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A VIKOR-Based Sequential Three-Way Classification Ranking Method

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Abstract: VIKOR uses the idea of overall utility maximization and individual regret minimization to afford a compromise result for multi-attribute decision-making problems with conflicting attributes. Many researchers have proposed corresponding improvements and expansions to make it more suitable for sorting optimization in their respective research fields. However, these improvements and extensions only rank the alternatives without classifying them. For this purpose, this text introduces the three-way sequential decisions method and combines it with the VIKOR method to design a three-way VIKOR method that can deal with both ranking and classification. By using the final negative ideal solution (NIS) and the final positive ideal solution (PIS) for all alternatives, the individual regret value and group utility value of each alternative were calculated. Different three-way VIKOR models were obtained by four different combinations of individual regret value and group utility value. In the ranking process, the characteristics of VIKOR method are introduced, and the subjective preference of decision makers is considered by using individual regret, group utility, and decision index values. In the classification process, the corresponding alternatives are divided into the corresponding decision domains by sequential three-way decisions, and the risk of direct acceptance or rejection is avoided by putting the uncertain alternatives into the boundary region to delay the decision. The alternative is divided into decision domains through sequential three-way decisions, sorted according to the collation rules in the same decision domain, and the final sorting results are obtained according to the collation rules in different decision domains. Finally, the effectiveness and correctness of the proposed method are verified by a project investment example, and the results are compared and evaluated. The experimental results show that the proposed method has a significant correlation with the results of other methods, and is effective and feasible, and is simpler and more effective in dealing with some problems. Errors caused by misclassification is reduced by sequential three-way decisions.

Keywords: ideal solution; sequential three-way decisions; VIKOR method; domain; multi-attribute decision making



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1. Introduction

Multi-attribute decision making is a key part of modern decision science, which is used to solve the problem of optimal selection and ranking. It is widely used in technology, engineering, management, and other fields, and mainly deals with evaluation and selection. The VIKOR method is a powerful tool to help decision makers find the best compromise solution between multiple attributes or objectives. It takes into account the concepts of compromise and balance, as well as the subjective tendencies of relevant decision makers, which can provide valuable solutions in complex decision problems. Many researchers have developed various extensions and improvements of the VIKOR method, which can play a more effective role in various application scenarios. Chiranjibe et al. [1] proposed an improved

version of the improved type-2 fuzzy interval VIKOR method (IT2FVIKOR) in order to obtain better ranking results for the MCDM problem. Alhadidi et al. [2] introduced the FAHP weight into the VIKOR and proposed a new FAHP-VIKOR method. Zhang et al. [3] proposed a VIKOR method According to regret theory. Yu et al. [4] proposed a novel T-SF VIKOR method for tradeoff ranking in multicriteria analysis to adapt to complex real-world environments. Arunodaya et al. [5] proposed an innovative multi-attribute decision making method by combining FHFSS and improved VIKOR. Although these extended VIKOR methods have their own characteristics, they all follow the basic principle of calculating individual regret value, group utility value, and decision index value, so as to obtain the final ranking. These methods improve the steps of VIKOR and introduce other concepts. At the same time, some studies have pointed out that the traditional MCDM method can only rank schemes, but not classify them. Zhang et al. [6] saw unity in multi-attribute and three-way decisions as a viable research path, proposing a new TOPSIS method based on three-way decisions. Bisht et al. [7] also believed that three-way decision making increases delayed decision and can efficiently cope with multi-attribute decision making by reducing decision risk. Therefore, the VIKOR method can be considered to combine with the three-way decisions to take into account both classification and ranking.

Based on human cognitive processes, a three-way decision model is constructed. People often put off making decisions that require deliberation. Three-way decisions solve complex problems by dealing with three different parts, so that the VIKOR method can deal with decision complexity, consider cost and uncertainty, and complete classification and ranking. Wang et al. [8] proposed a three-way decision model on account of the third-generation prospect theory, introduced the α model and β model optimization of digital threshold solution, and simplified the three-way decision rules. Yang et al. [9] generalize the related work of three-way decisions through complex network analysis. Zhan et al. [10] combined regret theory with IIS prospect theory and proposed a new three-way decision method to further combine three-way decisions with behavioral decisions. Subhashini et al. [11] proposed an opinion classification method combining three-way decisions with fuzzy concepts and semantic concepts to solve the problem of reducing the boundary area and maintaining high accuracy. In order to deal with the stability of the conflict system being affected by different preferences of decision makers, Jiang et al. [12] combined three-way decisions and conflict analysis with a grey system to establish a new conflict analysis model. In order to reduce the risk of decision-making errors and express linguistic uncertainty, Luo et al. [13] combined double-layer hesitant fuzzy term sets with multi-attribute three-way decision making. Recently, researchers focused on making reasonable decisions with limited info to reduce decision risks. Qi et al. [14] combined the fuzzy attribute concept with the three-way decision model and utility theory and proposed the three-way utility decision model. In multilevel data, obtaining acceptable decisions at various granularities is crucial. Qian et al. [15] proposed a multi-granularity hierarchical sequential three-way decision model to advance three-way decisions. Yi et al. [16] combined Sigmoid utility with three-way decisions using Pythagorean fuzzy sets to solve related problems. Wang et al. [17] constructed three fuzzy hesitant three-way decision methods to calculate the contingent probability of the target in the fuzzy hesitant information decision system without class labels. Pan et al. [18] combined three-way decisions with interval values to propose a new method for solving multi-attribute decision-making problems in uncertain environments. In order to reduce the influence of different attribute types on decision results, Yan et al. [19] integrated TOPSIS and regret theory into three-way decisions to settle multi-attribute decision-making difficulties in fuzzy environments without changing attribute types. The integration of three-way decisions and granular computing offered a befitting idea and method for the thinking of cognitive science and the trialization of information processing. Sequential three-way decisions are a classic representation of three-way granular computing, which makes multi-stage decisions through a series of transaction-optimization (TAO) models. Yang et al. [20] integrated three-way decisions and multiple granularities to enhance the domain system's hierarchy and structure. Creating a sequential three-way decisions model that combines three-way

decisions and granular computing is becoming more and more important in classification. Pei et al. [21] proposed four three-way sequential classifiers in order to solve the issue of the coarse and fine granularity of objects potentially causing conflict and thus affecting the decision accuracy. S3WD uses coarse-to-fine information granularity. Yu et al. [22] addressed granularity construction and decision cost in the three-way sequential decision model, verified the model's correctness and algorithm's feasibility, and solved the double constraint problem. The sequential three-way decision method, enhanced by multi-level granularity, is proven effective for solving human problems. Ju et al. [23] introduced a novel rational subspace classifier that outperforms advanced classifiers with fewer attributes. Combining granularity and multi-levels, this method efficiently handles uncertainty and dynamics in decision making.

The aforementioned researchers proposed improvements and extensions to the VIKOR method and three-way decisions to tailor them to their research fields. However, these enhancements did not integrate VIKOR with three-way decisions; VIKOR improvements focused solely on prioritization without classification. Similarly, three-way decision enhancements neglected ranking alternatives. This paper combines sequential three-way decisions with an improved VIKOR method, incorporating decision-makers' subjective tendencies and considering both maximizing team utility and minimizing individual regret. It argues that relying solely on team utility or individual regret for classification and judgment can lead to significant errors. Instead, it constructs a sequential three-way decision model considering both values. VIKOR classifies schemes into three regions using three-way sequential decisions. Ranking rules and formulas based on intradomain and cross-domain comparisons are proposed, and their correctness and applicability are demonstrated through an example.

Section 2 introduces the basics of three-way sequential decisions and multi-attribute decisions. Section 3 proposes the sequential three-way VIKOR method based on individual regret or group utility values, studying ordering rules within and across different areas. Section 4 introduces the sequential three-way VIKOR method considering both individual regret and team utility values, verifying its feasibility. Section 5 assesses the method's reliability and usefulness using a real-world engineering investment case study. Section 6 concludes the paper.

2. Preliminaries

In this chapter, we introduce the basic concepts of three-way sequential decisions and multi-attribute decision making.

2.1. Sequential Three-Way Decisions

Multi-step three-way decisions are three-way sequential decisions. The main concept of three-way sequential decisions is to obtain preliminary decisions at a coarse-grained level, and gradually obtain the final decisions by adding information and updating the universe. It is well known that multi-step classical sequential three-way decisions are gradually improved to form a three-way sequential decision model [24,25]. Next, we briefly introduce the contents of three-way sequential decisions.

Definition 1 ([26]). For the decision table $S = \{U, At = C \cap D, V, I\}$, a dynamic threshold is given $(\alpha, \beta) = \{(\alpha^1, \beta^1), (\alpha^2, \beta^2), \dots, (\alpha^l, \beta^l)\}$, E is a relation of equivalence. The lower approximation of the l -level granularity $\underline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l)$ and upper approximation $\overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l)$ ($i = 1, 2, \dots, s, l = 1, 2, \dots, L$) as defined below:

$$\begin{aligned}\overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) &= \left\{x \in U^l \mid p(D_i^l \mid [x_E]) \geq \alpha^l\right\} \\ \underline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) &= \left\{x \in U^l \mid p(D_i^l \mid [x_E]) > \beta^l\right\}\end{aligned}\quad (1)$$

where $D_i^l, [x_E]$ denotes the object, x is contained in the partitions by the equivalence classes $U^l/D = \{D_1^l, D_2^l, \dots, D_s^l\}$ and U^l/E , respectively. Dynamic updating of universe is an important part of sequential three-way decisions.

Unlike the seven different combinations for three regions proposed by Yang et al. [27], our next sequential decision uses boundary regions as the universe of discourse. $U^1 = U$, $U^{l+1} = \overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) - \underline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l)$. By upper and lower approximation of the l -rank granularity, we can obtain three probability regions as:

$$\begin{aligned} POS_E^{(\alpha^l, \beta^l)}(D_i^l) &= \overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) \\ BND_E^{(\alpha^l, \beta^l)}(D_i^l) &= \overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) - \underline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) \\ NEG_E^{(\alpha^l, \beta^l)}(D_i^l) &= U - \overline{apr}_E^{(\alpha^l, \beta^l)}(D_i^l) \end{aligned} \quad (2)$$

2.2. Multiple Attribute Decision Making

Multi-attribute decision making is very common in daily life, as shown in Table 1. The n alternatives are represented by the variable $A = \{a_1, a_2, \dots, a_n\}$, and the evaluation of each alternative involves m criteria. The set of m criteria is denoted by the variable $V = \{v_1, v_2, \dots, v_m\}$. The value of the object a_i under the criterion v_j ($1 \leq j \leq m$) is represented by the variable e_{ij} ($e_{ij} \in [0, 1]$). The importance of each criterion is represented by the corresponding weight, which can reduce the impact of the differences of different criteria on the final decision. The variable $W = \{w_1, w_2, \dots, w_m\}$ denotes the set of m weights, where $w_j \in [0, 1]$, $\sum_{j=1}^m w_j = 1$. The decision maker needs to rank each objective and combine the corresponding ranking information in the selection of the optimal scheme.

Table 1. Multi-criteria evaluation of information systems $S = (A, V)$.

	v_1	v_2	$\dots$	v_m
a_1	e_{11}	e_{12}	$\dots$	e_{1m}
a_2	e_{21}	e_{22}	$\dots$	e_{2m}
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
a_n	e_{n1}	e_{n2}	$\dots$	e_{nm}
W	w_1	w_2	$\dots$	w_m

The VIKOR method mainly includes five steps: calculating weight, calculating positive and negative ideal solutions, calculating individual regret value and group utility value, calculating decision index value, and sorting. The Lpmetric aggregation function is integrated into the calculation of “negative ideal solution” and “positive ideal solution” to obtain the compromise solution for each alternative. In the sorting process, the stability conditions and advantages are considered to obtain the ranking results according to the specific measurement between the ideal solution and each scheme. In recent years, many researchers [1–5] have carried out various types of expansion and improvement of the VIKOR method and combined it with other ideas and methods, showing better application effects compared with past methods. However, in addition to improving the decision results, these methods still have shortcomings in other aspects. In this paper, by introducing the VIKOR method into the three-way sequential decisions, a new VIKOR method combined with the sequential three-way decisions is proposed, which can classify and rank the alternatives.

In order to easily compare and process the attribute information of the alternatives, it is necessary to normalize the attribute evaluation value of the multi-attribute information table.

$$Y = (y_{ij})_{m \times n} \quad (3)$$

where $y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}}$, x_{ij} is the element of set in the multi-attribute information table.

3. VIKOR Model of Sequential Three-Way Decisions Based on Individual Regret Value or Group Utility Value

Group utility value is a quantitative representation of decision-makers' views and attitudes towards gain and loss effects, which is used to measure decision-makers' subjective tendencies and preferences for certain things. The loss caused by the decision maker's mistake is represented by the individual regret value, which is a special measure of the optimal value compared to other values in the scheme. In the VIKOR method, the individual regret value and group utility value are obtained by calculating the "specific measure" between the ideal solution and each alternative. The smaller value of the group utility value means the greater group utility of the corresponding alternative, and the larger value of the individual regret value means the greater individual regret of the corresponding alternative. In practical decision problems, more values can be deduced and calculated based on one or two ideal alternatives to rank all alternatives, but the ranking results must follow certain rules.

For example, if the group utility value of scheme A_i is smaller than scheme A_j , it means that scheme A_i has greater group utility than scheme A_j , and then A_i should be better than A_j . Conversely, if the individual regret value of A_j is less than that of A_i , the result should be that A_i is inferior to A_j .

In order to easily determine the importance degree of an alternative, this study first considers a single individual regret value or group utility value. It classifies the alternatives by the single value calculated for each alternative, as well as the ideal alternative using three-way sequential decisions. Different regions are assigned corresponding alternatives according to the maximum individual regret value or minimum group utility value among the alternatives. Finally, all the schemes are ranked based on the ranking rules of the different region or same regions. Next, the three-way sequential VIKOR model according to the minimum group utility value and the three-way sequential VIKOR model according to the maximum individual regret value are introduced.

3.1. Sequential Three-Way VIKOR Model Based on Minimum Group Utility Value

The key step of this model is to derive the group utility value corresponding to each alternative through alternatives, NIS, and PIS, and then classify the alternatives through three-way sequential decisions. The decision index value is calculated according to the measurement formula, and the ultimately ranking result is obtained through the ranking regulations of different regions and the same region. Therefore, the next step is to give the definition of PIS and NIS, as well as the measurement formula and the corresponding scheme comparison process.

Definition 2. Worst solution (NIS) and optimal solution (PIS) of the alternatives attribute matrix.

Where j is the condition attribute, the cost attribute set is J , the benefit attribute set is I , and the alternative and attribute sets are i and j .

According to the calculation results of the above formula, the NIS and PIS of all alternatives can be obtained, and then the group utility value of each alternative can be obtained on the basis of NIS and PIS. A smaller group utility value means that the corresponding scheme has better group utility.

Definition 3. Let there be m alternatives, $PIS = \{e_1^*, e_2^*, \dots, e_n^*\}$, $NIS = \{e_1^-, e_2^-, \dots, e_n^-\}$, and a weight set $W = \{w_1, w_2, \dots, w_n\}$, then the group utility value of alternative A_i is defined as follows:

$$S_i = \sum_{j=1}^n w_j \frac{e_j^* - e_{ij}}{e_j^* - e_j^-} \quad (4)$$

where S_i is the group utility value of the i th scheme.

Theorem 1. *The schemes in the negative region are inferior to the schemes in the positive region, that is, the ranking rules belonging to the schemes in different regions are derived from the group utility values of all the schemes calculated.*

Proof of Theorem 1. The alternatives in the positive region have relatively small group utility values and relatively large group utility. The alternatives that are in the negative region have larger group utility values and smaller group utility values. Therefore, the scheme for the negative region is inferior to the scheme for the positive region. To sum up, $POS\{A\} \succ NEG\{A\}$. $\square$

Since three-way sequential decisions can only classify the alternatives, it is necessary to introduce an integral operator to rank the schemes in the different region and same regions. Opricovi et al. [28] used the decision index value combining individual regret value and group utility value to rank, and the decision-making mechanism coefficient could help realize the minimization of single regret and the maximization of group utility. The larger the mechanism coefficient of decision-making process is, the more attention should be paid to the group maximum utility rather than the single regret. A smaller value of the decision index indicates a better alternative. By maximizing the team utility and minimizing the single regret, the compromise solution of each attribute of the alternative is obtained. The decision index value is defined as follows:

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)} \quad (5)$$

where, Q_i denotes the decision index value of the i th alternative, $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, the weights of maximum team utility and decision index coefficients are v , and $v = 0.5$ is usually assumed. According to Equation (5), the magnitude of Q_i value is related to the difference between S_i and S^* , R_i and R^* . S_i is closer to S^* and R_i is closer to R^* . That is, the single regret value of the scheme is closer to the minimum value in the set of single regret values, and the team utility value is also closer to the minimum value in the set of team utility values. Currently, the team utility of the alternative is larger, and the single regret is smaller, and the value of Q_i is smaller. This indicates that the individual regret value and group utility value of the better decision alternative are always as small as possible.

The steps of the three-way sequential VIKOR model according to minimum group utility value are as follows (where Algorithm 1 is used to Step 3):

Step 1. According to Definition 2, the NIS and PIS of the schemes are deduced from the multiple property message table.

Step 2. Calculate the group utility value for each alternative based on Definition 3.

Step 3. Algorithm 1 is used to classify the decision regions of the scheme.

Step 4. The decision index values of each scheme are calculated according to Equation (5).

Step 5. Sort the schemes belonging to the identical decision area in ascending order of their decision index values.

Step 6. The final ranking result is obtained by sorting the ranking rules about different regions shown in Figure 1.



Figure 1. Connections between decisions regions, $\rightarrow$ denotes $\succ$.

We will illustrate the time sophistication of Algorithm 1 as follows:

$A_{\max}$ and $A_{\min}$ are obtained in step 4 with a corresponding time sophistication of $O(|S|)$. Adding $A_{\max}$ and $A_{\min}$ to POS and NEG in steps 5 to 8, respectively, has a corresponding time sophistication of $O(\frac{|S|}{2})$. Steps 9 and 10 are the sequential process of processing S and BND .

This time sophistication is $O(|k|)$. Lastly, $O(|k||S|)$ is the time complexity of Algorithm 1. Next, the working process of the proposed model is briefly explained by an example.

Example 1. Suppose there are 8 alternatives $A_m = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$. The attribute set $C = \{C_1, C_2, C_3\}$ of alternatives consists of two benefit attributes $\{C_1, C_2\}$ and one cost attribute $\{C_3\}$, and the corresponding weight set is $w = \{0.3, 0.5, 0.2\}$. The normalized attribute evaluation value table is shown in Table 2, and the uniformized determination matrix is expressed as $X = ((X_{mn})_{8 \times 3})(m = 1, \dots, 8; n = 1, 2, 3)$.

Table 2. Normalized decision matrix.

	C_1	C_2	C_3
A_1	0.9000	0.5000	0.4000
A_2	0.2000	0.1000	0.8255
A_3	0.7465	0.9000	0.1000
A_4	0.4525	0.6000	0.4000
A_5	0.6000	0.3565	0.6000
A_6	0.4000	0.4000	0.5575
A_7	0.3000	0.2000	0.7000
A_8	0.1000	0.1500	0.9000

(1) calculate PIS and NIS according to Definition 2 from the nature assessing values of all schemes in Table 2, which can be obtained by $PIS = \{0.9, 0.9, 0.1\}$, $NIS = \{0.1, 0.1, 0.9\}$.

(2) obtain the team utility value $S_i (i = 1, 2, 3, 4, 5, 6, 7, 8)$ of each alternative by PIS and NIS according to Definition 3. $S_1 = 0.3250$, $S_2 = 0.9434$, $S_3 = 0.0576$, $S_4 = 0.4303$, $S_5 = 0.5772$, $S_6 = 0.6144$, $S_7 = 0.8125$, $S_8 = 0.9688$.

(3) classify all alternatives regionally according to Algorithm 1.

$POS^4 = \{A_3, A_1, A_4, A_5\}$, $NEG^4 = \{A_8, A_2, A_7, A_6\}$.

(4) calculate the decision index values of all schemes according to Equation (5) where Q_i represents the decision index value of the i th scheme, which can be obtained by $Q_1 = 0.3642$, $Q_2 = 0.9861$, $Q_3 = 0$, $Q_4 = 0.3513$, $Q_5 = 0.6039$, $Q_6 = 0.5936$, $Q_7 = 0.8436$, and $Q_8 = 0.9647$.

Algorithm 1 Compute the decision region based on the lowest sequential three-way VIKOR of the group utility value

input: Group utility values for m alternatives

k^{th} -level, $k = 0$.

output: Three decision regions, POS^k and NEG^k ;

1 POS = $\emptyset$, BND = $\{A_m\}$, NEG = $\emptyset$, $k = 0$, $S = \{S_m\}$;

2 while BND^k $\neq \emptyset$

3 $k++$, POS_{new} = NEG_{new} = $\emptyset$;

4 $A_{\max} = \min(S)$, $A_{\min} = \max(S)$;

5 Add $A_{\max}$ to POS_{new}

6 POS^k = POS^{k-1} $\cup$ POS_{new};

7 Add $A_{\min}$ to NEG_{new}

8 NEG^k = NEG^{k-1} $\cup$ NEG_{new};

9 BND^k = BND^{k-1} $\cup$ (POS_{new} $\cup$ NEG_{new});

10 $S = S - (A_{\max} \cup A_{\min})$;

11 end

12 Output POS^k(A_m), NEG^k(A_m).

(5) sort the alternatives in the same region according to the decision index value in ascending order, so that the ranking result in the POS domain is $\{A_3 \succ A_4 \succ A_1 \succ A_5\}$ and the ranking result in NEG domain is $\{A_6 \succ A_7 \succ A_8 \succ A_2\}$.

(6) through the ranking rules of different regional alternatives, $POS\{A_m\} \succ BND\{A_m\} \succ NEG\{A_m\}$, $\{A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_6 \succ A_7 \succ A_8 \succ A_2\}$ is the final sorting result.

3.2. Sequential Three-Way VIKOR Model Based on Minimum Individual Regret Value

Like the three-way sequential VIKOR model according to the minimum group utility value, the key steps of the ordinal three-way VIKOR model according to the minimum single regret value are to calculate the individual regret value for all alternatives, classify the alternatives using the sequential three-way decision method, calculate the decision indicator value of each scheme, and then sort schemes in the identical area. The final ranking results are acquired through the alternative ranking rules of different regions.

Definition 4. Let there be m alternatives $A = \{A_1, A_2, \dots, A_m\}$, $PIS = \{e_1^*, e_2^*, \dots, e_n^*\}$, $NIS = \{e_1^-, e_2^-, \dots, e_n^-\}$, and a weight set is $W = \{w_1, w_2, \dots, w_n\}$, then the individual regret value of alternative A_i is defined as follows:

$$R_i = \max_j w_j \frac{e_j^* - e_{ij}}{e_j^* - e_j^-} \quad (6)$$

where R_i denotes the individual regret value of the i th alternative.

The ordinal three-way VIKOR model based on minimum individual regret value is illustrated as follows:

Step 1. Derive the NIS and PIS of schemes from the multi-attribute information table based on Definition 2.

Step 2. The individual regret value is calculated for each scheme according to Definition 4.

Step 3. Classify the alternatives by a similar procedure as in Algorithm 1.

Step 4. The decision index values of each scheme are calculated according to Equation (5).

Step 5. The schemes belonging to the identical decision area are sorted by the decision index value in ascending order.

Step 6. According to the ordering regulation of different areas, the schemes of different areas are ordered to obtain the final ranking results.

Example 2 (like Example 1). The alternatives in Table 2 are calculated according to the above steps, and the final ranking result of the three-way sequential VIKOR model on account of the minimum individual regret value is $\{A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5 \succ A_7 \succ A_8 \succ A_2\}$. It can be acquired through the ranking solutions of all alternatives in Table 2 by the two models, which are different. The final ranking result of the three-way sequential VIKOR model on account of the minimum group utility value is $\{A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_6 \succ A_7 \succ A_8 \succ A_2\}$. However, the final ranking result of the three-way sequential VIKOR model on account of the minimum individual regret value is $\{A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5 \succ A_7 \succ A_8 \succ A_2\}$. In the three-way sequential VIKOR model on account of the minimum group utility value, A_5 is divided into the POS and A_6 is divided into the NEG, but in the three-way sequential VIKOR model on account of the minimum individual regret value, A_5 is divided into the NEG and A_6 is divided into the POS, which eventually leads to diversities in the ultimate classification of results for the two models. At the same time, the diversity between the final ordering consequences of the two models also verifies this paper's idea that only considering the single regret value or group utility value to classify and judge the alternatives will lead to large errors, and a single individual regret value and group utility value can only afford incomplete decision information. Thus, to make the ordering consequences of all schemes more accurate, this paper will simultaneously consider the single regret value and group utility value to sort through the three-way sequential decision method.

4. Sequential Three-Way VIKOR Model Based on Group Utility Value and Individual Regret Value

The two models mentioned above mainly classify the alternatives based on the minimum single regret value or group utility value, and then rank the same and different regions by calculating the integral operator formula. Therefore, in the model proposed next, the single regret value and group utility value of alternatives will be considered simultaneously. The three-way sequential VIKOR model will be established on account of the maximum individual regret value and the minimum group utility value. Finally, the sequential three-way VIKOR model will be established on account of the minimum single regret value and the maximum group utility value.

A good scheme should have a small single regret and a large group utility. The previous and the following models are classified on account of the three-way sequential decision method by calculating the single regret value and the group utility value of the scheme. Through the idea of three-way sequential decisions, all the schemes are reasonably and effectively separated into positive, boundary, and negative areas. The alternative is separated into the positive area, which signifies that it should be a good decision scheme and easy to be accepted by the decision maker. Dividing an alternative into a boundary region means that it has a large individual regret value and a small group utility value in the sequential three-way decision process, or on the contrary, that the decision maker needs to delay acceptance or rejection. The alternatives are classified into the negative region, which indicates that they should be inferior decision alternatives and easy to be rejected by decision makers. In the following part, all alternatives are separated into three areas—negative area, boundary area, and positive area—through three-way sequential decisions. Uncertain alternatives are put into the boundary region and the decision is delayed until more basis and support is obtained, to avoid misclassification and avoid the risk caused by directly accepting or rejecting alternatives. After that, the final sorting result is calculated according to the integral operator and the sorting rules of the different region and same regions. Next, the sequential three-way VIKOR model, based the maximum individual regret and the minimum group utility value, and the sequential three-way VIKOR model, based on the minimum individual regret value and the maximum group utility value, will be discussed.

4.1. Sequential Three-Way VIKOR Model Based on Minimum Group Utility Value and Maximum Individual Regret Value

Unlike the previously proposed model according to a single value, the proposed model comprehensively considers the single regret value and the group utility value. In most cases, it is impossible to obtain an accurate decision scheme by only considering the partial decision information corresponding to a single value. The alternative with larger group utility does not necessarily have smaller individual regret, and the decision scheme with smaller individual regret does not necessarily have larger group utility. When dealing with these alternatives, a new method is needed to identify the classification results of the schemes by considering both the single regret value and the group utility value of the alternatives.

Theorem 2. *The alternatives with the largest single regret value and group utility value or vice versa are classified as boundary regions in the model. Based on the concept of three-way decisions, a ranking rule considering the alternatives in different regions of the boundary domain is proposed, as shown in Figure 2.*



Figure 2. Ranking rules for different decision regions.

Proof of Theorem 2. The individual regret of the alternatives divided into the positive region is relatively small and the group utility is relatively large, which is easy to be accepted by the decision maker. The schemes divided into the negative region have large single regret and relatively small group utility, which are easy for decision makers to reject. The single regret and group utility of the schemes separated into the boundary area are equivalent, so the decision maker needs to delay the decision. Therefore, the boundary region scheme is better than the negative region scheme and the boundary region scheme is inferior to the positive region scheme; this can be obtained by: $\square$

$$POS\{A\} \succ BND\{A\} \succ NEG\{A\}$$

The overall steps of the proposed model are the same as those of the model considering a single value, the main difference being that the proposed model considers both the single regret value and the group utility value. In the classification of the alternatives by each round of sequential three-way decisions, the scheme with the minimum group utility value is divided into the positive region, the alternative with the maximum individual regret value is separated into the negative region, and the alternative with the maximum individual regret value and the minimum group utility value is divided into the boundary region.

The steps of sequential three-way VIKOR model according to maximum individual regret value and minimum group utility value are as follows (where Algorithm 2 is used in Step 3):

Step 1. Derive the NIS and PIS of schemes from the multi-attribute information table based on Definition 2.

Step 2. Calculate the single regret value and group utility value for each alternative based on Definitions 3 and 4.

Step 3. Algorithm 2 is used to classify the alternatives.

Step 4. Calculate the decision index value for each alternative according to Equation (5).

Step 5. Sort the schemes belonging to the same decision area in ascending rank of their decision index values.

Step 6. According to the ordering regulations of different areas, the results of different areas are ordered to obtain the final ordering results.

We illustrate the time sophistication of Algorithm 2 as follows: In Step 4, the extraction of the minimum value in S is denoted as $A_{\max}$ and the extraction of the maximum value in R is denoted as $A_{\min}$, and the time sophistication is $O(|S|)$. Steps 6 to 11 add the alternatives to the corresponding domain, respectively, with time sophistication $O(|k|)$. Steps 12 to 13 are sequential processing of the boundary region, S and R , and the time complexity is $O(|k|)$. In general, $O(|k||S|)$ is the time sophistication of Algorithm 2.

Example 3. The process of using this model to process the normalized attribute matrix $X = ((X_{mn}))_{8 \times 3}$ ($m = 1, \dots, 8; n = 1, 2, 3$) and the corresponding weight set $W = \{0.3, 0.5, 0.2\}$ shown in Table 2 is as follows:

(1) derives the NIS and PIS of alternatives from the multiple message attribute information table on account of Definition 2.

(2) calculates the individual regret value and group utility value of each scheme on account of Definitions 3 and 4, where R_i denotes the individual regret value of the i scheme and S_i represents the group utility value of the i th scheme.

$S_1 = 0.3250, S_2 = 0.9434, S_3 = 0.0576, S_4 = 0.4303, S_5 = 0.5772, S_6 = 0.6144, S_7 = 0.8125, S_8 = 0.9688. R_1 = 0.2500, R_2 = 0.5000, R_3 = 0.0576, R_4 = 0.1875, R_5 = 0.3397, R_6 = 0.3125, R_7 = 0.4375, R_8 = 0.4688.$

(3) divides all the alternatives into corresponding regions according to Algorithm 2. $POS^3 = \{A_3, A_1, A_4\}, BND^3 = \{A_5, A_6\}, NEG^3 = \{A_2, A_8, A_7\}.$

(4) calculates the decision indicatrix value of each scheme according to Equation (5), where Q_i denotes the decision index value of the i th alternative.

$Q_1 = 0.3642$, $Q_2 = 0.9861$, $Q_3 = 0$, $Q_4 = 0.3513$, $Q_5 = 0.6039$, $Q_6 = 0.5936$, $Q_7 = 0.8346$, $Q_8 = 0.9647$.

(5) sorts alternatives for the same region in ascending order of decision index values based on the ordering regulations of alternatives in the identical area. The ordering result in the positive region is $\{A_3 \succ A_4 \succ A_1\}$, the ranking result in the boundary area is $\{A_6 \succ A_5\}$, and the ranking result in the negative area is $\{A_7 \succ A_8 \succ A_2\}$.

(6) Finally, the ranking rules use different regions when ranking the alternatives are shown in Figure 2. The final sorting result is $\{A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5 \succ A_7 \succ A_8 \succ A_2\}$.

4.2. Sequential Three-Way VIKOR Model Based on Maximum Group Utility Value and Minimum Individual Regret Value

In the sequential three-way decision classification, the scheme with the largest group utility value is separated into the negative area, and the scheme with the smallest individual regret value is divided into the positive region. If the alternative with the minimum single regret value and the maximum group utility value is also divided into the boundary region, the other steps of the proposed model are roughly like the sequential three-way VIKOR model according to the maximum individual regret value and the minimum group utility value.

Algorithm 2 Sequential three-way VIKOR model based on minimum group utility value and maximum individual regret value

Input: Group utility values and individual regret values for m alternatives

k^{th} -level, $k = 0$;

output: Three decision regions, POS^k , BND_{new}^k and NEG^k ;

1 $POS = \emptyset$, $BND = \{A_m\}$, $NEG = \emptyset$, $k = 0$, $S = \{S_m\}$, $R = \{R_m\}$

2 **while** $BND^k \neq \emptyset$ **do**

3 $k++$, $POS_{new} = NEG_{new} = \emptyset$;

4 $A_{max} = \min(S)$, $A_{min} = \max(R)$;

5 **if** $A_{max} = A_{min}$

6 $\text{Add } A_{max} \text{ to } BND_{new}$

7 $BND_{new}^k = BND_{new}^{k-1} \cup BND_{new}$;

8 **else** $\text{Add } A_{max} \text{ to } POS_{new}$

9 $POS_{new}^k = POS_{new}^{k-1} \cup POS_{new}$;

10 $\text{Add } A_{min} \text{ to } NEG_{new}$

11 $NEG^k = NEG^{k-1} \cup NEG_{new}$;

12 $BND^k = BND^{k-1} - (POS_{new} \cup BND_{new} \cup NEG_{new})$;

13 $S = S - (A_{max} \cup A_{min})$;

14 **end**

15 **Output** $POS^k(A_m)$, $BND_{new}^k(A_m)$, $NEG^k(A_m)$.

Example 4. (Similar to Example 3), Using this model to process Table 2 can acquire the final ordering result of the schemes of the three-way sequential VIKOR model according to the minimum single regret value and the maximum group utility value as $\{A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5 \succ A_7 \succ A_8 \succ A_2\}$. It can be seen that the ranking results of the sequential three-way VIKOR model according to the maximum individual regret value and the minimum group utility value and the sequential three-way VIKOR model according to the minimum single regret value and the maximum group utility value are both $\{A_3 \succ A_4 \succ A_1 \succ A_6 \succ A_5 \succ A_7 \succ A_8 \succ A_2\}$. It shows the effectiveness of considering the group utility value and single regret value and the stability of the final ranking result. Unlike the sequential three-way VIKOR model, according to a single individual regret value or group utility value, the three-way sequential VIKOR model based on individual regret value and group utility value separated the schemes into the negative area and positive area and divides the schemes that are difficult to judge into the boundary area.

From Example 1 and Example 2, it can be seen that there is A_5 difference in the final ranking results of A_5 and A_6 . Example 1 divides A_5 into the positive area and A_6 into the negative area, while Example 2 divides A_6 into the positive area and A_5 into the negative area, which eventually leads to the difference in the ranking results. Therefore, it is difficult to acquire an accurate ranking consequence by only considering the partial decision information of A_5 single individual regret value or group utility value. By comprehensively considering the single regret value and group utility value, the result that A_6 is better than A_5 is consistent with the result obtained by VIKOR method.

The alternatives for these two models to be separated into the positive area are consistent with the alternatives to be classified into the negative area, $\{A_3 \succ A_4 \succ A_1\}$ and $\{A_8 \succ A_2 \succ A_7\}$, respectively. It shows that these two models also follow the principle that the smaller the single regret, the larger the group utility, and the better the scheme, and the larger the single regret, the smaller group utility, and the worse the scheme. Through sequential three-way decisions and considering both single regret value and group utility value, decision makers can obtain more decision information to classify alternatives. The sequential three-way VIKOR model based on single regret value and group utility value reduces the range of the positive area and negative area, and divides the difficult solutions into the boundary area, which avoids the difference in ranking results due to misclassification to a certain extent.

5. Experiment and Analysis

In this chapter, the ranking results of the proposed two models in two examples are compared with other existing multi-attribute ranking methods to further illustrate the VIKOR-based sequential three-way model. The first example is from Zhang [6] and the second example is from Ye [29]. The selected examples are from other papers on the combination of multiple attribute decision-making methods and sequential three-way decisions, and papers on the combination of fuzzy information systems and sequential three-way systems. These are representative in terms of multiple attribute decision-making methods and sequential three-way decisions, and the selected examples are also classic in terms of decision ranking and investment decision problems. Therefore, the selected examples are representative.

Example 5. In order to the usefulness and feasibility of the newly proposed multiple attribute decision-making method related to TOPSIS, Zhang et al. [6] used the data related to an investment plan. The company needs to comprehensively consider the expected benefit, market saturation, environmental impact, social benefit, and energy saving degree of the eight investment plans proposed and select the best investment plan. The set of eight investment plans is $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$, the corresponding attribute set of investment plans is $C = \{C_1, C_2, C_3, C_4, C_5\}$, the benefit attribute is $\{C_1, C_4, C_5\}$, and the cost attribute is $\{C_2, C_3\}$. The set of weights corresponding to each attribute is $W = \{0.3, 0.1, 0.15, 0.25, 0.2\}$, and the uniformized decision matrix corresponding to the eight investment alternatives is $X = ((X_{mn}))_{8 \times 5}$ ($m = 1, \dots, 8; n = 1, 2, 3, 4, 5$), as shown in Table 3. The visualization of Example 5 is presented in Figure 3.

Table 3. Normalized decision matrix corresponding to investment alternatives.

	C_1	C_2	C_3	C_4	C_5
A_1	0.8	0.4	0.3	0.8	0.9
A_2	0.9	0.5	0.5	0.7	0.6
A_3	0.3	0.4	0.6	0.4	0.3
A_4	0.5	0.2	0.2	0.7	0.6
A_5	0.7	0.6	0.6	0.5	0.8
A_6	0.4	0.8	0.7	0.7	0.3
A_7	0.9	0.5	0.1	0.8	0.7
A_8	0.6	0.8	0.8	0.3	0.4

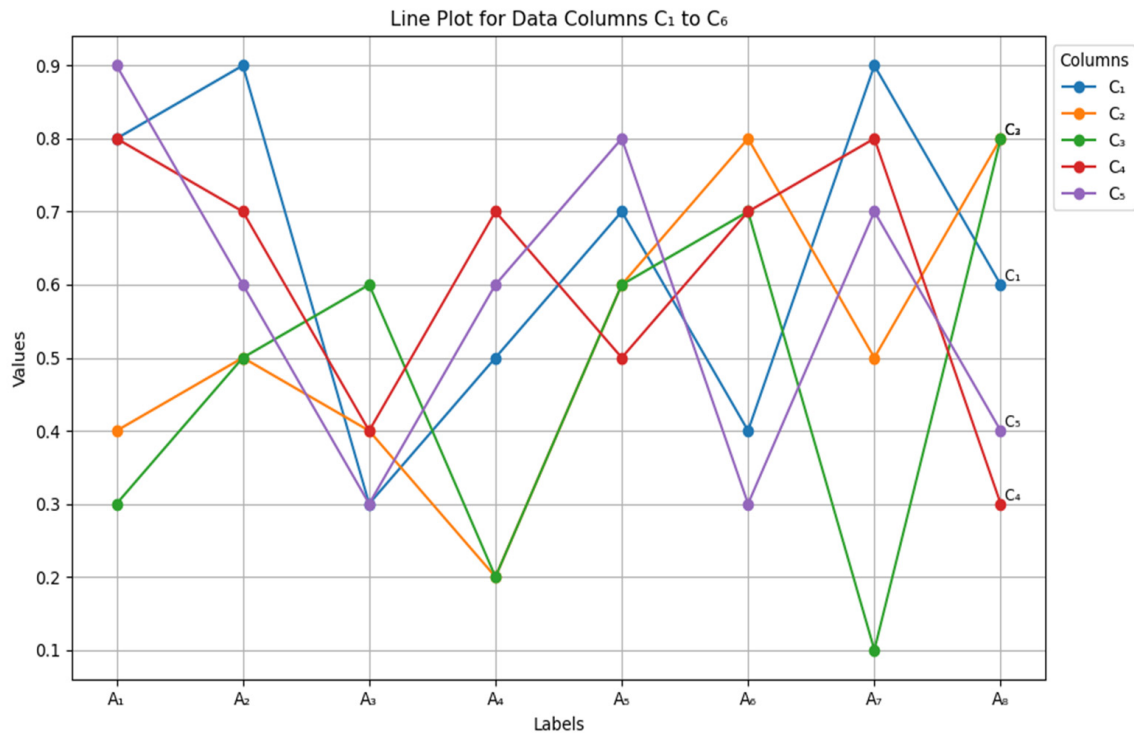


Figure 3. Visualization of Example 5.

Method 1. Sequential three-way VIKOR model according to maximum individual regret value and minimum group utility value:

Step 1. Calculate the PIS and NIS of all investment plans according to Table 3, the larger value of benefit attribute.

Step 2. Calculate the single regret value and group utility value of all schemes on account of (4) and (6), where R_i denotes the individual regret value of the i th scheme and S_i denotes the group utility value of the i th alternative.

$S_1 = 0.1262, S_2 = 0.2857, S_3 = 0.8404, S_4 = 0.5642, S_5 = 0.4571, S_6 = 0.6911, S_7 = 0.1167, S_8 = 0.8167. R_1 = 0.0500, R_2 = 0.1000, R_3 = 0.3000, R_4 = 0.2000, R_5 = 0.1500, R_6 = 0.2500, R_7 = 0.0667, R_8 = 0.2500.$

Step 3. Classify the alternatives based on Algorithm 2 by the maximum single regret value and the minimum group utility value, which can be obtained $POS^4 = \{A_7, A_1, A_2, A_5\}, NEG^4 = \{A_3, A_6, A_8, A_4\}$. The boundary region BND^4 is empty because there is no alternative that has both the maximum single regret value and the minimum group utility value when classifying the alternatives by sequential three-way decisions.

Step 4. Calculate the decision index values of all schemes according to Equation (5), and Q_i represents the decision index value of the i th scheme, which can be obtained by

$Q_1 = 0.0066, Q_2 = 0.2168, Q_3 = 1, Q_4 = 0.6092, Q_5 = 0.4352, Q_6 = 0.7969, Q_7 = 0.0334, Q_8 = 0.8836.$

Step 5. On the basis of the ordering regulations of alternatives in the identical area, the alternatives in the identical area are sorted in ascending rank of decision index values, and the ranking result in the positive region is $\{A_1 \succ A_7 \succ A_2 \succ A_5\}$, and the ranking result in the negative region is $\{A_4 \succ A_6 \succ A_8 \succ A_3\}$.

Step 6. The final ranking result $\{A_1 \succ A_7 \succ A_2 \succ A_5 \succ A_4 \succ A_6 \succ A_8 \succ A_3\}$ is obtained through the ordering regulations $POS\{A\} \succ BND\{A\} \succ NEG\{A\}$ of different regions.

Method 2. Sequential three-way VIKOR model based on minimum individual regret value and maximum group utility value:

Step 1. This step is the same as in Method 1.

Step 2. This step is the same as in Method 1.

Step 3. According to Algorithm 2, the alternatives are classified by the maximum individual regret value and the maximum group utility value. $POS^4 = \{A_1, A_7, A_2, A_5\}$, $NEG^4 = \{A_3, A_8, A_6, A_4\}$.

Step 4. is the same as in Method 1.

Step 5. Similar to Method 1, it can be observed that the ranking result in the positive domain is $\{A_1 \succ A_7 \succ A_2 \succ A_5\}$, and the ranking result in the negative domain is $\{A_4 \succ A_6 \succ A_8 \succ A_3\}$.

Step 6. The final ranking result $\{A_1 \succ A_7 \succ A_2 \succ A_5 \succ A_4 \succ A_6 \succ A_8 \succ A_3\}$ is obtained through the ranking rules of different regional alternatives.

Through the final ranking results of the above two models, it can be found that the ordering consequences of the two models are consistent. This shows that the comprehensive consideration of single regret value and group utility value makes the decision information considered by decision makers more complete and shows the effectiveness and the stability of the final ranking results. In three-way sequential decisions, the individual regret value and group utility value are used to provide more effective ranking information for the alternatives. Three-way sequential decisions divide the alternatives into different domains to obtain preliminary ranking information, which avoids some unreasonable ranking results to a certain extent.

The weight information of Example 5 is changed to $W = \{0.3, 0.1, 0.3, 0.2, 0.1\}$, and the final ranking result is changed to $\{A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3 \succ A_8\}$, as shown in Table 4 and in the visualization of comparison results with other methods shown in Figure 4. The ordering consequences of this method are consistent with other schemes or slightly different.

Table 4. Comparison results with other methods (Example 5).

Different Methods	Ranking Results	The Optimal Candidate	The Inferior Candidate
ours	$A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_7	A_8
Zhang et al.'s method [6]	$A_7 \succ A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_6 \succ A_8 \succ A_3$	A_7	A_3
Ye et al.'s method [29]	$A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_8 \succ A_6$	A_7	A_6
Jia et al.'s method [30]	$A_7 \succ A_1 \succ A_4 = A_2 \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_7	A_8
Wang et al.' method [31]	$A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_8 \succ A_3$	A_7	A_3
VIKOR method [28]	$A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3 \succ A_8$	A_7	A_8
TOPSIS method [32]	$A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_8 \succ A_3$	A_7	A_3

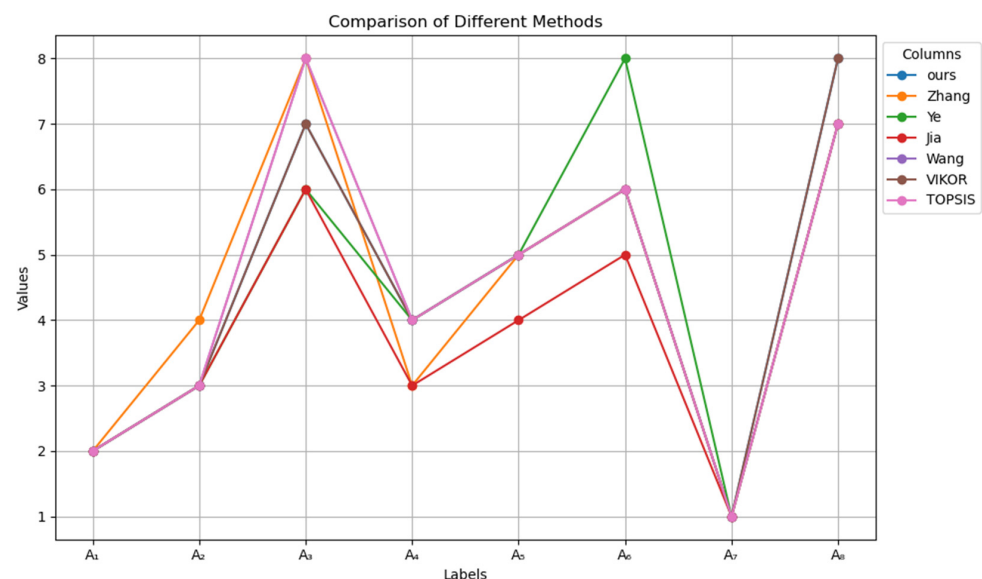


Figure 4. Visualization of comparison results of Example 5 with those of other methods.

The disparity between the proposed method and other comparison methods is that the positions of alternatives with similar rank are exchanged with each other, and the overall ranking information is only slightly different. On the premise of ensuring the accuracy of the experimental consequences, the usefulness and availability of the proposed method are verified.

Example 6. Ye et al. [29] considered the information of six manager candidates in the company and selected the manager candidates according to their health status, educational background, social ability, work style, writing ability and work experience, which were all benefit attributes. The set of manager candidates is $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$, the corresponding set of attributes is $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$, and the corresponding set of attribute weights is $W = \{0.1545, 0.1545, 0.19, 0.19, 0.1555, 0.1555\}$, as shown in Table 5 and Figure 5.

Table 5. Attribute evaluation values of manager candidates.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.68	0.74	0.64	0.83	0.65	0.75
A_2	0.48	0.83	0.62	0.78	0.48	0.72
A_3	0.83	0.48	0.78	0.58	0.75	0.48
A_4	0.74	0.68	0.83	0.64	0.75	0.65
A_5	0.76	0.66	0.48	0.62	0.72	0.83
A_6	0.66	0.76	0.58	0.48	0.83	0.75

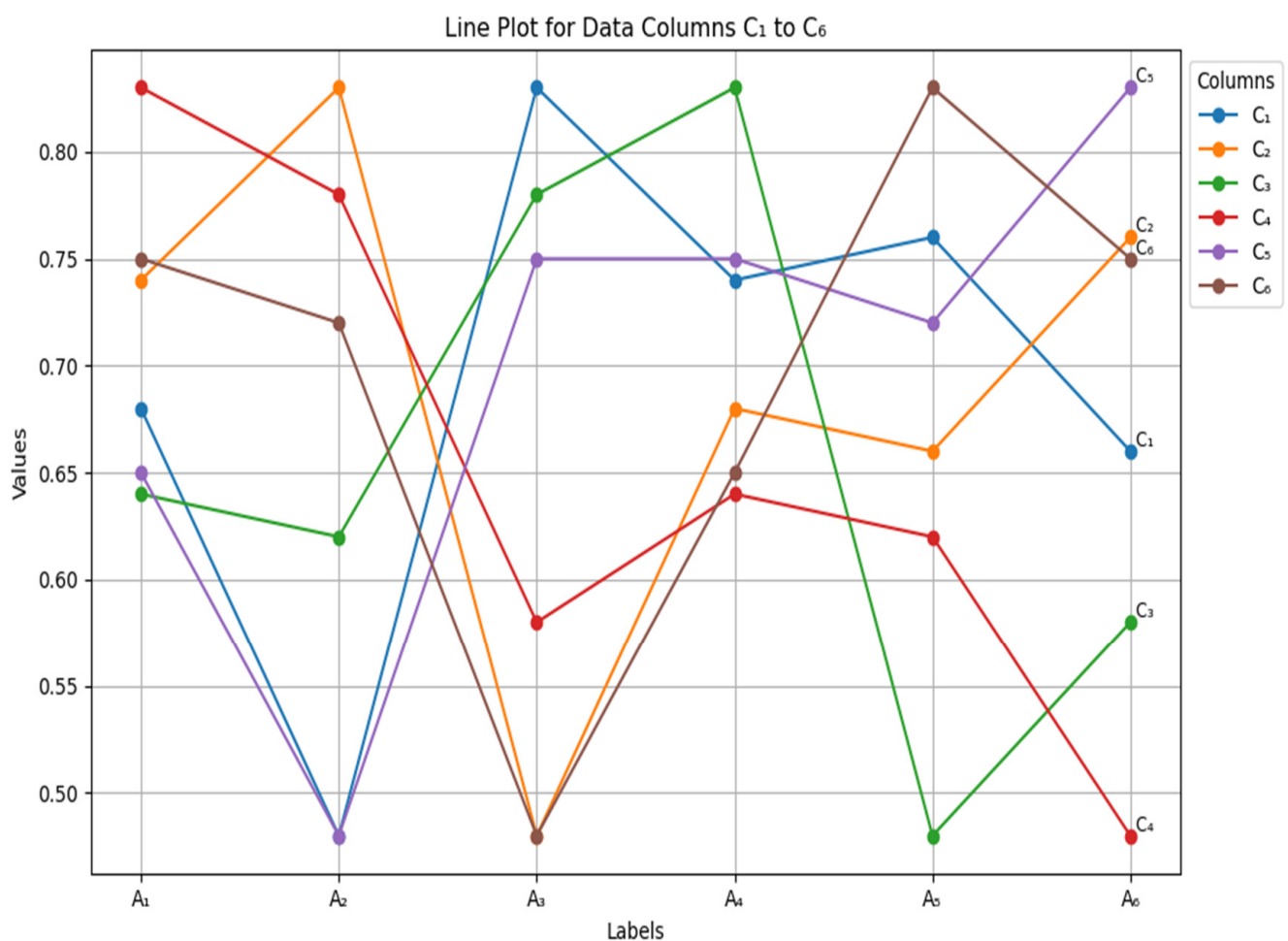


Figure 5. Visualization of Example 6.

The final ordering result acquired by the proposed method is $\{A_1 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3\}$ and is compared with other methods, as shown in Table 6 and visualization of comparison results with other methods shown in Figure 6.

Table 6. Comparison results with other methods (Example 6).

Different Methods	Ranking Results	The Optimal Candidate	The Inferior Candidate
ours	$A_1 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1	A_3
Ye et al.'s method [29]	$A_1 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1	A_3
TOPSIS method [32]	$A_1 \succ A_4 \succ A_5 \succ A_2 \succ A_6 \succ A_3$	A_1, A_4	A_3
WAA operator method [33]	$A_1 = A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1, A_4	A_3
EDAS method [34]	$A_1 = A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1, A_4	A_3
Jia et al.'s method [30]	$A_1 = A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1, A_4	A_3
Zhang et al.'s method [6]	$A_5 \succ A_6 \succ A_1 = A_4 \succ A_2 \succ A_3$	A_5	A_3
VIKOR method [28]	$A_1 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_3$	A_1	A_3

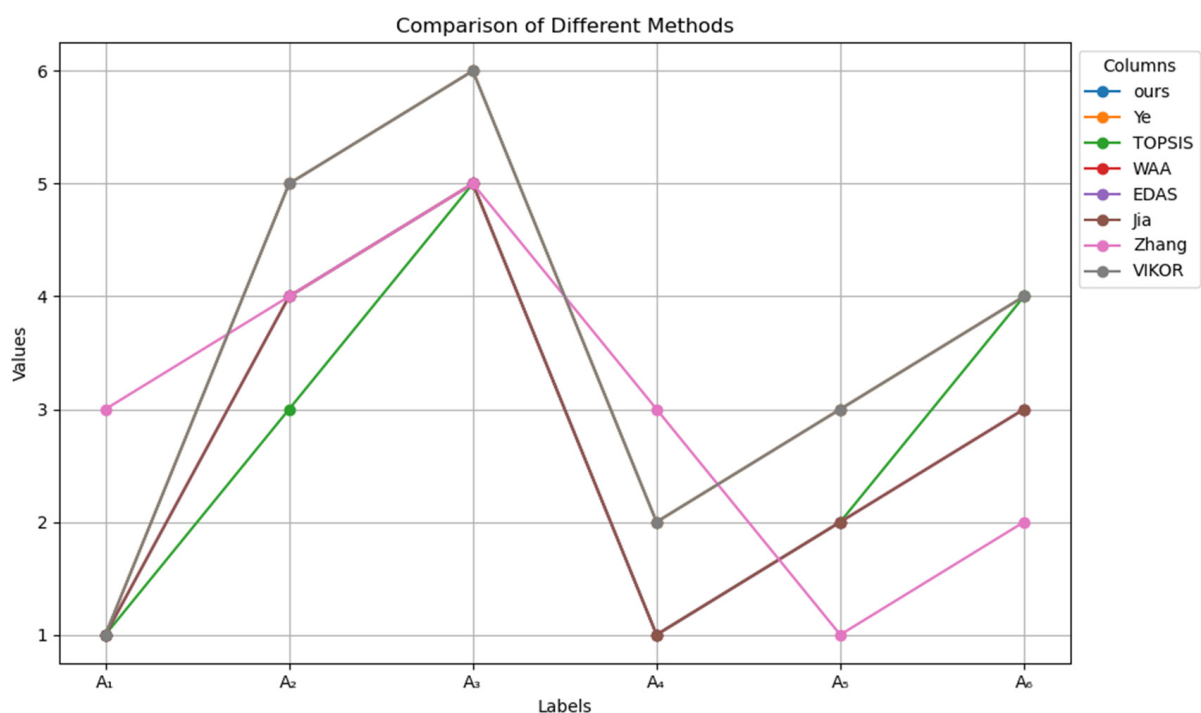


Figure 6. Visualization of comparison results with other methods of Example 6.

Table 6 shows that the ordering consequences of the proposed method are consistent with those of most methods. The ordering consequences of the proposed method are identical as those of Ye [29] and VIKOR method [28]. The disparity between the proposed method and other methods is that A_1 is used as the optimal solution in this paper. In addition to Zhang [6]'s method, other methods take A_1 and A_4 as optimal solutions at the same time. Table 6 shows that the proposed method has the same worst and optimal alternatives as most of the reference methods. According to Table 5, attributes $\{C_1, C_3, C_5\}$ and $\{C_2, C_4, C_6\}$ are evaluated by A_1 and A_4 with opposite values. Since the weights of $\{C_1, C_2\}$, $\{C_3, C_4\}$, and $\{C_5, C_6\}$ are the same, there is intuitively no difference between A_1 and A_4 . Other methods mainly use TOPSIS for ranking, which is based on the set of distances from the ideal solution, and a compromise scheme with dominance ratio is improved by VIKOR, which is the basis of the TOPSIS method. The VIKOR method makes the decision of decision makers more conservative or radical by adding the decision mechanism coefficient since TOPSIS method. The process of TOPSIS method does not incorporate any subjective factors, so there are slight differences in the priority of A_1 and A_4 . Besides that, most methods are consistent with the ranking results of the proposed

method. The consequences show that the proposed method is as effective as other methods in dealing with the above examples, and is more effective and simpler than other methods, which extends the application scope of sequential three-way decisions and VIKOR methods.

To further test the proposed method, Example 5 is used to quantitatively analyze the proposed method. Firstly, the sensitivity analysis of the decision-making mechanism coefficient v in Equation (5) is carried out, and the values in the interval $[0, 1]$ are taken in turn with a step of 0.1 to analyze the decision index value Q corresponding to the alternatives and the ranking order of the schemes. It can be observed from Figure 7 that with the continuous increase of v , the proportion of the relative distance calculated by the group utility value becomes larger and larger. This means that the decision index value corresponding to the alternative with the smaller group utility value and the decision index value corresponding to the alternative with the larger single regret value and the group utility value corresponding to the alternative with the larger group utility value becomes larger and larger.

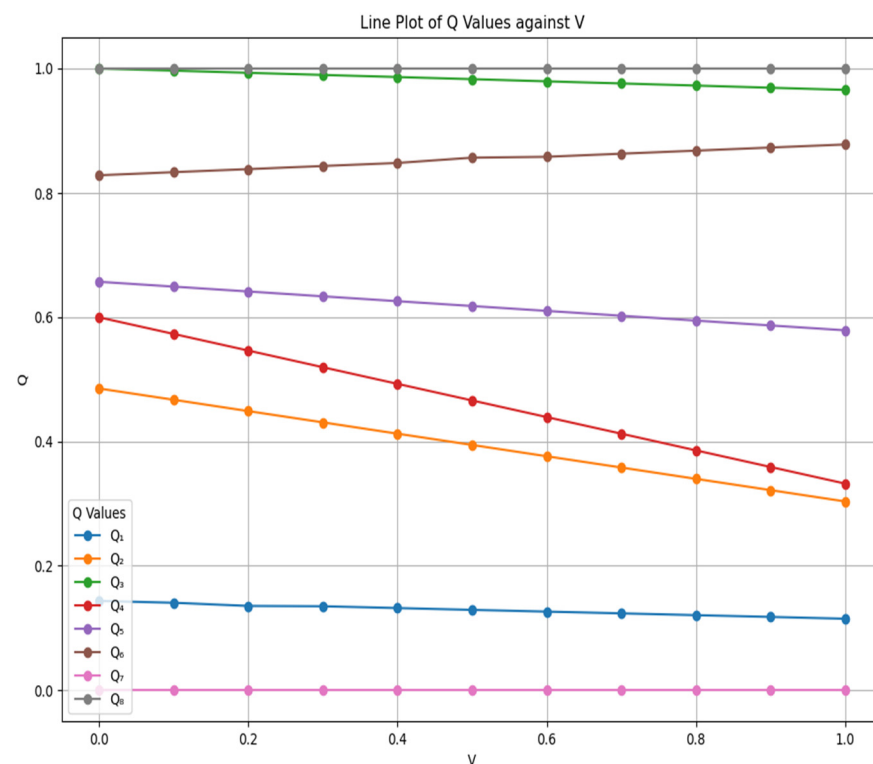


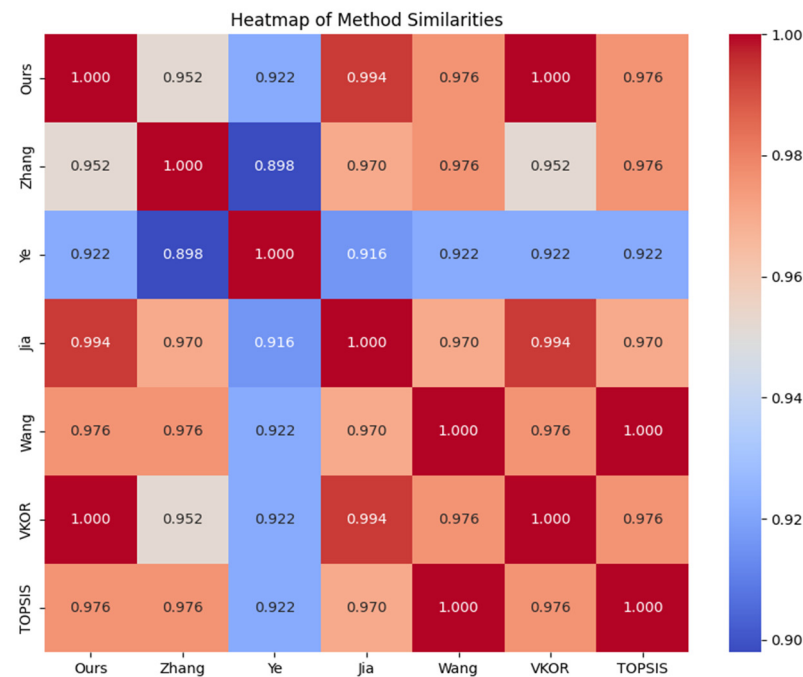
Figure 7. Influence of decision mechanism coefficient v on decision index value Q .

However, the relative order of all schemes is not changed, and only when $v = 0$, the ranking result is $\{A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3 = A_8\}$, and A_3 and A_8 are in the same priority. When $v = 0$, the group utility value is not considered in the calculation of the decision index value, and the individual regret values R_3 and R_8 of A_3 and A_8 are both 0.3 at this time. This possible decision error is caused by only referring to partial decision information, as mentioned in this paper. When $V \neq 0$, because the group utility values of A_3 and A_8 are not the same and the group utility value is considered in the calculation of the decision index value, the ordering consequence is $\{A_7 \succ A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3 \succ A_8\}$, which is consistent with the ordering consequences of other cases. Therefore, the proposed method is stable.

Next, the correlation between other methods and the ordering consequences of the proposed method is calculated by Example 5 as shown in Table 7 and Figure 8. It can be seen from Table 7 that the proposed method is significantly correlated with other methods and has high consistency in the ranking results.

Table 7. Spearman correlation coefficient between the proposed method and other methods.

Different Methods	Ours	Zhang [6]	Ye [29]	Jia [30]	Wang [31]	VIKOR [28]	TOPSIS [32]
ours	1	0.952	0.922	0.994	0.976	1	0.976
Zhang's method [6]	-	1	0.898	0.970	0.976	0.952	0.976
Ye's method [29]	-	-	1	0.916	0.922	0.922	0.922
Jia's method [30]	-	-	-	1	0.970	0.994	0.970
Wang's method [31]	-	-	-	-	1	0.976	1
VIKOR method [28]	-	-	-	-	-	1	0.976
TOPSIS method [32]	-	-	-	-	-	-	1

**Figure 8.** Heatmap of Spearman correlation coefficient.

In addition, it can be observed that the correlation coefficients between the method proposed in this study and other methods are higher than or equal to the correlation coefficients between other methods. This indicates that the method proposed in this study is comparable to the other methods to some extent. In summary, the proposed method demonstrates a certain level of superiority and effectiveness.

6. Conclusions

This paper proposes a three-way sequential classification ranking method based on the VIKOR method. Firstly, based on the single regret value and group utility value, two different sequential three-way VIKOR models based on single regret value or group utility value are proposed. One shortcoming of the two models is that referring to partial decision information may bring errors in classification and ranking. This paper proposed a three-way sequential VIKOR model according to individual regret value and group utility value to compensate for the errors in classification and ranking. Secondly, the VIKOR method is introduced into three-way sequential decisions to design a three-way VIKOR method that can deal with both ranking and classification. This can effectively solve the problem of classification ranking and divide alternatives that are difficult to judge into the boundary region, reducing the possibility of misclassification leading to differences in ranking results. This study extends the application scope of the VIKOR method and sequential three-way decisions. Based on the experimental results, the effectiveness and feasibility of the method are confirmed. In the decision-making process, the advantages of the two methods are combined. By comprehensively considering the ranking index of the VIKOR method and

the hierarchy and the division of decision domains for sequential three-way decisions, the possibility of misclassification and discrepancies in ranking results is reduced, leading to more scientific decision making.

Next, the related models were improved and supplemented in terms of weight allocation, multi-criteria preference, extension, and improvement of the VIKOR method. The VIKOR method based on sequential three-way decisions in more complex decision problems is further explored and applied to solve sequencing and investment decision problems. In addition, the effective integration of other multi-attribute decision-making methods and three-way decisions will continue to be explored.

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