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Abstract: In practical engineering, due to the lack of information, it is impossible to accurately determine the distribution of all variables. Therefore, time-variant reliability problems with both random and interval variables may be encountered. However, this kind of problem usually involves a complex multilevel nested optimization problem, which leads to a substantial computational burden, and it is difficult to meet the requirements of complex engineering problem analysis. This study proposes a decoupling strategy to efficiently analyze the time-variant reliability based on the mixed uncertainty model. The interval variables are treated with independent random variables that are uniformly distributed in their respective intervals. Then the time-variant reliability-equivalent model, containing only random variables, is established, to avoid multi-layer nesting optimization. The stochastic process is first discretized to obtain several static limit state functions at different times. The time-variant reliability problem is changed into the conventional time-invariant system reliability problem. First order reliability analysis method (FORM) is used to analyze the reliability of each time. Thus, an efficient and robust convergence hybrid time-variant reliability calculation algorithm is proposed based on the equivalent model. Finally, numerical examples shows the effectiveness of the proposed method.

Keywords: interval variables; random variables; time-variant reliability analysis; hybrid uncertain model

1. Introduction

Structural reliability is regarded as the ability of the device or structure to complete the required functions under the specified conditions within the prescribed design period [1–5]. Since the advent of this concept in engineering and industrial applications, structural reliability analysis in the probabilistic framework has been developed rapidly. Although a series of effective methods, including the first-order second-moment method [6,7], second-order second-moment method [8] and system reliability analysis [9], have been proposed. However, there are several time-variant uncertain parameters in practical structures such as external dynamic loads, degradation of material properties and changes of geometric characteristics that affect the time-variant reliability of structures [10]. Accordingly, it is significant importance to conduct to carry out a time-variant reliability analysis for the engineering structures.

Conventional time-variant reliability analyzing methods can be mainly divided into five categories [11–13], i.e. the first-passage method, numerical methods, extreme value density methods, surrogate methods and the quasi-static methods.

The time-variant reliability problem mainly originates from the first-passage method proposed by Rice [14] in the 1940s, which lays the foundation for the development of the subsequent crossing rate method. It should be noted that the calculation of the outcrossing rate is essential in Rice's method. Studies show that, although the Rice method has



Citation: Li, F.; Yan, Y.; Rong, J.; Zhu, H. An Efficient Time-Variant Reliability Analysis Method with Mixed Uncertainties. *Algorithms* **2021**, 14, 229. https://doi.org/10.3390/ a14080229

Academic Editors: Patrizia Beraldi and Frank Werner

Received: 17 June 2021 Accepted: 30 July 2021 Published: 31 July 2021

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remarkable advantages over other methods from the computational point of view, this is accounted for by the hypothesis that the outer crossover is statistically independent. However, this assumption results in the relatively low accuracy of the time-variant reliability approaches. In order to resolve this shortcoming, Andrieu Renaud et al. [15] transformed the calculation of the outcrossing rate into the reliability problem of a static parallel system and established the pHi2 method. Accordingly, they provided a good scheme to determine the span rate efficiently and analyze the time-variant reliability. On this basis, Sudret [16] presented the PHI2+ approach to achieve an analytical solution for the crossover rate.

The second scheme is the numerical simulation method. Mori and Ellingwood [17] and Singh et al. [18] developed sampling methods and subset simulation methods, respectively [19–21]. Moreover, studies show that the Monte Carlo simulation (MCS) method can be used to sample a random process or random variable in a limited state function of a structure and bring these random numbers into a function to compute time-variant reliability. Further investigations show that although the Monte Carlo simulation method has high accuracy, it has a relatively low computational efficiency.

The third method is the extreme value density method, which uses the probability distribution of the response extremum of the time-variant problem to change the time-variant problem into an invariant problem. In this fashion, Chen and Li [22] utilized an improved probability density evolution method. Furthermore, Hu and Du [23] transformed the time-variant reliability analysis into a reliability problem and proposed an innovative sampling method for the response extreme value distribution. It should be pointed out that the proposed method does not require time-variant parameters, thereby improving the calculation efficiency.

The fifth method is the surrogate model, which is based on constructing an analytical expression between the input variables and the structural response [24]. Then, an appropriate interpolation algorithm is applied to obtain the analytical expression that meets the accuracy requirements. Accordingly, Wang and Wang [25,26] embedded the efficient global optimization (EGO) method [27] in an extreme value surrogate model and proposed a nested extreme value response method to identify the extreme values. Moreover, Zhang et al. [28] proposed a new response approximation model for the time-dependent reliability analysis of uncertain structures under stochastic loads.

Finally, the main objective of the quasi-static method is to change the time-variant reliability issue into a constant reliability issue. In this method, time-variant uncertain parameters are discretized to transform the complex time-variant reliability problem into a problem independent of time-variant parameters. In this regard, Li et al. [29] and Cazuguel et al. [30] transformed the time-variant reliability model into a static reliability model through representing the Gaussian process as multiple independent normal distributions. Gong and Frangopol [31] proposed the NEWREL method based on stochastic discretization method. Jiang et al. [10,32] developed the reliability calculation method for discretizing time-variant uncertain parameters (TRPD) and extended it to system reliability. Subsequently, an improved version of TRPD [33] was proposed in which time-variant reliability analysis was performed only at the component level. This paper focuses on the TPRD approach.

Known by references aforementioned, the majority of investigations in the field of time-variant reliability are confined to conventional structural reliability problems with random variables [34]. This kind of time-variant reliability analysis requires a large number of experimental samples to construct accurate probability distributions of random variables. However, experimental sample data is limited in many engineering applications. In this case, the boundary of the uncertain variable is easy to determine, and the uncertain variable is suitable to be described by the non-probabilistic interval variable. For reliability analysis problems with interval variables, non-probabilistic methods can be used to measure reliability [34]. There may be both random and non-probability interval-uncertain variables in structural systems. Therefore, it is necessary to develop effective methods to solve the mixed uncertainty problems [35,36]. However, only a few researchers studied time-variant

reliability problems of mixed variable structures. Specifically, a multi-layer nested analysis approach has been adopted to solve mixed time-variant reliability problems [29,37]; the outer layer is used to discretize the stochastic process in the time-variant function, while the inner layer is nested optimized at each time step. Accordingly, the computational efficiency is extremely low, which adversely affects its practicability in industrial applications. To this end, it is vital to develop efficient algorithms to significantly decrease the computational burden of the time-variant reliability method with mixed variables.

In the present study, it is intended to presented a novel approach to analyze the structural time-variant reliability with mixed variables. To this end, the existing time-variant reliability theory is combined with the interval uncertainty analysis. The rest of this paper is organized as follows: the structural time-variant reliability model with random variables is introduced in Section 2. Then, the structural time-variant reliability model with mixed variable and traditional solution method are discussed in Section 3. In Section 4, a new method is proposed to analyze the structural time-variant reliability. For evaluating the effectiveness of the presented approach, it is applied to several numerical and engineering case studies and then the obtained results are presented in Section 5. Finally, Section 6 summarizes the main results and conclusions of this study.

2. Structural Time-Variant Reliability Model with Random Variables

Generally, time-variant reliability models are an account of the generalized force model and contain one random variable. Let g be the limit state function composed of the general random process of structural resistance R(t) and the stochastic process of structural load effect S(t). This function, which can be linear or nonlinear, can be given as:

$$g(R(t), S(t)) = R(t) - S(t)$$
 (1)

where *t* is the time variable. Generally, the stochastic process S(t) of structural load effect contains the permanent load **A** (usually *n*-dimensional random vector) and the stochastic process Q(t) of the variable load. Accordingly, Equation (1) can be rewritten as follows:

$$g(R(t), Q(t), \mathbf{A}) = R(t) - \mathbf{A} - Q(t)$$
⁽²⁾

In practical engineering problems, the common degradation forms of structural resistance stochastic process R(t) include exponential and logarithmic degradation forms, which can be mathematically expressed through Equations (3) and (4), respectively.

$$\varphi(R_0, t) = R_0 \exp(-\varepsilon t) \tag{3}$$

$$p(R_0, t) = R_0 [1 + \ln(1 - \varepsilon t)]$$
(4)

where R_0 and ε denote the initial resistance and the attenuation coefficient, respectively. Based on the definition of reliability, reliability $P_s(t)$ and failure probability $P_f(t)$ of the structure within the design service period *T* can be expressed in the form below:

$$P_s(t) = P\{g(R(t), Q(t), \mathbf{A}) > 0, t \in [0, T]\}$$
(5)

$$P_f(t) = P\{g(R(t), Q(t), \mathbf{A}) < 0, t \in [0, T]\}$$
(6)

3. Analysis Method of Structural Time-Variant Reliability Model with Random and Interval Variables

3.1. Time-Variant Reliability Model of Structures with Random and Interval Variables

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Studies show that structural models in the majority of practical engineering problems contain random and interval variables. Accordingly, the time-variant reliability model of structural mixed variables can be changed into the following expression:

$$g(R(t, \mathbf{Y}), S(t, \mathbf{Y})) = R(t, \mathbf{Y}) - S(t, \mathbf{Y})$$
(7)

$$\mathbf{Y} \in \left[\mathbf{Y}^{L}, \mathbf{Y}^{U}\right], \mathbf{Y}_{j} \in \left[\mathbf{Y}_{j}^{L}, \mathbf{Y}_{j}^{U}\right], j = 1, 2, \dots, n$$
(8)

where Y is an *n*-dimensional interval variable.

Under the generalized stress intensity model, the limit state function is expressed as

$$g(R(t, \mathbf{Y}), Q(t, \mathbf{Y}), \mathbf{A}) = R(t, \mathbf{Y}) - \mathbf{A} - Q(t, \mathbf{Y})$$
(9)

within the design service period *T*, instantaneous reliability and failure probability of the structure are as follows:

$$P_{s}(t) = P\{g(R(t, \mathbf{Y}), Q(t, \mathbf{Y}), \mathbf{A}) > 0, t \in [0, T]\}$$
(10)

$$P_f(t) = 1 - P_s(t) = 1 - P\{g(R(t, \mathbf{Y}), Q(t, \mathbf{Y}), \mathbf{A}) > 0, t \in [0, T]\}$$
(11)

3.2. Formulation of Stochastic Process Discretization with Mixed Variables

According to the definition of time-variant reliability [13,32,38], the design reference period *T* can be divided into *m* uniformly equal periods. Consequently, each period can be $\tau = T/m$. The resistance stochastic process *R*(*t*) and dynamic load *Q*(*t*) can be discretized to obtain *m* random variables. It should be noted that the median value of the resistance in the *i*-*th* period equals the *R_i* value. Figure 1 illustrates distributions of resistance *R*(*t*) and variable load *Q*(*t*) over time.



Figure 1. Distribution of resistance R(t) and variable load Q(t) over time.

The instantaneous value of the dynamic load Q_i can be obtained statistically in the period. Generally, Q_i can be regarded as an independent and identically distributed function. Based on the reliability theory of the series system, Equation (11) can be rewritten in the form below:

$$P_f(T) = 1 - P\left\{ \bigcap_{i=1}^{m} [g(R_i(t, \mathbf{Y}), Q_i(t, \mathbf{Y}), \mathbf{A}) \ge 0] \right\}$$
(12)

From the engineering point of view, Q' is Q_i 's s the maximum value. It should be indicated that Q_i is independently and evenly distributed. Based on the principle of mechanism statistics, the probability distribution function $F_{Q_T}(x)$ of the maximum load effect Q_T in the reference period T can be formulated as:

$$F_{Q_T}(x) = \left[F_{Q_T}(x)\right]^m \tag{13}$$

In practical problems, the loading effect Q_i is usually approximated to the extreme value type I distribution with parameters α_{τ} and u_{τ} , subjected to the extreme value type I distribution with parameters α_T and u_T . This can be mathematically expressed in the form below:

$$\begin{array}{c} \alpha_T = \alpha_\tau \\ u_T = u_\tau + \frac{\ln m}{\alpha_\tau} \end{array} \right\}$$
(14)

Then probability distribution function $F_{Q'}(q')$ and a new random variable Q' with a probability density function $f_{Q'}(q')$ and is defined. Accordingly, Equation (12) can be rewritten in the form below:

$$P_{f}(T) = 1 - P\left\{\bigcap_{i=1}^{m} Q_{i}(t, \mathbf{Y}) < R_{i}(t, \mathbf{Y}) - \mathbf{A}\right\}$$

$$= 1 - \int_{0}^{+\infty} \int_{0}^{+\infty} \dots \int_{0}^{+\infty} \prod_{i=1}^{m} F_{Q_{\tau}}(r_{i} - a) f_{R_{1}, R_{2}, \cdots, R_{m}}(r_{1}(\mathbf{Y}), r_{2}(\mathbf{Y}), \cdots, r_{m}(\mathbf{Y})) \qquad (15)$$

$$\times f_{\mathbf{A}}(\mathbf{a}) dr_{1} dr_{2} \cdots dr_{m} d\mathbf{a}$$

where $f_{R_1,R_2,...,R_m}(r_1, r_2, ..., r_m)$ denotes the joint probability density function of $R_1, R_2, ..., R_m$ variables, and $f_{\mathbf{A}}(\mathbf{a})$ is the probability density function of \mathbf{A} . Meanwhile, $F_{Q_{\tau}}$ denotes the probability distribution function of Q_i .

Equation (15) is a multi-dimensional integral problem, which requires a large amount of computation. In order to improve the computational efficiency, a new random variable Q' is introduced

$$P_{f}(T) = 1 - \int_{0}^{+\infty} \int_{0}^{+\infty} \dots \int_{0}^{+\infty} f_{Q'}(q') f_{R_{1},R_{2},\dots,R_{m}}(r_{1}(\mathbf{Y}), r_{2}(\mathbf{Y}), \dots, r_{m}(\mathbf{Y}))$$

$$\times f_{\mathbf{A}}(\mathbf{a}) dq' dr_{1} dr_{2} \cdots dr_{m} d\mathbf{a} \qquad (16)$$

$$= P\{F_{Q'}^{-1}[\prod_{i=1}^{m} F_{Q_{\tau}}(R_{i}(\mathbf{Y}) - \mathbf{A})] - Q' < 0\}$$

where Ω stands for the following integral field:

$$\Omega = \left\{ \left(Q, R, \mathbf{A}, \mathbf{Y} \right) | Q' < F_{Q'}^{-1} \prod_{i=1}^{m} F_{Q_{\tau}}(R_i(\mathbf{Y}) - \mathbf{A}) \right\}$$
(17)

 $F_{Q'}^{-1}$ is the inverse function of $F_{Q'}$. Accordingly, the limit state function can be expressed as:

$$g(R_1, R_2, \dots, R_m, Q', \mathbf{A}, \mathbf{Y}) = F_{Q'}^{-1} [\prod_{i=1}^m F_{Q_\tau}(R_i(\mathbf{Y}) - \mathbf{A})] - Q'$$
(18)

when the random variable Q' is introduced, the general distribution form of Q' is not specified. In other words, when Equation (18) is used as the limit state function to calculate the reliability index, the expression only contains the $F_{Q'}(Q')$ term. In this case, $F_{Q'}(Q')$ can be used as a variable so that the Q' distribution is not required anymore in the whole analysis. However, in order to coordinate with the current unified design standard of structural reliability and make Q' of engineering significance, Q' is the maximum Q_T of Q_i . Accordingly, Equation (18) can be rewritten as the following:

$$g(\mathbf{X}, \mathbf{Y}) = F_{Q^T}^{-1} [\prod_{i=1}^m F_{Q^T} (R_i(\mathbf{Y}) - \mathbf{A})] - Q_T$$
(19)

where $\mathbf{X} = [R_1, R_2, \dots, R_m, Q', \mathbf{A}]$ is a random variable.

3.3. Double-Layer Nesting Optimization Method

There are both random and interval variables in Equation (19). At present, some methods have been proposed to solve this kind of problem. It is necessary to standardize the random variables.

In the analysis and calculation, the random variables need to be standardized. Equation (19) can be converted into:

$$\phi(\mathbf{U}) = F_{\mathbf{X}}(\mathbf{X}, \mathbf{Y}), \mathbf{X} = F_{\mathbf{X}}^{-1}(\phi(\mathbf{U}), \mathbf{Y})$$
(20)

where ϕ is the standard normal distribution function and **U** is the standard normal space variable.

By substituting Equation (20) into Equation (19), the limit state function in standard normal space can be obtained:

$$G(\mathbf{U}, \mathbf{Y}) = g' \left(F_{\mathbf{X}}^{-1}(\phi(\mathbf{U}), \mathbf{Y}, t_i) \right)$$
(21)

As the coexistence of random and interval variables, the limit state function (21) turns into a band in the parameter space as shown in Figure 2. The lower and upper boundaries are as follows:

$$S_L : \min_{\mathbf{Y}} g(\mathbf{U}, \mathbf{Y}, t_i)$$

$$S_U : \max_{\mathbf{Y}} g(\mathbf{U}, \mathbf{Y}, t_i)$$
(22)

where S_L and S_U represent the lower and upper boundaries, respectively.



Figure 2. Limit state region.

The limit state region in Figure 2 shows failure probability P_f and reliability index β obtained from different **Y**-values of the limit state function on S_L and S_U planes. It should be indicated that both of these parameters are range values, in the form below:

$$\beta \in \left[\beta^L, \beta^U\right] \tag{23}$$

$$P_f \in \left[P_f^L, P_f^U\right] = \left[\phi\left(-\beta^U\right), \phi\left(-\beta^L\right)\right]$$
(24)

where β^{U} , β^{L} , P_{f}^{U} and P_{f}^{L} represent the upper and lower interfaces of the reliability index and the upper and lower boundaries of failure probability, respectively.

Generally, the following two optimization problems are set to calculate the values of β^L and $-\beta^U$:

$$\beta^{L} = \min_{\mathbf{U}} \|\mathbf{U}\|$$
s.t. $\min_{\mathbf{Y}} G(\mathbf{U}, \mathbf{Y}, t_{i}) = 0$
(25)

$$\beta^{U} = \min_{\mathbf{U}} \|\mathbf{U}\|$$

s.t. max $G(\mathbf{U}, \mathbf{Y}, t_{i}) = 0$ (26)

It should be reminded that time-variant failure probability falls in an interval. In most practical engineering problems, only the maximum failure probability is concerned. Consequently, only Equation (25) needs to be solved and the governing equations are reduced to the following expression:

Outer Optimization:

$$\beta^{L} = \min_{\mathbf{U}} \|\mathbf{U}\|$$

s.t. $\min_{\mathbf{Y}} G(\mathbf{U}, \mathbf{Y}^{*}, t_{i}) = 0$ (27)

Inner layer optimization:

$$G(\mathbf{U}, \mathbf{Y}^*, t_i) = \min_{\mathbf{Y}} G(\mathbf{U}, \mathbf{Y}, t_i)$$

s.t. $Y_{i,j}^L \le Y_{i,j} \le Y_{i,j}^U \ j = 1, 2, \dots, n$ (28)

Equation (27) is a two-level nested optimization problem with complicated calculations and high computational expenses. Therefore, the optimization efficiency is difficult to meet the needs of engineering. Although decoupling strategies have been successfully applied to static hybrid reliability analysis problems, such as sequential single-loop strategies. It is a difficult problem to solve the time-variant reliability problem, because of time discretization, and inner and outer layer iteration. Also, a rigorous mathematical proof for decoupling strategy is lacking. Therefore, it is challenging to develop time-variant hybrid reliability decoupling strategies with good robustness and convergence.

4. A New Analysis Method of Structural Time-Variant Reliability with Random and Interval Variables

4.1. Establishment of Equivalent Model

In time-variant reliability analysis, uncertainty is divided into two kinds of uncertain variables, namely the random variable X and interval variable Y. In the present study, a novel method is initially presented. The random variable X is kept stable, while the interval variable Y obeys the uniform distribution. Therefore, it can be transformed into the following conventional time-variant reliability problem with only random variables:

$$\begin{cases} \beta^{L} = \min_{\mathbf{U}, \mathbf{V}} \sqrt{\|\mathbf{U}\|^{2} + \|\mathbf{V}\|^{2}} \\ \text{s.t.} \quad G'(\mathbf{U}, \mathbf{V}, t_{i}) = 0 \end{cases}$$
(29)

where **U** and **V** denote the standard normal space, vectors transformed by **X** and **Y**, respectively. Moreover, *G* is the limit state function of the transformation in the **U**-**V** space. For the equivalent model after transformation, the most probable point (MPP) in the standard normal space random variable (**U** + **V**) and its corresponding optimal value (\mathbf{X}', \mathbf{Y}') can be calculated according to Equation (29).

4.2. Model Equivalence Proof

According to the studies of Breitung [39] on the asymptotic approximation of polynomial integrals, Madsen et al. [40] on first-order and second-order reliability analysis of series structures, and Madsen [41] on structural safety methods, it is seen that MPP has the highest probability density at all points on the limit state function. Therefore, $(\mathbf{X}', \mathbf{Y}')$ is also the optimal solution to the following problem:

$$\begin{cases} \max_{\mathbf{X},\mathbf{Y}} f_{\mathbf{X},\mathbf{Y}}(\mathbf{X},\mathbf{Y}) \\ \text{s.t. } g(\mathbf{X},\mathbf{Y},t_i) = 0 \end{cases}$$
(30)

where $f_{X,Y}$ denotes the probability density function (PDF) of random variables X and Y.

Since all variables in the limit state function are independent of each other, Equation (30) can be modified as the following equation, based on the processing approach of the series system:

$$\begin{cases} \max_{\mathbf{X},\mathbf{Y}} f_{\mathbf{X}}(\mathbf{X}) f_{\mathbf{Y}}(\mathbf{Y}) \\ \text{s.t. } g(\mathbf{X},\mathbf{Y},t_i) = 0 \end{cases}$$
(31)

Since the random variable Y is uniformly distributed, the probability density function $f_Y(Y)$ of the above-mentioned equation is a normal number. Therefore, Equation (31) can be rewritten into the following equation:

$$\begin{cases} \max_{\mathbf{X},\mathbf{Y}} f_{\mathbf{X}}(\mathbf{X}) \\ \text{s.t. } g(\mathbf{X},\mathbf{Y},t_{i}) = 0 \\ Y_{j}^{L} \leq Y_{j} \leq Y_{j}^{R}, j = 1,2,\dots,n \end{cases}$$
(32)

For the constrained optimization problem of Equation (32), Nocedal and Wright [42] proposed a numerical optimization approach according to the Karush-Kuhn-Tucker necessary conditions. Therefore, the optimal $(\mathbf{X}', \mathbf{Y}')$ must meet the following equation:

$$\begin{cases}
-\nabla_{X_k} f_X(\mathbf{X}) + \lambda_1 \nabla_{X_k} g(\mathbf{X}, \mathbf{Y}, t_i) = 0, k = 1, 2, ..., n \\
\lambda_1 \nabla_{Y_i} g(\mathbf{X}, \mathbf{Y}, t_i) + \lambda_{3j} - \lambda_{2j} = 0, j = 1, 2, ..., n \\
\lambda_{2j} \left(Y_j^L - Y_j \right) = 0, j = 1, 2, ..., n \\
\lambda_{3j} \left(Y_j - Y_j^R \right) = 0, j = 1, 2, ..., n \\
g(\mathbf{X}, \mathbf{Y}, t_i) = 0 \\
Y_j^L \le Y_j \le Y_j^R, j = 1, 2, ..., n
\end{cases}$$
(33)

where, λ_1 , λ_{2i} , λ_{3i} represent Lagrange multipliers.

Secondly, the reliability of the original mixed time-invariant model is investigated. As described above, in order to compute the maximum failure probability through the first-order second-moment approach, the MPP U' and the corresponding optimal value Xshould be obtained. Moreover, the following optimization issues should be resolved:

$$\begin{cases} \beta = \min_{\mathbf{U}} \|\mathbf{U}\| \\ \text{s.t.} \min_{\mathbf{Y} \in \mathbf{Y}^l} (\mathbf{U}, \mathbf{Y}, t_i) = 0 \end{cases}$$
(34)

Therefore, according to the properties of MPP, \mathbf{X}' is the optimal solution of the following problem:

$$\max_{\mathbf{X},\mathbf{Y}} f_{\mathbf{X}}(\mathbf{X})$$
s.t. ming(X, Y, t_i) = 0
(35)

where **Y** is the interval variable. In the above mentioned equation, there is a sub-optimization problem ming($\mathbf{X}, \mathbf{Y}, t_i$), represented by an interval variable \mathbf{Y} in the constraint, and the Y∈Yⁱ

Karush-Kuhn-Tucker necessary condition is mathematically expressed as follows:

$$\begin{cases} \nabla_{\mathbf{Y}_{i}g}(\mathbf{X}, \mathbf{Y}, t_{i}) + \lambda_{3i} - \lambda_{2i} = 0, i = 1, 2, \dots, n \\ \lambda_{2i}(\mathbf{Y}_{i}^{L} - \mathbf{Y}_{i}) = 0, i = 1, 2, \dots, n \\ \lambda_{3i}(\mathbf{Y}_{i} - \mathbf{Y}_{i}^{R}) = 0, i = 1, 2, \dots, n \\ \mathbf{Y}_{j}^{L} \leq \mathbf{Y}_{j} \leq \mathbf{Y}_{j}^{R}, j = 1, 2, \dots, n \end{cases}$$

$$(36)$$

By substituting Equation (36) into Equation (35), the following optimization problem can be obtained:

$$\max_{\mathbf{X}} f_{\mathbf{X}}(\mathbf{X})$$

s.t. $g(\mathbf{X}, \mathbf{Y}, t_i) = 0$
 $\nabla_{Y_i}g(\mathbf{X}, \mathbf{Y}, t_i) + \lambda_{3i} - \lambda_{2i} = 0, i = 1, 2, ..., n$
 $\lambda_{2i} \left(\mathbf{Y}_i^L - \mathbf{Y}_i \right) = 0, i = 1, 2, ..., n$
 $\lambda_{3i} \left(\mathbf{Y}_i - \mathbf{Y}_i^R \right) = 0, i = 1, 2, ..., n$
 $Y_j^L \le Y_j \le Y_j^R, j = 1, 2, ..., n$
(37)

Then, the Karush–Kuhn–Tucker necessary condition of Equation (37) is obtained, which is expressed the same as Equation (33). Therefore, it is mathematically proved that the original problem and the equivalent problem have the same solution when calculating reliability in this form. It should be indicated that the interval variable is treated with the uniform distribution, which is regarded as a random variable. in this situation, the limit state function of the time-variant reliability model with mixed uncertainties only contains random variable. Complex nested optimization problems for time variance can be successfully avoided.

4.3. Procedure

Based on the equivalent analysis above, the original structural time-variant reliability model of mixed variables is transformed into the equivalence model of the structural time-variant reliability model of mixed variables. As shown in the Figure 3, the specific steps are as follows:

- (1) The uncertainty in the structure is simply divided into two types according to the information given, including the probability random vector **X** and the non-probability vector **Y** (interval vector).
- (2) The interval variable **Y** is treated as a uniform distribution, which is regarded as a random variable.
- (3) The limit state function of the time-variant reliability model of the mixed uncertainties only contains the random variables X and Y. Then, the equivalent substitution model of the mixed reliability analysis is established.
- (4) The time parameters are discretized into m periods t_i by the static transformation method. Meanwhile, the generalized resistance random process R(t) and the variable load Q(t) can be discretized. Therefore, the time-variant problem is changed into a time-invariant problem.
- (5) FORM is adopted to compute and solve the transformed equivalent mode in Equation (19). Then, an optimum $(\mathbf{X}', \mathbf{Y}')$ can be obtained in each period.
- (6) The minimum reliability and the maximum failure probability of the model in each period is obtained.



Figure 3. Algorithm flow chart.

5. Numerical Examples

In this section, the presented approach is applied to four examples, which include a mechanical part, a reinforced concrete short column of a structure, a roof truss structure, and a wing structure. In addition, the Monte Carlo Simulation approach (MCS) is employed to verify the feasibility of the presented method. The number of random numbers of each parameter generated by MCS in this study is $ns = 1 \times 10^6$.

5.1. Time-Variant Reliability Analysis of the Structure of a Mechanical Part

The change rule of the resistance of a mechanical part [10] with time is $R(t) = R_0 \exp(-at)$ MPa. Where a is the attenuation coefficient and interval variable, and R_0 is the initial resistance, obeying the lognormal distribution of $(\mu_{R_0}, \sigma_{R_0})$ MPa. μ_{R_0} and σ_{R_0} are 336.3 MPa and 33.63 MPa, respectively. It is worth noting that the permanent load effect A follows the normal distribution of (μ_A, σ_A) MPa. μ_A and σ_A are 66.25 MPa and 4.738 MPa. Taking $\tau = 1000$ h as a period, the statistical value Q_i of the dynamic load Q(t) in the *i*th period follows the extreme value I distribution. Parameters α_{τ} and u_{τ} equal $\alpha_{\tau} = 0.04493$ MPa⁻¹ and $u_{\tau} = 49.77$ MPa, respectively. Attenuation coefficient a^{-1} is interval variables. $a^{-1} \in [1.9, 2.1] \times 10^{-5}$.

As can be seen from the Table 1, the reliability index when the sample number is 20,000, 50,000 and 100,000 is gradually close to the result when the sample number is 3,000,000. When the sample is 500,000, 1,000,000, 2,000,000 and 3,000,000, the calculation results in convergence. The sample number is 100,000, which can meet the accuracy requirement. However, in order to make the results more accurate and stable, this paper selected 1,000,000 times. With the increase of samples, the calculation time increases gradually. The calculation time is proportional to the number of samples. For example, when the number of samples is 3,000,000 and 1,000,000, the calculation time is 870.69 s and

	Design Reference Period (h)											
ns	ns <u>1000 2000 3000</u>	4000	5000 6000		7000	7000 8000		10,000	(s)			
20,000	3.481	3.390	3.156	2.989	2.903	2.776	2.669	2.628	2.530	2.457	5.26	
50,000	3.673	3.470	3.279	3.179	3.062	2.921	2.802	2.708	2.624	2.556	15.78	
100,000	3.719	3.414	3.291	3.102	2.970	2.870	2.794	2.694	2.604	2.519	30.37	
500,000	3.580	3.350	3.211	3.104	2.999	2.901	2.793	2.697	2.603	2.515	149.68	
1,000,000	3.622	3.393	3.219	3.094	2.979	2.876	2.781	2.690	2.600	2.510	300.82	
2,000,000	3.612	3.388	3.223	3.099	2.984	2.877	2.782	2.688	2.597	2.508	561.31	
3,000,000	3.599	3.370	3.215	3.094	2.981	2.880	2.786	2.692	2.597	2.509	870.69	

300.82 s, respectively. The number of samples is three times larger, and the calculation time is nearly three times longer.

Table 1. Calculation results of Monte Carlo method under different sample num	bers.
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Figure 4 and Table 2 show the reliability index curve of a mechanical part in the design reference period. When the design reference period is 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000 and 10,000 respectively, the reliability indices are 3.600, 3.379, 3.227, 3.102, 2.991, 2.889, 2.793, 2.699, 2.608 and 2.518 respectively. Therefore, it is observed that as the design base period extends, the reliability of this part decreases gradually. Correspondingly, the failure probability will gradually increase, which is consistent with the actual situation.

Table 2. Reliability index of the mechanical part in each design reference period.

	Design Reference Period (h)										
Methods	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	
MCS	3.622	3.393	3.219	3.094	2.979	2.876	2.781	2.690	2.600	2.510	
Proposed method	3.600	3.379	3.227	3.102	2.991	2.889	2.793	2.699	2.608	2.518	
Deviation/%	0.62	0.41	0.25	0.27	0.40	0.45	0.43	0.34	0.31	0.32	



Figure 4. The reliability index curve of a mechanical part in the design reference period.

MCS is a more accurate method to solve structural reliability problems, which is generally adopted as an accurate result to verify the accuracy of other methods. In Table 2, when t = 1000, the maximum error is 0.62%, and when t = 4000, the minimum error is 0.27%, which is within the engineering allowable range. The results of the proposed method are very close to those of MCS method, which indicates that the proposed method is effective and feasible. In this study, the limit state function only needs to be called 72 times, while it needs to be called 9,984,164 times through MCS method. The running time of the MCS method is 300.82 s, while the running time of the current method is 0.089s. The total number of calls to limit state functions in this method is many orders of magnitude less than that in the MCS method.

5.2. A Short Column of the Reinforced Concrete of a Structure

The example discussed in this section is modified according to the literature [43]. The width and height of the cross-section of the reinforced concrete short column of a structure are denoted as B and h, respectively. Moreover, the parameter values of the width and height of the cross-section are interval values, whose parameter values are [270 mm, 330 mm] and [315 mm, 380 mm], respectively.

The strength grade of the concrete is C30 and the area of the steel bar in the column is 1811.28mm², which is a grade II steel bar. Moreover, R(t), G and Q denote the random resistance process of the reinforced concrete short column, the permanent load effect and the variable load effect, respectively. Table 3 presents the specific parameters and distribution.

Table 3. Variable distribution of reinforced concrete short columns of a structure.

Parameter	Mean μ	Standard Deviation σ	Type of Distribution
G(kN)	530	37.1	normal
$Q_T(kN)$	700	203	Extreme I type
$f_c(N/mm^2)$	26.1	4.437	normal
$f_y(N/mm^2)$	384	28.591	normal

R(t) is mathematically defined as:

$$R(t) = bhf_c(t) + A_s f_y(t) = bhf_c \varphi_c(t) + A_s f_y \varphi_y(t)$$
(38)

where f_c and f_y denote the compressive strength of the concrete and the yield strength of reinforcement, respectively.

The resistance variation coefficients $\varphi_c(t)$ and $\varphi_y(t)$ are:

$$\begin{cases} \varphi_c(t) = 1 - 8.0 \times 10^{-7} t^3 \\ \varphi_y(t) = 1 - 2.2 \times 10^{-6} t^3 \end{cases}$$
(39)

The limit state function is mathematically expressed as:

$$g = R(t) - G - Q \tag{40}$$

From Table 4, it is proved once again that the results of MCS method are convergent. With the increase of the number of samples, the calculation time increases linearly. Figure 5 illustrates that as the design reference period exceeds, the reliability of the structure or product gradually decreases. As can be seen from Table 5, the errors of the current method and MCS method are as follows: 1.02%, 1.33%, 1.36%, 1.67%, 1.99%, 2.02%, 2.06%, 1.80%, 2.15% and 1.89%. The maximum error is 2.02 and the minimum error is 1.02. The error can meet the needs of engineering, moreover the accuracy of the present method is proved again. From viewpoint of computational efficiency, the total number of calls to limit state functions in this method is only 171, while it is 9,997,832 times for the MCS method. The MCS method runs in 673.8 s, and the current method runs in 0.12 s. Therefore, it is proved that the computational efficiency of the presented approach is much higher than that

of the MCS method. The results of an example show that the proposed method is an effective approach.



Figure 5. Reliability index curves of reinforced concrete short column of a structure in design reference period.

	Design Reference Period (Year)										
115	5	10	15	20	25	30	35	40	45	50	(s)
30,000	3.72	3.59	3.54	3.40	3.38	3.35	3.31	3.29	3.23	3.18	8.5
100,000	4.10	3.81	3.69	3.61	3.54	3.49	3.43	3.37	3.32	3.22	32.4
500,000	3.94	3.78	3.68	3.58	3.50	3.46	3.40	3.34	3.27	3.18	157.4
1,000,000	3.94	3.77	3.67	3.59	3.52	3.46	3.40	3.34	3.26	3.17	673.8
2,000,000	3.94	3.78	3.68	3.60	3.53	3.47	3.41	3.35	3.27	3.17	734.8
3,000,000	3.94	3.77	3.66	3.58	3.52	3.46	3.39	3.34	3.26	3.16	1153.8

 Table 4. Calculation results of Monte Carlo method under different sample numbers.

Table 5. Reliability index of each design reference period of the short column.

	Design Reference Period (Year)									
Methods	5	10	15	20	25	30	35	40	45	50
MCS	3.94	3.77	3.67	3.59	3.52	3.46	3.40	3.34	3.26	3.17
Proposed method	3.98	3.82	3.72	3.65	3.59	3.53	3.47	3.40	3.33	3.23
Deviation/%	1.02	1.33	1.36	1.67	1.99	2.02	2.06	1.80	2.15	1.89

5.3. A Roof Truss Structure

Figure 6 shows that the bottom and tension members are made of steel, and the top and pressure members are reinforced with concrete [44,45].



Figure 6. Roof truss model.

The roof is subjected to evenly distributed load q(t), where q(t) can be converted to the equivalent nodal force P = q(t)l/4. Moreover, the vertical displacement at node C can be expressed as follows:

$$\Delta C = \frac{q(t)l^2}{2} \left(\frac{3.81}{A_C E_C} + \frac{1.13}{A_S E_S} \right)$$
(41)

where A_C and A_S denote the cross-sectional areas of the cement and steel rods, respectively. It should be indicated that A_C and A_S are interval variables with the ranges of [0.0323 m², 0.0357 m²] and [8.93 × 10⁻⁴ m², 9.87 × 10⁻⁴ m²], respectively. Moreover, E_C and E_S denote their elastic modulus, respectively. During the loading process, the vertical displacement of node C should be less than D(t) that decays with time. The decay rule is: $D(t)=D_0[1 + \ln(1 - 0.0002t)]$, and D_0 is the initial displacement. Table 6 lists the specific situation of the random variable in this problem. The design base period is assumed to be T = 10 years. The limit state function is defined as:

$$g(t) = D(t) - \Delta C \tag{42}$$

 Table 6. Distribution of random variables of the roof truss structure.

Parameter	Mean μ	Standard Deviation σ	Type of Distribution
q(N/m)	20,000	1600	Extreme I type
$D_0(\mathbf{m})$	0.022	0.001	normal
<i>l</i> (m)	12	0.24	normal
$E_S(N/m^2)$	2×10^{11}	1.4×10^{10}	normal
$E_C(N/m^2)$	3×10^{10}	2.4×10^{9}	normal

It can be seen again from Table 7 that the calculation results based on MCS method are convergent, and the calculation time increases linearly with the increase of the number of samples. As shown in Figure 7 and Table 8, the reliability indices are 3.35, 3.15, 3.04, 2.95, 2.88, 2.83, 2.78, 2.74, 2.70 and 2.67, which decreases significantly with time. Table 7 presents that the maximum error between the results obtained by this method. It can be seen that the errors are 1.52%, 1.29%, 1.67%, 1.72%, 1.77%, 1.80%, 1.83%, 2.24%, 2.27% and 2.30%, respectively. Compared with MCS method, the maximum error is 2.30%. Once again, it should be indicated that the results obtained by this method are very accurate. From the

viewpoint of computational efficiency, this method only calls the limit state function 511 times, while MCS method calls the limit state function 9,999,463 times. The running time of MCS method is 384.4 seconds, while the running time of the current method is 0.115 s. This shows the effectiveness of the presented method.



Figure 7. Reliability index curve of roof structure in design reference period.

Table 7. Calculation results of Monte Carlo method under different sample nu	imbers.
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	Design Reference Period (Year)										
ns	1	2	3	4	5	6	7	8	9	10	(s)
30,000	3.41	3.20	3.10	2.94	2.88	2.82	2.76	2.72	2.68	2.66	9.9
100,000	3.34	3.15	3.04	2.94	2.85	2.78	2.73	2.69	2.65	2.62	38.4
500,000	3.33	3.13	3.02	2.92	2.83	2.78	2.73	2.68	2.65	2.62	205.5
1,000,000	3.30	3.11	2.99	2.90	2.83	2.78	2.73	2.68	2.64	2.61	384.4
2,000,000	3.31	3.11	2.99	2.91	2.83	2.78	2.73	2.68	2.64	2.61	763.8
3,000,000	3.31	3.11	2.99	2.90	2.83	2.77	2.72	2.68	2.64	2.61	1153.8

Table 8. Reliability index of the roof truss structure in each design reference period.

	Design Reference Period (Year)									
Methods	1	2	3	4	5	6	7	8	9	10
MCS	3.30	3.11	2.99	2.90	2.83	2.78	2.73	2.68	2.64	2.61
Proposed method Deviation/%	3.35 1.52	3.15 1.29	3.04 1.67	2.95 1.72	2.88 1.77	2.83 1.80	2.78 1.83	2.74 2.24	2.70 2.27	2.67 2.30

5.4. A Wing Structure

The reliability problem of the wing structure [45] is adopted to verify the application of the presented approach in actual engineering. Figure 8 shows that the wing structure consists of three ribs and a skin. The load F is applied to the left side of the wing, and the other side is constrained in three directions.



Figure 8. A wing structure.

The young's modulus of the rib as an interval variable, the Poisson's ratio and the density, are E_1 , 0.35 and 4.0 × 10³ kg/m³, respectively. Moreover, the young's modulus of the skin as an interval variable, the Poisson's ratio and the density are E_2 , 0.35 and 1.8 × 10³ kg/m³, respectively. It is worth noting that the rib thickness and skin thickness are recorded as *th* and *s*, respectively.

The variables {*th*, *s*, *E*₁, *E*₂}^{*T*} are independent from each other: *th*~*N*(3.0, 0.02²), *s*~*N*(3.0, 0.02²) *E*₁∈[42,750 MPa, 4,7250 MPa] *E*₂∈[28,500 MPa, 31,500 MPa]. The wing structure requires that during the design baseline period *T* = 10 years, the vertical deformation *D*(*t*) of the wing surface during the loading process should not exceed $D(t) = D_0[1 + \ln(1 - 0.0002t)]$, and D_0 is the initial displacement. D_0 ~*N*(85, 7.5²). When the design life is 5 years, the time-variant load is the *F*, and the maximum variable load *F*_T is subjected to an extreme value I distribution, $\mu_{F_T} = 3000N$, $\sigma_{F_T} = 350N$. Therefore, the limit state function of the wing structure can be defined as:

$$g(t) = D(t) - D_{\max}(E_1, E_2, F(t), s, th)$$
(43)

where D_{max} () denotes the maximum displacement of the structure, which can be achieved through the finite element method (FEM).

Figure 9 shows that the wing structure adopts the finite element model composed of hexahedron elements and four-node shell elements. The model has 2899 elements and 2869 nodes, respectively. Opstruct software is used to calculate this finite element model. Then, the second-order response surface model is constructed through the Latin square experimental design, which is mathematically expressed as follows:

$$D_{\max}(E_1, E_2, F(t), s, th) = 45.8996263360021 + 4.26848205843078 \times 10^{-5} \times E_1$$

- $-0.00148621547290688 \times E_2 + 0.038913641543905 \times F(t)$
- $-14.8883552537885 \times s 0.91139115939223 \times th$
- $-3.02398909744693 \times 10^{-10} \times E_1^2 + 1.59448056099034 \times 10^{-8} \times E_2^2$

$$+4.91682235848162 \times 10^{-7} \times F(t)^{2} + 1.66001878817579 \times s^{2}$$

 $+0.0239216893925098 \times th^{2} + 1.30009641471656 \times 10^{-10} \times E_{1} \times E_{2}$ (44)

$$-1.57829880943195 \times 10^{-8} \times E_1 \times F(t) - 4.75441415091528 \times 10^{-7} \times E_1 \times s$$

 $-2.09147436555016 \times 10^{-6} \times E_1 \times th - 4.38849189706437 \times 10^{-7} \times E_2 \times F(t)$

 $+0.00016322598362916 \times E_2 \times {\it s} + 1.10267118769858 \times 10^{-5} \times E_2 \times {\it th}$

- $+0.00450034073967354 \times F(t) \times s + 0.000245652575761249 \times F(t) \times th$
- $+0.0447098069737952\times s\times th$

Five sampling points are randomly chosen in the design space to verify the accuracy of the approximate model. As shown in Table 9, the relative errors between the response surface results based on these points and the simulation model results are 0.0077%, 0.0026%,

0.0162%, 0.0214% and 0.0035%. The maximum error is much less than 0.1%, and the response surface approximation model has high accuracy, which is completely acceptable in engineering.



Figure 9. Finite element model of the wing structure.

$\mathbf{T}_{\mathbf{r}} \in \mathbf{D}_{\mathbf{r}}$	Relative Error of Finite Element Model
lest Point $(E_1, E_2, F(t), s, th)$	D _{max}
(42,750, 29,500, 1130.833, 2.9832, 2.9832)	0.0077%
(43,250, 28,833.3333, 1156.389, 3.1167, 2.9499)	0.0026%
(43,750, 31,500, 1169.167, 3.15, 3.15)	0.0162%
(44,250, 30,833.3333, 1207.5, 2.8833, 3.0834)	0.0214%
(45,250, 30,166.6667, 1092.5, 3.0168, 3.1167)	0.0035%

Figure 10 and Table 10 present reliability indices under the design reference period for a certain wing surface structure. The reliability indexes calculated by the current method are 3.63, 3.45, 3.34, 3.26, 3.20, 3.14, 3.10, 3.06, 3.02 and 2.99. The calculation results show that due to the attenuation of dynamic load and maximum displacement, the reliability index is no longer a fixed value, but gradually decreases from 3.63 to 2.99 with the design lifetimes increase. Comparing the obtained results with calculations through the MCS demonstrates that the maximum error between the two methods is 0.91%, indicating the high accuracy of the presented approach. In terms of computational efficiency, the total number of calls to the response surface approximation model of limit state function is only 83, while MCS needs to call 9,991,818.



Figure 10. Reliability index curve of machine wing surface structure in design reference period.

Methods	Design Reference Period (Year)									
	1	2	3	4	5	6	7	8	9	10
Proposed method	3.63	3.45	3.34	3.26	3.20	3.14	3.10	3.06	3.02	2.99
MCS	3.63	3.44	3.32	3.24	3.17	3.12	3.07	3.03	3.00	2.97
Deviation/%	0.02	0.29	0.45	0.56	0.71	0.74	0.79	0.88	0.91	0.84

 Table 10. Reliability index of the wing surface structure for different design periods.

6. Conclusions

In the present article, a time-variant reliability analysis method is proposed for mixedvariable structures. The time-variant reliability problem of structures with mixed variables is transformed into a time-variant reliability problem of structures with random variables. Then the stochastic process is discretized and the static limit state functions of different periods are obtained. After the original problem is changed into an invariant reliability problem, the first-order second-moment method is applied to solve the problem. Finally, the reliability index under the design base period is obtained.

In order to evaluate the performance of the presented approach, it is applied to three numerical and one engineering case studies. Obtained results show that in the studied cases, the performance of the presented approach is very close to that of the Monte Carlo method. Moreover, it is found that the proposed algorithm avoids multi-layer nesting optimization and greatly improves computational efficiency.

The present method can be extended to time-variant reliability analysis problems with convex set variables and probability variables, and to multidisciplinary reliability analysis problems. When the range of interval parameters are big, the computational expenses of the proposed method cannot be justified. In order to resolve this shortcoming, improving the efficiency of time-variant hybrid reliability analyses will be discussed in the near future.

Author Contributions: Conceptualization, F.L.; methodology, F.L.; validation, J.R. and H.Z.; formal analysis, Y.Y.; investigation, Y.Y; data curation, H.Z.; writing—review and editing, J.R. and H.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was partially supported by the Science and Technology Program of Guangzhou (202102010428), National Natural Science Foundation of China (11772070,11672104,11902085), Hunan Provincial Natural Science Foundation of China (2019JJ40296) and the Open Fund of Key Laboratory of Bridge Engineering Safety Control by Department of Education (19KB06).

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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