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A Risk-Aware Supply Function Nash Equilibrium Framework for Strategic Bidding in Day-Ahead Electricity Markets

Muhammad Muzammal Islam ^{1,*}, Tianyou Yu ², Massimo La Scala ¹, Sergio Bruno ¹, Ziqiang Wang ³,
Cosimo Iurlaro ¹ and Andrea Altamura ¹

¹ Department of Electrical and Information Engineering (DEI), Politecnico di Bari, 70126 Bari, Italy; massimo.lascala@poliba.it (M.L.S.); sergio.bruno@poliba.it (S.B.); cosimo.iurlaro@poliba.it (C.I.); a.altamura12@studenti.poliba.it (A.A.)

² School of Electronic, Information and Electrical Engineering (SEIEE), Shanghai Jiao Tong University, Shanghai 200240, China; tianyouyu@sjtu.edu.cn

³ Department of Electrical and Electronic Engineering, The Hong Kong Polytechnic University, Hong Kong; polyu-ziqiang.wang@polyu.edu.hk

* Correspondence: muhammadmuzammal.islam@poliba.it

Abstract

Strategic bidding in day-ahead electricity markets requires generation companies to maximize expected profits while managing financial risks arising from market uncertainty and competitors' strategic behavior. This paper proposes a game-theoretic risk-aware strategic bidding framework based on a Supply Function Nash Equilibrium (SFNE) for dominant market operators in a uniform-pricing day-ahead market. Each operator strategically determines cluster-level bidding markups for its heterogeneous generation portfolio while anticipating competitors' strategies. Demand uncertainty is represented by a finite scenario set, and Conditional Value-at-Risk (CVaR) of profit shortfall is incorporated into each operator's expected-profit objective. The resulting non-cooperative equilibrium is solved using a relaxed Nikaido–Isoda (NI) best-response algorithm with convergence criteria based on the relative NI gap, strategy variation, and utility variation. The framework is validated using publicly available Italian day-ahead market offer data, where technology-oriented clustering reduces the strategic decision dimension while preserving portfolio heterogeneity. The proposed algorithm satisfies all convergence criteria within approximately 54 iterations. Numerical results show that the proposed risk-aware SFNE reduces aggregate downside-profit risk by approximately 7% compared with the risk-neutral SFNE while maintaining comparable expected profitability and slightly lowering market-clearing prices and procurement costs. A realistic 24 h market simulation further confirms the robustness and applicability of the proposed framework under time-varying market conditions. Overall, the proposed framework provides an economically interpretable and computationally tractable benchmark for risk-aware strategic bidding in day-ahead electricity markets.

Keywords: day-ahead electricity market; market-clearing mechanism; game theory; strategic bidding; supply function Nash equilibrium (SFNE); conditional value-at-risk (CVaR)



Academic Editor: Bo Yang

Received: 19 July 2026

Revised: 5 August 2026

Accepted: 7 August 2026

Published: 9 August 2026

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1. Introduction

In recent years, the European electricity market has experienced significant structural stress, particularly following geopolitical events such as the Russia–Ukraine conflict, which triggered a severe energy crisis across Europe. This crisis intensified discussions on electricity market reform owing to extreme natural gas price volatility and the increasing transition

toward low-carbon energy systems. Since gas-fired units frequently set marginal prices in several European markets, natural gas price shocks can be transmitted to day-ahead electricity prices. These developments have intensified debate regarding the distributional and price-formation effects of market-clearing mechanisms during periods of elevated gas prices. Reducing the dominant influence of gas on electricity price formation has therefore become a key objective of ongoing market reform initiatives [1–4]. Under these evolving market conditions, dominant market operators, each owning multiple generation units across different generation technologies, have stronger incentives to strategically adjust their bidding decisions to maximize profits, giving rise to complex strategic interactions in the day-ahead electricity market. Since each participant's optimal bidding strategy depends on the anticipated decisions of its competitors, non-cooperative game theory provides a rigorous mathematical framework for modeling these interdependent decision-making processes and characterizing stable market outcomes through Nash equilibrium analysis.

Over the past few years, strategic bidding has attracted increasing attention owing to electricity market reforms and the need to better understand market operator behavior in competitive electricity markets. Since bidding incentives depend on the market-clearing mechanism, strategic bidding analysis must account for the adopted settlement rules. In European day-ahead markets, the uniform-pricing Pay-as-Clear (PaC) mechanism remunerates all accepted offers at a common market-clearing price determined by the marginal accepted offer. While promoting economically efficient dispatch, this mechanism makes market outcomes highly sensitive to the bidding behavior of marginal generating units [5,6]. Alternatively, under the Pay-as-Bid (PaB) mechanism, accepted generators are remunerated according to their individual bid prices, creating different incentives for strategic bid shading and price formation [7,8]. Comparative studies, such as [9], have shown that these market-clearing mechanisms lead to fundamentally different strategic bidding behaviors, influencing market-clearing prices, procurement costs, producer profits, and overall market efficiency. Given that the uniform-pricing mechanism remains the dominant market design in European day-ahead electricity markets, this work examines strategic bidding under this framework using game theory to model the interactions among dominant market operators.

From a methodological perspective, strategic bidding in day-ahead electricity markets has been extensively investigated using mathematical optimization, heuristic and metaheuristic algorithms, and game-theoretic approaches. For instance, the study in [10] proposed an AI-enhanced bidding framework for virtual power plants by integrating a non-dominated sorting genetic algorithm with long short-term memory forecasting to determine optimal bidding strategies, while the study in [11] presented an enhanced Benders decomposition framework with a privacy-preserving decentralized optimization model for virtual power plant bidding. Moreover, the research study in [12] developed a Lyapunov optimization-based real-time bidding strategy for electric vehicle aggregators under uncertain operating conditions. Beyond optimization-based approaches, game theory has also been widely applied to model strategic interactions in transactive energy systems and electricity markets through both cooperative and non-cooperative formulations. For example, the study in [13] proposed a cooperative transactive energy framework for PV-based multi-microgrids, whereas the research study in [14] introduced a master–slave Stackelberg game for multi-microgrid energy trading to minimize operational costs while ensuring fair profit allocation. More recently, the study in [15] developed a Stackelberg game-theoretic bidding framework for energy storage in multi-microgrid systems to balance the interests of multiple stakeholders and to improve overall operational efficiency. Furthermore, the research study in [16] provided a comprehensive survey of evolutionary game theory applications for analyzing the strategic behavior of power generation companies and electricity purchasers in competitive electricity markets. Although game-theoretic approaches have

been widely applied to strategic bidding in electricity markets, limited attention has been given to technology-oriented cluster-level bidding for heterogeneous generation portfolios. Consequently, the effects of technology-specific bidding on market equilibrium and price formation remain largely unexplored. Furthermore, integrated frameworks combining technology-based clustering with equilibrium analysis remain scarce, highlighting the need to explicitly model technology-dependent strategic bidding behavior.

In recent years, risk aversion has received increasing attention in research on competitive electricity markets owing to rising fuel price volatility, the growing penetration of intermittent renewable energy sources, and uncertainties in energy demand and electricity market conditions [17,18]. Consequently, numerous studies have investigated risk-aware methodologies to better characterize rational market behavior under uncertainty. For instance, the study in [19] developed a risk-aware strategy for the optimal participation of parking lots in the day-ahead electricity market, enabling them to buy and sell electricity in the wholesale market while maximizing expected profit. Similarly, the work in [20] incorporated Conditional Value-at-Risk (CVaR) into the electricity market optimization framework for energy storage system (ESS) operators to mitigate the financial risk arising from uncertainty in imbalance prices. Likewise, the study in [21] proposed a two-stage stochastic risk-aware framework for the coordinated day-ahead and real-time balancing of wind, solar, and pumped hydro storage systems to maximize total profit while reducing the expected renewable energy curtailment rate. Furthermore, the review in [22] presented a comprehensive overview of recent advances in risk-averse bidding methodologies for virtual power plants under multiple sources of uncertainty. Despite these advances, limited attention has been devoted to incorporating downside financial risk into equilibrium-based strategic bidding models that capture strategic interactions among competing market operators. Risk-neutral formulations may expose operators to low-profit scenarios and inadequately represent risk-averse behavior. Incorporating CVaR into a Supply Function Nash Equilibrium (SFNE) framework therefore provides a systematic means of analyzing the profit–risk trade-off and its effects on equilibrium markups, market prices, and procurement costs.

Motivated by the above research gaps, this work adopts an equilibrium-based game-theoretic framework that provides an economically interpretable and computationally tractable approach for strategic bidding analysis. SFNE is particularly well suited to day-ahead electricity markets because it explicitly captures the strategic interactions among competing market operators within a uniform-pricing market-clearing mechanism. Since the dominant market operators collectively own multiple heterogeneous generation units, account for a substantial share of the selected market, and exert the greatest influence on uniform market-clearing prices, the proposed framework focuses on the five dominant market operators to capture the principal strategic interactions while maintaining computational tractability. To highlight the research gap and clearly distinguish the proposed framework from existing studies, Table 1 compares the main characteristics of representative strategic bidding approaches reported in the literature.

The main contributions of this paper are summarized as follows:

- A scenario-based risk-aware SFNE formulation is developed for cluster-level strategic bidding by dominant operators under a uniform-pricing day-ahead market. Unlike conventional risk-neutral equilibrium models, the proposed formulation incorporates the CVaR-based downside financial risk directly into each operator's non-cooperative decision problem.
- A technology-oriented clustering representation is proposed and integrated with the equilibrium model to preserve heterogeneous bidding behavior across generation portfolios while reducing the dimensionality of the operator-level strategy space.

- An enhanced relaxed Nikaido–Isoda (NI) best-response solution procedure is proposed for the resulting risk-aware game, together with a multi-criterion equilibrium verification scheme based on the relative NI gap, strategy updates, and risk-adjusted utility variations.
- A fixed strategy versus reoptimized equilibrium assessment is introduced to distinguish the intrinsic effect of increasing demand uncertainty from the downside-risk mitigation achieved through strategic reoptimization.
- The proposed framework is validated using publicly available Italian day-ahead electricity market offer data. Comparative and sensitivity studies quantify the effects of truthful, risk-neutral, and risk-aware bidding on operator profits, downside-profit CVaR, equilibrium markups, market-clearing prices, and procurement costs.

Table 1. Comparison of representative strategic bidding studies with the proposed framework.

Ref.	Data Processing & Clustering	Heterogeneous Portfolio Representation	Technology-Level Strategic Bidding	Explicit Risk Aversion	SFNE-Based Framework	Real-Market Validation
[10]	✓	✓	—	—	—	✓
[11]	✓	✓	—	✓	—	✓
[12]	✓	✓	—	—	—	✓
[15]	—	✓	—	—	—	✓
[16]	✓	✓	—	✓	—	✓
[18]	✓	✓	—	✓	—	✓
[19]	✓	✓	—	—	—	✓
[20]	✓	✓	—	✓	—	✓
[21]	✓	✓	—	✓	—	✓
This paper	✓	✓	✓	✓	✓	✓

The remainder of this paper is organized as follows: Section 2 presents the problem statement and the mathematical formulation of the market-clearing mechanism. Section 3 introduces the proposed non-cooperative game-theoretic risk-aware SFNE framework. Section 4 describes the electricity market data preprocessing and technology-oriented clustering procedures. Section 5 presents the numerical results, discusses the strategic bidding performance in the day-ahead electricity market, demonstrates the superiority of the proposed framework through a 24 h market simulation under realistic operating conditions, and outlines the modeling assumptions considered in this study. Finally, Section 6 concludes this paper and outlines future research directions.

2. Problem Statement and Mathematical Formulation

2.1. Problem Statement

Strategic bidding in day-ahead electricity markets requires market participants to determine profit-maximizing bidding strategies under competitive conditions. Since the market-clearing price and accepted generation depend on all submitted offers, each operator's profit is affected by competitors' bidding decisions. In addition, uncertainty in forecast day-ahead demand influences market-clearing prices, accepted generation, and operator profits, increasing the complexity of strategic bidding. This paper considers a pool-based day-ahead electricity market with rigid demand under a uniform-pricing mechanism. Each dominant market operator owns heterogeneous generation units that are aggregated into representative supply-offer clusters according to generation technology and marginal cost. Strategic bidding is implemented through cluster-level markups that adjust offer prices while preserving portfolio characteristics. The market-clearing model minimizes the total accepted-offer cost subject to the demand constraint. This section presents the

mathematical formulation of the market-clearing mechanism and operator profit model, while the proposed risk-aware SFNE framework is developed in the following section. Figure 1 illustrates the overall architecture of the proposed game-theoretic risk-aware SFNE framework for strategic bidding in the day-ahead electricity market.

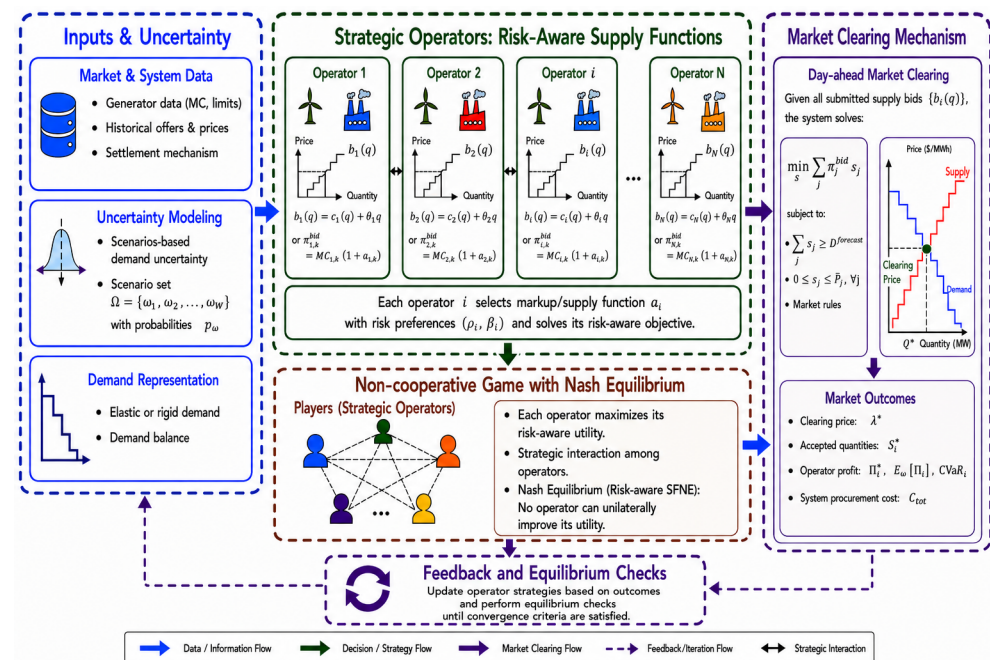


Figure 1. Game-theoretic risk-aware SFNE framework for day-ahead electricity market bidding.

2.2. Day-Ahead Uniform-Pricing Market-Clearing Mechanism

Let $\mathcal{J}$ denotes the set of submitted supply offers participating in the day-ahead electricity market. Each offer $j \in \mathcal{J}$ is characterized by its submitted offer price b_j [€/MWh], available generation quantity Q_j [MWh], and accepted quantity s_j [MWh]. Assuming an inelastic demand D [MWh], the day-ahead market is cleared through a uniform-pricing mechanism by solving the following optimization problem:

$$\begin{cases} \min_{\{s_j\}} & \sum_{j \in \mathcal{J}} b_j s_j \\ \text{s.t.} & \sum_{j \in \mathcal{J}} s_j = D, \\ & 0 \leq s_j \leq Q_j, \quad \forall j \in \mathcal{J}. \end{cases} \quad (1)$$

The optimization model in (1) determines the least-cost market-clearing solution by selecting the accepted generation quantities from all submitted offers. The objective function minimizes the total accepted-offer cost, while the first constraint guarantees the system-wide power balance by enforcing complete demand satisfaction. The second constraint limits the accepted quantity of each submitted offer to its available generation capacity, thereby ensuring the physical feasibility of the market-clearing solution. The market is modeled as a single, network-unconstrained bidding zone; therefore, transmission constraints, congestion, and zonal price separation are neglected.

For the deterministic market-clearing problem in (1), the corresponding uniform market-clearing price is determined as

$$\lambda = \max_{j \in \mathcal{J}} \{b_j \mid s_j > 0\}, \quad (2)$$

$$C = \lambda D, \quad (3)$$

where λ [€/MWh] denotes the uniform market-clearing price, and C [€] denotes the total market procurement cost. Accordingly, all accepted supply offers are remunerated at the uniform market-clearing price, and the corresponding procurement cost is computed using (3). Equations (2) and (3) determine the market-clearing price and the associated procurement cost under the uniform-pricing market-clearing mechanism.

The total accepted energy of operator i is

$$S_i = \sum_{k \in \mathcal{K}_i} \sum_{j \in \mathcal{J}_{ik}} s_j, \quad (4)$$

where S_i [MWh] denotes the total accepted energy of operator i , $\mathcal{K}_i$ denotes the set of generation clusters owned by operator i , and $\mathcal{J}_{ik}$ denotes the set of submitted offer blocks associated with cluster $k \in \mathcal{K}_i$.

Since the market is cleared through a uniform-pricing mechanism, the corresponding market revenue is

$$R_i = \lambda S_i, \quad (5)$$

where R_i [€] denotes the total market revenue of operator i .

The total production cost is given by

$$G_i = \sum_{k \in \mathcal{K}_i} \sum_{j \in \mathcal{J}_{ik}} c_j^{\text{mc}} s_j, \quad (6)$$

where G_i [€] denotes the total production cost of operator i , and c_j^{mc} [€/MWh] denotes the marginal production cost associated with offer block j .

Accordingly, the operating profit of operator i is computed as

$$\Pi_i = R_i - G_i, \quad (7)$$

where Π_i [€] denotes the operating profit of operator i . Equations (4)–(7) quantify the economic performance of each strategic operator. Specifically, (4)–(6) compute the accepted generation, market revenue, and production cost, respectively, while (7) defines the operating profit used as the economic performance metric in the proposed game-theoretic risk-aware strategic bidding framework.

3. Non-Cooperative Game Theory and Risk-Aware Supply Function Nash Equilibrium

3.1. Non-Cooperative Strategic Bidding Game

Building upon the day-ahead market-clearing formulation presented in Section 2, strategic bidding is formulated as a non-cooperative game in which dominant market operators independently determine their bidding strategies to maximize their individual economic objectives. Since the uniform market-clearing price and accepted generation quantities depend on the submitted offers of all market participants, the economic performance of each operator is influenced by the strategic decisions of its competitors.

Let $\mathcal{I} = \{1, \dots, I\}$ denote the set of strategic market operators, where I is the total number of strategic operators participating in the day-ahead electricity market. The strategic interaction is represented by the game

$$\mathcal{G} = (\mathcal{I}, \{\mathcal{A}_i\}_{i \in \mathcal{I}}, \{U_i\}_{i \in \mathcal{I}}), \quad (8)$$

where $\mathcal{G}$ denotes the non-cooperative strategic bidding game, $\mathcal{A}_i$ is the feasible strategy set of operator i , and U_i is its risk-adjusted utility function. The cluster-level bidding strategy of operator i is defined as

$$\alpha_i = [\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{i|\mathcal{K}_i|}]^\top, \quad (9)$$

where α_i denotes the strategy vector of operator i , α_{ik} [%] is the bidding markup assigned to cluster $k \in \mathcal{K}_i$, $|\mathcal{K}_i|$ is the number of clusters owned by operator i , and $(\cdot)^\top$ denotes vector transposition.

The feasible strategy set of operator i is

$$\mathcal{A}_i = \{\alpha_i \mid 0 \leq \alpha_{ik} \leq \bar{\alpha}, \quad \forall k \in \mathcal{K}_i\}, \quad (10)$$

where $\bar{\alpha}$ [%] denotes the maximum admissible cluster-level bidding markup.

The joint strategy profile of all strategic operators is

$$\alpha = [\alpha_1^\top, \alpha_2^\top, \dots, \alpha_I^\top]^\top, \quad \alpha \in \mathcal{A} = \prod_{i \in \mathcal{I}} \mathcal{A}_i, \quad (11)$$

where α denotes the joint bidding strategy, $\mathcal{A}$ is the corresponding joint strategy space, and $\prod$ denotes the Cartesian product of the individual strategy sets.

Equations (8)–(11) define the strategic operators, their cluster-level bidding strategies, individual feasible strategy sets, and joint strategy profile. The latter constitutes the strategy profile of the proposed SFNE formulation.

3.2. Cluster-Level Supply Function Bidding Model

To reduce the computational burden associated with unit-level strategic bidding while preserving the heterogeneity of generation portfolios, the generating units owned by each strategic operator are aggregated into representative supply-offer clusters according to their generation technologies and marginal production costs. All offer blocks belonging to the same cluster are assigned a common strategic markup, yielding the following cluster-level supply function:

$$b_j(\alpha) = c_j^{\text{mc}} \left(1 + \frac{\alpha_{ik}}{100} \right), \quad j \in \mathcal{J}_{ik}, \quad i \in \mathcal{I}, \quad k \in \mathcal{K}_i, \quad (12)$$

where $b_j(\alpha)$ [€/MWh] denotes the strategic offer price of block j under the joint strategy profile α . Equation (12) preserves the original offer quantities and block structure while allowing the cluster-level markups to modify the submitted merit order.

3.3. Scenario-Based Demand Representation

To account for demand uncertainty, the proposed framework considers a finite scenario set Ω generated around the forecast day-ahead demand using predefined demand multipliers γ^ω . The corresponding occurrence probabilities satisfy $\sum_{\omega \in \Omega} p_\omega = 1$, providing a computationally tractable representation of demand uncertainty for the proposed CVaR-based risk assessment.

$$D^\omega = \gamma^\omega \bar{D}, \quad \forall \omega \in \Omega, \quad (13)$$

where D^ω [MWh] denotes the demand realization in scenario ω , $\bar{D}$ [MWh] is the forecast day-ahead demand, and γ^ω is the corresponding dimensionless demand multiplier.

Let p_ω denote the dimensionless occurrence probability of scenario ω . The scenario probabilities satisfy

$$\sum_{\omega \in \Omega} p_\omega = 1, \quad p_\omega \geq 0, \quad \forall \omega \in \Omega. \quad (14)$$

To ensure the feasibility of the market-clearing problem under every scenario, each demand realization is restricted by

$$0 < D^\omega \leq \sum_{j \in \mathcal{J}} Q_j, \quad \forall \omega \in \Omega. \quad (15)$$

Equations (13)–(15) define the scenario-dependent demand realizations, their occurrence probabilities, and the associated market-clearing feasibility condition. In particular, (15) ensures that the aggregate available offered quantity is sufficient to satisfy demand in every scenario. For each scenario $\omega \in \Omega$, the uniform-pricing market-clearing problem in (1) is solved by replacing D with D^ω . This yields the scenario-dependent accepted quantities s_j^ω [MWh], market-clearing price λ^ω [€/MWh], and operating profit Π_i^ω [€]. The superscript ω denotes the realization of the corresponding market quantity under scenario ω , whereas the available generation quantities Q_j remain deterministic.

3.4. Risk-Aware Utility Function

Each strategic operator seeks to maximize its expected operating profit while limiting its exposure to adverse profit realizations arising from demand uncertainty. Building upon the profit formulation in Section 2, the CVaR measure is employed to quantify the downside shortfall in profit for each operator.

The expected operating profit of operator i is

$$\bar{\Pi}_i(\alpha) = \mathbb{E}[\Pi_i(\alpha)] = \sum_{\omega \in \Omega} p_\omega \Pi_i^\omega(\alpha), \quad (16)$$

where $\bar{\Pi}_i(\alpha)$ [€] denotes the expected operating profit of operator i under the joint strategy profile α , $\mathbb{E}[\cdot]$ is the expectation operator, and $\Pi_i^\omega(\alpha)$ [€] is the corresponding scenario-dependent operating profit.

Let Π_i^{tar} [€] denote the operator-specific reference profit, defined as the expected operating profit obtained under the corresponding risk-neutral equilibrium and held fixed during the subsequent risk-aware optimization. The downside-profit shortfall in scenario ω is defined as

$$L_i^\omega(\alpha) = [\Pi_i^{\text{tar}} - \Pi_i^\omega(\alpha)]^+ = \max\{0, \Pi_i^{\text{tar}} - \Pi_i^\omega(\alpha)\}, \quad \forall \omega \in \Omega, \quad (17)$$

where $L_i^\omega(\alpha)$ [€] denotes the downside-profit shortfall of operator i in scenario ω , and $[x]^+ = \max\{0, x\}$ denotes the positive-part operator. The reference profit Π_i^{tar} is defined as the expected profit under the corresponding risk-neutral SFNE, providing a consistent benchmark for evaluating the trade-off between expected profit and downside-risk mitigation. Thus, a positive shortfall is incurred only when the scenario-dependent profit falls below this reference profit.

For a confidence level $\beta \in (0, 1)$, the downside financial risk is quantified using the Rockafellar–Uryasev representation:

$$\text{CVaR}_{i,\beta}(L_i(\alpha)) = \min_{\eta_i \in \mathbb{R}} \left[\eta_i + \frac{1}{1-\beta} \sum_{\omega \in \Omega} p_\omega (L_i^\omega(\alpha) - \eta_i)^+ \right], \quad (18)$$

where $\text{CVaR}_{i,\beta}(L_i(\alpha))$ [€] denotes the CVaR of the downside-profit shortfall of operator i , $L_i(\alpha)$ is the random downside-shortfall variable whose realization in scenario ω is $L_i^\omega(\alpha)$, β is the CVaR confidence level, and η_i [€] is an auxiliary threshold corresponding to the $\text{VaR}_{i,\beta}$ of the downside shortfall at the optimum.

For numerical implementation, the CVaR measure in (18) is equivalently represented as

$$\text{CVaR}_{i,\beta}(L_i(\alpha)) = \eta_i + \frac{1}{1-\beta} \sum_{\omega \in \Omega} p_\omega z_i^\omega, \quad (19)$$

subject to

$$z_i^\omega \geq L_i^\omega(\alpha) - \eta_i, \quad \forall \omega \in \Omega, \quad (20a)$$

$$z_i^\omega \geq 0, \quad \forall \omega \in \Omega, \quad (20b)$$

where z_i^ω [€] denotes the nonnegative excess of the scenario-dependent downside shortfall above the threshold η_i . The formulation in (18)–(20b) evaluates the conditional expected downside shortfall within the upper $(1 - \beta)$ tail of its probability distribution. Increasing β places greater emphasis on progressively more extreme low-profit scenarios.

Based on the expected profit in (16) and the downside-risk measure in (18), the risk-adjusted utility function of operator i is defined as

$$U_i(\alpha_i, \alpha_{-i}) = \bar{\Pi}_i(\alpha) - \rho_i \text{CVaR}_{i,\beta}(L_i(\alpha)), \quad (21)$$

where α_{-i} denotes the joint strategy profile of all operators other than operator i , and $\rho_i \geq 0$ is the dimensionless risk-aversion coefficient of operator i . Equation (21) balances expected profitability against exposure to extreme downside-profit shortfalls. Setting $\rho_i = 0$ recovers the risk-neutral SFNE, whereas increasing ρ_i assigns greater weight to downside-risk mitigation and may induce stronger adjustments in the operator's equilibrium bidding markups.

3.5. Individual Risk-Aware Strategic Bidding Problem

Given the bidding strategies of its competitors, operator i determines its optimal cluster-level bidding markups by maximizing its risk-adjusted utility. For each candidate strategy profile, the uniform-pricing market-clearing problem in (1) is solved under every demand scenario to obtain the corresponding accepted quantities, market-clearing prices, and scenario-dependent profits. Accordingly, the individual risk-aware strategic bidding problem of operator i is formulated as

$$\left\{ \begin{array}{ll} \max_{\alpha_i, \eta_i, \{z_i^\omega\}_{\omega \in \Omega}} & \sum_{\omega \in \Omega} p_\omega \Pi_i^\omega(\alpha_i, \alpha_{-i}) \\ & - \rho_i \left[\eta_i + \frac{1}{1-\beta} \sum_{\omega \in \Omega} p_\omega z_i^\omega \right] \\ \text{s.t.} & 0 \leq \alpha_{ik} \leq \bar{\alpha}, \quad \forall k \in \mathcal{K}_i, \\ & z_i^\omega \geq L_i^\omega(\alpha_i, \alpha_{-i}) - \eta_i, \quad \forall \omega \in \Omega, \\ & z_i^\omega \geq 0, \quad \forall \omega \in \Omega, \\ & \eta_i \in \mathbb{R}. \end{array} \right. \quad (22)$$

The scenario-dependent downside-profit shortfall appearing in (22) is defined by (17), with $\alpha = (\alpha_i, \alpha_{-i})$.

Equation (22) maximizes the expected operating profit defined in (16) while penalizing the CVaR of the operator's downside-profit shortfall. The first constraint enforces the admissible bounds on the cluster-level bidding markups. The second and third constraints provide the auxiliary-variable CVaR representation in (20a) and (20b), respectively, while the final condition specifies the domain of the threshold variable η_i . The scenario-dependent profits are determined endogenously through the uniform-pricing market-clearing problem in (1), using the submitted offers generated by (12) under the joint strategy profile (α_i, α_{-i}) .

The optimal strategies associated with (22) define the best-response correspondence of operator i as

$$\text{BR}_i(\alpha_{-i}) = \arg \max_{\alpha_i \in \mathcal{A}_i} U_i(\alpha_i, \alpha_{-i}), \quad (23)$$

where $\text{BR}_i(\alpha_{-i}) \subseteq \mathcal{A}_i$ denotes the set of feasible bidding strategies that maximize the risk-adjusted utility of operator i for a fixed strategy profile of its competitors. The auxiliary variables η_i and $\{z_i^\omega\}_{\omega \in \Omega}$ in (22) are implicitly optimized when evaluating the utility in (23). Since the market-clearing prices and accepted quantities depend jointly on the submitted offers of all strategic operators, the individual best responses are mutually coupled. A fixed point of these best-response correspondences therefore constitutes the risk-aware SFNE defined in the following subsection.

3.6. Supply Function Nash Equilibrium

The proposed strategic bidding game reaches a risk-aware SFNE when no operator can improve its risk-adjusted utility by unilaterally changing its cluster-level bidding strategy while the strategies of all competitors remain fixed. A feasible joint strategy profile

$$\alpha^* = \left[(\alpha_1^*)^\top, \dots, (\alpha_I^*)^\top \right]^\top \in \mathcal{A}$$

is a risk-aware SFNE if

$$U_i(\alpha_i^*, \alpha_{-i}^*) \geq U_i(\alpha_i, \alpha_{-i}^*), \quad \forall \alpha_i \in \mathcal{A}_i, \quad \forall i \in \mathcal{I}, \quad (24)$$

where the superscript $(\cdot)^*$ denotes an equilibrium strategy or strategy profile. Equation (24) states that each operator's equilibrium strategy maximizes its risk-adjusted utility given the equilibrium strategies of its competitors. Therefore, no operator has an incentive to deviate unilaterally.

Equivalently, using the best-response correspondence defined in (23), the equilibrium strategy of each operator satisfies

$$\alpha_i^* \in \text{BR}_i(\alpha_{-i}^*), \quad \forall i \in \mathcal{I}. \quad (25)$$

Thus, α^* is a fixed point of the operators' best-response correspondences. Figure 2 illustrates the computational procedure of the proposed risk-aware SFNE framework. The relaxed NI best-response algorithm iteratively solves the day-ahead market-clearing problem, evaluates the CVaR-based risk-adjusted utilities, updates the operators' cluster-level bidding strategies, and repeats this process until the prescribed convergence criteria are satisfied.

The risk-aware SFNE is computed using the iterative procedure summarized in Algorithm 1. At each iteration, each operator solves its risk-aware optimization problem for fixed competitors' strategies, after which the aggregate strategy profile is updated through a relaxed best-response step using the relaxation parameter κ . The uniform-pricing market-clearing problem is then solved for all demand scenarios to obtain the corresponding market-clearing prices, accepted quantities, operator profits, and CVaR values. The relative NI gap defined in (38) is used as the equilibrium residual, and the algorithm terminates when the relative NI gap, strategy update norm, and utility variation simultaneously satisfy the prescribed convergence criteria for the required number of consecutive iterations and the subsequent verification stage.

Algorithm 1 Risk-aware SFNE computation via relaxed NI best-response update

Initialize: Set the operator and cluster sets, offer data, demand scenarios, reference profits, risk parameters, markup bound, relaxation parameter $\kappa \in (0, 1]$, convergence tolerances, persistence requirement N_{pers} , verification length N_{ver} , and maximum iteration count R_{max} . Initialize $\alpha^{(0)} \in \mathcal{A}$, the iteration index $r \leftarrow 0$, and the consecutive-convergence counter $n_{\text{conv}} \leftarrow 0$.

Define: For a joint strategy profile α , construct the submitted offer prices according to (12).

While $r < R_{\text{max}}$

1. Market clearing: For each scenario $\omega \in \Omega$:

- Construct the demand realization D^ω using (13);
- Construct the submitted offer prices using (12);
- Solve the uniform-pricing market-clearing problem in (1) with $D = D^\omega$;
- Compute λ^ω , $\{s_j^\omega\}_{j \in \mathcal{J}}$, and $\{\Pi_i^\omega\}_{i \in \mathcal{I}}$.

2. Risk-aware utility evaluation: For each operator $i \in \mathcal{I}$:

- Compute the expected operating profit $\bar{\Pi}_i(\alpha^{(r)})$ using (16);
- Evaluate the scenario-dependent downside-profit shortfalls using (17);
- Compute $\text{CVaR}_{i,\beta}(L_i(\alpha^{(r)}))$ using (19)–(20b);
- Evaluate the risk-adjusted utility $U_i(\alpha_i^{(r)}, \alpha_{-i}^{(r)})$ using (21).

3. Best-response computation: For each operator $i \in \mathcal{I}$:

- Solve the individual risk-aware strategic bidding problem in (22);
- Obtain $\hat{\alpha}_i^{(r)} \in \text{BR}_i(\alpha_{-i}^{(r)})$ according to (23).

4. Relaxed best-response update: Construct the simultaneous best-response profile $\hat{\alpha}^{(r)} = [(\hat{\alpha}_1^{(r)})^\top, \dots, (\hat{\alpha}_I^{(r)})^\top]^\top$, and update $\alpha^{(r+1)} = (1 - \kappa)\alpha^{(r)} + \kappa\hat{\alpha}^{(r)}$.

5. Convergence check: Evaluate the relative NI gap defined in (38), the relative strategy update norm, and the relative utility variation. Update n_{conv} accordingly. If all convergence criteria are satisfied for N_{pers} consecutive iterations and remain satisfied for an additional N_{ver} verification iterations, terminate. Otherwise, set $r \leftarrow r + 1$.

Return: α^* , $\{\lambda^{\omega,*}\}_{\omega \in \Omega}$, $\{s_j^{\omega,*}\}_{j \in \mathcal{J}, \omega \in \Omega}$, $\{\bar{\Pi}_i(\alpha^*)\}_{i \in \mathcal{I}}$, $\{\text{CVaR}_{i,\beta}(L_i(\alpha^*))\}_{i \in \mathcal{I}}$, and the convergence metrics.

3.7. Equilibrium Existence and Stability Analysis

This subsection establishes the theoretical basis of the proposed risk-aware SFNE formulation using the bounded strategy sets in (10), the risk-adjusted utility function in (21), the best-response correspondence in (23), and the equilibrium condition in (24).

Proposition 1. Suppose that, for every market operator $i \in \mathcal{I}$, the feasible strategy set $\mathcal{A}_i$ is nonempty, compact, and convex. Furthermore, assume that the risk-adjusted utility function $U_i(\alpha_i, \alpha_{-i})$ is continuous in the joint strategy profile α , and that the corresponding best-response correspondence $\text{BR}_i(\alpha_{-i})$ is nonempty, convex-valued, and upper hemicontinuous for every $\alpha_{-i} \in \mathcal{A}_{-i}$. Then, the proposed risk-aware strategic bidding game admits at least one SFNE.

Proof. From (10), each feasible strategy set $\mathcal{A}_i$ is defined by bounded linear constraints on the cluster-level bidding markups and is therefore nonempty, compact, and convex. The corresponding joint strategy set is

$$\mathcal{A} = \prod_{i \in \mathcal{I}} \mathcal{A}_i. \quad (26)$$

Since the operator set $\mathcal{I}$ is finite, the Cartesian product $\mathcal{A}$ is also nonempty, compact, and convex.

The best-response correspondence of operator i , introduced in (23), is

$$\text{BR}_i(\alpha_{-i}) = \arg \max_{\alpha_i \in \mathcal{A}_i} U_i(\alpha_i, \alpha_{-i}). \quad (27)$$

By assumption, for every feasible α_{-i} , the correspondence $\text{BR}_i(\alpha_{-i})$ is nonempty, convex-valued, and upper hemicontinuous. Consequently, the aggregate best-response correspondence is defined as

$$\text{BR}(\alpha) = \prod_{i \in \mathcal{I}} \text{BR}_i(\alpha_{-i}),$$

which maps the compact and convex set $\mathcal{A}$ into itself while preserving these properties. Therefore, by Kakutani's Fixed-Point Theorem [23], there exists at least one fixed point $\alpha^* \in \mathcal{A}$ such that

$$\alpha_i^* \in \text{BR}_i(\alpha_{-i}^*), \quad \forall i \in \mathcal{I}. \quad (28)$$

By the definition of the best-response correspondence, (28) implies

$$U_i(\alpha_i^*, \alpha_{-i}^*) \geq U_i(\alpha_i, \alpha_{-i}^*), \quad \forall \alpha_i \in \mathcal{A}_i, \quad \forall i \in \mathcal{I}. \quad (29)$$

Hence, no market operator can improve its risk-adjusted utility through a unilateral deviation, and the fixed point α^* satisfies the SFNE condition in (24). Therefore, the proposed strategic bidding game admits at least one risk-aware SFNE. $\square$

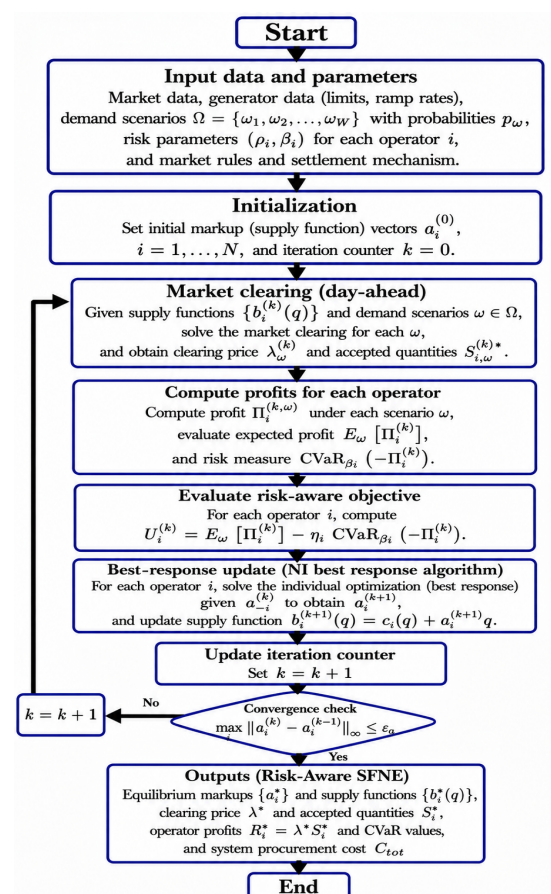


Figure 2. Flowchart of the proposed risk-aware SFNE solution procedure.

Proposition 2. A feasible joint strategy profile $\alpha^* \in \mathcal{A}$ is an SFNE if and only if its corresponding NI gap is zero.

Proof. For any feasible strategy profile $\alpha \in \mathcal{A}$, let $\beta = (\beta_1, \dots, \beta_I) \in \mathcal{A}$ denote a feasible deviation profile, where $\beta_i \in \mathcal{A}_i$ is a candidate unilateral strategy of operator i . Following the generalized NI framework in [24], the NI function is defined as

$$\Psi(\alpha, \beta) = \sum_{i \in \mathcal{I}} [U_i(\beta_i, \alpha_{-i}) - U_i(\alpha_i, \alpha_{-i})], \quad (30)$$

where each summand represents the utility change resulting from a unilateral deviation by operator i while its competitors retain α_{-i} . The corresponding NI gap is defined as

$$V(\alpha) = \max_{\beta \in \mathcal{A}} \Psi(\alpha, \beta), \quad (31)$$

where $V(\alpha)$ denotes the maximum aggregate incentive for unilateral deviation from α .

Selecting $\beta = \alpha$ yields $\Psi(\alpha, \alpha) = 0$; therefore,

$$V(\alpha) \geq 0, \quad \forall \alpha \in \mathcal{A}. \quad (32)$$

To establish the necessity of the equilibrium condition, suppose that $\alpha^* \in \mathcal{A}$ is a SFNE. By the equilibrium condition in (24), no operator can improve its risk-adjusted utility through a unilateral deviation. Hence,

$$U_i(\beta_i, \alpha_{-i}^*) - U_i(\alpha_i^*, \alpha_{-i}^*) \leq 0, \quad \forall \beta_i \in \mathcal{A}_i, \quad \forall i \in \mathcal{I}. \quad (33)$$

Summing (33) over all operators gives

$$\Psi(\alpha^*, \beta) \leq 0, \quad \forall \beta \in \mathcal{A}. \quad (34)$$

Because choosing $\beta = \alpha^*$ yields $\Psi(\alpha^*, \alpha^*) = 0$, (31), (32), and (34) imply

$$V(\alpha^*) = 0. \quad (35)$$

Conversely, suppose that

$$V(\alpha^*) = 0. \quad (36)$$

By the definition of the maximum in (31), it follows that

$$\Psi(\alpha^*, \beta) \leq 0, \quad \forall \beta \in \mathcal{A}. \quad (37)$$

Consider any operator $i \in \mathcal{I}$ and choose a deviation profile β such that $\beta_i \in \mathcal{A}_i$ is arbitrary, while

$$\beta_\ell = \alpha_\ell^*, \quad \forall \ell \in \mathcal{I} \setminus \{i\}.$$

Under this construction, all terms in (30), except the term associated with operator i , are zero. Consequently, (37) implies

$$U_i(\alpha_i^*, \alpha_{-i}^*) \geq U_i(\beta_i, \alpha_{-i}^*), \quad \forall \beta_i \in \mathcal{A}_i, \quad \forall i \in \mathcal{I}. \quad (38)$$

Equation (38) is precisely the SFNE condition in (24). Therefore, a feasible joint strategy profile is a SFNE if and only if its corresponding NI gap is zero.

For numerical implementation, the relative NI gap is defined as

$$\epsilon_{\text{NI}}(\alpha) = \frac{V(\alpha)}{\epsilon_0 + \sum_{i \in \mathcal{I}} |U_i(\alpha_i, \alpha_{-i})|}, \quad (39)$$

where $\epsilon_{\text{NI}}(\alpha)$ denotes the relative NI gap and $\epsilon_0 > 0$ [€] is a sufficiently small regularization constant that prevents division by zero and preserves dimensional consistency. A sufficiently small value of $\epsilon_{\text{NI}}(\alpha)$ indicates a negligible aggregate incentive for profitable unilateral deviation and is therefore used as the numerical equilibrium residual and convergence criterion for the proposed algorithm. $\square$

Proposition 3. Suppose that the aggregate best-response correspondence admits a continuously differentiable local selection $\mathcal{B} : \mathcal{A} \rightarrow \mathcal{A}$ in a neighborhood of an equilibrium α^* , with the selected branch satisfying $\mathcal{B}(\alpha^*) = \alpha^*$. If the spectral radius of the Jacobian matrix of the relaxed best-response mapping satisfies

$$\rho(\mathbf{J}_{\mathcal{T}_\kappa}(\alpha^*)) < 1, \quad (40)$$

α^* is a locally asymptotically stable fixed point of the relaxed best-response iteration.

Proof. Stability analysis of equilibrium solutions is commonly used to investigate the convergence properties of iterative equilibrium algorithms in electricity market games [25]. Let $\mathcal{B}_i(\alpha_{-i})$ denote a continuously differentiable local selection from the best-response correspondence of operator i , satisfying

$$\mathcal{B}_i(\alpha_{-i}) \in \text{BR}_i(\alpha_{-i}).$$

The corresponding selected best-response mapping is defined as

$$\mathcal{B}(\alpha) = [\mathcal{B}_1(\alpha_{-1})^\top, \mathcal{B}_2(\alpha_{-2})^\top, \dots, \mathcal{B}_I(\alpha_{-I})^\top]^\top. \quad (41)$$

The relaxed best-response iteration employed to compute the equilibrium is expressed as

$$\alpha^{(r+1)} = \mathcal{T}_\kappa(\alpha^{(r)}) = (1 - \kappa)\alpha^{(r)} + \kappa\mathcal{B}(\alpha^{(r)}), \quad 0 < \kappa \leq 1, \quad (42)$$

where $\mathcal{T}_\kappa : \mathcal{A} \rightarrow \mathcal{A}$ denotes the relaxed best-response mapping, κ is the relaxation parameter, and r is the iteration index. In (40), $\rho(\cdot)$ denotes the spectral radius, while $\mathbf{J}_{\mathcal{T}_\kappa}(\alpha^*)$ is the Jacobian matrix of $\mathcal{T}_\kappa$ evaluated at the equilibrium.

By assumption, the selected local branch of the aggregate best-response mapping satisfies

$$\alpha^* = \mathcal{B}(\alpha^*). \quad (43)$$

Substituting (43) into the relaxed update rule in (42) gives

$$\mathcal{T}_\kappa(\alpha^*) = (1 - \kappa)\alpha^* + \kappa\mathcal{B}(\alpha^*) = \alpha^*, \quad (44)$$

where α^* is also a fixed point of the relaxed best-response mapping. Let the strategy perturbation around the equilibrium at iteration r be defined as

$$\Delta\alpha^{(r)} = \alpha^{(r)} - \alpha^*. \quad (45)$$

A first-order Taylor expansion of $\mathcal{T}_\kappa$ around α^* yields

$$\Delta\alpha^{(r+1)} = \mathbf{J}_{\mathcal{T}_\kappa}(\alpha^*)\Delta\alpha^{(r)} + \mathcal{O}(\|\Delta\alpha^{(r)}\|_2^2), \quad (46)$$

where $\mathcal{O}(\|\Delta\alpha^{(r)}\|_2^2)$ represents the higher-order terms neglected in the local linearization.

From the definition of the relaxed mapping in (42), its Jacobian evaluated at the equilibrium is

$$\mathbf{J}_{\mathcal{T}_\kappa}(\alpha^*) = (1 - \kappa)\mathbf{I} + \kappa\mathbf{J}_B(\alpha^*), \quad (47)$$

where $\mathbf{I}$ is the identity matrix of the appropriate dimension, and $\mathbf{J}_B(\alpha^*)$ denotes the Jacobian matrix of the selected best-response mapping evaluated at the equilibrium.

If the spectral-radius condition in (40) holds, every eigenvalue of $\mathbf{J}_{\mathcal{T}_\kappa}(\alpha^*)$ lies strictly inside the unit circle. The linearized perturbation dynamics are therefore asymptotically stable. Consequently, the standard local stability result for continuously differentiable discrete-time mappings implies

$$\lim_{r \rightarrow \infty} \|\alpha^{(r)} - \alpha^*\|_2 = 0, \quad (48)$$

for every initial strategy profile sufficiently close to α^* , where $\|\cdot\|_2$ denotes the Euclidean norm. Hence, the spectral-radius condition in (40) is sufficient for the local asymptotic stability of the relaxed best-response iteration. $\square$

Figure 3 illustrates the theoretical properties of the proposed risk-aware SFNE framework. The intersection of the selected best-response mappings defines the equilibrium strategy profile α^* , at which no operator can improve its risk-adjusted utility through a unilateral deviation. The dashed trajectory illustrates the convergence of the relaxed best-response iteration from a feasible initialization toward the equilibrium through successive strategy updates. Moreover, the NI gap $V(\alpha)$ defined in (31) measures the aggregate incentive for unilateral deviation and becomes zero at equilibrium, as established in Proposition 2. Collectively, Propositions 1–3 establish the existence of the proposed risk-aware SFNE, characterize equilibrium through the NI gap, and provide a sufficient condition for the local asymptotic stability of the relaxed best-response iteration.

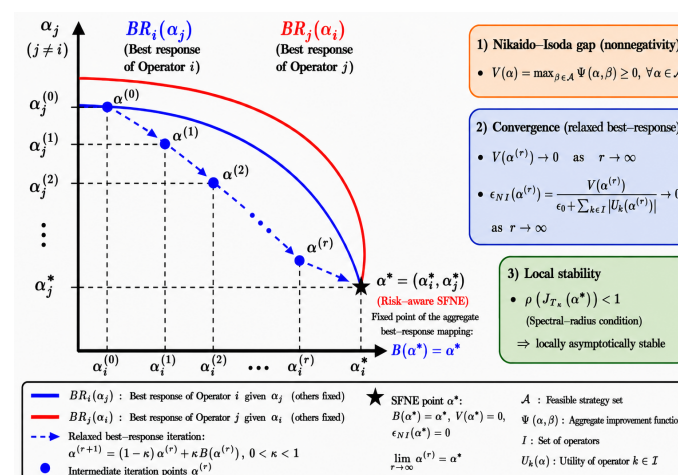


Figure 3. Graphical illustration of the relaxed best-response dynamics, equilibrium convergence, and stability of the proposed risk-aware SFNE.

3.8. Algorithmic Parameter Configuration

The proposed risk-aware SFNE framework for strategic bidding relies on several algorithmic parameters governing the equilibrium computation and the CVaR-based risk formulation. Table 2 summarizes the adopted parameter values, selected to ensure stable convergence.

Table 2. Algorithmic parameters of the proposed SFNE framework.

Parameter	Value
Best-response relaxation factor (κ)	0.25
Relative NI-gap tolerance (ϵ_{NI})	10^{-3}
Maximum iterations (R_{max})	1000
CVaR confidence level (β)	0.95
Risk-aversion coefficient range (ρ_i)	$[0, 1]$
Relative strategy tolerance (ϵ_{α})	10^{-6}
Relative utility tolerance (ϵ_U)	10^{-4}

4. Data Analysis and Preprocessing

The proposed framework is validated using publicly available 2024 day-ahead offer data from the Italian electricity market operator (GME) [26]. Since the GME dataset does not explicitly identify generation technologies for each market operator, a technology-oriented classification is constructed in this study using engineering-based assumptions to represent heterogeneous generation portfolios for strategic bidding analysis. After filtering the market records, only accepted and rejected sell offers are retained. The analysis focuses on the five dominant market operators within a representative bidding zone to ensure a consistent market environment while avoiding congestion-induced price variations. Techno-economic and bidding features, including the marginal cost, offer price, and offered quantity, are then extracted for each generation unit. To reduce the computational complexity of the proposed game-theoretic framework while preserving bidding heterogeneity, the generation units are clustered using the K-means clustering algorithm [27,28]. The strategic bidding markup is limited to 0–20%, reflecting realistic bidding behavior under the uniform-pricing market-clearing mechanism. This preprocessing preserves the essential characteristics of the aggregated supply curve while substantially improving the computational tractability of the proposed SFNE framework.

Figure 4a presents the clustering quality indices evaluated for different numbers of clusters. The Calinski–Harabasz and Silhouette indices attain their highest values at approximately $k = 5$, whereas the Davies–Bouldin and Xie–Beni indices reach comparatively lower values, indicating improved cluster compactness and inter-cluster separation. Although the Dunn index exhibits moderate fluctuations owing to the heterogeneous generation portfolios, the overall agreement among the five complementary clustering validation metrics [29] supports the selection of five clusters as an appropriate trade-off between clustering quality and computational efficiency. To further validate the selected clustering approach, Figure 4b compares the clustering structures produced by K-means and density-based spatial clustering of applications with noise (DBSCAN) [30] in the $(Q_{\text{mean}}, P_{w,\text{mean}})$ feature space. The differently colored dots represent the different clusters, denoted as $C_1, C_2, \dots, C_5$. Although both methods capture the principal bidding patterns, K-means provides more compact and balanced cluster partitions with clearer separation among representative bidding groups than DBSCAN. Therefore, K-means is adopted to construct representative cluster-level generation portfolios, which are subsequently aggregated at the operator level while preserving renewable and thermal technology classifications. Table 3 summarizes the final clustered operator portfolios, which are processed from [26], including the aggregated renewable and thermal capacities and the corresponding numbers of generation units for the five dominant operators, representing a total installed capacity of 22,753 MW. The renewable and thermal capacity shares reported in Table 3 denote each operator’s proportion of the total installed renewable and thermal capacities, respectively, considered in the case study.

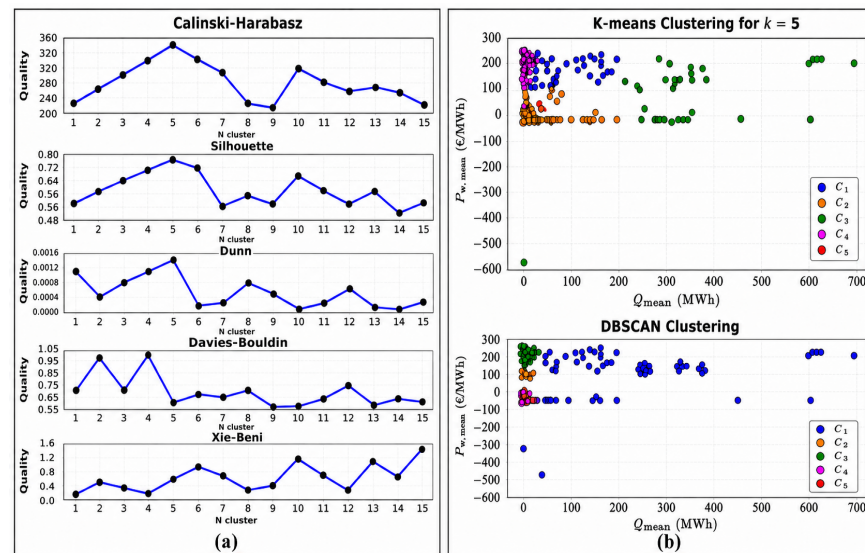


Figure 4. (a) Quality indices for K-means clustering with different numbers of clusters. (b) Clustering results obtained using K-means and DBSCAN, projected onto the $(Q_{mean}, P_{w,mean})$ feature space.

Table 3. Representative operator portfolios used in the case study after technology-based clustering and aggregation.

Operator	Aggregated Capacity (MW)	Renewable Capacity (MW)	Thermal Capacity (MW)	Renewable Fleet Share (%)	Thermal Fleet Share (%)	No. of Generation Units
Op_1	13,767	530	13,237	13.35	70.47	143
Op_2	3688	435	3253	10.96	17.32	64
Op_3	3639	2062	1577	51.94	8.40	119
Op_4	1004	500	504	12.59	2.68	72
Op_5	655	443	212	11.16	1.13	79
Total	22,753	3970	18,783	100	100	477

5. Numerical Results and Discussion

This section evaluates the proposed game-theoretic risk-aware SFNE framework for strategic bidding under a uniform-pricing market-clearing mechanism. The framework is implemented in Python 3.12, and the equilibrium is computed using the relaxed NI best-response algorithm, with the market-clearing problems solved using Gurobi. The simulations are performed on a workstation equipped with 32 GB of RAM and a 12th-generation Intel® Core™ i9-12900F processor. The numerical analysis examines algorithmic convergence and evaluates the effects of strategic bidding and risk aversion on equilibrium market outcomes. Sensitivity analyses are subsequently conducted with respect to the risk-aversion coefficient and the magnitude of demand uncertainty. Finally, the superiority of the proposed framework is demonstrated through a realistic 24 h market simulation.

5.1. Convergence Analysis of the Risk-Aware SFNE

The proposed game-theoretic risk-aware SFNE framework is solved using a relaxed NI best-response algorithm, in which dominant market operators iteratively update their cluster-level bidding markups while accounting for competitors' strategic responses under uncertainty. Since the framework incorporates a CVaR measure into a non-cooperative game, demonstrating numerical convergence is essential for verifying that the computed strategy profile satisfies the adopted numerical equilibrium criteria. Figure 4 presents the convergence behavior using four complementary indicators: the relative NI gap, the

relative strategy update norm, the relative utility variation, and the evolution of the equilibrium markups.

As illustrated in Figure 5a, the relative NI gap decreases during the iterative process and falls below the prescribed tolerance after approximately 54 iterations. This indicates that the remaining utility improvements achievable through unilateral deviations are numerically negligible. Likewise, Figure 5b shows that the relative strategy update norm decreases by more than two orders of magnitude and remains below its prescribed tolerance, indicating stabilization of the successive relaxed best-response updates. Figure 5c presents the relative variation in the operators' risk-adjusted utilities. Despite a temporary increase associated with changes in the set of accepted offers, the utility variation subsequently decreases and satisfies the adopted stopping criterion. Figure 5d illustrates the evolution of the strategic bidding decisions. The mean markup converges to a stable value, while the maximum markup remains below the imposed bidding cap, indicating that the computed equilibrium does not lie on the upper boundary of the admissible strategy set. The absence of persistent oscillations indicates that the relaxation mechanism stabilizes the iterative best-response updates and supports reliable numerical convergence to a locally stable equilibrium.

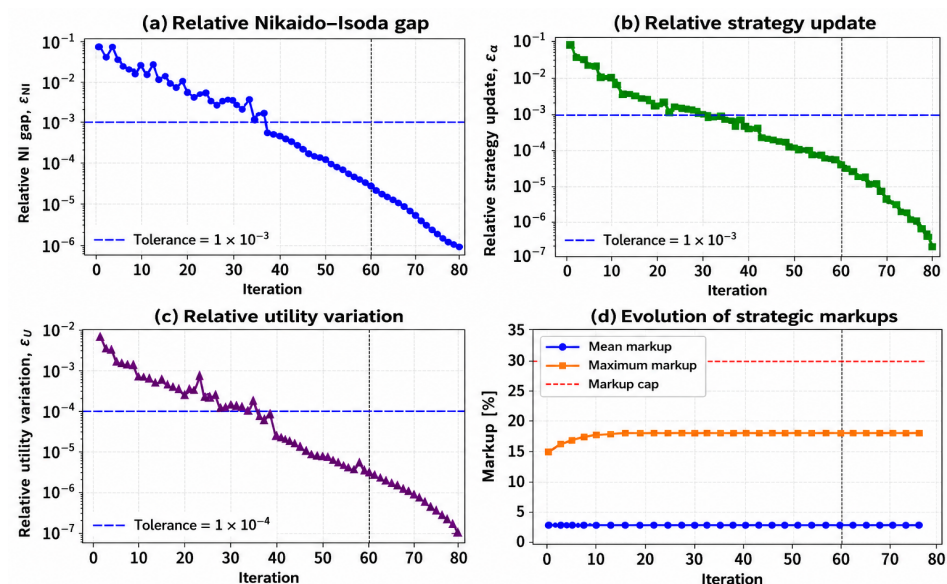


Figure 5. Convergence behavior of the proposed relaxed NI best-response algorithm: (a) relative Nikaido–Isoda gap, (b) relative strategy update, (c) relative utility variation, and (d) evolution of equilibrium markup statistics.

To avoid premature termination during the initial transient stage, convergence assessment is activated only after a prescribed number of iterations. Termination occurs when the relative NI gap, relative strategy update norm, and relative utility variation simultaneously remain below their respective tolerances for the required persistence period and subsequently pass the verification stage. This multi-criterion rule provides a more reliable numerical equilibrium certificate than any single stopping metric.

5.2. Baseline Comparison of Market Equilibrium Outcomes

To evaluate the effectiveness of the proposed framework, the market equilibrium outcomes of the proposed risk-aware SFNE are compared with two benchmark strategies: truthful bidding under the conventional uniform-pricing market-clearing mechanism and the risk-neutral SFNE. The truthful PaC benchmark assumes that operators bid at their marginal production costs, whereas the risk-neutral SFNE determines equilibrium

strategies by maximizing expected profit without explicitly accounting for downside risk. In contrast, the proposed framework incorporates a CVaR-based risk measure into the strategic bidding formulation, enabling operators to balance expected profitability and downside-risk exposure under uncertainty.

As shown in Figure 6a, strategic bidding significantly increases the expected profits of all market operators compared with truthful PaC bidding. The proposed risk-aware SFNE achieves expected profits that remain close to those of the risk-neutral SFNE, indicating that incorporating risk aversion incurs only a marginal reduction in expected profitability. Figure 6b,c show that truthful PaC yields the lowest expected procurement cost and expected market-clearing price because offers are submitted at marginal production costs. In contrast, strategic bidding under both SFNE formulations increases equilibrium markups, resulting in higher procurement costs and market-clearing prices. Nevertheless, the proposed risk-aware framework produces slightly lower procurement costs and market-clearing prices than the risk-neutral SFNE, reflecting more conservative bidding behavior under uncertainty. Most importantly, Figure 6d demonstrates that incorporating the CVaR-based risk measure substantially reduces the aggregate downside-profit risk compared with the risk-neutral SFNE while preserving nearly the same level of expected profit. Overall, the proposed risk-aware SFNE provides a more balanced market equilibrium by effectively improving the trade-off between profitability and financial risk, thereby establishing a strong foundation for the subsequent operator-level and sensitivity analyses.

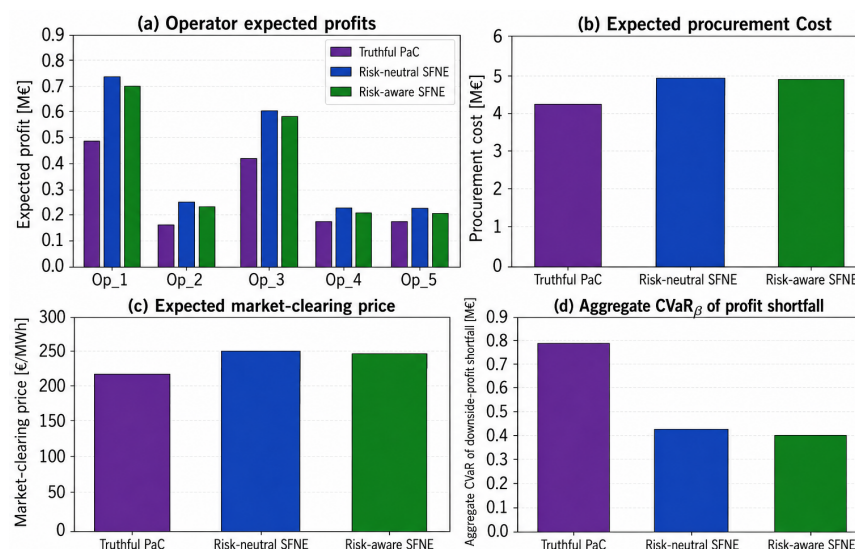


Figure 6. Comparison of equilibrium outcomes under truthful PaC, risk-neutral SFNE, and risk-aware SFNE: (a) operator expected profits, (b) expected procurement cost, (c) expected market-clearing price, and (d) aggregate CVaR _{β} of profit shortfall.

5.3. Cluster-Level Strategic Markup Analysis

To further investigate the influence of risk awareness on strategic bidding, Figure 7 compares the cluster-level equilibrium markups obtained under the risk-neutral and risk-aware SFNE formulations. In the non-cooperative game, each operator determines cluster-level markups while accounting for competitors' strategic responses. Consequently, the equilibrium bidding behavior varies across operators, generation clusters, and demand levels.

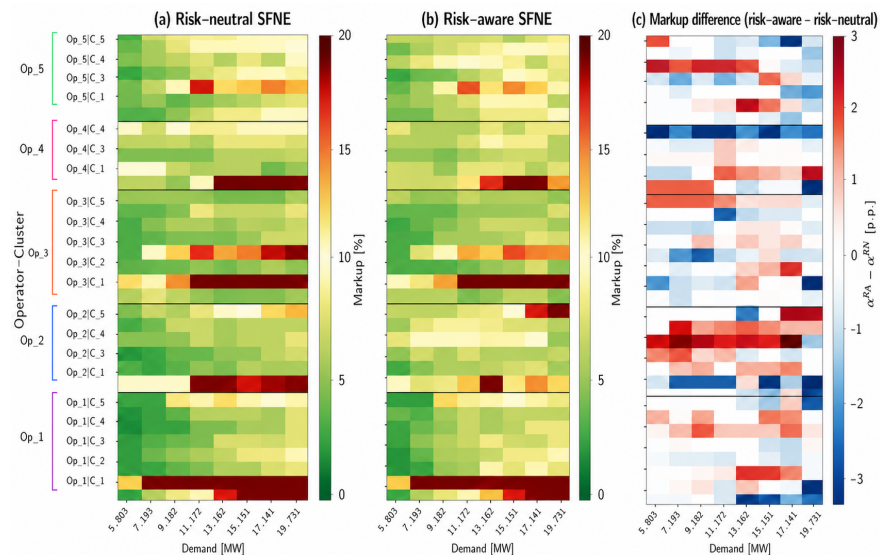


Figure 7. Comparison of cluster-level equilibrium markups across representative demand levels: (a) risk-neutral SFNE, (b) proposed risk-aware SFNE, and (c) risk-aware-minus-risk-neutral markup differences.

As shown in Figure 7a, the risk-neutral SFNE yields relatively high markups for selected clusters of Op_1, Op_2, and Op_3, particularly under medium- and high-demand conditions. This indicates that, when downside-profit risk is not explicitly considered, the expected-profit objective leads to relatively aggressive bidding for these operator-cluster combinations.

After incorporating the CVaR-based downside-risk measure, the equilibrium markups in Figure 7b become more selectively adjusted rather than uniformly reduced. While selected clusters of Op_1, Op_2, and Op_3 continue to exhibit relatively high equilibrium markups under the risk-aware formulation, Op_4 and Op_5 generally display more moderate or mixed bidding behavior, depending on the generation cluster and demand level. These differences reflect the interaction among offer price adjustments, dispatch probability, accepted quantities, market-clearing prices, and scenario-dependent profit shortfalls. The differential heatmap in Figure 7c further illustrates the markup differences between the two equilibrium formulations, showing that risk awareness redistributes strategic markups across operator-cluster combinations rather than uniformly increasing or decreasing bidding aggressiveness. Overall, the proposed cluster-level formulation captures heterogeneous strategic behavior more effectively than an operator-level model based on a single common markup.

5.4. Sensitivity Analysis of Risk Aversion

The risk-aversion coefficient (ρ) determines the relative importance assigned to downside-risk mitigation in the proposed CVaR-based risk-aware SFNE framework. While the confidence level (β) specifies the tail of the profit distribution considered in the CVaR measure, ρ controls the weight assigned to the CVaR term in each operator's strategic optimization problem. For this sensitivity analysis, all strategic market operators are assigned a common risk-aversion coefficient, which is systematically varied to investigate how different levels of risk aversion influence equilibrium bidding strategies and overall market outcomes.

Figure 8 illustrates the impact of the common risk-aversion coefficient on the equilibrium market outcomes. As shown in Figure 8a, the aggregate expected profit gradually decreases as ρ increases, indicating that more risk-averse operators willingly sacrifice a small portion of expected profit to reduce their exposure to adverse market outcomes. Con-

sistently, Figure 8b shows that the aggregate downside-profit risk, measured by $\text{CVaR}_{0.95}$, decreases monotonically with increasing ρ , demonstrating that the proposed CVaR-based formulation effectively mitigates extreme profit shortfalls. Together, Figure 8a,b reveal the expected risk–return trade-off, whereby improved financial robustness is achieved at the expense of a modest reduction in expected profitability.

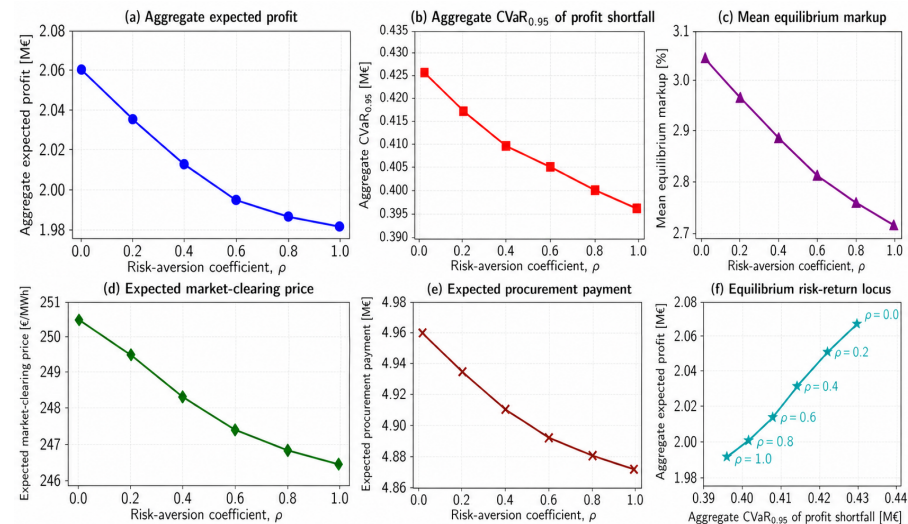


Figure 8. Sensitivity analysis with respect to the risk-aversion coefficient ρ : (a) aggregate expected profit, (b) aggregate downside-profit $\text{CVaR}_{0.95}$, (c) mean equilibrium markup, (d) expected market-clearing price, (e) expected procurement payment, and (f) equilibrium risk–return locus.

The influence of risk aversion on equilibrium bidding behavior is illustrated in Figure 8c. As the risk-aversion coefficient increases, the mean equilibrium markup decreases steadily, indicating that strategic market operators adopt progressively more conservative bidding strategies to reduce downside-risk exposure. The reduction in equilibrium markups consequently lowers both the expected market-clearing price and the expected procurement payment, as shown in Figures 8d and 8e, respectively. These results indicate that increasing the degree of risk aversion not only improves financial risk management but also slightly moderates market prices and total procurement expenditure under the investigated market conditions. The equilibrium risk–return relationship is further illustrated in Figure 8f, where each point corresponds to the market equilibrium obtained for a particular value of the common risk-aversion coefficient. As ρ increases, the equilibrium moves toward lower aggregate downside-profit risk while experiencing only a modest reduction in aggregate expected profit. This behavior confirms that the proposed framework provides a systematic mechanism for navigating the trade-off between expected profitability and downside-risk exposure through the adjustment of the operators’ risk preferences.

Table 4 summarizes the equilibrium outcomes for different values of the common risk-aversion coefficient ρ . As ρ increases from 0 to 1, the aggregate expected profit decreases slightly from 2.060 M€ to 1.982 M€, whereas the aggregate $\text{CVaR}_{0.95}$ decreases from 0.426 M€ to 0.396 M€, indicating improved downside-risk mitigation. The corresponding reductions in equilibrium markups, market-clearing prices, and procurement payments reflect more conservative strategic bidding. Overall, the proposed CVaR-based risk-aware SFNE framework provides a stable and computationally robust trade-off between expected profitability, financial resilience, and market efficiency under uncertainty.

Table 4. Quantitative economic analysis of equilibrium market outcomes under different risk-aversion coefficients.

Risk-Aversion Coefficient ρ	Aggregate Expected Profit [M€]	Aggregate CVaR _{0.95} [M€]	Expected Market-Clearing Price [€/MWh]	Expected Procurement Payment [M€]	Mean Equilibrium Markup [%]
0.0	2.060	0.426	250.5	4.960	3.04
0.2	2.035	0.417	249.5	4.935	2.96
0.4	2.012	0.409	248.3	4.910	2.88
0.6	1.995	0.405	247.4	4.892	2.81
0.8	1.987	0.400	246.8	4.881	2.76
1.0	1.982	0.396	246.5	4.872	2.72

5.5. Sensitivity Analysis with Respect to Demand Uncertainty

To evaluate the robustness of the proposed risk-aware SFNE framework under increasingly uncertain market conditions, the dispersion of the scenario-based demand distribution is systematically varied while the CVaR confidence level and risk-aversion coefficients are held fixed at their baseline values. The horizontal axis reports the realized standard deviation of the generated demand scenarios, denoted by σ_D^{real} , after applying the prescribed feasibility bounds to the demand multipliers. This analysis therefore isolates the effect of demand uncertainty on equilibrium bidding strategies, downside-profit exposure, and resulting market outcomes.

Figure 9 presents the equilibrium outcomes obtained under increasing levels of realized demand uncertainty. As shown in Figure 9a, aggregate expected profit increases with σ_D^{real} , reflecting the combined effects of greater demand dispersion together with the endogenous adjustments of equilibrium markups, accepted quantities, and market-clearing prices. This result should be interpreted as an equilibrium response over the investigated uncertainty range rather than as a general implication that uncertainty necessarily improves profitability. The downside-risk comparison in Figure 9b distinguishes between the intrinsic effect of uncertainty and the risk-mitigation capability of the proposed framework. The fixed-equilibrium curve is obtained by retaining the bidding strategy determined at the lowest uncertainty level and evaluating it under progressively wider demand distributions. The substantial increase in this curve indicates that greater demand uncertainty naturally amplifies exposure to adverse profit realizations when operators do not adapt their strategies. In contrast, the reoptimized risk-aware SFNE is resolved independently at each uncertainty level and yields consistently lower aggregate downside-profit CVaR_{0.95}. Although the reoptimized downside risk initially increases as the demand distribution widens, it subsequently declines at higher uncertainty levels, indicating that equilibrium strategy adjustments partially offset the additional downside exposure.

This adaptation mechanism is further illustrated in Figure 9c, where the mean equilibrium markup increases with demand uncertainty. The equilibrium solutions exhibit progressively higher cluster-level markups to reduce exposure to adverse profit realizations under increasing demand uncertainty. Consequently, the expected market-clearing price also increases, as shown in Figure 9d. The combined results reveal the economic consequence of downside-risk mitigation: reoptimization improves financial robustness relative to a fixed bidding strategy, but the resulting strategic adjustments increase market prices.

Overall, the results demonstrate that the proposed CVaR-based risk-aware SFNE framework captures how dominant market operators adapt their equilibrium strategies to changing demand uncertainty. Comparing the fixed and reoptimized equilibrium strategies separates the intrinsic effect of increasing uncertainty from the benefit of strategic adaptation, thereby providing a clearer assessment of the framework's robustness.

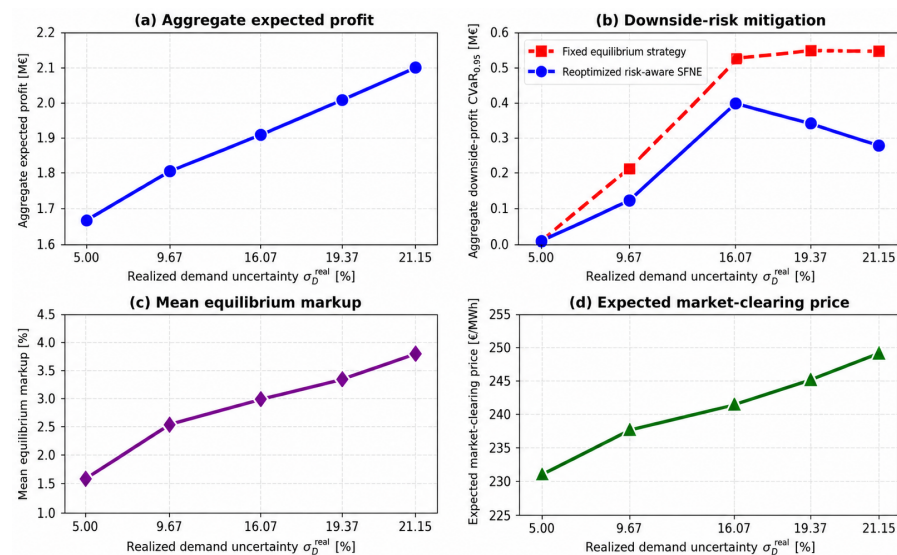


Figure 9. Sensitivity of the proposed risk-aware SFNE framework to demand uncertainty: (a) aggregate expected profit, (b) aggregate downside-profit CVaR_{0.95} under the fixed reference equilibrium strategy and the reoptimized risk-aware SFNE, (c) mean equilibrium markup, and (d) expected market-clearing price.

5.6. Performance Evaluation Under a 24 H Market Simulation

The preceding analyses evaluate the proposed framework at representative demand levels and under controlled variations in demand uncertainty. To further examine its applicability over a realistic intra-day demand trajectory, an additional case study is conducted using a 24 h load profile derived from publicly available data provided by the Italian transmission system operator, Terna. The original profile is normalized and rescaled to remain within the feasible supply range of the modeled generation portfolio. Let $\bar{D}_t$ denote the forecast demand at hour $t \in \mathcal{T} = \{1, \dots, 24\}$. The corresponding demand scenarios are generated as

$$D_t^\omega = \gamma^\omega \bar{D}_t, \quad t \in \mathcal{T}, \quad \omega \in \Omega, \quad (49)$$

where γ^ω is a scenario-specific multiplicative demand factor applied consistently across the hourly profile. The scenario probabilities, CVaR confidence level, and risk-aversion coefficients are held fixed at their baseline values.

For each hour, the relaxed NI best-response algorithm independently computes the risk-neutral and risk-aware SFNEs using identical forecast demand and uncertainty scenarios. The resulting hourly equilibria yield the corresponding bidding strategies, market-clearing prices, accepted quantities, operator profits, procurement payments, and downside-profit risk measures. This comparison isolates the effect of CVaR-based risk preferences along the same intra-day demand trajectory. Because intertemporal operating constraints are not considered, the analysis represents 24 independent hourly day-ahead auctions rather than an intertemporally coupled scheduling problem.

Figure 10a presents the rescaled Terna-based demand profile, which preserves the characteristic intra-day shape of the original load curve, including low nighttime demand and the morning and evening peak periods. As shown in Figure 10b, the expected market-clearing price generally follows the demand trajectory. Prices remain relatively low during the initial low-demand hours, rise during the morning ramp, and attain their highest values near the evening peak, consistent with the acceptance of higher-priced offer blocks as demand increases. The hourly procurement-payment trajectory exhibits a similar peak-period pattern because it depends jointly on accepted demand and the market-clearing price; it is therefore omitted for brevity.

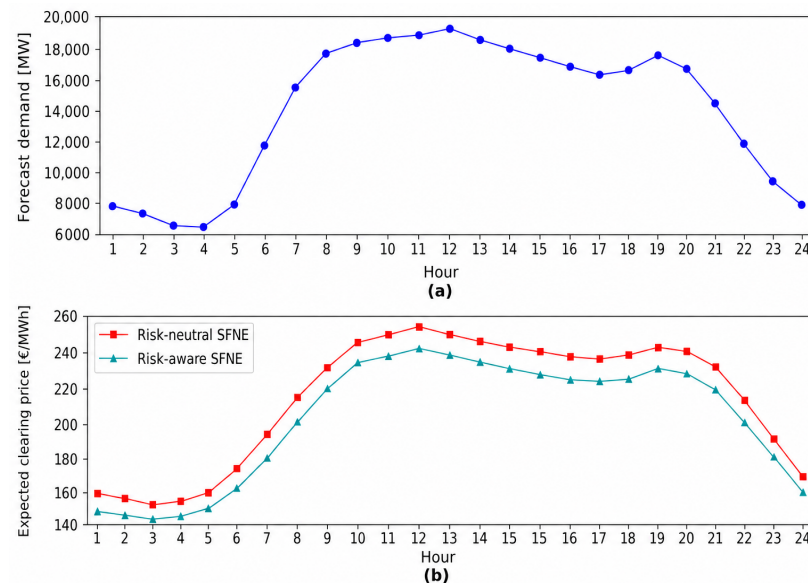


Figure 10. A 24 h market simulation: (a) a rescaled forecast demand profile and (b) expected market-clearing prices under the risk-neutral and risk-aware SFNE formulations.

The risk-neutral and risk-aware SFNEs produce similar overall price patterns, indicating that the inclusion of CVaR-based risk preferences does not materially alter the fundamental demand–price relationship. Nevertheless, the risk-aware equilibrium produces modest price adjustments during selected hours, particularly during the morning ramp and peak-demand periods. These differences arise from endogenous cluster-level markup adjustments obtained at equilibrium in response to scenario-dependent profit-shortfall exposure.

Figure 11a shows that aggregate expected profit generally increases with hourly demand, reflecting the combined effects of higher accepted generation, changes in marginal-unit selection, and higher market-clearing prices. The risk-neutral and risk-aware profit trajectories remain close throughout the day, although the risk-aware equilibrium yields slightly higher or lower profits during selected hours. This indicates that the proposed formulation preserves expected profitability while modifying the scenario-dependent distribution of operator profits. Figure 11b compares the aggregate hourly downside-profit $\text{CVaR}_{0.95}$. The risk-aware SFNE reduces downside exposure during most medium- and high-demand hours, with particularly visible reductions during the morning ramp and several daytime periods. The risk-mitigation effect is not uniform across all hours because each operator optimizes its own risk-adjusted utility within a non-cooperative game, and changes in marginal offer acceptance may redistribute risk across operators. Overall, the results indicate that CVaR-based strategic adaptation can mitigate downside-profit exposure without a substantial loss of expected profitability.

The daily expected profit of operator i is obtained by aggregating its scenario-weighted hourly profits over the 24 h horizon. As shown in Figure 12, Op_1 and Op_3 attain the highest daily profits, consistent with their larger market participation and accepted generation. The remaining operators obtain lower profits according to their portfolio capacities, marginal costs, and dispatch frequencies. The risk-aware and risk-neutral results remain close for all operators, indicating that the proposed formulation largely preserves the operator-level profitability ranking. Small profit differences arise from heterogeneous equilibrium markup adjustments across generation portfolios. Accordingly, the principal benefit of the risk-aware formulation is the reduction in downside-profit exposure while maintaining comparable daily profitability.

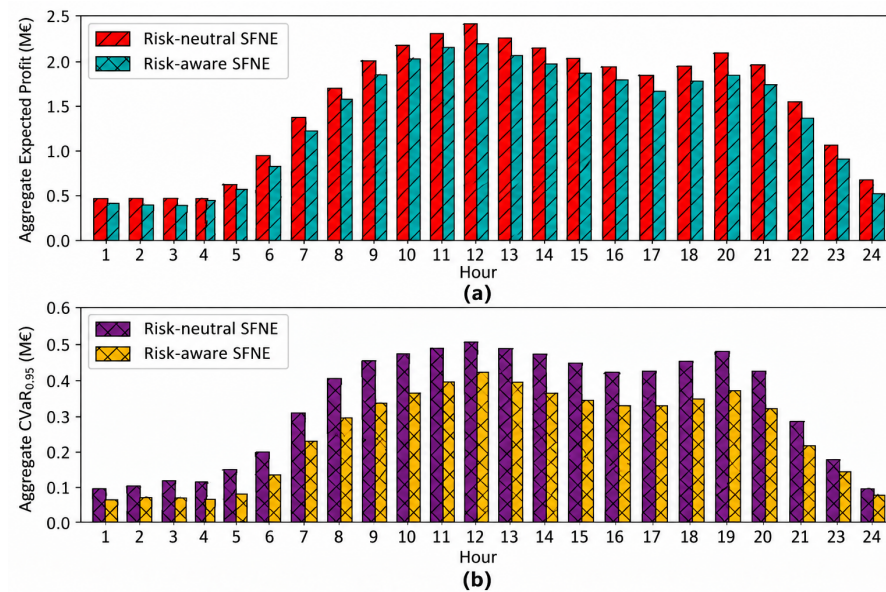


Figure 11. Hourly economic and downside-risk outcomes under the 24 h market simulation: (a) aggregate expected profit and (b) aggregate CVaR_{0.95} of profit shortfall under the risk-neutral and risk-aware SFNE formulations.

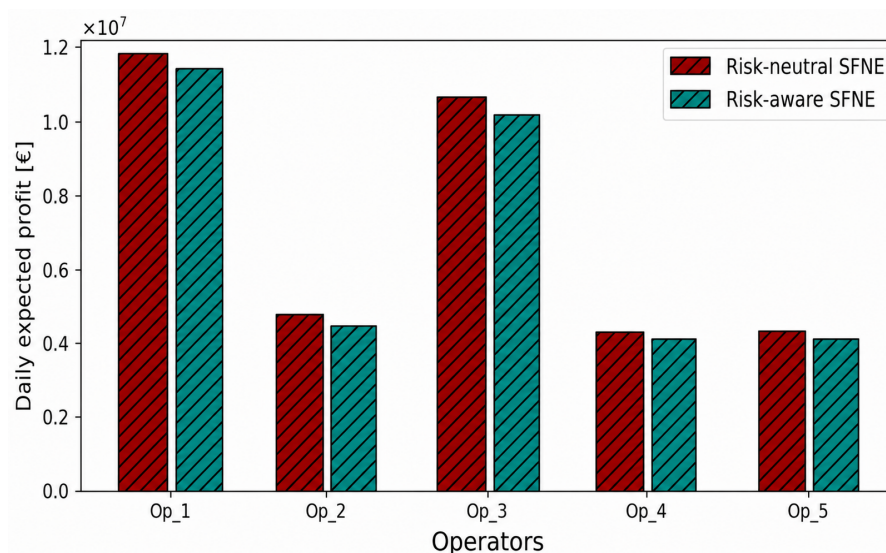


Figure 12. Operator-level expected daily profits under the risk-neutral and risk-aware SFNE formulations for the 24 h market simulation.

Overall, the 24 h market simulation complements the representative demand-level and uncertainty-sensitivity studies by evaluating the proposed framework under realistic intra-day demand conditions. The risk-aware SFNE generally reduces downside-profit exposure while maintaining expected profitability close to the risk-neutral benchmark, although modest changes in market-clearing prices and procurement costs occur during selected hours.

5.7. Modeling Assumptions and Limitations

The proposed framework is developed under several modeling assumptions that define the scope of the present study and should be considered when interpreting the numerical results:

- Since the GME dataset does not explicitly identify generation technologies at the operator level, a technology-oriented classification is developed through data preprocessing,

feature construction, clustering, and engineering-based modeling assumptions. Only positive bidding values are retained, whereas negative bids are excluded to reduce computational complexity while preserving representative heterogeneous generation portfolios for strategic bidding analysis. Consequently, all numerical results are based on this approximate technology-oriented representation derived from the available market data.

- The case study considers a single bidding zone of the Italian day-ahead electricity market. Therefore, transmission network constraints, inter-zonal congestion, and network-induced price separation are not explicitly modeled. Although this assumption enables the strategic interactions among dominant market operators to be analyzed in a computationally tractable manner, extending the proposed framework to network-constrained and multi-zonal electricity markets remains an important direction for future research.
- The strategic bidding analysis focuses on the dominant market operators because they account for a substantial share of the market and exert the greatest influence on market-clearing prices. In addition, market demand is assumed to be inelastic during the day-ahead market-clearing process. These assumptions reduce the computational complexity of the equilibrium problem while preserving the principal strategic interactions among the dominant market participants.

6. Conclusions and Future Work

This paper proposed a game-theoretic, risk-aware SFNE framework for strategic bidding in uniform-pricing day-ahead electricity markets, in which dominant generation companies determine cluster-level bidding markups while accounting for competitors' behavior and downside financial risk under demand uncertainty. A technology-oriented clustering approach reduced the dimensionality of the equilibrium problem while preserving representative portfolio heterogeneity, and the equilibrium was computed using a relaxed NI best-response algorithm. The numerical results demonstrated the computational tractability of the proposed framework, with all convergence criteria satisfied within approximately 54 iterations. Compared with the risk-neutral SFNE, the risk-aware framework reduced aggregate downside-profit risk by approximately 7% while maintaining comparable expected operator profits and slightly lowering market-clearing prices and procurement costs. The comparative analysis further demonstrated the influence of risk aversion on equilibrium bidding strategies and market outcomes, whereas the realistic 24 h market simulation confirmed consistent performance under time-varying demand conditions. Overall, the proposed framework provides a computationally tractable and theoretically well-founded benchmark for risk-aware strategic bidding in the investigated single-zone, uniform-pricing electricity market. Future research will extend the framework to network-constrained and multi-zonal electricity markets, adaptive multi-agent learning, alternative market-clearing mechanisms, intertemporal operational constraints, renewable and price uncertainties, and larger populations of strategic market participants.

Author Contributions: Conceptualization, M.M.I. and M.L.S.; methodology, M.M.I., T.Y. and M.L.S.; software, M.M.I. and T.Y.; validation, M.M.I. and S.B.; formal analysis, M.M.I., Z.W., C.I. and A.A.; investigation, M.M.I. and T.Y.; resources, M.M.I., S.B. and Z.W.; writing—original draft preparation, M.M.I.; writing—review and editing, M.M.I. and T.Y.; visualization, M.M.I., Z.W., C.I. and A.A.; supervision, M.L.S.; project administration, M.L.S.; funding acquisition, M.L.S. All authors have read and agreed to the published version of the manuscript.

Funding: The research of Muhammad Muzammal Islam is supported by the Italian National Recovery and Resilience Plan (PNRR), Mission 4 “Education and Research”, Component 2, “From Research to Business”, Investment 1.2 “Funding Projects Presented by Young Researchers”, under

CDR.DEI.DM47.2025 (CUP: D93C25000310006), Department of Electrical and Information Engineering, Politecnico di Bari, Italy.

Data Availability Statement: The data supporting the conclusions of this article will be made available by the authors upon request.

Conflicts of Interest: The authors declare no conflicts of interest.

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