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Handling Exponentially Growing Strategies in Spatial Cooperative Games: The Case of the European Union

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Abstract: This paper introduces a comprehensive cooperative game theory framework to measure the significance of location and neighborhood relations in conjunction with the magnitude of players/parties. The significances of these relations are measured over the EU geography. In this case, there are (i) the test of availability of a core solution that satisfies all associated parties/players; (ii) the measurement of players'/parties' rational minimal and maximal return expectations from the grand coalition regarding their all individual and sub-group strategies and associated return rationalities; (iii) the determination of the critical players/parties in the grand coalition. The study's main contributions are the provision of a methodology that identifies spatially/geographically critical players/parties and the design of an algorithm for handling exponentially growing strategies alongside increasing numbers of players/parties. In sum, a comprehensive cooperative game theory framework is introduced to measure the significance of location and neighborhood relations in conjunction with the magnitude of the players/parties. The case of the EU has revealed the union's geographically critical countries, with Germany being found to be the most influential.

Keywords: game theory; spatial cooperative games; linear programming; multinational coalitions; The European Union

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1. Introduction

"Geography and topography play a considerable role in the destiny of men" [1]. A number of national and international issues are found to have arisen from both the constraints and contributions that geographies present. By the same token, the spatial characteristics of a geography can also prepare a basis for cooperation and coordination among the stakeholders who share this geography. While some players take the primary acting position in this process, geography pushes other players to secondary or tertiary places. Thus, from the micro to the macro levels, geographic locations have been perceived to shape destiny.

Having certain locational characteristics at the global macro scale, some countries may have strategically advantageous locations for controlling and commanding multinational flows and routes. Similar to the pieces of a jigsaw puzzle, some countries may have a very critical location that either prevents or enables all links and flows of factors, goods, and services among many other countries. To illustrate, the Suez Canal is the most critical line between European and Asian trade routes. Many stakes occur regarding the control of the Suez Canal, especially for large-scale trading countries. Therefore, many army bases are found in and around the Red Sea. The potential of the Suez Canal has simultaneously brought many opportunities and threats to Egypt. Similar situations have been observed for the Panama Canal in Panama, for the Strait of Gibraltar in the territories of Spain and the UK, and the Dardanelles and the Bosporus in Türkiye. These specific geographic locations have been sources of direct or indirect conflicts and cooperation. These are some

of the extreme cases illustrating strategic locations. However, each country's geographic characteristics can be rationally evaluated and sorted. Based on this perspective, a rational technique is needed for measuring countries' geostrategic characteristics.

This research utilizes a cooperative game theory framework in order to measure the benefits of various parties' geographic characteristics. The main geographic characteristics can be identified as having direct or indirect neighborhood relations that enable an uninterrupted flow of humans, goods and services, and factors of production throughout spatially contiguous coalitions. In the meantime, these relations also provide the potential power to disable and control the cooperation of multiparty geographic systems. This research systematically lists and evaluates these geographic relations and coalitions in order to present the parties' roles with regard to locale. Later these coalitions are formulated using strategies found under the spatial cooperative game model, with the outputs from the listed coalitions being used later as the core constraints in a cooperative game model.

This study makes two major contributions to spatial cooperative game analyses and their applications. The first major contribution pertains to cooperative game theory problems and the exponential increase in the number of strategies per player involved in the game causing difficulties for calculations in large systems [2]. For example, while a 2-player game ($N = 2$) has three strategies ($2^N - 1 = 2^2 - 1$), this number goes up to 1023 ($=2^{10} - 1$) for a 10-player ($N = 10$) and 1,048,575 ($=2^{20} - 1$) for a 20-player game. The literature shows three methods that can be generally used in order to solve the computational problem against an exponential increase in the number of strategies: The first is to develop concise forms of representation by clustering/grouping the players [3–10] under a maximum of five groups, the second is to pull each player out of the system and calculate that player's marginal contribution to the grand coalition [11], and the third is to work on different algorithms to reduce the number of strategies [12]. Apart from these three categories, other studies [13–16] have focused on hypothetical models or theoretical discussions without specifying the number of players and coalitions.

Many international coalition problems inherently require coalitions among large numbers of parties/players. In order to pursue a cooperative game theory analysis, all possible coalition formations need to be listed and processed systematically. Rapid developments in computer technology have allowed new parameters that had been previously ignored to be added using appropriate software to arrive at more accurate solutions [17]. This research involves a technical solution for handling large numbers of strategies regarding large numbers of players, as well as the associated coalition benefits. As a consequence of the total number of players increasing, more extensive spatial areas contain larger number of players, and so wider geographic entities are included into the spatial cooperative game theory analyses. The practical use of spatial game theory models has become applicable for nearly $N = 27$ players/parties using this technical solution, which is sufficiently large for covering the European Union geography. In this way, the more critical and spatially strategic players are able to be identified succinctly and systematically.

The second major contribution to this study pertains to cooperative game theory studies and how they usually take the spatial proximity or neighborhood effect into account when identifying players without using this as a game rule. A very limited number of studies have used the neighborhood effect as a game rule. De Frutos and Martín-Herrán analyzed a transboundary pollution differential game for two hypothetical regions by including both spatial and temporal dimensions in order to be able to identify the geographical relationships among regions [18]. Wang et al. formulated local cooperation between neighboring small cellular base stations (SBSs) within the non-cooperative game model by adding the total throughput of neighboring SBSs to the game [19]. Zhao et al. used cooperative game theory regarding image classification by taking the pixels of the image as the players and the labels as the strategies [20]. According to the algorithm they introduced, the coalition is formed by a merge rule associated with the labels of adjacent pixels. William et al. stressed the role of players' spatial positioning within a storm water network in their study within a subset of Baltimore, Maryland [9]. Arce and Sandler's stud-

ies on military cooperation emphasized the influence of border neighborhoods [21]. They examined spatial incentives for alliance formation by testing various scenarios for three hypothetical allies using cooperative game theory solutions when prospective allies are contiguous [22] and when they are not contiguous [21]. The authors stated that the neighborhoods bordering the alliance members reduce military defense costs, thus neighbors' contributions to the costs can increase or decrease according to their spatial location within the alliance. Abolhasani et al. employed a strategic form game as a decision-making tool in urban land-use planning [23]. They considered neighborhood effects as players' criteria in the decision-making process in terms of compatibility, dependency, and compactness regarding various land-use types. The current study makes another important contribution to the literature by producing a spatial connectivity matrix for up to 27 players and defining the neighborhood factor as a game rule. This matrix, together with a script, enables us to identify and remove the redundant strategies based on the algorithm presented in the methodology section. Although any number of players may choose to be in a coalition, geographic direct or indirect disconnections do not provide fruitful coalitions for players. Thus, those types of coalitions need to be removed from equation listings later making up LP models. The removal enables us to utilize hardware and software capabilities in favor of increasing the number of players and larger geographic extents. Remaining valid strategies are transformed into the economic return values requested from the grand coalition via linear constraints for those who make up the coalition strategies. Because all constraints are linear, LP is the mathematical model to find the core solution that satisfies all participating members of coalition.

In both local (i.e., building block level) and global (i.e., multinational union level) coalitions, some coalition members are more distinctive compared to others due to how their strategic locations enable or inhibit larger and more extensive coalition formations, which thus gives them greater benefits. The following sections firstly present a literature review, secondly a mathematical demonstration of the developed game theoretic models, thirdly case studies, and lastly the discussion and conclusion. The case study covers the European Union countries.

2. The Literature

Game theory is the study of situations where conflict and cooperation are the possible strategies [24]. Decision makers are called players, and the objective function is called the payoff function. The players can be individuals, firms, nations, or the like, and they choose their strategies with respect to their payoffs. The literature sees international relations to have generally been analyzed by means of non-cooperative games, with the usual approach being zero-sum games. This approach is usually limited to 2×2 strategies between two opposing sides and does not consider cooperative approaches [25]. The usual game theory applications at the international level can be characterized by games that are defined with respect to the characteristics of payoff function (i.e., zero-sum, nonzero-sum, and constant difference games). According to Intriligator, cooperative games based on pre-play negotiations can also be utilized, especially with regard to multinational security issues [25]. In this respect, he illustrated the potential use of the Shapley value method for allocating coalition benefits [25].

The current research constructs a model based on the concept of cooperative games as classified under games defined by the nature of pre-play negotiations. The studies by Giglio and Wrightington, Heaney and Dickinson, Rogers, Kucukmehmetoglu, Wu and Whittington, Tsang and Jim, Küçükmehmetoğlu and Büyükgöz, and Cubukcu are a series of academic studies that use cooperative game theory with regard to a variety of cases [26–33]. Giglio and Wrightington, and Heaney and Dickinson wrote pioneering articles that introduced various rational cost apportionment techniques for common projects, with the cooperative game theory being one among many proposed and illustrated techniques [26,27]. The studies by Rogers, Kucukmehmetoglu, and Wu and Whittington involve research on effectively utilizing cooperative games with regard to water resources

and allocating scarce water resources and benefits among multiple stakeholders, while Tsang and Jim's study aimed at allocating green-roof benefits among the residents of a building [28–31].

Tlemsani et al. analyzed the costs and gains of strategic alliances in the global education industry with regard to the case of a British–Russian partnership [34]. Liu et al. applied cooperative game theory to analyze the relationship between airline ticket prices and optimal flight frequency using three hypothetical airlines [35]. Seo and Sakava used an *n*-person cooperative game involving five players for a hypothetical international conflict analysis [36]. Gonzalez and Rostom created a hypothetical cooperative game model where countries can form coalitions in order to optimize the long-term discounted consumption flows within their natural resource stocks [16]. Bosetti et al. analyzed the profitability and stability of international climate coalitions in reducing GHG emissions by grouping countries into 12 macro-regions [4]. Since the number of possible coalitions is very large (4095 coalitions) for 12 players, they took into account a smaller number of coalitions that they assessed as potentially meaningful in order to be able to facilitate computation. Rogna employed the cooperative game theoretical approach while analyzing the constitution of an international environmental agreement [5]. In the scenario development phase, they considered only five countries according to their wealth and vulnerability. Liu et al. examined the potential effects of expanding the Panama Canal in terms of competition, cooperation, and market share by including three groups (Ocean Carrier, East Coast Players, West Coast Players) in the game [6]. Yu et al. presented an approach for analyzing the impact of the cascade reservoir system operation on transboundary cooperation over the Lancang–Mekong River [7]. They divided the six riparian countries into three representatives (i.e., China, Myanmar, and the Mekong River Commission [MRC] countries). Liu et al. introduced a fuzzy coalition game model for multinational water resource allocation and presented the optimal allocation strategy for these six riparian countries within the same river basin [37].

The literature has two articles that focused on spatial cooperative games: Küçükmehmetoğlu and Büyükgöz, and Cubukcu, which focused on coalitions in urban building blocks [32,33]. Küçükmehmetoğlu and Büyükgöz focused on coalitions formed to attain higher floor area ratio (FAR) bonuses in a development block [32]. The achieved FAR benefits were allocated among lot owners using a spatial cooperative game. Cubukcu utilized the Shapley value allocation model, a form of cooperative game, to allocate the benefits that emerged from avoiding setback constraints by amalgamating small and irregular lots and then tested whether the Shapley value allocation was in the core solution [33]. Both studies used theoretical formulations to provide some clues and instances of potential solutions for urban development problems. Nevertheless, both models fall short of handling larger numbers of players. The current research increases the number of players up to the technical limits of the software and/or personal computers' hardware and furthermore extends the spatial context of analyses to multinational scales.

This study focuses on coalitions that require geographical contiguity to enable free flow of factors, intermediate inputs, goods, and services. To illustrate, energy grids, highways, pipelines, railways, free flight zones over countries, and even maritime routes do not tolerate any spatial discontinuity or interruptions but instead require geographic unity. This technical necessity makes some of the member countries with strategic locations more prominent and enables them to request more returns from geographic unions and cooperation as critical players. This research identifies and sorts the critical players in geographic organizations by means of spatial cooperative games.

In order to facilitate a trade between two regions for a homogenous product, having a sufficiently large price difference between regions is an absolute necessity. This difference is necessarily greater than the transport cost between these two regions. Once trade is actualized, the price difference declines to a level that equals the transport cost. After trade, additional consumer and producer surpluses become sources of extra welfare gains in these regions. Samuelson mathematically modeled and presented this simple trade model, which Takayama and Judge later improved and to which Richardson referred in a regional

economics book [38–40]. The scope of transport costs can be extended to include all forms of transaction costs. The current research uses these transport and transaction costs to refer to the physical and governmental links between all sides. When the number of trading parties (i.e., countries) increases, these links refer to a complete uninterrupted network that enables all physical flows. Trade, in general, is a form of coalition that provides mutual and/or multiparty benefits once physical flows are rationally actualized. Therefore, trade unions can be spatially modeled under a form of cooperative game. Every country simultaneously creates its own unilateral, bilateral, and multilateral coalition strategies while establishing trade links with its counterparts. The grand coalition is an ultimate extension of coalitions covering all union members. The United States of America (USA) and the European Union (EU) are some of the most extensive forms of these unions. Different forms of unions can be established through the Silk Road countries under the One Belt, One Road Project, such as the New Silk Road Economic Belt. These settings have complex benefit and cost relations among the competing and cooperating parties that are essentially based on uninterrupted link infrastructures. This research methodologically presents a new form of cooperative game applications under spatial cooperative games illustrated over the European Union geography and identifies the spatially critical players in the union geography using this application. In other words, if a player does not agree to join a union or choses to act on their own, this player's magnitude in affecting the functionality of the union's economic geography is measured by means of these spatial cooperative games.

3. Methodology and Cooperative Game Theory Models

The cooperative game models used in this research are adopted versions of the models applied by Küçükmehmetoğlu and Büyükgöz, and Küçükmehmetoğlu et al. [32,41]. The components of the cooperative games are introduced under two headings: (i) introduction of mathematical forms of the cooperative game models, and (ii) determination of coalition strategies and computations of their associated payoffs.

3.1. Mathematical Forms of the Cooperative Game Models

This section introduces four different cooperative game models. The first model is a base model that has an explicit list of all the necessary equations. The other three models are other versions of the base model that have been modified for different purposes.

3.1.1. Model I

Model I is a linear programming optimization model that has been designed to check whether a core solution exists in a cooperative game setting and contains all the constraints, some of which are selectively used in the subsequent models (Models II–IV). Let i be the indices for involved parties/players ($1 \rightarrow N$), S be all possible sub-coalition formations ($\forall\ S \in N$). Next, b_i is the individual benefit derived from i's unilateral action; b_S is the sub-coalition benefit derived from the cooperative action of S subset of N players; and b_N is the grand coalition benefit derived from the cooperation of all players. All b_i, b_S, and b_N are the parameters externally computed and loaded into the models. Next, x_i is the benefit apportioned to the ith member of the grand coalition from the grand coalition benefit b_N. Moreover, Z is an additional party benefit or cost from the formation of the coalition in the form of a tax or subsidy. In addition to N players, this party Z can be a moderating and/or facilitating supreme authority (e.g., government).

Given these sets, parameters, and variables, F is the objective function and is a function of either x_i or Z (see Equation (1)). Equations (2)–(5) are the constraints defining the individual (Equation (2)), subgroup (Equation (3)), and grand coalition (Equations (4) and (5)) rationalities. Individual rationalities are the constraints for each player i expecting x_i amount of return from the grand coalition benefit b_N whose least expected benefit is b_i (Equation (2)). Next, the sub-coalition rationalities are the constraints for multilaterally acting subgroup S expecting an $\sum_{i \in S} x_i$ amount of return from the grand coalition benefit b_N whose least expected benefit is b_S (Equation (3)). These are the necessary conditions

for being a part of a cooperative game, but these are not sufficient conditions. Finally, the total of allocations, $\sum_{i \in N} x_i$, are equal to the grand coalition benefit, b_N (Equation (4)), with Equations (2)–(4) being used to calculate the core constraints. When all these constraints are satisfied in an optimization model, one or multiple core solutions are present. When adding the supreme authority to the model through the variable Z, the total of allocations, $\sum_{i \in N} x_i + Z$, equals the grand coalition benefit, b_N (Equation (5)). Using Z as a maximized objective function (i.e., using Equation (5) instead of Equation (4) as the constraint in the model) relaxes (or contracts) the core constraints minimally by guaranteeing the existence of a core solution [42]. The succeeding Models II–IV are forms of game theoretical linear programming (LP) models that differ from Model I in terms of their objective functions.

Model I

$$\text{Maximize} \quad F(x_i, Z) \tag{1}$$

Subject to

$$x_i \geq b_i \; \forall i \in N \; (\text{Individual Rationality}) \tag{2}$$

$$\sum_{i \in S} x_i \geq b_S \; \forall S \subset N \; (\text{Sub-Group Rationality}) \tag{3}$$

$$\sum_{i \in N} x_i \; = \; b_N \; (\text{Grand Coalition}) \tag{4}$$

$$\sum_{i \in N} x_i + Z \; = \; b_N \; (\text{Grand Coalition with tax/subsidy}) \tag{5}$$

Model II

$$\text{Maximize } F = Z \tag{6}$$

$$\text{Subject to Equations (2), (3) and (5)} \tag{7}$$

Model III (repeats for every i)

$$\text{Maximize } F = x_i \tag{8}$$

$$\text{Subject to Equation (7)}$$

Model IV (repeats for every i)

$$\text{Minimize } F = x_i \tag{9}$$

$$\text{Subject to Equation (7)}$$

3.1.2. Model II

For some reasons beyond the individual, sub-group, and grand coalition rationalities, a supreme authority may want all players to participate in the grand coalition and satisfy any unhappy parties with a minimal cost or may want to extract a certain amount of benefit from the grand coalition without violating any of the rationalities listed above. For this purpose, Model II, as defined by Equations (6) and (7), has been developed as a specific version of Model I by replacing Equations (1) and (4) with Equations (5) and (6), respectively. Model II has been developed in order to (i) test for the presence of a core solution and its extent if present, or (ii) measure the minimal external subsidy Z needed for the grand coalition benefit, $b_N + Z$ if no core solution exists. Model II provides optimal solutions for the Z and all x_i in this regard.

While x_i is the optimal player benefits, the optimal Z may have three different values and meanings: (i) if $Z = 0$, then one unique core solution exists that satisfies all parties and rationalities, (ii) if $Z > 0$, then an infinite number of core solutions exist within the flexibility of deductible Z, and this Z is the maximum value that can be taxed without violating the core constraints (i.e., players' rationalities), and (iii) if $Z < 0$, then no core solutions exist at all, in this case at least $|Z|$ amount of subsidy (i.e., -tax) is required to reach a core solution [32,42].

3.1.3. Models III and IV

These models have been designed to measure countries' maximum (Model III using Equations (7) and (8)) and minimum (Model IV using Equations (7) and (9)) rational range of benefit expectations [$x_{i\text{-}max}$, $x_{i\text{-}min}$] from the grand coalition within the core. After applying Model II, the obtained optimal Z value is utilized as a constant parameter in the equations for Models III and IV instead of as a variable. Upon successfully running a single optimization of Models I and II, Models III and IV are then operated for each player i separately and independently. Mathematically, Models III and IV have the same constraints and objective functions, with the only difference being the selection of the *maximization* and *minimization* of the objective function. While maximizing Model III, the objective function (Equation (8)) maximizes the benefit of individual i in order to determine the upper bound of its individual benefit, $x_{i\text{-}max}$. While minimizing Model IV, the objective function (Equation (9)) minimizes the benefit of individual i in order to determine the lower bound of its individual benefit, $x_{i\text{-}min}$.

3.2. Listing Cooperative Game Strategies and Computing Their Returns

The goal of cooperation is to achieve greater benefits/returns. In order to proceed with the analyses, cooperative game models require all possible strategies be listed and all associated benefits/returns be found. In other words, before applying Models I–IV, the prerequisite steps necessary to proceed are: (1) initially listing all possible individual (i), subgroup (S), and grand coalition (N) strategies and then (2) computing the associated benefit parameters of these strategies, namely the individual (b_i), subgroup (b_S), and grand coalition (b_N) benefits. These parameters are used to determine the availability and extent (Z) of the core, as well as the upper and lower bounds of player benefits ($x_{i\text{-}max}$, $x_{i\text{-}min}$) by applying Models II–IV. The following sub-sections involve the techniques designed to list the strategies and procedures for computing their associated returns.

3.2.1. Listing Cooperative Game Strategies

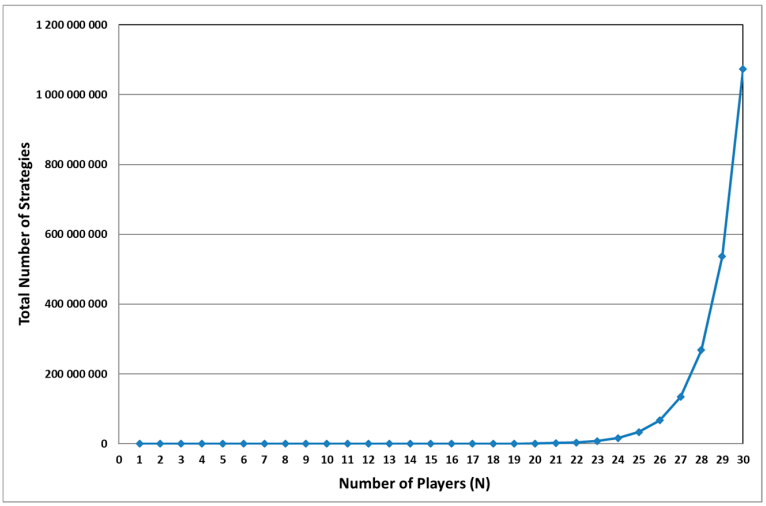
Given the sum of all players with N, the total number of cooperative game theory strategies is computed using the formula $2^N - 1$. Table 1 has been prepared to show the total number of strategies and the extent of the problem size. And its accompanying graph shows the total number of strategies to grow exponentially as the number of players increases. After a certain N value (≈7), the strategies first can no longer practically be listed by hand; later on, listing the number of strategies using personal computer programs becomes technically impossible for a higher N value (≈19). Büyükgöz utilized a hand-based approach for just six participants [43].

The current research has prepared an algorithm for handling large numbers of players (N) and the associated strategies ($2^N - 1$). Some strategies are observed to be equivalent to earlier narrower coalition strategies due to spatial disconnections. Spatial disconnections and disunities neither allow fruitful coalitions to take advantage of spatially uninterrupted flows nor, as a result, coalitions. In order to remove these redundant strategies based on the algorithm presented in Figure 1, a script has been developed for extensive multiparty geographic analyses as the technical base and contribution of this research. The flowchart demonstrates the sequential coalition listing and elimination procedures for redundant strategies: Step 1 involves generating and listing coalition strategies ($2^N - 1$) by combining the procedures starting from the individual (i) to the larger subgroup (S) levels and ultimately to the grand coalition (N). Step 2 involves evaluating a strategy in line whether it is redundant or not. Step 3 involves the decision phase where, if the strategy in question is redundant (Yes), the strategy gets discarded before moving on to the next strategy in the list; if the strategy is not redundant (No), the associated coalition benefit (b_S) is computed and the associated LP constraint for the models is prepared. Step 4 also involves a decision phase that checks the availability of further strategies in the list where if it is not the last coalition strategy (grand coalition), then the script goes back to the next strategy in the list for further evaluations. All evaluation processes are noteworthily pursued, from the least

to the most comprehensive coalition strategies. This algorithm is designed to list the LP constraints in a way where the generated constraints text can be easily read and solved by an optimization solver within the General Algebraic Modeling System (GAMS).

Table 1. The association between the number of players (N) and the total number of strategies.

Number of Players (N)	Total Number of Strategies ($2^N - 1$)
1	1
2	3
3	7
4	15
5	31
6	63
7	127
…….	…….
19	524,287
…….	…….
24	16,777,215
25	33,554,431
26	67,108,863
27	134,217,727
28	268,435,455
29	536,870,911
30	1,073,741,823



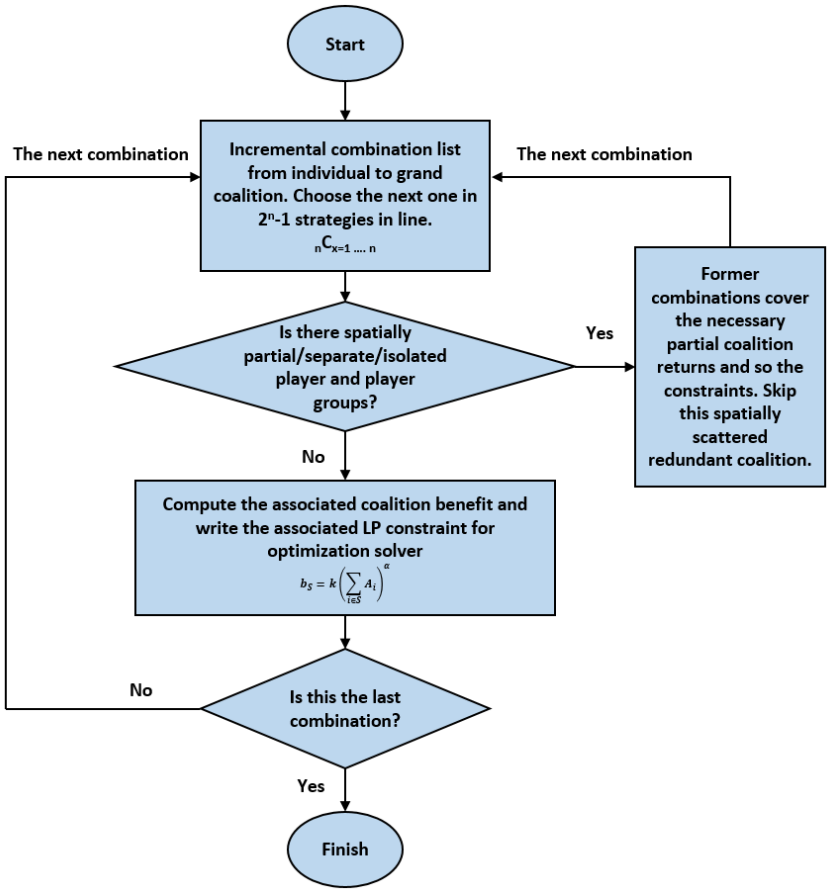


Figure 1. An algorithm to eliminate redundant coalitions as well as the associated allocation constraints.

For the practical application of the optimization models, having more players provides a better scope of analysis. The total number of achievable players becomes $N = 27$, which is much better for extensive and multiparty geographical analyses. This research selected 24 European Union countries for one case study, which means the number of players is slightly less than $N = 27$. Nonetheless, any continental or complete global analyses require many more players.

3.2.2. Computing the Returns for the Game Strategies

With respect to the nature of cooperative games, player rewards are either minimized costs or maximized benefits. This research basically builds the provided cases and associated models with regard to the benefits provided to the players. In order to build a strong and sustainable grand coalition, an incrementally increasing coalition reward/bonus scheme needs to occur alongside the increasing size of the coalition. Otherwise, forming a grand coalition becomes technically impossible without an external subsidy. In other words, the individual and/or sub-coalition returns for some cases may entail higher returns to their associated players compared to the grand coalition allocations. In those cases, no core solution at all is present.

In the case study, all return/bonus schemes are designed to encourage more extensive coalitions, ending with the grand coalition by means of the highest benefit multiplier. The ultimate goal is to measure the geographic advantages/returns of players along with their sizes/scales rather than to measure their actual empirical parametric benefits, which are mostly unavailable.

In order to establish the core constraints, the benefits for the large number of coalition strategies need to be computed. The benefit calculations can be modeled in two different ways:

- *Discrete incremental bonus calculation:* This is based on discrete reward thresholds that define the level that triggers bonus rewards as a multiplier. When any valid coalition size reaches the threshold level, an associated bonus multiplier is applied to the benefit calculation. These types of bonus schemes have been extensively used in various urban development/regeneration projects in Istanbul in the form of floor area ratio (FAR) bonuses.
- *Continuous incremental bonus calculation:* This is based on the continuous reward function defined in Equation (10), where A_i is the size of player i, $\sum_{i \in S} A_i$ is the sum of the sizes of all players in the sub-coalition S, k and α are the critical parameters determining the incremental return based on coalition size in $\sum_{i \in S} A_i$, and b_S is the sub-coalition benefit/return of the subgroup S. This reward function has been designed to compute the coalition benefits of multinational systems. In this paper, the selected multinational system is the European Union geography, whose spatial economic cooperation occurs in the form of free trade as the source of incremental returns.

$$b_S = k\left(\sum_{i \in S} A_i\right)^{\alpha} \tag{10}$$

Equation (10) can be remodeled in order to incorporate additional parameters such as distance decays. One advantage of the continuous reward function is that the relations can be later adjusted to any other mathematical forms that take the distance between masses into consideration, such as gravity models.

4. Spatial Cooperative Game Models and Their Applications in Terms of Locales

Initial versions of spatial cooperative games in urban development/regeneration were presented in Küçükmehmetoğlu and Büyükgöz, Büyükgöz, and Küçükmehmetoğlu et al. [32,41,43]. The numbers of players and associated strategies in these studies were within the limit and capabilities of hand processing; however, once the number of players exceeds six, the hand-based capabilities of spatial analyses become quite limited and even technically impossible. As part of this research, once the necessary scripts were completed,

the number of players was raised to 27, after which the number of strategies needing to be listed for the cooperative game models was increased to the hundreds of thousands. In this way, two different levels of analyses (i.e., local and global) have become technically practical. These analyses take into account both the size and scale of the players as well as their spatial neighborhood relations. The first analysis occurs over the urban development/regeneration building blocks [41] and the second one over the multinational coalitions/unions.

In the context of the two selected characteristic case studies, the cooperative game models measure: (i) the significance of geographic locations and their associated neighborhood relations, (ii) the possible impacts of individual sizes and scales and their collective contributions to coalitions, and (iii) the unilateral and/or combined effects of players on the allocations from the grand coalition benefits. The critical players in a geographic setting are identified and sorted in this way. The model applications have been pursued over the following two different case areas:

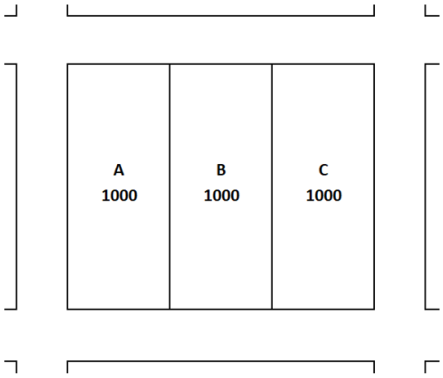
1. A three-player ($N = 3$) hypothetic cooperative game case on a three-lot urban building block. The selected minimal number of players allows the proposed models to be presented succinctly. The model presentation contains every detail concerning the individual, sub-coalition, and grand coalition rationalities in the forms of equations, tables, and graphs.
2. A 24-player multinational cooperative game designed to evaluate the European Union countries possessing spatial contiguity and unity. The developed model determines varying critical positions of players in the union for the selected case area.

The second characteristic case for the EU utilizes two different approaches to measure the significance of players' geographic locations and neighborhood relations alongside their relative sizes/scales. Given the geographic setting presenting all locations and neighborhood relations, the analyses are based on (i) utilizing the players' actual sizes/scales for the incremental multipliers, and (ii) assuming and utilizing equal sizes/scales for all players in order to eliminate the size/scale factor and determine the isolated impacts based on the players' geographic characteristics. In other words, the first approach observes the combined effects of players' geographic and size/scale aspects, while the second approach only observes the isolated effects of their geographic characteristics. In order to have comparable allocation results from these two different approaches, the second approach uses equal size/scale values for all players as obtained from the base grand coalition size factor ($\sum_{i \in N} A_i$) divided by the number of players (N). This way the grand coalition benefits/returns can be ascertained to be the same for the two different approaches. The allocation values provide results that can be compared among the players and between the approaches.

Furthermore, for better understanding and in order to obtain comparable results, all players' returns (x_i) have been normalized with their original sizes/scales (A_i) by dividing the game theoretical allocation values to their original actual or assumed sizes, (x_i/A_i). In this way, the returns from spatial cooperative game are computed for each per unit of size and presented for every player.

4.1. Case I: The Three Player (N = 3) Hypothetical Spatial Cooperative Game

A hypothetical urban regeneration area exists that has one building block consisting of three neighboring parcels (*PA*, *PB*, and *PC*; see Figure 2). Each parcel/lot is owned separately by one player in this urban spatial cooperative game. If each player prefers unilateral action, they may only use the limited base floor area ratio (FAR), which is equal to 1. The municipal government endorses comprehensive projects rather than three unilateral parcel-based projects. If possible, even one single block-based project would be preferred in order to solve various development and inefficiency problems such as common usage of green areas, car parking, and security management. One endorsement method is to provide larger FAR values, which is a strategy extensively utilized in İstanbul for solving the *Gecekondu* [squatter housing] problem.



Neighborhood	PA	PB	PC
PA	1	1	
PB	1	1	1
PC		1	1

Figure 2. Hypothetical three-player neighborhood design and the associated matrix [41].

Table 2 provides the decision rules regarding FAR bonuses. For this hypothetical case, the density bonus decision rules provide two cooperating neighboring parties a FAR of 1.5, which is a 50% increase in development rights compared to unilateral strategies. In the same direction, the FAR value reaches 2.0 for the grand coalition containing all three parties, and this is a 100% increase in development rights compared to unilateral strategies. This model setting has a sufficiently large FAR subsidy for encouraging the grand coalition. Overall, an increase in the FAR schedule is seen to occur that encourages larger and more extensive coalitions.

Table 2. Floor Area Ratio (FAR) thresholds for a building block in an urban regeneration area [41].

Lower Threshold (m^2)		Coalition Project Area (m^2)		Upper Threshold (m^2)	Floor Area Ratio (FAR)
0	<	A	≤	1000	1.0
1000	<	A	≤	2000	1.5
2000	<	A			2.0

The fact that cooperating parties are required to be spatially contiguous neighborhoods needs to be mentioned. To illustrate, the sub-coalition strategy of PA and PC is an unproductive coalition that cannot have a unified project able to benefit from scale economies; therefore, they cannot receive a FAR greater than the unilateral FAR values. The benefit calculations (b_i, b_S, b_N) with FAR bonuses are clearly illustrated in Table 3. In this table, the row with a gray background represents an unproductive coalition strategy. To overcome the technical limitations and thus increase the number of players, these types of neutral strategies are found to be unnecessary with regard to the optimization constraints and were later eliminated from the strategy list.

Table 3. Computation of returns for the listed coalition strategies using FAR bonuses [41].

Strategies		Returns from Strategies
Individual	b_A	A (1000 m^2) without bonus
	b_B	B (1000 m^2) without bonus
	b_C	C (1000 m^2) without bonus
Sub-Group	b_{AB}	AB (1000 + 1000 = 2000 m^2) 50% bonus (2000 × 1.5 = 3000 m^2)
	b_{AC}	AC (1000 + 1000 = 2000 m^2) unproductive coalition 0% bonus (2000 × 1 = 2000 m^2)
	b_{BC}	BC (1000 + 1000 = 2000 m^2) 50% bonus (2000 × 1.5 = 3000 m^2)
Grand	b_{ABC}	ABC (1000 + 1000 + 1000 = 2000 m^2) 100% bonus (3000 × 2.0 = 6000 m^2)

Next, Table 4 has been prepared in order to list the possible block-based regeneration coalition scenarios in columns containing all the possible coalition strategies listed in the rows in Table 3. To illustrate, Scenario I is a three-player unilateral scenario, Scenario IIa involves one player unilateral (PC) and one bilateral (2-player; PA and PB) coalition. The row totals show the maximum attainable FAR benefits for each scenario (Scenarios I–III) in this development block. These totals also imply the critical party's involvements in coalition strategies. As can be seen from Table 4, Scenarios I and IIb have the lowest block return totals, while Scenario III has the highest block return total. The difference between Scenarios I and III can be attributed to the involvement of player PB because player PB has a strategic location that enables the coalition of players PA and PC. The same impact can also be seen in Scenarios IIa, IIb, and IIc, where the coalition with player PB provides higher FAR values. In summary, player PB's critical geographic role in such a minimal development block is clearly identified in Table 4, and this provides valuable clues about the extensive use of the model in larger development blocks, namely in multinational geographical settings with large numbers of countries/players. The same results are later observed in the cooperative game optimization model applications.

Table 4. Coalitional strategies and possible scenario return totals (FAR m^2) [41].

Strategies	**Parcels**	**Return Strategies**	**Individual Strategies**	**Individual and Bilateral Coalition Strategies**			**Grand Coalition Strategy**
	Scenarios:		I	IIa	IIb	IIc	III
Individual	*PA*	b_A	1000			1000	
	PB	$\boldsymbol{b_B}$	**1000**		**1000**		
	PC	b_C	1000	1000			
Sub-Group	*PA and PB*	b_{AB}		3000			
	PA and PC	b_{AC}			2000		
	PB and PC	b_{BC}				3000	
Grand	*PA, PB, and PC*	b_{ABC}					6000
Return totals for the building block:			**3000**	4000	**3000**	4000	6000

Before performing the optimization operations, Table 5 was designed to present the simplest form of cooperative game linear programming model constraints in equation form. Table 5 provides graphic representations of all possible coalition strategies and their associated model constraints alongside the equation numbers consistent with the optimization equations for Models I–IV. In Table 5, due to Equations (2.1) and (2.3) having the same role as Equation (3.2) in the equation system, Equation (3.2) has been specifically highlighted in gray. Because of its neutral effect in determining objective value, this equation can be considered unnecessary and has thus been removed from the constraint list. Though seeming to be a minor intervention, this removal implies a mathematical solution for technical limitations when the number of players reaches the twenties, and software programs and computers fall short of being able to handle the strategy listing and associated benefit calculation assignments. The number of redundant equations also exponentially increases alongside increases in the number of players. Removing these redundant equations allows for larger numbers of players to be included in the models and better spatial analyses to be made, especially in multiparty/multinational settings. This procedure has been algorithmically presented in Figure 1. Finally, because this representative case only has three players, all constraints and possible allocation solutions for any maximization problems can be graphically presented in a triangular graph (Figure 3).

Table 5. Core constraints in explicit equation forms and the returns from all strategies * [41].

Strategies	Allocation Variables (m^2)					Returns from Strategies (m^2) **	Equations	Strategies in Graphs
Individual (unilateral)	X_A				$\geq$	$b_A = 1000$	Equation (2.1)	
		X_B			$\geq$	$b_B = 1000$	Equation (2.2)	
			X_C		$\geq$	$b_C = 1000$	Equation (2.3)	
Sub-Group Coalition (Bilateral)	X_A	$+ X_B$			$\geq$	$b_{AB} = 3000$	Equation (3.1)	
	X_A		$+ X_C$		$\geq$	$b_{AC} = 2000$	Equation (3.2) **	
		X_B	$+ X_C$		$\geq$	$b_{BC} = 3000$	Equation (3.3)	
Grand coalition	X_A	$+ X_B$	$+ X_C$		$=$	$b_{ABC} = 6000$	Equation (4)	
Grand coalition with (Z)	X_A	$+ X_B$	$+ X_C$	$+ Z$	$=$	$b_{ABC} = 6000$	Equation (5)	

* Values are obtained from Table 3. ** Equation (3.2) is a redundant strategy as its role in the equation system is strictly played by Equations (2.1) and (2.3); therefore, its removal does not change the optimal value of any designed objective function in the optimization problem.

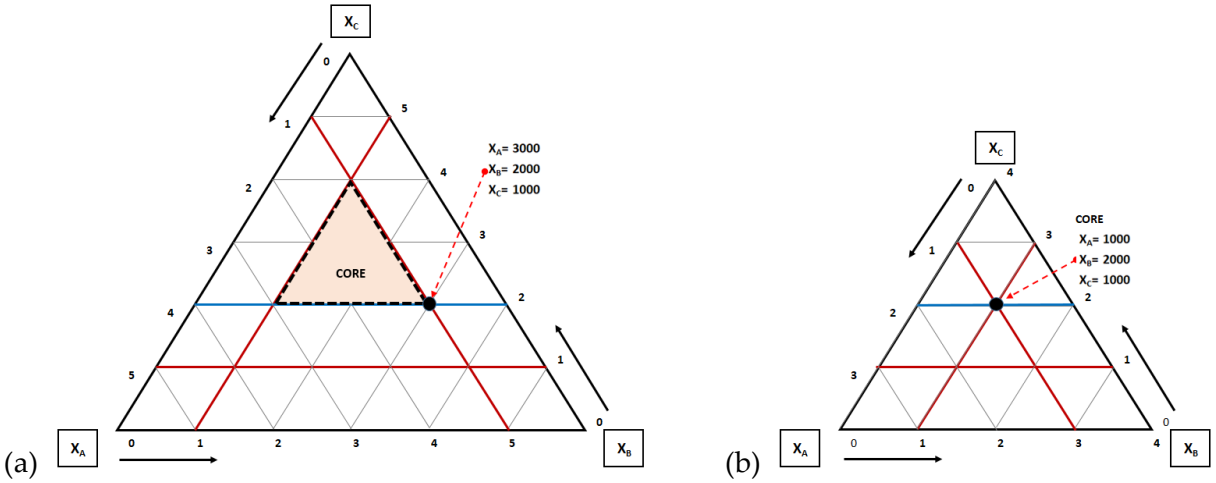


Figure 3. Model III application with the core and maximized X_A (10^3): (**a**) Tax is zero ($Z = 0$): (**b**) Tax is two thousand ($Z = 2000$) [41].

Core Existence Test and Core Size

Figure 3 provides the triangular graph solution prior to the solver-based optimization model applications for the Model IV application maximizing player *PA*'s return (X_A) from the grand coalition without violating individual (red lines), sub-group, or grand coalition rationalities. Figure 3a shows the maximized X_A to be 3000 m^2 and the remaining allocations X_B and X_C to be 2000 and 1000 m^2, respectively. This solution is presented by a dot at the right corner of the highlighted core area. For Figure 3b, when a 2000 m^2 benefit is retained as tax in the model without violating the base core constraints from Table 5, one single core solution occurs. Next, the allocation values for X_A, X_B, and X_C are found to be 1000, 2000, 1000 m^2, respectively. When the number of players is greater than three, graphical solutions and presentations become technically impossible. Thus, the solver-based model applications become a technical necessity.

Model II is the first one used in the computer-based model applications and has been designed to test the availability of a core solution. The objective function maximizes Z, given the associated constraints in Model II. The maximized objective value Z_{Max} is found to be 2000 in Table 6, which is consistent with the graphed solution presented in Figure 3. This value of Z_{Max} = 2000 shows that a core solution exists, with the grand collation providing a 2000 m^2 taxable return that can be retained or later allocated to the players.

Table 6. Results from the cooperative game optimization models [41].

Initial Base Values		Model III (Z = 2000)			Model IV (Z = 2000)		
Parcel	**Area (m^2)**	**PA_{Max}**	**PB_{Max}**	**PC_{Max}**	**PA_{Min}**	**PB_{Min}**	**PC_{Min}**
PA	1000	**1000**	1000	1000	**1000**	1000	1000
PB	1000	2000	**2000**	2000	2000	**2000**	2000
PC	1000	1000	1000	**1000**	1000	1000	**1000**

Later on, the Z_{Max} (2000) is used as a constant in Equation (5), with Models III and IV then being operated separately for each player's return maximization and minimization given the same constraints. Table 6 presents the obtained maximum and minimum values (P_{Max} and P_{Min}) for each player. Because this is a problem limited to three players, both the maximum and minimum values are found be equal for each player (PA_{Max} = PA_{Min} = 1000; PB_{Max} = PB_{Min} = 2000; PC_{Max} = PC_{Min} = 1000) after deducting Z_{Max} = 2000 from the grand coalition. For such a small problem, this single core vector solution is not an unexpected outcome; however, for the coming case studies with larger numbers of players (N = *24* and N = *25*), the P_{Max} and P_{Min} values may be different and may provide valuable insights. The single core vector solutions show player PB to receive twice as high a return compared to players PA and PC. This difference reflects the importance of players having a strategic location in this simple development block, and the models designed in a simple spatial cooperative game framework technically uncover these characteristics. Table 7 uses the optimization results provided in Table 6 and normalizes the maximum and minimum return values by dividing them by the base area sizes. The normalized values show that, for each unit of base parcel area, player PB receives two units of return. On the other hand, players PA and PC receive only a single unit of return. Player PB technically can say, "If I do not cooperate, you or the government cannot receive any bonus reward at all; therefore, I rationally expect to receive a 2000 m^2 return from this cooperative game theory formation". The same results can be observed in Figures 4 and 5. Instead of withdrawing Z_{Max} = 2000 from the core, if it is allocated to the players, their maximum return values would be 2000 m^2 units more than the values presented in Table 7 (PA_{Max} = 3000; PB_{Max} = 4000; PC_{Max} = 3000), and the per unit returns can be tripled for the edge parcels and quadrupled for the interim parcel.

Table 7. Normalized cooperative game optimization results [41].

Initial Base Values		Model III–IV (Z = 2000)		Normalized Values	
Parcel	**Area (m^2)**	**Minimum**	**Maximum**	**P_{Min}/P_{Area}**	**P_{Max}/P_{Area}**
PA	1000	1000	1000	1.0	1.0
PB	1000	2000	2000	2.0	2.0
PC	1000	1000	1000	1.0	1.0

All allocation tables and figures show that the developed spatial cooperative game models and associated analyses have unearthed the critical roles players' strategic locations have in the formation of spatially extensive coalitions. The simplest form of the linear programming models technically works quite well in identifying and sorting the critical players. The models' extensive uses in the coming section will provide more insights for

prospective research. The same presentation procedures and techniques are used in the coming sections for the extensive cases with larger numbers of players.

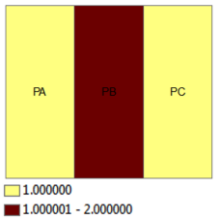


Figure 4. After tax (Z), the minimum expected individual benefits per unit base square meter area ($P_{_Min}/P_{_Area}$) [41].

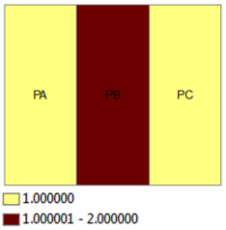


Figure 5. After tax (Z), the maximum expected individual benefits per unit base square meter area ($P_{_Max}/P_{_Area}$) [41].

5. Measuring the Spatial Importance of Countries' Geographic Locations and Economic Sizes in Multinational Coalitions

The urban regeneration problem has clear incremental bonus rules and regulations regarding building blocks and the associated parcels, and municipal authorities use these practically [32]. However, at the international level, no such clear-cut return/bonus rules exist for use among cooperating countries. Conversely, absolute benefits and welfare gains certainly exist among trading and cooperating multinational parties regarding the efficient use of resources. Multinational organizations such as the EU, NAFTA, NATO, and BRICS generate benefits from eased trade and common operations by taking advantage of various scale and external economies. The major source of coalition benefits comes from spatial continuity enabling a frictionless flow of factors, goods, and services between demanding and supplying countries. A functional spatial continuity requires common infrastructures (e.g., roads, pipelines, railroads, energy grids) and institutional settings. In contrast to the cases presented in urban regeneration, finding clear cut and exact parameters for computing the returns from coalitions is technically impossible in a multinational setting. Therefore, a hypothetical incrementally increasing return function has been developed in Equation (10) as a function of countries' economic sizes (i.e., gross domestic product [GDP]) for presenting the capability of spatial cooperative game models. This study classifies countries' economic sizes both before and after the coalition (Table 8).

Table 8. European union countries and their gross domestic product values.

Country	Country Code	Base GDP Before Grand Coalition ($\$A_i \cdot 10^9$)	GDP by EU Countries ($\$10^9$) *
Austria	AT	159	441
Czechia	CZ	136	376
Germany	DE	1515	4199
Hungary	HU	104	290
Italy	IT	836	2317
Slovakia	SK	65	180
Slovenia	SI	26	71
Poland	PL	406	1126
Belgium	BE	191	529

Table 8. *Cont.*

Country	Country Code	Base GDP Before Grand Coalition ($\$A_i \cdot 10^9$)	GDP by EU Countries ($\$10^9$) *
Denmark	DK	104	288
France	FR	1030	2856
Luxembourg	LU	22	62
Netherlands	NL	333	924
Croatia	HR	37	102
Romania	RO	174	483
Lithuania	LT	33	91
Sweden	SE	187	518
Spain	ES	641	1778
Bulgaria	BG	55	154
Latvia	LV	19	54
Finland	FI	88	245
Portugal	PT	113	314
Greece	GR	108	299
Estonia	EE	15	42
Total	EU	A_N = 6398	b_N = 17,739

* Source: https://www.cia.gov/the-world-factbook/ (accessed on 1 October 2020) [44].

5.1. The European Union (EU) as a Case Area—Number of Players and Scenarios

The 24 EU countries with connecting borders have been selected as one case study area (see Figure 6 and Table 8). The total number of strategies is estimated at 16,777,215 (or 2^{24}–1). This value is beyond the capability of an ordinary office program and should be reduced to a manageable number. The algorithm developed for this study eliminates the unnecessary scenarios from the strategy list and brings the number of strategies down to a manageable scale. After this reduction, the total number of useful strategies is only 156,244, which is less than 1% of the total strategies, and this is small enough for software programs and personal computers to handle.



Figure 6. *Cont.*

	AT	BE	BG	HR	CZ	DK	EE	FI	FR	DE	GR	HU	IT	LV	LT	LU	NL	PL	PT	RO	SK	SI	ES	SE
AT	1				1					1		1	1								1	1		
BE		1							1	1						1	1							
BG			1								1									1				
HR				1								1										1		
CZ	1				1					1								1			1			
DK						1				1														1
EE							1							1										
FI								1																1
FR		1							1	1			1			1							1	
DE	1	1			1	1			1	1						1	1	1						
GR			1								1													
HU	1			1								1								1	1	1		
IT	1								1				1									1		
LV							1							1	1									
LT														1	1			1						
LU		1							1	1						1								
NL		1								1							1							
PL					1					1					1			1			1			
PT																			1				1	
RO			1									1								1				
SK	1				1							1						1			1			
SI	1			1								1	1									1		
ES									1										1				1	
SE						1		1																1

Figure 6. European Union countries and their neighborhood relations.

5.2. Return/Benefit Rules for Coalition Strategies

Coalition benefits emerge when the barriers between countries are removed and connections (physical and institutional) are established for allowing the free flow of goods and services among the countries. The sources of trade benefits can be attributed to such things as scale economies, positive external economies, agglomeration economies, trade through comparative advantages, intra-industry trades, and efficient factor markets. This study focuses on the generation of coalition benefits requiring spatial connectedness and multinational border connections, similar to the cases in urban regeneration projects that take advantage of FAR bonuses. As a case study area, the EU countries' common intra-union physical and institutional projects and policies are the sources of the cooperating union members' long-term economic returns.

In order to pursue a multinational spatial cooperative game analysis, having a return function to compute the returns of country coalition strategies is necessary. Unfortunately, the literature has no return function that can be effectively used. Because the goal of the study is to present a developed spatial cooperative game model for analyzing and identifying the spatial aspects of country locations at the international level, a hypothetical return function (Equation (11)) has been designed for illustrative purposes.

In a spatial setting, the countries with critical geographic locations, large numbers of neighborhood relations, and larger economic sizes are expected to be more likely to have relatively higher returns from any form of coalition, with higher returns being expected from larger coalition sizes. Otherwise, adding new members to a union is meaningless. All coalitions are rewarded by incrementally increasing returns until the highest reward is achieved from the grand coalition. The highest return is generated from the largest coalition of all EU members, which is the grand coalition. In order to measure the spatially and economically critical players in the EU, the return function (Equation (11)) and its parameters have been specifically designed to endorse multinational coalitions. Equation (11) is a modified version of the reward function presented in Equation (10). Letting A_i be the country GDP size (Table 8), $\sum_{i \in S} A_i$ be the sub-coalition GDP size, and k and α be the constant and the power parameters, respectively, allows b_S to be computed as per Equation (11). All returns are computed as a function of the accumulated base GDP sizes ($\sum_{i \in S} A_i$) from Table 8.

$$b_S = k\left(\sum_{i \in S} A_i\right)^{\alpha} \rightarrow b_S = \begin{Bmatrix} A < 215,\ 1.0\left(\sum_{i \in S} A_i\right)^{1.0} \\ A \geq 215,\ 0.2\left(\sum_{i \in S} A_i\right)^{1.3} \end{Bmatrix} \quad (11)$$

$$\sum_{i \in N} A_i = 6398 \implies b_N = 17,739 = 0.2(6398)^{1.3} \quad (12)$$

The reward function (Equation (11)) has two segments. In the first segment, any qualified coalition size below the GDP threshold value of \$215 $\times 10^9$ does not provide any additional return to the associated country economic size as per the assumed $k = 1$ and $\alpha = 1$ parameters. In the second segment, for any qualified coalition size above the threshold GDP, the coefficients and power parameters are assumed to be equal to $k = 0.2$ and $\alpha = 1.3$ in order to reward more to larger coalitions. In other words, the second segment in the function is based on the assumption that larger coalitions have greater payoffs that arise from the efficient use of resources, trade, and scale economies as an outcome of countries' connectedness and coalitions. Better parameters can be derived from further extensive econometric research measuring the contributions from free trade and custom unions.

Table 8 shows two GDP figures in columns, with the first being used as a basis for coalition formations and the second being the most recent GDPs for the EU countries [44]. When the total base GDP of \$6398 billion is used as a grand coalition size and applied to the return function in Equation (11), the computed GDP size equals the sum of the EU GDP size of \$17,739 billion. Equation (12) simply illustrates the numerical calculation of the grand coalition benefit. The base country GDPs are estimated in proportion to the countries' GDPs.

The return function is designed to provide incrementally increasing coalition benefits to the union countries. As opposed to the totally disunited European Union countries with base GDPs, a grand coalition has nearly 2.8 times (b_N/A_N) higher rewards. The interim sub-coalition returns are computed using Equation (11), converted to multipliers (b_S/A_S). The return function presented in Equation (11) and the base country GDP sizes in the first numerical column of Table 8 have been used to estimate all 156,244 different coalition benefits.

5.3. The Scope of the Analyses

The EU is one of the most extensive multinational coalitions and has the main objective of promoting its citizens' well-being and thus member states' welfare. Among EU members, some have higher weights than others. At the macro-multinational scale, spatial cooperative game theory models are used to determine (i) geographically critical players with key locations and important neighborhood connections in forming spatial coalitions, and (ii) economically critical players that provide significant economic returns to cooperating parties as a function of their size and scale.

The following sub-sections have two different scenarios regarding EU countries. In the first one, the base GDP sizes are unequal (i.e., differentiated; see Table 9). In the second one, the base GDP sizes are assumed to be the same (Table 10). In both cases, the country base GDP size totals are the same, and their grand coalitions generate the same economic returns as per Equations (11) and (12).

Table 9. Optimization results—Differentiated GDP.

Start			Models III and IV * (Z = 39.04)		Normalized Values	
Code	**Country**	***GDP* (\$$A_i \cdot 10^9$)**	***X_Min***	***X_Max***	***X_Min*/GDP_**	***X_Max*/GDP_**
AT	Austria	159	159	533	1.00	3.40
CZ	Czechia	136	136	449	1.00	3.30
DE	Germany	1515	2738	6011	1.80	4.00
HU	Hungary	104	104	1147	1.00	11.00
IT	Italy	836	1258	2912	1.50	3.50
SK	Slovakia	65	65	195	1.00	3.00
SI	Slovenia	26	26	54	1.00	2.10
PL	Poland	406	492	1581	1.20	3.90
BE	Belgium	191	191	645	1.00	3.40
DK	Denmark	104	104	1019	1.00	9.80

Table 9. *Cont.*

Start			Models III and IV * (Z = 39.04)		Normalized Values	
Code	**Country**	***GDP*** **($A_i \cdot 10^9$)**	$X_{_Min}$	$X_{_Max}$	$X_{_Min}/GDP_$	$X_{_Max}/GDP_$
FR	France	1030	1657	5003	1.60	4.90
LU	Luxembourg	22	22	39	1.00	1.80
NL	Netherlands	333	379	1153	1.10	3.50
HR	Croatia	37	37	93	1.00	2.50
RO	Romania	174	174	1001	1.00	5.80
LT	Lithuania	33	33	166	1.00	5.00
SE	Sweden	187	187	858	1.00	4.60
ES	Spain	641	891	2517	1.40	3.90
BG	Bulgaria	55	55	440	1.00	8.00
LV	Latvia	19	19	69	1.00	3.60
FI	Finland	88	88	279	1.00	3.20
PT	Portugal	113	113	369	1.00	3.30
GR	Greece	108	108	351	1.00	3.20
EE	Estonia	15	15	15	1.00	1.00

* Each minimum and maximum value is derived from a separate optimization operation.

Table 10. Optimization results—Equal GDP.

Start			Models III and IV * (Z = 408.16)		Normalized Values	
Code	**Country**	***GDP*** **($A_i \cdot 10^9$)**	$X_{_Min}$	$X_{_Max}$	$X_{_Min}/GDP_$	$X_{_Max}/GDP_$
AT	Austria	266.6	545	545	2.00	2.00
CZ	Czechia	266.6	545	545	2.00	2.00
DE	Germany	266.6	915	2148	3.40	8.10
HU	Hungary	266.6	915	2148	3.40	8.10
IT	Italy	266.6	545	545	2.00	2.00
SK	Slovakia	266.6	545	545	2.00	2.00
SI	Slovenia	266.6	545	545	2.00	2.00
PL	Poland	266.6	915	2148	3.40	8.10
BE	Belgium	266.6	545	545	2.00	2.00
DK	Denmark	266.6	284	1719	1.10	6.40
FR	France	266.6	932	1719	3.50	6.40
LU	Luxembourg	266.6	545	545	2.00	2.00
NL	Netherlands	266.6	545	545	2.00	2.00
HR	Croatia	266.6	545	545	2.00	2.00
RO	Romania	266.6	284	1719	1.10	6.40
LT	Lithuania	266.6	284	1719	1.10	6.40
SE	Sweden	266.6	284	1202	1.10	4.50
ES	Spain	266.6	284	1202	1.10	4.50
BG	Bulgaria	266.6	284	1202	1.10	4.50
LV	Latvia	266.6	284	1202	1.10	4.50
FI	Finland	266.6	284	545	1.10	2.00
PT	Portugal	266.6	284	545	1.10	2.00
GR	Greece	266.6	284	545	1.10	2.00
EE	Estonia	266.6	284	545	1.10	2.00

* Each minimum and maximum value is derived from a separate optimization operation.

The following sub-sections compare and present the impacts of altered case assumptions, initially for in-scenario evaluations and then for intra-scenario evaluations.

5.4. In-Scenario Evaluations

Two scenarios exist regarding the EU case. The first one is designed to measure both locations and neighborhood relations alongside their varying economic sizes, and the second one is designed to disregard countries' relative sizes and scales in order to focus only on locations and neighborhood relations using the assumed equal base GDPs. In both analyses, the critical players in the EU are identified using spatial cooperative game models.

- *Models with varying country economic sizes:* In the cooperative game model applications, Model II was first applied to 24 European Union countries (see Table 8 and Figure 6) to test the availability of a core solution. The maximized tax value (Z_{Max}) was found to

be \$39.04 billion; therefore, the availability of a core solution is assured and thus so is a withdrawable return from the grand coalition. In other words, without violating individual and subgroup coalition rationalities (i.e., constraints), various allocation vectors exist for member countries ($X_{AT}, \ldots X_{EE}$) with or without allocating \$39.04 billion to participating countries.

Furthermore, after deducting the tax (\$39.04 billion), a unique vector solution can be had in the core solution. Slightly in contrast to the availability of a unique solution, some benefit exchanges can still be seen among countries within the limit of core constraints for some players. Therefore, determining the ranges of the changes in allocations would be useful. In order to do this, the Z in Equation (5) is set to \$39.04 billion as a deducted tax, which is then included as a parameter in Models III and IV, respectively. Models III and IV, respectively, maximize ($X_{_Max}$) and minimize ($X_{_Min}$) every member country's allocation benefits without violating the core constraints. This means 48 separate optimization operations occur: two for each country (one for the maximum and another for the minimum value). Table 9 summarizes the results. These values are later divided by the associated country base GDP in order to normalize and present them as returns in per unit base economic size. In this way, countries' geographic and economic size factors become comparable and presentable in the tables and graphs. These extreme results are presented in Table 9 in two columns: one for the country minimums ($X_{_Min}$) and one for the country maximums ($X_{_\mathrm{Max}}$). The normalized minimum ($X_{_Min}/GDP_$) and maximum ($X_{_Max}/GDP_$) values are also presented in the last two columns of the same table. Afterward, Figures 7 and 8 are generated from the normalized values in Table 9.

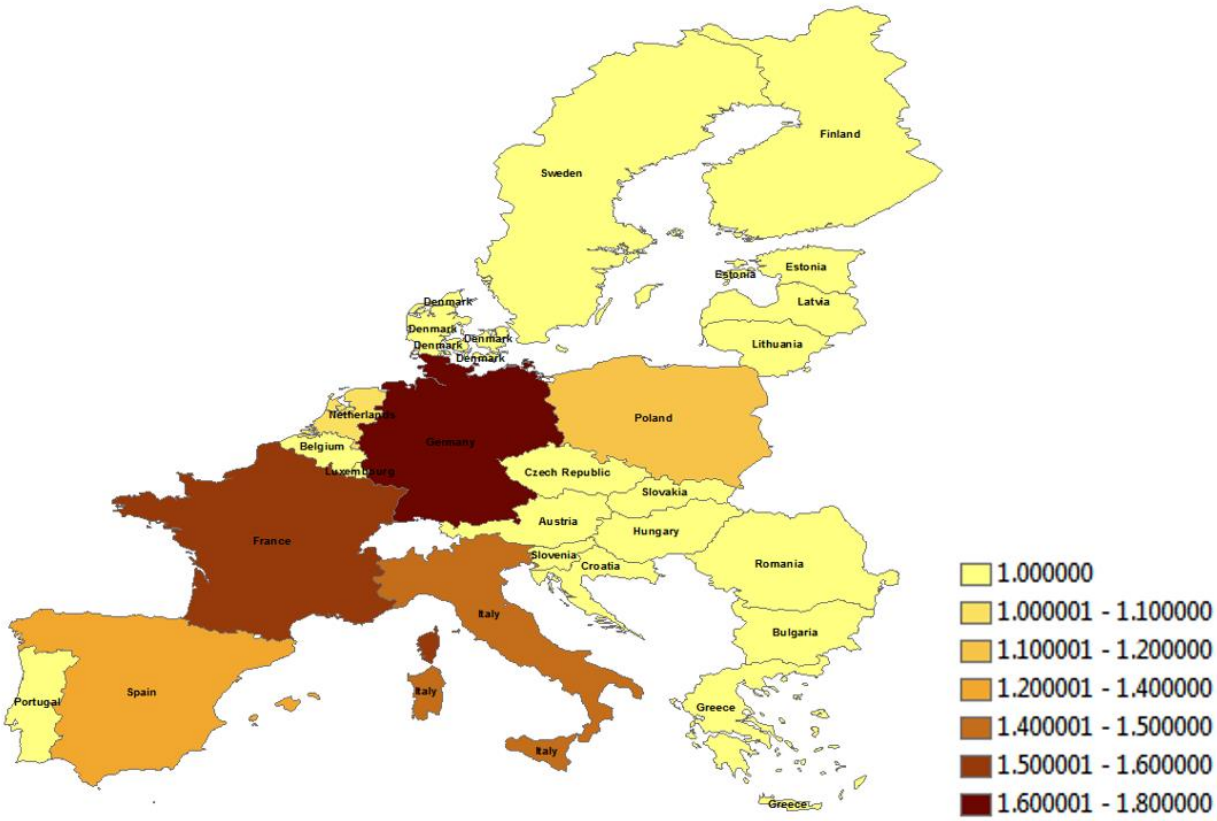


Figure 7. After tax (Z), the minimum expected country benefits per unit GDP size ($X_{_Min}/GDP_$)—Differentiated base GDP.

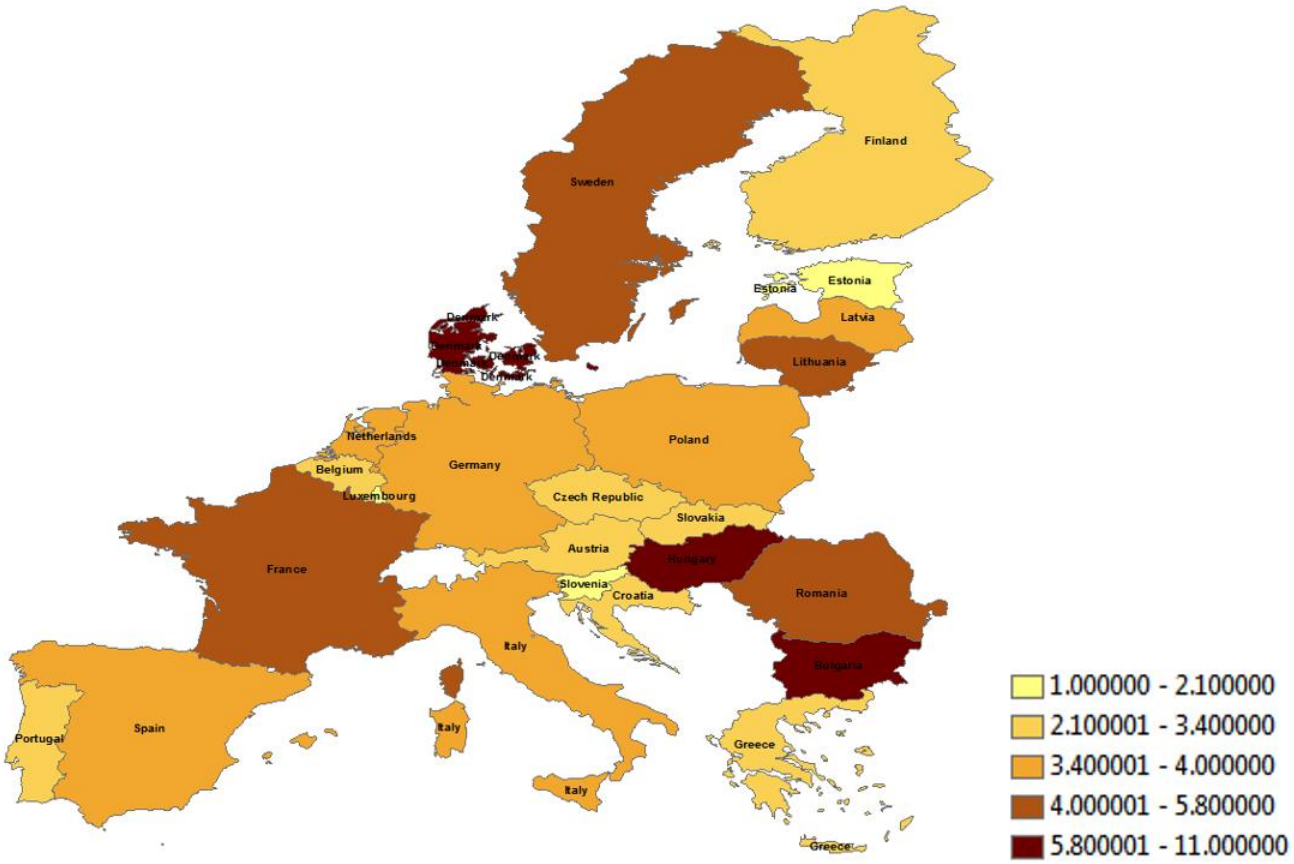


Figure 8. After tax (Z), the maximum expected country benefits per unit GDP size ($X_{_Max}/GDP_$)—Differentiated base GDP.

Germany, France, Italy, and Spain are seen to have significance in the EU regarding the returns, as per their unit base GDP sizes in Table 9 and Figure 7, which are 1.8, 1.6, 1.5, and 1.4, respectively. Germany's geostrategic locational and economic scale characteristics in particular are perfectly reflected in the model applications. The last column of Table 9 and Figure 8 show the reachable country maximums per unit of base GDP sizes to imply that having contact with large-scale countries provides significant returns to the neighboring countries. In other words, in addition to strategic locations, having physical contact and a coalition with a large-scale county triggers the higher return multipliers implied in Equation (11). In this respect, Hungary, Denmark, and Bulgaria were able to, respectively, reach 11, 9.8, and 8 times more returns, in reference to their per unit GDP sizes in Table 9 and Figure 8.

After tax, the differences in the minimum and maximum values represent the range of countries' returns and imply the flexibility of exchange benefits among each other. That flexibility can only be an advantage to a country if the other players agree. On the other hand, country minimums are certainly the absolute returns to the relevant player as arising from their complex role in the spatial cooperative game strategies in the geography. These min–max ranges are prominent for Denmark (min = 1; max = 9.8 times the normal return) and Bulgaria (min = 1; max = 8 times the normal return). Conversely, the critical values are the minimums, which are the lower limits determined by the core constraints. Regarding the minimums, Table 9 and Figure 7 show the critical players to be Germany, France, Italy, Spain, Poland, and Netherlands.

- *Models with assumed equal country economic sizes:* In the second scenario, given the same geographic setting in Figure 6, the country base GDPs (Table 10) are assumed to be \$266.6 billion for each country, with their total equaling the sum of the base GDPs (Table 9) in the first scenario. By eliminating the relative country sizes from the analyses in this way, only the impacts of strategic locations and neighborhood relations are kept in the focus of the analyses. In other words, the equal sizes allow for a controlled experiment to be performed focusing on spatial geography.

In the first step, Model II was applied to test the availability of a core solution and to measure the size of the tax return (Z_{Max}). Given the same core constraints, the maximized tax value (Z_{Max}) is found to be \$408.16 billion, which is much higher than the earlier scenario. That implies that among the lopsided players, the larger a country's GDP, the larger the scale economies and in-country free trade within the largest country allow reward benefits to be retained alongside its largest A_i. On the other hand, when the countries have uniform equal economic base GDP sizes, they earn much higher grand coalition benefits, which can then be allocated to players or be retained as tax.

Keeping the estimated maximum tax (Z_{Max} = \$408.16 billion) as a constant (Z) in Equation (5), some of the countries' benefits from optimizations may slightly vary. In order to estimate the range of these changes, Models III and IV are separately applied to each country (X_{AT}, ... X_{EE}), Model III for the maximum and Model IV for the minimum (X_{Max} and X_{Min}). The results are presented in Table 10. Later, these extreme values are normalized by dividing them by the base GDP sizes in order to allow them to be compared in terms of per unit economic size base. These normalized values are also added in the last two columns of Table 10. In order to present the significance of geographic locations and neighborhood relations, these values are then used in Figures 9 and 10.

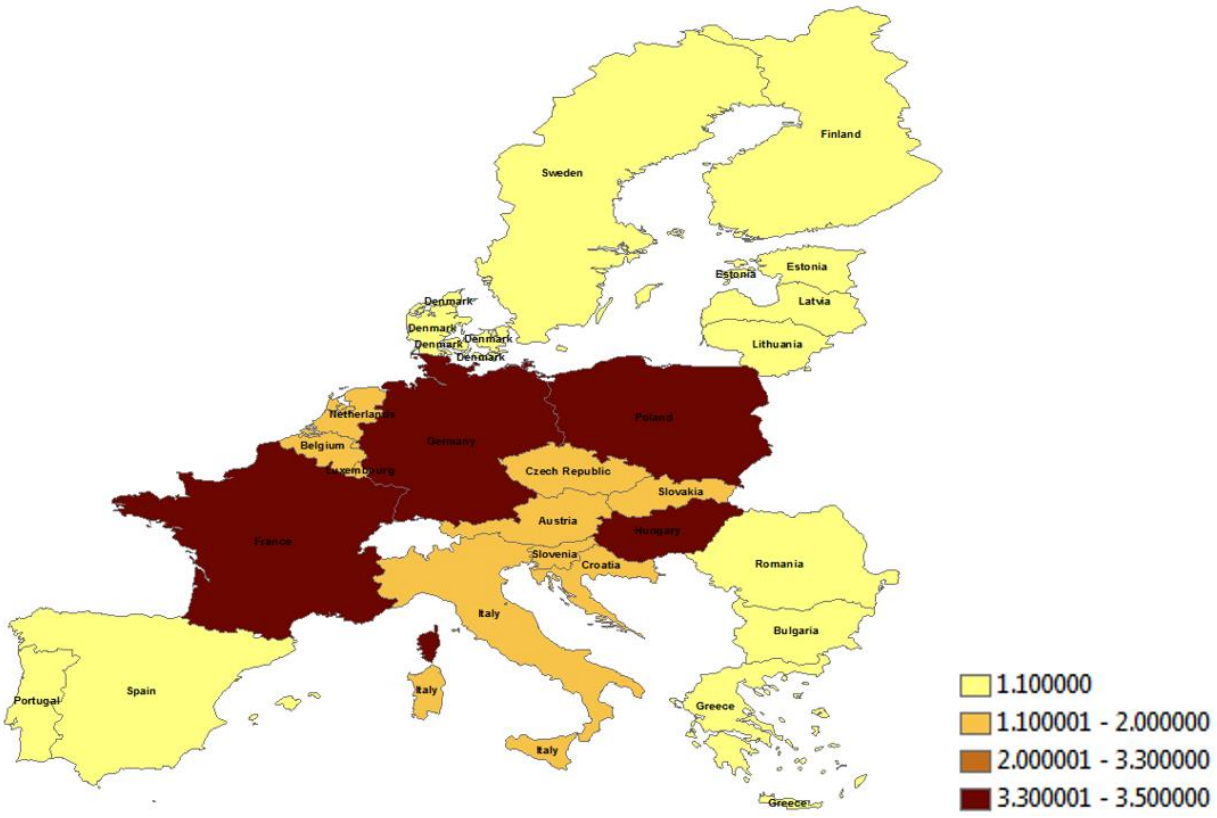


Figure 9. After tax (Z), the minimum expected country benefits per unit GDP size ($X_{Min}/GDP_$)—Equal base GDP.

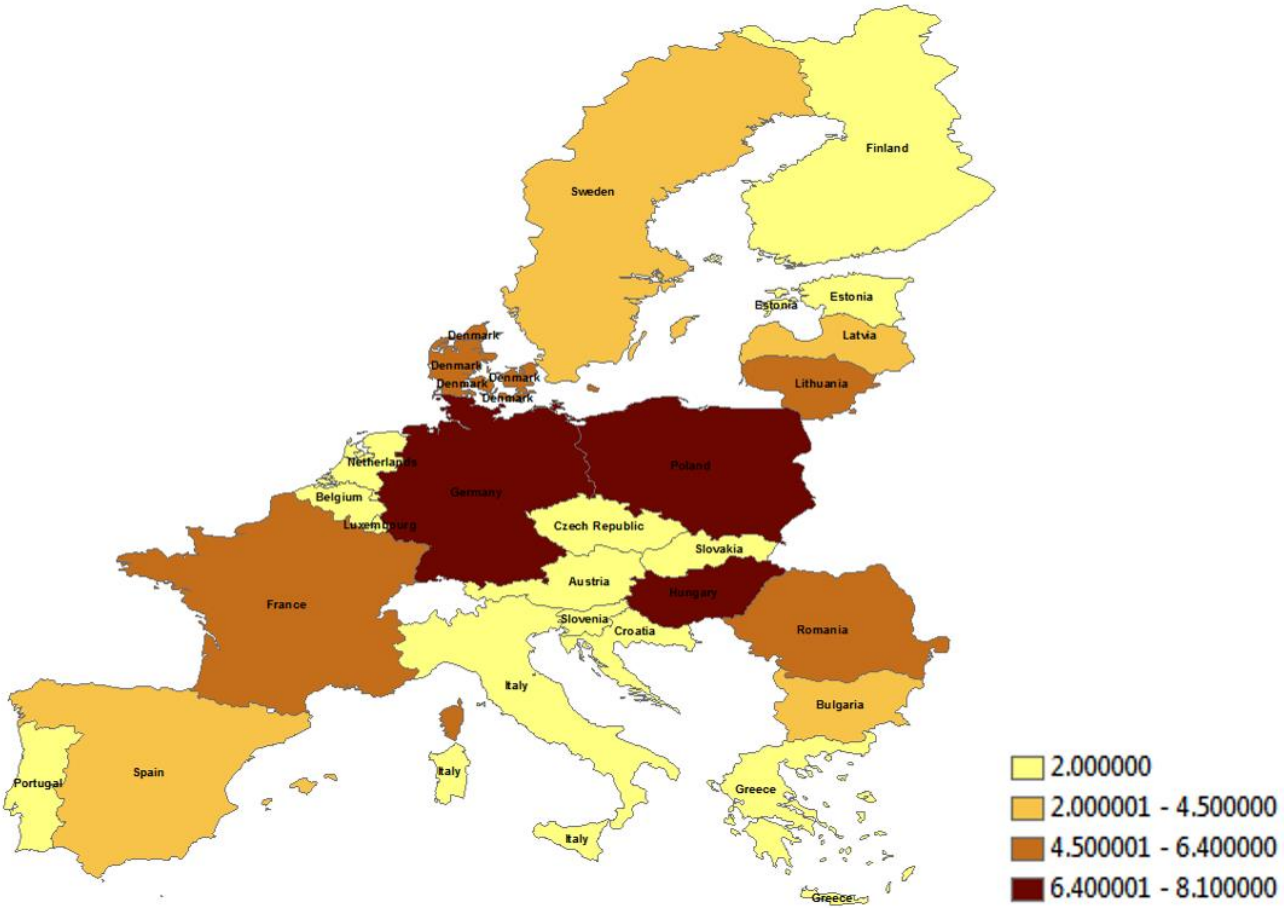


Figure 10. After tax (*Z*), the maximum expected country benefits per unit GDP size ($X_{_Max}/GDP_$)—Equal base GDP.

Among these minimization ($X_{_Min}$) scenarios, Table 10 shows the country returns per unit economic base size ($X_{_Min}/GDP_$) to vary between 1.1 and 3.5. The highest minimum returns are observed for France, Germany, Hungary, and Poland, and these are presented as a graph in Figure 9. Their returns are 3.5–3.4 times larger than their base unit economic sizes. Figure 9 clearly presents the geographic key aspects of these countries, showing them to have direct neighborhood connections to Central Europe and also to provide a bridge between the peripheral and central European countries. Consequently, by excluding the size and scale factors, the EU countries' geographically distinctive aspects and relative powers are clearly reflected in the spatial cooperative game analyses.

Among the maximization ($X_{_Max}$) scenarios, Table 10 shows the country returns per unit economic base size ($X_{_Max}/GDP_$) to vary between 2.0 and 8.1. The highest returns are seen for Germany, Hungary, and Poland, at 8.1 times their base economic size, followed by France, Denmark, Romania, and Lithuania at 6.4 times their base economic size (see Table 10 and Figures 9 and 10).

The minimum values represent the least expected grand coalition benefits reflecting the significance of countries' geographic locations. Because the base economic sizes are assumed to be equal, higher return for a country implies a more critical location in the EU geography. In this respect, France, Germany, Hungary, and Poland are distinctively strong countries. The maximum values are the maximum economic benefits reachable from the grand coalition given the constraints of all individual and sub-group rationalities. If the minimum and maximum values are the same, then individual and sub-coalition constraints do not permit any margin of change in the maximized and minimized allocations. However, some players may have some room to gain a little more without violating the core constraints at the expense of others' benefits. This feature can be seen in the allocations

for Denmark, Romania, and Lithuania. Their range of change is between 1.1 and 6.4 in Table 10.

5.5. Evaluations Between Scenarios

The previous section had two separate spatial game scenarios: the first involving the comprehensive effects of both the economic and geographic aspects of countries regarding allocating the grand coalition benefits, and the second involving the isolated effects of countries' geographic aspects in terms of their locations and neighborhood relations regarding allocating the grand coalition benefits. In the first one, countries' base GDPs are proportional to their actual sizes, and in the second one countries' base GDPs are all the same size. This current section makes analysis-based comparisons of their inter-scenarios.

- *Comparing the minimum returns between two different base GDP size scenarios:* After retaining Z_{Max}, the normalized minimum returns ($X_{Min}/GDP__$) presented in Tables 9 and 10 and graphed in Figures 7 and 9 show EU countries' relative powers/magnitudes. The countries' economic size and scale factors as well as geographic aspects bring Germany to the primary position in the EU (Table 9 and Figure 7). This is quite consistent with current EU politics and practices. Germany is followed by France, Italy, and Spain, with Poland and Netherlands being the other significant countries in the EU. Upon excluding economic size factors from the analyses by assuming an equal base GDP for all countries, Figure 9 clearly reflects the geographic factors. The results show France, Germany, Hungary, and Poland to be the four most distinctive players in the EU geography. In the first scenario, the economic scale factor dominates the relative weight of countries' geographic aspects. To illustrate, Hungary's normalized benefit falls from 3.4 to 1.0 (Tables 9 and 10). Technically, this result can also be attributed to the characteristics of the reward function (Equation (11)), which provides size and scale benefits when base size A_S reaches \$215 billion.
- *Comparing the maximum returns between two different base GDP size scenarios:* After retaining Z_{Max}, the normalized maximum returns ($X_{Max}/GDP__$) presented in Tables 9 and 10 and graphed in Figures 8 and 10 show the potential for extra benefit from being in the cooperative game framework. The estimated country minimum values come from their own characteristics; however, the maximum values come from the constraint structures based on other countries' individual and sub-group strategies and associated benefit expectations. If the constraints are permissive, some members of the grand coalition may attain higher benefits as long as other players have concessions. In this respect, some of the countries that have connections with large size/scale countries or connections with large numbers of countries may attain extra benefits, such as Hungary and Denmark. Germany, Poland, France, Romania, Lithuania, Sweden, Spain, and Bulgaria, despite not being the countries receiving the highest returns, do receive sufficiently large benefits (Table 9 and Figure 8). In the second scenario assuming equal base GDP sizes, the highest returns are provided to those countries that enable a connection to Central Europe. Denmark enables a connection with the Scandinavian countries of Sweden and Finland; Hungary enables the link with the Balkan countries of Greece, Bulgaria, and Romania; and Poland enables a connection with the Baltic countries of Estonia, Latvia, and Lithuania (Table 10, Figures 9 and 10). After these, France, Denmark, Romania, and Lithuania are other countries that provide peripheral access to Central Europe.

6. Conclusions

For real world issues, coalitions are sustainable only if the participants are not adversely affected and some positive returns are actualized. In other words, under the Pareto superiority principle, in order to actualize cooperation, no one is to be harmed while providing benefits to other members of coalitions [45]. The scope of sides can be either unilateral or multilateral. Unilateral sides are limited to the number of players; however, multilateral sides refer to all sub-coalition formations, the total number of which exponentially increases

alongside the growing number of players. In order to perform a spatial cooperative game analysis, not only do unilateral and grand coalition formations need to be listed, but so do all forms of sub-coalition formations and the associated returns systematically computed. In this respect, the research has found a technical solution for this problem using designed algorithms and codes. This technical solution also eliminates redundant coalitions from the list and enables the manageable number of players to increase. Consequently, the ranges of rational expectations from a grand coalition have been systematically and efficiently presented.

First off, the research has theoretically presented the technical aspects of spatial cooperative game theory and its associated strategies. The conditions needed for a sustainable cooperative game model are presented by Model I, which is based on the linear programming (LP) optimization model. Model I defines the necessary and sufficient conditions for the presence of core solutions and allows its availability to be tested. Furthermore, three additional models (Models II, III, and IV) have been put forth. Model II specifically focuses on the availability of a core solution: When one is present, Model II shows the amount of the estimated deductible tax, and when a core solution is not present, the model assesses the necessary side payments (-tax) needed to actualize a grand coalition. Model III has been used to estimate the lowest expected benefit from being a part of the grand coalition after deducting the maximum tax from the grand coalition. Model IV has been used to find the maximum benefits players may attain after maximum tax from the grand coalition in the same way Model III.

In order to show the capability of cooperative game models with regard to characteristics of geographic areas, the models were applied to the European Union (EU) geography. Economically and geographically critical countries in the EU were revealed using the designed spatial cooperative game models. The payoffs from the coalitions are based on a hypothetical mathematical function that provides higher returns to larger economic coalitions. In this respect, Germany's indispensable position has been identified by means of cooperative game models. Then, after eliminating the economic size aspects from the analyses by assuming equal economic sizes for every country, Germany, Hungary, France, and Poland were found to be the countries with critical geographic locations and neighborhood connections that enable peripheral links to Central Europe.

For future research, the spatial cooperative game models introduced in this study can be easily modified and elaborated upon for studies on urban geography at the micro-level of urban development/regeneration problems and global geography at the macro-level of multinational coalitions. For the coalitions requiring border connections and route continuity, a strong spatial cooperative game theory methodology can be utilized. This methodology will be able to measure countries' geographic capabilities and power when forming spatial unions. In complex vibrant global coalition settings, weak and strong players can be efficiently identified, and their reasonable expectations can be succinctly presented. On the other hand, well-known critically strategic locations in the geographic setting of the current world can be revisited, and sources of significance can be presented by means of spatial cooperative game theory models.

Apart from cases requiring geographic contiguity and continuity, other players with no direct spatial connection may also be included in the models by adding a "1" to the neighborhood matrices. In this way, although no spatial connection exists, other forms of coalitions may also be modeled by means of cooperative games.

Finally, the number of players can be increased to the level that covers the most populous, extensive, and comprehensive coalitions once the necessary model parameters have been empirically estimated and the necessary hardware and software infrastructure have been provided; after achieving that, the quality of geographic cooperative game analyses can be significantly improved. Furthermore, various forms of gravity-based analyses can be pursued by including the distance parameter into the cooperative game models. These are the avenues that prospective studies can take.

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