   | Error  | n   | Min. $V_h$ (cm/hr) | Max. $V_h$ (cm/hr) | $V_{m\_critical}$ Method | $V_{m\_critical}$ Value |
|--|---------------|----------|-------|----------------|--------|--------|-----|--------------------|--------------------|--------------------------|-------------------------|
| SRFM ( $k_{Hogg}$ )                    |               | Hogg     | 0.366 | 0.458          | 31.149 | 63.4 % | 300 | 0.020              | 32.383             |                          |                         |
| SRFM ( $k_{Vand}$ )                    |               | Vand     | 0.509 | 0.458          | 27.453 | 49.1 % | 300 | 0.028              | 45.063             |                          |                         |
| Tmax_Coh ( $k_{Hogg}$ )                | Cohen's       | Hogg     | 1.213 | 0.841          | 19.754 | 21.3 % | 221 | 4.256              | 145.039            |                          |                         |
| Tmax_Coh ( $k_{Vand}$ )                | Cohen's       | Vand     | 1.005 | 0.807          | 12.816 | 0.5 %  | 151 | 4.419              | 135.880            |                          |                         |
| Tmax_Klu ( $k_{Hogg}$ )                | Kluitenberg's | Hogg     | 1.355 | 0.861          | 24.340 | 35.5 % | 225 | 2.049              | 168.578            |                          |                         |
| Tmax_Klu ( $k_{Vand}$ )                | Kluitenberg's | Vand     | 1.178 | 0.885          | 14.300 | 17.8 % | 158 | 7.960              | 159.667            |                          |                         |
| DMA_Coh ( $k_{Hogg}$ ) <i>a priori</i> | Cohen's       | Hogg     | 1.148 | 0.842          | 17.839 | 14.8 % | 300 | 0.020              | 145.039            | <i>a priori</i>          | 16.928 (cm/hr)          |
| DMA_Coh ( $k_{Vand}$ ) <i>a priori</i> | Cohen's       | Vand     | 0.984 | 0.888          | 11.896 | 1.6 %  | 292 | 0.028              | 135.880            | <i>a priori</i>          | 22.817 (cm/hr)          |
| DMA_Klu ( $k_{Hogg}$ ) <i>a priori</i> | Kluitenberg's | Hogg     | 1.285 | 0.852          | 22.130 | 28.5 % | 300 | 0.020              | 168.578            | <i>a priori</i>          | 15.812 (cm/hr)          |

|  |               |      |       |       |        |        |     |       |         |                   |  |
|--|---------------|------|-------|-------|--------|--------|-----|-------|---------|-------------------|--|
| DMA_Klu<br>( $k_{Vand}$ )<br><i>a priori</i>   | Kluitenberg's | Vand | 1.107 | 0.887 | 14.208 | 10.7 % | 298 | 0.028 | 159.667 | <i>a priori</i>   | 24.588<br>(cm/hr)                                |
| DMA_Coh<br>( $k_{Hogg}$ )<br><i>posteriori</i> | Cohen's       | Hogg | 1.116 | 0.935 | 11.375 | 11.6 % | 300 | 0.020 | 145.039 | <i>posteriori</i> | 32.540<br>(cm <sup>3</sup> /cm <sup>2</sup> /hr) |
| DMA_Coh<br>( $k_{Vand}$ )<br><i>posteriori</i> | Cohen's       | Vand | 0.988 | 0.918 | 10.446 | 1.2 %  | 300 | 0.028 | 135.880 | <i>posteriori</i> | 32.540<br>(cm <sup>3</sup> /cm <sup>2</sup> /hr) |
| DMA_Klu<br>( $k_{Hogg}$ )<br><i>posteriori</i> | Kluitenberg's | Hogg | 1.240 | 0.940 | 15.526 | 24.0 % | 300 | 0.020 | 168.578 | <i>posteriori</i> | 32.540<br>(cm <sup>3</sup> /cm <sup>2</sup> /hr) |
| DMA_Klu<br>( $k_{Vand}$ )<br><i>posteriori</i> | Kluitenberg's | Vand | 1.110 | 0.932 | 11.526 | 11.0 % | 300 | 0.028 | 159.667 | <i>posteriori</i> | 32.540<br>(cm <sup>3</sup> /cm <sup>2</sup> /hr) |