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Assessment of Distributed PV Hosting Capacity in Distribution Areas Based on Operating Region Analysis

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Abstract

With the high penetration of distributed photovoltaics (PV) in distribution areas, transformer capacity limits and source–load fluctuations have become key factors constraining PV accommodation. To accurately assess the PV hosting capacity under energy storage regulation, this paper proposes an assessment method based on operating region analysis. First, a coordinated operation model for the distribution area is established, incorporating the transformer capacity, energy storage constraints, and power balance. On this basis, the calculation boundaries for the PV hosting capacity are discussed in two scenarios: Model 1 ignores power curve uncertainty, characterizing the geometry of the conventional operating region to find the maximum deterministic hosting capacity (S_1) that keeps the region non-empty. Model 2 introduces box-type uncertainty sets for the source and load, proposes the concept of a “Self-Balanced Operating Region”, and constructs a robust feasibility determination model (f_3) based on a Min–Max–Min structure. To solve this multi-layer nested non-convex model, an iterative algorithm based on duality theory and Benders decomposition is employed to determine the robust hosting capacity under uncertainty (S_2) at the critical point where f_3 shifts from zero to non-zero. Case studies show that source–load uncertainty leads to a significant contraction of the operating region, and the robust hosting capacity under uncertainty requirements is strictly less than the deterministic hosting capacity ($S_1 > S_2$). This method quantifies the reduction effect of uncertainty on the accommodation capability, providing a theoretical basis for planning high-renewable penetration distribution areas and energy storage configuration.

Keywords: distributed PV; hosting capacity assessment; operating region; self-balancing; uncertainty; Benders decomposition



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1. Introduction

1.1. Background and Significance

Driven by the “Carbon Peaking and Carbon Neutrality goals [1]”, distributed renewable energy (DRE), represented by photovoltaics (PV), has exhibited an explosive growth trend on the distribution network side, particularly within distribution transformer areas. As the terminal link connecting the power system to end-users, the distribution transformer area has gradually transformed from a traditional passive network into an active network characterized by source–load interaction. However, with the large-scale integration of PV into these areas, its inherent volatility and intermittency pose threats to the safe and stable operation of the distribution area.

The distribution transformer acts as the connection point between the distribution area and the upper-level grid, and its rated capacity constitutes the maximum constraint limiting the power exchange between them. When the PV output significantly exceeds the load demand, substantial reverse power flow is generated within the system, potentially leading to issues such as reverse overload of the transformer and nodal voltage limit violations. To alleviate these issues, Energy Storage Systems (ESS) have been widely introduced into distribution areas [2]. As a flexible resource, ESS plays a crucial role in smoothing power fluctuations and performing peak shaving and valley filling. However, even with the introduction of ESS as a flexible regulation resource, a theoretical upper limit remains for the PV capacity that the distribution area can accommodate. Traditional hosting capacity assessment methods are mostly based on typical deterministic scenarios or consider only the static power balance, failing to accurately describe the dynamic operating boundary of the system under ESS regulation. Consequently, accurately assessing the maximum PV output that the system can accommodate—defined as the “distributed PV hosting capacity of the distribution area”—under the triple constraints of “transformer capacity, ESS regulation, and PV integration” has become a critical research issue in distribution network planning and operation.

1.2. Literature Review

The operating region (or feasible region) refers to the set formed by key state variables (such as tie-line exchange power) of a power system satisfying all physical and security constraints. In recent years, operating region theory has been widely applied in the coordinated dispatch of microgrids and the main grid to describe the dispatchability of microgrids [3]. Using polyhedral projection algorithms or the Minkowski Sum method [4–7], high-dimensional constraints of internal resources can be mapped into low-dimensional interactive power boundaries at the Point of Common Coupling (PCC). However, the existing studies primarily focus on characterizing “deterministic” boundaries [4,8]. A systematic quantitative analysis framework is still lacking, regarding the deformation of operating region boundaries caused by the randomness and volatility of PV output, as well as the impact of PV output on hosting capacity limits.

Currently, assessment methods for the distributed PV hosting capacity are mainly divided into index-constraint-based analysis methods and optimization-model-based calculation methods [9,10]. Early research often employed streamlined methods or iterative power flow methods, using voltage deviation and thermal stability limits as constraints to calculate the maximum permissible capacity at each grid node [11]. With the development of Active Distribution Network (ADN) technology, optimization assessment methods considering the coordinated regulation of On-Load Tap Changers (OLTC), reactive power compensation devices, and ESS have become mainstream [12]. However, these methods mostly focus on static maximization at a single time instant or a typical daily profile, rarely analyzing the dynamic compression effect of source–load uncertainty on hosting capacity boundaries from the perspective of continuous system operation [13–16].

Addressing the uncertainty and volatility of the PV output, Robust Optimization provides an effective solution [17,18]. By constructing an Uncertainty Set to describe the range of source–load fluctuations and seeking the optimal operating strategy under the worst-case scenario, the robust security of the system can be guaranteed. However, traditional robust dispatch models generally focus on “minimizing operating costs,” whereas when assessing the PV hosting capacity, the core question is one of existence: “does a feasible solution exist?” [19–22]. Combining the judgment of the “self-balancing” capability in robust optimization with the geometric characteristics of the operating region to define the robust hosting capacity remains one of the challenges in current research. Overall, the

existing literature has provided important foundations for feasible-region characterization, distributed PV hosting-capacity assessment, and uncertainty-aware dispatch. However, most studies treat these three aspects separately. A unified framework is still lacking for linking the geometric evolution of the operating region, the existence of recourse under uncertainty, and the identification of the robust hosting-capacity boundary in distribution transformer areas with flexible resources. This gap motivates the method proposed in this paper.

1.3. Main Contribution

This paper proposes a distributed PV hosting capacity assessment method based on operating region analysis. The main contributions are:

1. A coordinated operation model for the source–storage–load in the distribution area is constructed. Considering constraints such as distribution transformer capacity, ESS charging/discharging power and energy state, and system power balance, a coupling model of the interactive power between the distribution area and microgrid and internal flexible resources is established, laying the foundation for subsequent analysis.
2. A hosting capacity assessment method based on the deterministic operating region is proposed. Without considering power curve uncertainty, the Adaptive Constraint Generation (Ad-CG) algorithm is used to characterize the interactive power operating region of the distribution area. The PV output accommodated when the operating region becomes exactly an empty set is defined as S_1 , which represents the maximum PV output that the system can accommodate at that point.
3. A robust hosting capacity assessment model based on the self-balanced operating region is constructed. A source–load box uncertainty set is introduced, requiring the distribution area to maintain stable operation under any uncertainty disturbance. A feasibility judgment model (f_3) with a Min–Max–Min structure is established. Using duality theory and the Benders decomposition algorithm, the problem is transformed into a solvable Mixed-Integer Linear Programming (MILP) problem. The critical point where f_3 shifts from zero to non-zero is defined as the robust hosting capacity under uncertainty (S_2).
4. The difference in hosting capacity between the two scenarios is compared to demonstrate the impact of uncertainty. Case studies analyzing the difference between S_1 and S_2 verify that uncertainty reduces the PV hosting capacity, providing a reference for planning distribution areas with high reliability requirements.

2. Fundamental Coordinated Operation Model of Source–Storage–Load in Distribution Area

2.1. Physical Topology and Basic Assumptions

With the deepening interaction between sources and loads, the modern distribution transformer area no longer refers merely to a power supply area served by a single distribution transformer. Instead, it has gradually evolved into a micro-energy system comprising distributed generation (DG), energy storage systems (ESS), and diversified loads. The physical topology of the distribution area studied in this paper is shown in Figure 1.

To facilitate the analysis, the following assumptions are made:

1. To focus on transformer-area-level hosting-capacity assessment and to support the aggregated PV power interaction analysis of a larger distribution system, this paper adopts an aggregated steady-state active-power model. Under this modeling scope, the main concern is the overall active-power exchange capability between the distribution transformer area and the upstream grid, rather than detailed AC power-flow behavior at each node. Therefore, the reactive power, voltage regulation details, and

network losses are neglected, and the upstream medium-voltage grid is treated as an equivalent stiff grid. This simplification allows for the dominant coupling among PV output, transformer/interface capacity, energy storage regulation, and source–load uncertainty to be explicitly captured, while detailed voltage/reactive-power analysis is left for future work.

2. The operation of the distribution transformer area is modeled in discrete time, and the scheduling horizon is divided into multiple time intervals of equal length Δt .
3. Distributed PV power is preferentially consumed within the local distribution transformer area, and only the surplus power is transmitted to the upstream grid through the transformer interface.

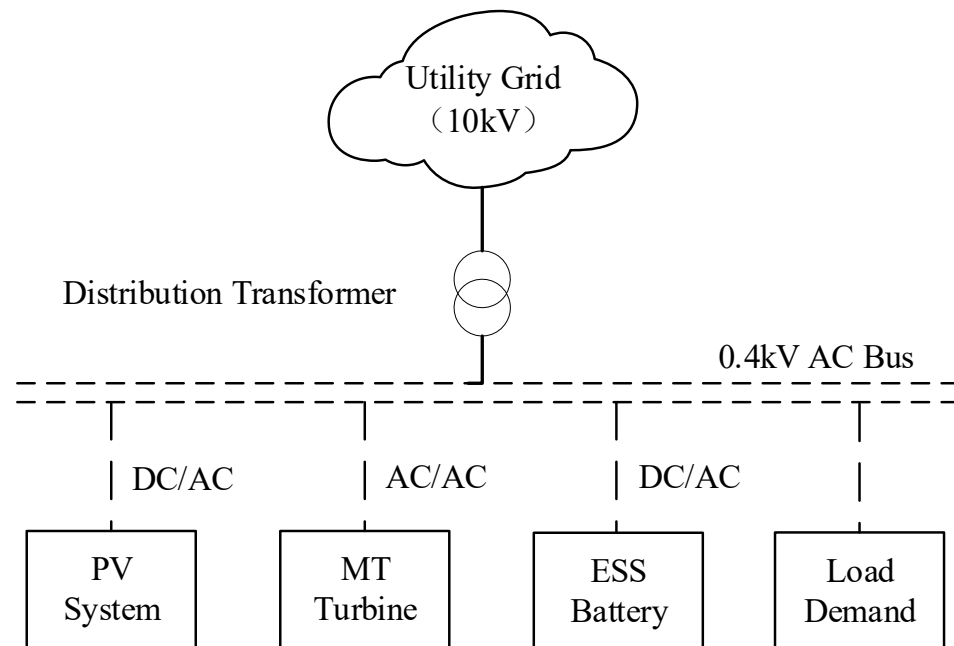


Figure 1. Physical topology of the distribution area.

2.2. Modeling

2.2.1. Distribution Transformer Constraints

The rated capacity of the distribution transformer constitutes a rigid constraint, limiting the power exchange between the distribution area and the upper-level grid. Defined as the net interactive power on the low-voltage side of the transformer at time t (where $P_t^{grid} > 0$ indicates power import from the grid, and $P_t^{grid} < 0$ indicates power export to the grid), it must satisfy the following constraint:

$$-P_{eq}^{max} \leq P_t^{grid} \leq P_{eq}^{max}, \forall t \in T \quad (1)$$

where $P_{eq}^{max} = \min(S_{cap}, P_l^{max})$ is the equivalent interface transfer limit, jointly determined by the transformer rated capacity S_{cap} and the long-term thermal carrying capability of the feeder/cables P_l^{max} , and T denotes the set of scheduling time intervals in the discrete-time scheduling horizon, i.e., $T = \{1, 2, \dots, N_T\}$, where N_T is the total number of time periods.

2.2.2. Distributed Generation Constraints

The distributed generation in the distribution area is primarily composed of micro gas turbines (MT). Their active power output and ramping constraints are considered as follows:

$$P_k^{MT,min} \leq P_{k,t}^{MT} \leq P_k^{MT,max} \quad (2)$$

$$P_{k,t+1}^{MT} - P_{k,t}^{MT} \leq R_k^{MT,up} \cdot \Delta t \quad (3)$$

$$P_{k,t}^{MT} - P_{k,t+1}^{MT} \leq R_k^{MT,down} \cdot \Delta t \quad (4)$$

where $P_{k,t}^{MT}$ is the active power output of the k -th MT unit at time t ; $P_k^{MT,max}$ and $P_k^{MT,min}$ are the upper and lower limits of the active power output of the k -th MT unit, respectively; $R_k^{MT,up}$ and $R_k^{MT,down}$ are the ramp-up and ramp-down rates of the k -th MT unit, respectively; and Δt is the time interval, which is typically set to 15 min or 1 h.

2.2.3. Energy Storage System Constraints

The ESS in the distribution area must satisfy constraints regarding charging/discharging power, logical operations, energy capacity, initial/final energy balance, and state-of-charge limits. These constraints are expressed as follows:

$$0 \leq P_{k,t}^{ch} \leq P_k^{ch,max} \quad (5)$$

$$0 \leq P_{k,t}^{dis} \leq P_k^{dis,max} \quad (6)$$

$$P_{k,t}^{ch} \cdot P_{k,t}^{dis} = 0 \quad (7)$$

$$E_k^{ESS,min} \leq E_{k,0}^{ESS} + \eta_k^{ch} \sum_1^t P_{k,t}^{ch} \cdot \Delta t - \frac{1}{\eta_k^{dis}} \sum_1^t P_{k,t}^{dis} \cdot \Delta t \leq E_k^{ESS,max} \quad (8)$$

$$\eta_k^{ch} \sum_{t=1}^T P_{k,t}^{ch} \cdot \Delta t - \frac{1}{\eta_k^{dis}} \sum_{t=1}^T P_{k,t}^{dis} \cdot \Delta t = 0 \quad (9)$$

$$E_k^{ESS,max} \times 20\% \leq E_{k,0}^{ESS} + \eta_k^{ch} \sum_1^t P_{k,t}^{ch} \cdot \Delta t - \frac{1}{\eta_k^{dis}} \sum_1^t P_{k,t}^{dis} \cdot \Delta t \leq E_k^{ESS,max} \times 90\% \quad (10)$$

where $P_{k,t}^{ch}$ and $P_{k,t}^{dis}$ are the charging and discharging power of the k -th ESS unit at time t , respectively; $P_k^{ch,max}$ and $P_k^{dis,max}$ are the upper limits for the charging and discharging power of the k -th ESS unit; η_k^{ch} and η_k^{dis} are the charging and discharging efficiencies of the k -th ESS unit; $E_k^{ESS,min}$ and $E_k^{ESS,max}$ are the lower and upper energy limits of the k -th ESS unit, respectively; and $E_{k,0}^{ESS}$ is the initial stored energy of the k -th ESS unit.

Since the charging/discharging logical constraint in (7) is a nonlinear complementary constraint, it involves the product of two decision variables. Its physical role is to ensure that the ESS cannot charge and discharge simultaneously. It also complicates the characterization of the subsequent operating region. Therefore, for the sake of computational tractability, this paper adopts a linear relaxation for the ESS model. Specifically, constraint (7) is approximated by the following linear constraint:

$$\frac{P_{k,t}^{ch}}{P_k^{ch,max}} + \frac{P_{k,t}^{dis}}{P_k^{dis,max}} \leq 1 \quad (11)$$

2.2.4. PV and Load Uncertainty Modeling

The distributed PV output P_t^{PV} and conventional load demand P_t^{Load} are both composed of a “forecasted value” and an “uncertainty deviation”:

$$P_{k,t}^{PV} = P_{k,t}^{PV,pre} + \Delta P_{k,t}^{PV} \quad (12)$$

$$\Delta P_{k,t}^{PV,min} \leq \Delta P_{k,t}^{PV} \leq \Delta P_{k,t}^{PV,max} \quad (13)$$

$$P_t^{Load} = P_t^{Load,pre} + \Delta P_t^{Load} \quad (14)$$

$$\Delta P_t^{Load,min} \leq \Delta P_t^{Load} \leq \Delta P_t^{Load,max} \quad (15)$$

where $P_{k,t}^{PV,pre}$ and $\Delta P_{k,t}^{PV}$ are the forecasted baseline value and power fluctuation deviation of the k -th PV unit at time t , respectively; $\Delta P_{k,t}^{PV,min}$ and $\Delta P_{k,t}^{PV,max}$ are the minimum and maximum PV output deviations at time t ; $P_t^{Load,pre}$ and ΔP_t^{Load} are the forecasted baseline value and fluctuation deviation of the conventional load at time t ; and $\Delta P_t^{Load,min}$ and $\Delta P_t^{Load,max}$ are the minimum and maximum load demand deviations at time t .

2.2.5. System Power Balance

As a micro-energy balance system, the distribution area must satisfy the power balance at any time instant. The nodal power balance equation is expressed as follows:

$$P_t^{grid} + P_t^{PV} + P_t^{MT} + P_t^{dis} = P_t^{ch} + P_t^{Load}, \forall t \in T \quad (16)$$

Rearranging the terms yields the expression for the interactive power between the distribution area and the upper-level grid:

$$P_t^{grid} = P_t^{ch} + P_t^{Load} - P_t^{dis} - P_t^{MT} - P_t^{PV}, \forall t \in T \quad (17)$$

Formulas (16) and (17) describe the active-power balance relationship in the distribution transformer area and show how the interactive power with the upstream grid is determined by internal adjustable resources, PV generation, and load demand.

2.3. Constraint Aggregation and Linearization

The overall model is generalized into a compact matrix form:

$$\Omega^{xyw} = \{x, y, \Delta w | \mathbf{A}x + \mathbf{B}y + \mathbf{C}\Delta w \leq \mathbf{b}\} \quad (18)$$

where Ω^{xyw} represents the high-dimensional operating region formed by the set of all internal constraints; x is the aggregated variable describing the physical quantity obtained after aggregation (typically the interactive power P_t^{grid}); y is the adjustment variable used to adjust the operating point within the constraint set Ω^{xyw} under uncertain conditions; Δw is the uncertainty variable arising from forecast errors due to various random factors in the actual operation, affecting the range of the aggregated variable; $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are the coefficient matrices for variables x , y , and Δw , respectively; and $\mathbf{b}$ is the constant vector of the system.

To facilitate the subsequent operating region analysis, the original scheduling problem is rewritten into the compact form of (18), in which the interaction power x is treated as the aggregated variable of interest, while y collects all internal recourse variables, including the MT output, ESS charging/discharging power, and flexible load adjustment. The uncertainty variable u represents the source–load forecast deviation that perturbs the feasible range of x through the coupling constraints. Therefore, the essence of the hosting-capacity problem is to determine how large the PV capacity can be before the feasible set of x shrinks to an empty set. From this perspective, the hosting-capacity boundary can be interpreted geometrically as the critical point at which the operating region loses its non-empty interior. This matrix model (Equation (18)) constitutes a polytope for the distribution area system. The discussion about the “operating region” in subsequent chapters essentially explores the geometric projection relationships among variables x , y , and Δw under the constraints of this polytope.

3. Assessment of PV Hosting Capacity Without Considering Uncertainty

In the first scenario of assessing the PV hosting capacity for the distribution area, the randomness of the PV output and load demand fluctuations is temporarily disregarded. It is assumed that the system operates exactly according to the forecasted curves. This represents an idealized scenario, where the assessment results reflect the theoretical maximum level of PV accommodation achievable under “perfect forecast” conditions. This chapter introduces the concept of the “deterministic operating region” to analyze the feasible space of the distribution area’s interactive power from a geometric perspective. By analyzing the changes in the operating region as the PV capacity increases, a calculation model for the maximum deterministic hosting capacity (S_1) is established based on the criterion of a non-empty operating region.

3.1. Distribution Area Operating Region

Based on the polytope expression of the distribution area system derived in Section 2, under the first scenario, let the uncertainty variable $\Delta w = 0$. The polytope expression for the distribution area in the first scenario is obtained as follows:

$$\Omega^{xy} = \{x, y | \mathbf{A}x + \mathbf{B}y \leq \mathbf{b}\} \quad (19)$$

According to convex set projection theory, the essence of operating region projection is projecting the high-dimensional space Ω^{FR} , composed of variables x and y , onto a low-dimensional space $\phi^{\text{FR}}(x)$ that is independent of y and solely related to x . This can be expressed as follows:

$$\phi^{\text{FR}}(x) = \text{Pr}_x(\Omega^{\text{FR}}) = \{x | \exists y : \mathbf{A}x + \mathbf{B}y \leq \mathbf{b}\} \quad (20)$$

For any point x in the projected space ϕ^{FR} , the following holds:

$$Y = \{y | \mathbf{B}y \leq \mathbf{b} - \mathbf{A}x\} \quad (21)$$

where Y is the set of all adjustment variables.

The deterministic operating region can be described as follows: for any interactive power point x within the operating region ϕ^{FR} , there exists at least one corresponding y such that the system can maintain stable operation; for points x outside the operating region, no adjustment scheme y exists to maintain system stability. Generally, the operating region represents the maximum range of interactive power that the regional grid can provide. In mathematical terms, the operating region is a set $\phi^{\text{FR}}(x) = \{x | \mathbf{A}x + \mathbf{B}y \leq \mathbf{b}\}$, satisfying that for all $\forall x \in \phi^{\text{FR}}, Y \neq \emptyset$; conversely, for all $\forall x \notin \phi^{\text{FR}}, Y = \emptyset$.

This projection step is the key to reducing the original high-dimensional feasibility problem into a low-dimensional boundary-identification problem. After projection, the internal variables, y , are no longer explicitly tracked, and all information relevant to the interaction capability of the distribution area is encoded into the boundary of the operating region in x -space. In this way, the hosting-capacity assessment problem is transformed from a direct capacity-search problem into an operating region existence problem.

Since the projection process is a linear mapping, the projection of a high-dimensional convex polytope remains a convex polytope. Therefore, the operating region obtained above can be expressed as a set of linear inequalities:

$$\phi^{\text{FR}} = \{x | \mathbf{D}x \leq \mathbf{d}\} \quad (22)$$

where $\mathbf{D}$ and $\mathbf{d}$ are the boundary information of the deterministic operating region. The essence of characterizing the deterministic operating region is to solve for these two boundary parameters.

3.2. Characterization of the Operating Region

Before characterizing the whole operating region, it is necessary to determine whether an arbitrary candidate point x_0 belongs to the projected feasible set. For this purpose, an auxiliary optimization problem is constructed by introducing an external slack variable. The role of this variable is to measure the minimum boundary relaxation required for the candidate point to become feasible. Therefore, the sign of the optimal slack value provides a direct feasibility certificate for the operating point.

$$\begin{aligned} f &= \min_s \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x_0 \\ s &\geq 0 \end{aligned} \quad (23)$$

where s is an external slack variable, which is geometrically equivalent to the external displacement from the operating region boundary.

The objective of optimization problem (23) is to find the minimum value of the external slack variable that satisfies the constraints, i.e., to alter the constraints as little as possible while satisfying the operating point x_0 . If the optimization result $f > 0$, then $s > 0$, indicating that no adjustment variable y exists to make the operating point satisfy the constraint $\mathbf{B}y \leq \mathbf{b} - \mathbf{A}x_0$, meaning $Y = \emptyset$, which proves that the point lies outside the sub-high-dimensional operating region ϕ^{FR} . Conversely, if the optimization result $f = 0$, then $s = 0$, indicating that there exists an adjustment variable y enabling the operating point to satisfy the constraint $\mathbf{B}y \leq \mathbf{b} - \mathbf{A}x_0$, meaning $Y \neq \emptyset$, which proves that the point lies within the sub-high-dimensional operating region ϕ^{FR} . In summary, it can be concluded that $f = 0$ is a necessary and sufficient condition for the operating point x_0 to be within the sub-high-dimensional operating region ϕ^{FR} .

After obtaining a pointwise feasibility criterion, the next step is to determine whether an entire candidate space is contained in the operating region. Since the operating region is convex, it is sufficient to examine the extreme boundary points of the current space. If the farthest point of the current space is still feasible, then all points inside this space are also feasible. Otherwise, the current space must intersect the infeasible region, and an additional cut is required to shrink the candidate space.

Drawing on the idea of robust optimization, the relationship between the current space and the operating region can be determined by judging whether the farthest point in the current space lies within the operating region. The following Max–Min bi-level optimization problem is designed:

$$\begin{aligned} f &= \max_{x \in R^c} \min_{y, s} s \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x \\ s &\geq 0 \end{aligned} \quad (24)$$

The purpose of optimization problem (24) is to determine whether the farthest point in the current space lies within the range of the operating region. If the farthest point in R^c

is also included in the operating region, then $R^C \in \phi^{FR}$. To solve this bi-level optimization problem, the inner minimization problem is converted into its dual problem.

$$\begin{aligned} f &= \max_{z,x} z^T(\mathbf{b} - \mathbf{A}x) \\ s.t. & \mathbf{B}^T z = 0 \\ & -1 \leq z \leq 0 \\ & \mathbf{D}^C x \leq \mathbf{d}^C \end{aligned} \quad (25)$$

where z is the dual variable, and $\mathbf{D}^C$ and $\mathbf{d}^C$ are the boundary parameters of the current space R^C , respectively. Due to the weak duality property, the maximum value of the dual problem (25) is less than or equal to the minimum value of the original problem (24). Therefore, when $f = 0$, the following inequality holds:

$$z^T(\mathbf{b} - \mathbf{A}x) \leq 0 \quad (26)$$

When x varies within the deterministic operating region R^C , i.e., for all $\forall x \in R^C$, the optimal solution z of problem (25) will take an extreme value in the dual space, which corresponds to a vertex z_v in the space $Z = \{\mathbf{B}^T z = 0, -1 \leq z \leq 0\}$. The polytope form of the operating region can be expressed as follows:

$$\phi^{FR} = \left\{ x \mid z_v^T \mathbf{A}x \geq z_v^T \mathbf{b} \right\} \quad (27)$$

However, the objective function of problem (25) is not linear, making it difficult to solve directly. Moreover, the constraints for variables z and x are decoupled. Therefore, problem (25) can be decomposed into two parts, based on the optimization variables:

$$\begin{aligned} f &= \max_{z \in Z} (z^T \mathbf{b} + \max_x (-z^T \mathbf{A}x)) \\ s.t. & \mathbf{D}x \leq \mathbf{d} : v^d \end{aligned} \quad (28)$$

where v^d is the dual variable of the operating region constraints.

The Karush–Kuhn–Tucker (KKT) conditions provide necessary optimality conditions for constrained optimization problems, including stationarity, primal feasibility, dual feasibility, and complementary slackness conditions.

At this stage, the operating region characterization problem has been reformulated as a bilevel optimization problem. However, such a form is still difficult to solve directly. Since the inner problem is convex, its optimality can be equivalently characterized by the KKT conditions. This allows the original bilevel problem to be converted into a single-level problem with complementarity conditions, which can then be linearized through the Big-M technique and solved by standard MILP solvers [23]:

$$\begin{cases} \mathbf{A}^T z + \mathbf{D}^T v^d = 0 \\ v^d \geq 0 \\ \mathbf{D}x \leq \mathbf{d} \\ v^d(\mathbf{d} - \mathbf{D}x) = 0 \end{cases} \quad (29)$$

Equation (29) represents the Karush–Kuhn–Tucker (KKT) optimality conditions of the inner optimization problem. By introducing the KKT conditions, the inner optimization problem can be equivalently replaced by its optimality conditions. This transformation converts the original bilevel optimization problem into a single-level problem with complementarity constraints, which facilitates the subsequent reformulation and solution.

To handle the nonlinear constraint in (29), the Big-M method can be introduced to split $v^d(\mathbf{d} - \mathbf{D}x) = 0$ into the following two constraints:

$$\begin{aligned} 0 &\leq v^d \leq M\alpha \\ 0 &\leq \mathbf{d} - \mathbf{D}x \leq M(\alpha - 1) \end{aligned} \quad (30)$$

where M is a sufficiently large constant, and α is a 0–1 variable.

At this point, (25) has been converted into a Mixed-Integer Linear Programming (MILP) problem that can be solved by commercial solvers:

$$\begin{aligned} f &= \max_{z, x, v^d} z^T \mathbf{b} + \mathbf{D}^T v^d \\ \text{s.t. } &\mathbf{B}^T z = 0 \\ &\mathbf{A}^T z + \mathbf{D}^T v^d = 0 \\ &-1 \leq z \leq 0 \\ &0 \leq v^d \leq M\alpha \\ &0 \leq \mathbf{d} - \mathbf{D}x \leq M(\alpha - 1) \end{aligned} \quad (31)$$

The optimal solution of problem (31) corresponds to a vertex in the dual space of the original problem, which is the farthest point in the current space. When $f > 0$, the farthest point in the current space is outside the operating region; when $f = 0$, the farthest point in the current space is inside the operating region.

Next, an operating region solution algorithm based on the outer approximation cutting method is proposed. According to formula (27), for all $\forall x \in \phi^{\text{FR}}$, inequality (26) holds. Conversely, if $x' \notin \phi^{\text{FR}}$, then the following formula holds:

$$\exists z' \in Z \quad z'^T \mathbf{A}x' < z'^T \mathbf{b} \quad (32)$$

Thus, for all $\forall x \in \phi^{\text{FR}}$, the following equality can separate the operating region from the non-operating region, which is termed as a feasibility cut:

$$z'^T \mathbf{A}x = z'^T \mathbf{b} \quad (33)$$

Feasibility cuts generate constraints that separate the feasible space from the infeasible space, based on boundary nodes, i.e., the boundaries of the operating region. According to the polytope mathematical expression of the operating region, the optimal solution z' of problem (25) takes an extreme value in the dual space of the original problem, i.e., a vertex in space $Z = \{\mathbf{B}^T z = 0, -1 \leq z \leq 0\}$. Therefore, each vertex in the dual space corresponds to an operating region boundary. However, due to the excessive number of vertices in the dual space of the power system mathematical model, it is difficult to enumerate all vertices, and the generated feasibility cuts contain a large number of redundant constraints. Therefore, an outer approximation cutting method based on the improved Ad-CG (Adaptive Constraint Generation) algorithm is proposed below to adaptively generate vertices, thereby obtaining feasibility cut constraints and finally solving for the deterministic operating region. The logic is as follows:

1. First, initialize and generate a sufficiently large initial space, R^C .
2. Solve problem (31) in this space to find a vertex z_v in the dual space.
3. Generate an operating region boundary $z'^T \mathbf{A}x \geq z'^T \mathbf{b}$ using this vertex.
4. Update the current space R^C with the newly generated boundary information.
5. Repeat the above steps until the optimal value of problem (31) is $f = 0$, at which point the output space R^C is the operating region.

Although the Ad-CG algorithm can accurately characterize the shape of the operating region, to calculate S_1 precisely, it is not necessary to traverse all capacities. Instead, a single-layer linear programming model with the objective of maximizing the PV capacity can be constructed. In this model, the PV hosting capacity S_{PV} is transformed from a model parameter into a decision variable. First, the constant term $\mathbf{b}$ in the distribution area operation constraint matrix is decomposed into a base vector $\mathbf{b}_{base}$ unrelated to PV and a sensitivity vector $\mathbf{b}_{unit}$ describing the characteristics of unit PV output. Then, with maximizing S_{PV} as the objective function, the following linear programming model is constructed:

$$\begin{aligned} S_1 = & \max_{x,y,S_{PV}} S_{PV} \\ \text{s.t. } & \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} - S_{PV} \cdot \mathbf{b}_{unit} \leq \mathbf{b}_{base} \\ & -S_{cap} \leq P_t^{grid} \leq S_{cap} \\ & S_{PV} \geq 0 \end{aligned} \quad (34)$$

where $\mathbf{b} = S_{PV} \cdot \mathbf{b}_{unit} + \mathbf{b}_{base}$. $\mathbf{b}_{base}$ is the base constant vector, representing all fixed physical boundaries and load demands faced by the system when no PV is connected to the distribution area, and $\mathbf{b}_{unit}$ is the unit PV impact vector. This model searches for the upper limit of PV capacity that the system's physical boundaries can accommodate by synergistically optimizing the interactive strategy x between the distribution area and the microgrid and the internal resource adjustment strategy y . The optimal solution of this model is the deterministic hosting capacity S_1 in the first scenario.

4. Assessment of PV Hosting Capacity Considering Power Uncertainty

In the previous chapter, the theoretical upper limit of the distribution area's deterministic hosting capacity, S_1 , was calculated based on deterministic forecast curves. However, in actual operation, the distributed PV output exhibits significant volatility and randomness, due to factors such as irradiance and temperature. Simultaneously, users' load behavior also involves uncertainty. If planning is based solely on the forecasted power, the distribution area system may face limit violation accidents due to insufficient regulation capability in extreme scenarios like "sudden PV increase and sudden load decrease." Therefore, this chapter considers power uncertainty, proposes the concept of the "Self-Balanced Operating Region," and constructs a robust feasibility judgment model (f_3) based on a Min–Max–Min structure to assess the maximum robust hosting capacity under uncertainty S_2 of the distribution area system under the premise of robust security.

In physical terms, the self-balanced operating region describes the maximum interaction range that the distribution area can sustain while preserving internal balance under all admissible source–load fluctuations. It is therefore not merely a feasible set under one operating scenario, but a robust feasible set that already accounts for the recourse capability of internal flexible resources. This is also why the robust hosting capacity obtained in Case 2 must be lower than or equal to the deterministic hosting capacity obtained in Case 1.

Compared with Case 1, the essential difference in Case 2 is that the feasibility of the interaction power x is no longer judged under a single forecasted operating condition, but under all admissible uncertainty realizations within the prescribed box uncertainty set. Therefore, the operating region concept must be strengthened accordingly: a point x is only regarded as feasible if, for every uncertainty realization u in the uncertainty set, there exists at least one corresponding internal adjustment strategy y that restores system balance. This stricter requirement leads to the concept of the self-balanced operating region.

It should be emphasized that the term "self-balanced" in this paper does not mean autonomous islanded operation. Instead, it refers to the ability of the distribution transformer area to maintain a steady-state active-power balance under grid-connected operation

through internal flexible-resource adjustment and power exchange with the upstream grid. Accordingly, issues that are specific to autonomous island operation, such as short-circuit behavior, protection coordination, and dynamic voltage/frequency support, are beyond the scope of this study.

4.1. Self-Balanced Operating Region

Based on the analysis in Section 2, the simplified operation model of the distribution area system considering uncertainty is as follows:

$$\Omega^{xyw} = \{x, y, \Delta w | \mathbf{A}x + \mathbf{B}y + \mathbf{C}\Delta w \leq \mathbf{b}\} \quad (35)$$

From the perspective of the adjustment variable y , for any uncertainty variable Δw in the uncertainty variable domain Ω^w and any aggregated variable x in the self-balanced operating region Ω^x , there exists a set Y satisfying the following constraint:

$$Y = \{y | \mathbf{B}y \leq \mathbf{b} - \mathbf{A}x - \mathbf{C}\Delta w\} \quad (36)$$

where Y represents the set of all adjustment variables y , i.e., all adjustment measures available to cope with any uncertainty variable Δw in Ω^w and any aggregated variable x in Ω^x .

Therefore, theoretically, the self-balanced operating region implies that for any aggregated variable x within Ω^x and any uncertainty variable Δw within Ω^w , an adjustment variable y can be found to execute x ; conversely, for any aggregated variable x outside Ω^x , there exists an uncertainty variable Δw within Ω^w , for which no adjustment variable y can be found to execute x .

Practically, the self-balanced operating region of the distribution area signifies the maximum range of aggregated active power that the distribution area can provide to cope with any uncertainty variable, i.e., the maximum range of interactive power between the distribution area and the upper-level grid.

Consistent with the analysis in Section 3, the uncertain operating region can still be expressed as a set of linear inequalities:

$$\Omega^x = \{x | \mathbf{D}x \leq \mathbf{d}\} \quad (37)$$

4.2. Characterization of the Self-Balanced Operating Region

To characterize the self-balanced operating region, the key is to determine its boundary information. First, for a certain operating point $(x_0, \Delta w_0)$ in the sub-high-dimensional space, the following optimization problem is designed to judge whether the point is within the sub-high-dimensional operating region Ω^{xw} :

$$\begin{aligned} f_1 &= \min_{y, s} s \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x_0 - \mathbf{C}\Delta w_0 \\ s &\geq 0 \end{aligned} \quad (38)$$

where s is an external slack variable that is geometrically equivalent to the external displacement from the operating region boundary.

The objective of optimization problem (38) is to find the minimum value of the external slack variable satisfying the constraints, i.e., to alter the constraints as little as possible while satisfying the operating point $(x_0, \Delta w_0)$. If the optimization result $f_1 > 0$, then $s > 0$, indicating that no adjustment variable y exists to make the operating point satisfy the constraint $\mathbf{B}y \leq \mathbf{b} - \mathbf{A}x_0 - \mathbf{C}\Delta w_0$, meaning $Y = \emptyset$, which proves that the point is

outside the sub-high-dimensional operating region Ω^{xw} . Conversely, if $f_1 = 0$, then $s = 0$, indicating that there exists an adjustment variable y enabling the operating point to satisfy the constraint, meaning $Y \neq \emptyset$, which proves that the point is within Ω^{xw} . In summary, $f_1 = 0$ is a necessary and sufficient condition for the operating point $(x_0, \Delta w_0)$ to be within the sub-high-dimensional operating region Ω^{xw} .

Next, a Max–Min problem is designed to determine the most severe operating condition in the current uncertainty space R^w for a certain operating point x_0 :

$$\begin{aligned} f_2 &= \max_{\Delta w \in R^w} \min_{y, s} s \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x_0 - \mathbf{C}\Delta w \\ s &\geq 0 \end{aligned} \quad (39)$$

where the uncertainty space R^w can be represented by constructing a box uncertainty set to describe the fluctuation range of the uncertainty variable Δw . In this case, R^w is equivalent to the determined uncertainty variable Ω^w :

$$R^w = \{\Delta w | \Delta w \in [\underline{\Delta w}, \overline{\Delta w}]\} \quad (40)$$

where $\underline{\Delta w}$ and $\overline{\Delta w}$ are the lower and upper limits of the maximum allowable fluctuation deviation for the uncertainty variable Δw , respectively.

Since the box uncertainty set is a convex polytope, it can also be described by a set of linear inequalities:

$$R^w = \{\Delta w | \mathbf{D}_w \Delta w \leq \mathbf{d}_w\} \quad (41)$$

Furthermore, a max–Max–Min three-level optimization problem is designed to determine whether the current aggregated space R^x is inside the self-balanced operating region Ω^x :

$$\begin{aligned} f_3 &= \max_{x \in R^x} \max_{\Delta w \in R^w} \min_{y, s} s \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x - \mathbf{C}\Delta w \\ s &\geq 0 \end{aligned} \quad (42)$$

The three-level structure of (42) has a clear physical interpretation. The outer optimization searches for the most critical interaction-power point in the current candidate space. The middle optimization selects the worst uncertainty realization within the box uncertainty set. The inner optimization then attempts to recover system feasibility through internal resource adjustment. Therefore, the model explicitly captures the sequence of “interaction strategy selection–worst-case disturbance realization–internal recourse response,” which is the core mechanism of robust self-balancing analysis.

If the objective function of Equation (42) $f_3 > 0$, it indicates that there exists an operating point x in the current aggregated space R^x and an uncertainty variable Δw in the uncertainty space R^w , for which no adjustment variable y can be found to satisfy the constraints, i.e., $Y = \emptyset$. This means that some operating points in R^x are outside Ω^x . If $f_3 = 0$, it indicates that for any operating point x in R^x and any uncertainty variable Δw in R^w , an adjustment variable y can be found to satisfy the constraints, i.e., $Y \neq \emptyset$. This means that all operating points in R^x are inside Ω^x . Thus, $f_3 = 0$ is a necessary and sufficient condition for the current aggregated space R^x to be inside the self-balanced operating region Ω^x .

The constructed self-balanced operating region model for the distribution area f_3 presents a Min–Max–Min three-level nested structure. The outer optimization involves the interaction strategy (distribution transformer interactive power) between the distribution area and the upper-level grid, while the inner two levels involve the robust adjustment of internal resources (ESS, flexible loads) under PV and load uncertainty.

Benders decomposition can be used to solve f_3 , rewriting it as [24–26]:

$$\begin{aligned} f_3 = \min_x \max_{\Delta w \in R^w} \min_{y,s} s &= \min_x (0 + \max_{\Delta w \in R^w} \min_{y,s} s) \\ \text{s.t. } \mathbf{B}y - s &\leq \mathbf{b} - \mathbf{A}x - \mathbf{C}\Delta w \\ s &\geq 0 \end{aligned} \quad (43)$$

Although the above model can be expressed in compact form, its direct solution remains computationally demanding because the interaction-power decision and the worst-case uncertainty verification are strongly coupled. To exploit the problem structure, Benders decomposition is adopted to separate the decision of x from the feasibility verification under uncertainty. In this way, the original problem is decomposed into a master problem that searches for candidate interaction strategies and a sub-problem that checks their robust feasibility.

Using Benders decomposition, f_3 is decomposed into a master problem and a sub-problem. The master problem optimizes the interactive power x , while the sub-problem verifies the impact of the worst-case uncertainty scenario Δw for a given x and feeds back verification information to the master problem via Benders cuts, achieving coordinated convergence.

First, the master problem is constructed to find the optimal interactive strategy x under relaxed uncertainty constraints, introducing an auxiliary variable θ to approximate the risk value from the sub-problem. The mathematical form is as follows:

$$\begin{aligned} \min_{x,\theta} \quad & \theta \\ \text{s.t. } \quad & \theta \geq 0 \\ & -S_{cap} \leq x_t \leq S_{cap}, \quad \forall t \in T \\ & -R_{ramp} \leq x_t - x_{t-1} \leq R_{ramp}, \quad \forall t \in T \\ & \theta \geq \lambda_{(i)}^T (\mathbf{b} - \mathbf{A}x - \mathbf{C}\Delta w_{(i)}), \quad \forall i \in I \end{aligned} \quad (44)$$

where x is the decision variable representing the net interactive power through the transformer at each moment; S_{cap} is the rated capacity of the transformer, the rigid physical boundary for power interaction; R_{ramp} is the maximum ramp rate for interactive power; the last constraint is the Benders feasibility cut, where $i \in \{1, 2, \dots, I\}$ denotes the iteration index of the Benders decomposition algorithm, with each iteration corresponding to a newly generated feasibility cut; and $\lambda_{(i)}$ and $\Delta w_{(i)}$ are the dual variable and worst-case uncertainty scenario fed back by the i -th sub-problem, respectively.

Then, the sub-problem is constructed to verify whether the MTs and ESS in the distribution area can balance the system under the worst-case PV and load fluctuations for the interactive strategy x given by the master problem. If not, the minimum system violation s is calculated.

The original sub-problem is a bi-level programming (Max–Min) form, denoted as $Q(x')$:

$$Q(x') = \max_{\Delta w \in \Omega^w} \min_{y,s \geq 0} \{ \mathbf{B}y - s \leq \mathbf{b} - \mathbf{A}x' - \mathbf{C}\Delta w \} \quad (45)$$

To solve this Max–Min problem, the strong duality principle of linear programming is used to convert the inner minimization problem into its dual maximization problem. The sub-problem can be expressed as follows:

$$\begin{aligned} Q(x') &= \max_{\xi, x', \Delta w} \xi^T (\mathbf{b} - \mathbf{A}x' - \mathbf{C}\Delta w) \\ \text{s.t. } \quad & \mathbf{B}^T \xi = 0 \\ & -1 \leq \xi \leq 0 \\ & \mathbf{D}_w \Delta w \leq \mathbf{d}_w \end{aligned} \quad (46)$$

where ξ is the dual variable corresponding to the constraints, and $\mathbf{D}_w$ and $\mathbf{d}_w$ are the boundary parameters of the current uncertainty space. The sub-problem can be rewritten as follows:

$$\begin{aligned} Q(x') &= \max_{\xi, x', \Delta w} \xi^T (\mathbf{b} - \mathbf{A}x') - \xi^T \mathbf{C} \Delta w \\ \text{s.t. } &\mathbf{B}^T \xi = 0 \\ &-1 \leq \xi \leq 0 \\ &\mathbf{D}_w \Delta w \leq \mathbf{d}_w \end{aligned} \quad (47)$$

The objective function contains a bilinear term $\xi^T \mathbf{C} \Delta w$, making the sub-problem non-convex. The KKT optimality conditions can be used to transform Δw . Introducing the dual variable λ_w , the stationarity condition gives the following:

$$\nabla_{\Delta w} \mathcal{L} = \mathbf{C}^T \xi + \mathbf{D}_w^T \lambda_w = 0 \quad (48)$$

The complementary slackness condition gives the following:

$$\lambda_w (\mathbf{d}_w - \mathbf{D}_w \Delta w) = 0 \quad (49)$$

Based on the strong duality property, the original bilinear term can be exactly replaced at the optimal solution:

$$-\xi^T \mathbf{C} \Delta w = \lambda_w^T \mathbf{d}_w \quad (50)$$

Although the objective function is linearized, nonlinear complementary slackness constraints are introduced. Introducing binary auxiliary variables $\mu_w \in \{0, 1\}$ and a constant M , the Big-M method converts the complementary slackness conditions into mixed-integer linear constraints:

$$0 \leq \lambda_w \leq M \cdot \mu_w \quad (51)$$

$$0 \leq \mathbf{d}_w - \mathbf{D}_w \Delta w \leq M \cdot (1 - \mu_w) \quad (52)$$

Thus, the sub-problem is transformed into a Mixed-Integer Linear Programming (MILP) problem that is solvable by commercial solvers:

$$\begin{aligned} &\max_{\xi, \Delta w, \lambda_w, \mu_w} \xi^T (\mathbf{b} - \mathbf{A}x') + \lambda_w^T \mathbf{d}_w \\ \text{s.t. } &\mathbf{B}^T \xi = 0 \\ &-1 \leq \xi \leq 0 \\ &\mathbf{C}^T \xi + \mathbf{D}_w^T \lambda_w = 0 \\ &0 \leq \lambda_w \leq M \cdot \mu_w \\ &0 \leq \mathbf{d}_w - \mathbf{D}_w \Delta w \leq M \cdot (1 - \mu_w) \\ &\mu_w \in \{0, 1\} \end{aligned} \quad (53)$$

The specific solution process for f_3 , based on Benders decomposition, is as follows:

1. Set iteration counter $k = 1$, convergence tolerance $\varepsilon = 10^{-4}$, and initialize the risk variable bounds for the master problem: $UB = +\infty$, $LB = -\infty$.
2. Solve the master problem under constraints to obtain the optimal interactive strategy $x^{(i)}$ and risk assessment value $\theta^{(i)}$, updating the lower bound $LB = \theta^{(i)}$.
3. Fix $x^{(i)}$ and solve the sub-problem to obtain the worst-case scenario $\Delta w^{(i)}$, the corresponding dual multiplier $\lambda_w^{(i)}$, and the maximum violation $s^{(i)} = Q(x')$, updating the upper bound $UB = \min(UB, s^{(i)})$.
4. Calculate the gap between the upper bound and zero. If the gap $\leq \varepsilon$, it indicates that under strategy $x^{(i)}$, the distribution area can maintain system balance even under the

worst power fluctuations, i.e., $f_3 = 0$; the self-balanced operating region is non-empty; and the algorithm stops. If the gap $> \varepsilon$, proceed to (5).

5. Construct a Benders feasibility cut using the sub-problem solution:

$$\theta \geq \xi^{(i)T}(\mathbf{b} - \mathbf{A}x - \mathbf{C}\Delta w^{(i)}) \quad (54)$$

Add this constraint to the master problem, set $i = i + 1$, and return to (2) to continue iteration. If the iteration count exceeds the limit, stop and determine the current self-balanced operating region as empty.

In Case 2, which accounts for source–load uncertainty, the maximum PV hosting capacity, denoted as S_2 , is defined as the maximum permissible PV integration capacity that ensures that the Self-Balanced Operating Region remains non-empty ($f_3 = 0$) while accommodating any power fluctuation within the box uncertainty set. This paper proposes the following iterative procedure to determine S_2 :

1. Initialize the PV capacity scanning multiplier, $\alpha \in [\alpha_{\min}, \alpha_{\max}]$, and the step size, $\Delta\alpha$.
2. Update the forecasted PV output and the corresponding uncertainty boundaries based on the current scanning multiplier α_i .
3. Substitute the updated parameters into the model to solve for the objective function value $f_3(\alpha_i)$ under the current state.
4. If $f_3(\alpha_i) \leq \varepsilon$, the self-balanced operating region is non-empty; update $\alpha_{i+1} = \alpha_i + \Delta\alpha$ and proceed to the next iteration. Conversely, if $f_3(\alpha_i) > \varepsilon$, the self-balanced operating region becomes empty; terminate the calculation. The capacity point immediately preceding the transition of f_3 from zero to a positive value is identified as S_2 .

Theoretically, since S_2 must cope with additional worst-case scenarios, it must be that $S_1 > S_2$. The difference $\Delta S = (S_1 - S_2)$ is the hosting capacity sacrificed to ensure system robustness, also known as the “robustness cost.”

5. Case Study Analysis

A typical distribution area is selected with a 1500 kVA transformer. The ESS capacity is 400–1800 kWh (500 kW). The MT output is 80–800 kW. The PV peak is 900 kW, and the load peak is 1000 kW. For Case 2, a box uncertainty set is used with PV fluctuation $[-11\%, +17\%]$ and load $[-8\%, +12\%]$.

The algorithm is developed on the MATLAB R2023b platform, using the YALMIP toolbox to call the CPLEX solver, running on a system with a 2.60 GHz processor and 16 GB of RAM.

5.1. Case 1

Using the Ad-CG algorithm proposed in Section 3, the deterministic operating region of the distribution area under the baseline PV capacity is characterized. Figure 2 displays the three-dimensional operating region of the interactive power between the distribution area and the upper-level grid ($P_{14}^{grid}, P_{15}^{grid}, P_{16}^{grid}$).

As seen in the figure, the operating region presents itself as an irregular convex polytope. Any point within this polytope represents an interactive power strategy that satisfies all constraints. Evidently, the volume of the operating region is greater than zero at this time, indicating that the distribution area system has sufficient capability to accommodate the PV.

As the PV connection capacity increases, the net load curve shifts downward, causing changes in the right-hand-side term $\mathbf{b}$ of the constraint equations. Geometrically, this manifests as the boundaries of the operating region polytope continuously contracting

inward, resulting in a gradual decrease in volume. Figure 3 shows the operating region volumes corresponding to different PV capacities.

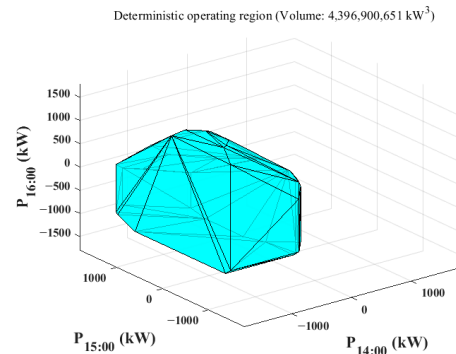
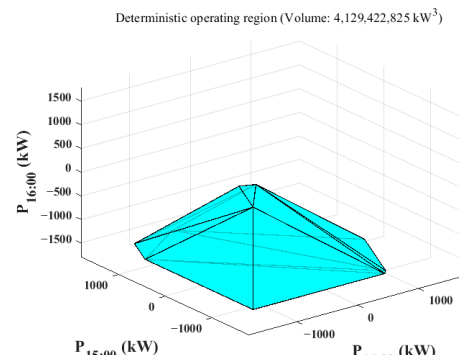
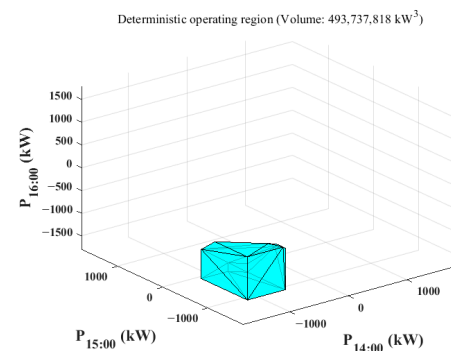


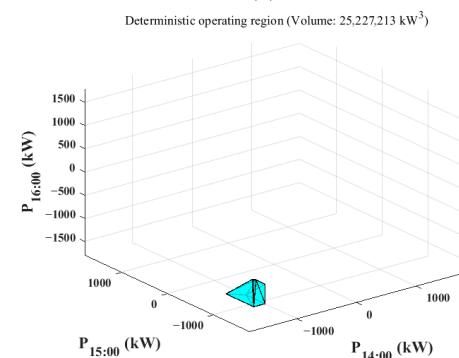
Figure 2. Three-dimensional operating region of the interactive power between the distribution area and the upper-level grid.



(a)



(b)



(c)

Figure 3. The operating region volumes corresponding to different PV capacities. (a) $P^{PV,pre} = [1800; 1560; 1600]$; (b) $P^{PV,pre} = [2250; 1950; 2000]$; and (c) $P^{PV,pre} = [2700; 2340; 2400]$.

It can be concluded from Figure 3 that the volume of the operating region shrinks sharply as the PV capacity increases continuously. This is because, at low PV levels, the distribution area can either import power from or export power to the upper-level grid, allowing for a wide range of x values, while the ESS can freely choose to charge, discharge, or remain idle. At high PV levels, the PV output far exceeds the load demand, and surplus power must be transmitted to the upper-level grid. Consequently, the transformer is forced to remain in a high-power reverse transmission state for extended periods. P_t^{grid} is forcibly locked near -1500 kW, drastically compressing the variable range. To prevent transformer overload, the ESS must also charge at full capacity for peak shaving.

By solving the linear programming model derived in Section 3, the maximum deterministic hosting capacity S_1 for the first scenario is obtained, as shown in the Figure 4.

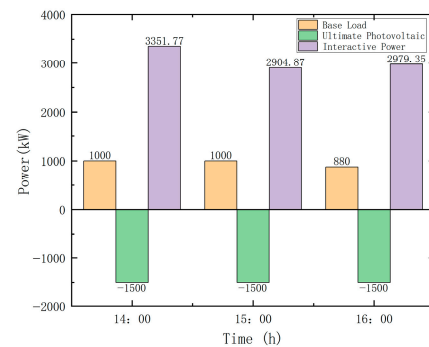


Figure 4. The maximum deterministic hosting capacity S_1 for the first scenario.

The calculation results show that the maximum deterministic hosting capacity (S_1) in the limit state is 3351.77 kW. In this limit state, the ESS charges at full power during peak PV moments, the MT output is adjusted to the minimum, and the transformer reverse power transmission reaches the limit of -1500 kW, indicating that all flexible regulation resources in the distribution area system are occupied.

To further verify the correctness of the deterministic hosting-capacity boundary, additional operating region tests were carried out at two representative PV capacities. When the PV capacity was set to 3200 kW, a non-empty operating region could still be generated, as shown in Figure 5a, which indicates that the system remained feasible below the calculated boundary. In contrast, when the PV capacity was increased to 3800 kW, no feasible operating region could be obtained, as shown in Figure 5b, indicating that the system constraints could no longer be simultaneously satisfied. These results are consistent with the calculated deterministic hosting capacity of 3351.77 kW.

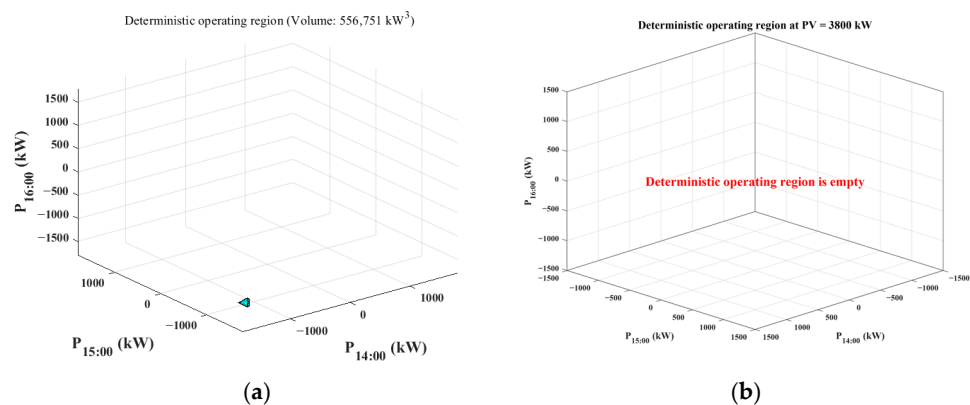


Figure 5. Additional validation of the deterministic hosting capacity boundary. (a) Deterministic operating region at PV = 3200 kW and (b) deterministic operating region at PV = 3800 kW.

5.2. Case 2

Next, the self-balanced operating region considering uncertainty is verified. Based on the Benders decomposition algorithm mentioned in Section 4, f_3 is calculated. By scanning the PV capacity multiples, the corresponding self-balancing criterion f_3 is computed. The results are shown in Table 1 and Figure 6.

Combining the table and the figure, it can be seen that the state of the distribution area system exhibits a significant two-stage characteristic as the PV capacity increases:

1. Safe stage (0~2160 kW):

In this interval, the value of the objective function f_3 remains zero. This indicates

that for any PV/load fluctuation within the box uncertainty set, the distribution area system can achieve an internal power balance through its internal flexible resources and interactive power with the upper-level grid. At this time, the self-balanced operating region of the distribution area system is non-empty.

2. Regulation Failure Stage (>2160 kW):

When the PV capacity exceeds 2160 kW, the value of f_3 begins to deviate from zero. When the PV capacity is 2250 kW, $f_3 = 0.83$, indicating that the system begins to experience a slight regulation deficit. As the PV capacity continues to increase, f_3 exhibits an approximately linear growth. At this point, the system's flexible resources are exhausted, and the system cannot accommodate the newly added PV capacity.

According to the definition of the second scenario, the robust hosting capacity under uncertainty S_2 is the critical capacity at which the self-balanced operating region transitions from non-empty to empty. Based on the calculation results, $S_2 = 2160$ kW.

Table 1. Corresponding scenarios for different PV capacities.

PV Multiple	PV Capacity (kW)	f_3	Operation Domain State
2.0	1800	0	non-empty set (feasible)
2.1	1890	0	non-empty set (feasible)
2.2	1980	0	non-empty set (feasible)
2.3	2070	0	non-empty set (feasible)
2.4	2160	0	non-empty set (feasible)
2.5	2250	0.83	empty set (infeasible)
2.6	2340	12.60	empty set (infeasible)
2.7	2430	26.48	empty set (infeasible)

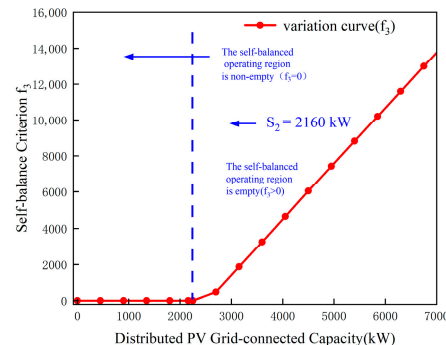


Figure 6. f_3 variation curve.

To further validate the robust hosting capacity under uncertainty, additional self-balanced operating region tests were conducted at two nearby PV capacities. When the PV capacity was set to 2000 kW, the self-balanced operating region remained non-empty, as shown in Figure 7a, indicating that the distribution area could still maintain feasible operation under all uncertainty realizations within the prescribed box uncertainty set. However, when the PV capacity was increased to 2300 kW, no self-balanced operating region could be generated, as shown in Figure 7b, implying that the system lost its robust self-balancing capability. These results are consistent with the calculated robust hosting capacity under uncertainty of 2160 kW.

Combining the calculation results obtained previously for the first scenario with those for the second scenario, a comparison is made:

$$\Delta S = S_1 - S_2 \approx 3351.77 - 2160 = 1191.77 \text{ kW}.$$

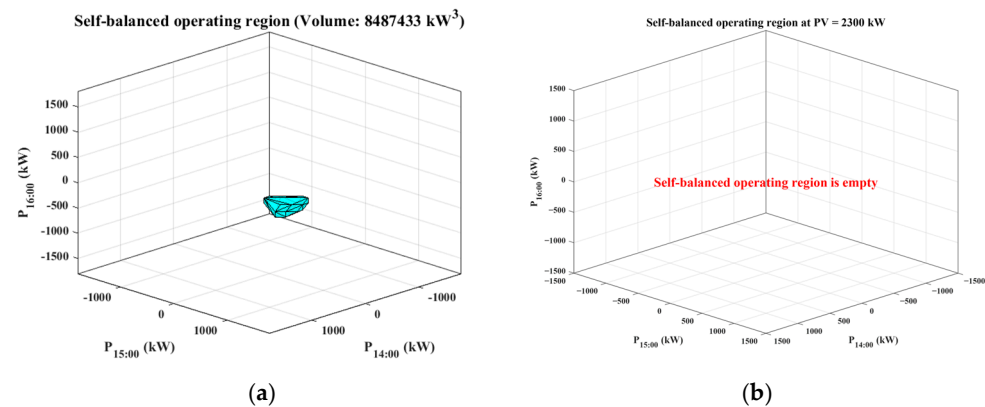


Figure 7. Additional validation of the robust hosting capacity under uncertainty boundary. (a) Self-balanced operating region at PV = 2000 kW and (b) self-balanced operating region at PV = 2300 kW.

5.3. Result Analysis

1. Significant reduction in hosting capacity: S_2 is only about 64.44% of S_1 . This indicates that once the source–load volatility is considered, the PV hosting capacity of the distribution area system is greatly reduced.
2. Price of robustness: To cope with the potential “worst-case scenarios”, the distribution area system must reserve a regulation margin of approximately 1191.77 kW (including the transformer’s reverse transmission margin and the ESS’s reserve capacity). This portion of capacity represents the cost paid to ensure the robust security of the system.
3. Planning significance: If planning is based solely on S_1 , the system will face severe risks of limit violations as the PV capacity increases. When planning is based on S_2 , although the plan may be conservative, it ensures that the system operates with extremely high reliability.

6. Conclusions

Focusing on the challenge of PV accommodation in distribution areas under high penetration scenarios, and addressing the limitations of traditional methods in balancing system dynamic regulation capabilities with source–load uncertainty, this paper proposes a distributed PV hosting capacity assessment method based on operating region analysis. In the first deterministic scenario, the Adaptive Constraint Generation algorithm is utilized to characterize the operating region and calculate the theoretical maximum capacity (S_1) via a linear programming model. In the second uncertainty scenario, a Min–Max–Min robust model based on the “Self-Balanced Operating Region” is solved, using duality theory and Benders decomposition to determine the robust hosting capacity under uncertainty (S_2) at the critical feasibility limit. By establishing a coordinated source–storage–load operation model, the ultimate PV hosting capacity is analyzed under both deterministic and uncertain conditions.

Case studies indicate that the robust hosting capacity under uncertainty S_2 , which satisfies robust security requirements, is significantly lower than the deterministic hosting capacity S_1 ($S_1 > S_2$). This discrepancy reveals that to cope with source–load fluctuation risks, the system must reserve certain margins for transformer transmission and energy storage regulation capability, thereby sacrificing a portion of the PV hosting space. The proposed method effectively quantifies this “price of robustness”, demonstrating that under limited energy storage configuration, ignoring uncertainty leads to overly optimistic assessment results, imposing limit violation risks on system operation.

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