

Supplement A

## Detailed methodology for deriving a marginal Beta distribution from expert opinion of risk impact on cost and schedule.

**Step 1:** Define the root causes of a risk factor ( $N_i$ ) and the possible scenarios for each root cause.

**Step 2:** For each root cause or scenario ( $M_i$ ), subjectively evaluate the frequency of occurrence and adverse consequence using the corresponding fuzzy membership function.

**Step 3:** Using Equation 3, form a fuzzy relationship between the adverse consequence and frequency of occurrence for each root cause/scenario.

$$\mu_R(x_i, y_i) = \min[\mu_F(x_i), \mu_C(y_i)] \quad (3)$$

where  $x_i$  = an element of universe X

$y_i$  = an element of universe Y

$\mu_R(x_i, y_i)$  = the membership value of element  $(x_i, y_i)$  in the fuzzy relation R

$\min$  = the minimum values of both elements  $x_i$  and  $y_i$

$\mu_F(x_i)$  = the membership value of element  $x_i$  in fuzzy set F

$\mu_C(y_i)$  = the membership value of element  $y_i$  in fuzzy set C.

**Step 4:** Using Equations 4 and 5, develop a fuzzy union matrix between all relationships.

$$\mu_{SUZ}(x_i, y_i) = \max[\mu_s(x_i, y_i), \mu_z(x_i, y_i)] \quad (4)$$

where  $\max$  = the maximum value of both relations  $s$  and  $z$ .

Union U, between the fuzzy relation matrices R (F, C), is then computed as:

$$U = \max [(F_1 \times C_1) \cup (F_2 \times C_2) \cup (F_3 \times C_3) \dots \dots \dots \cup (F_k \times C_k)] \quad (5)$$

where  $\max$  = the maximum value of the two relations.

**Step 5:** Define the minimum and maximum (A, B) values for the risk impact.

**Step 6:** Divide the impact range (between minimum and maximum) into equal subsets.

**Step 7:** Evaluate the mapping value of each range (example provided as follows):

Impact range	Small impact	Medium impact	Large impact
Impact value	10 12.5 15.0 17.5 20.0	20.0 22.5 25.0 27.5 30.0	30.0 32.5 35.0 37.5 40
Mapping degree	1.0 0.9 0.8 0.7 0.6	0.7 0.85 1.0 0.85 0.7	0.5 0.7 0.8 0.9 1.0

**Step 8:** Using Equation 3, form a fuzzy relationship between the adverse consequence of a root cause/scenario and the range of a risk impact.

$$\mu_R(x_i, y_i) = \min[\mu_F(x_i), \mu_C(y_i)] \quad (3)$$

where  $x_i$  = an element of universe X

$y_i$  = an element of universe Y;

$\mu_R(x_i, y_i)$  = the membership value of element  $(x_i, y_i)$  in the fuzzy relation R

$\min$  = the minimum values of both elements  $x_i$  and  $y_i$

$\mu_F(x_i)$  = the membership value of element  $x_i$  in fuzzy set F

$\mu_C(y_i)$  = the membership value of element  $y_i$  in fuzzy set C.

**Step 9:** Using Equations 4 and 5, develop a fuzzy union matrix between all relationships in Step 8.

$$\mu_{S \cup Z}(x_i, y_i) = \max[\mu_s(x_i, y_i), \mu_z(x_i, y_i)] \quad (4)$$

where  $\max$  = the maximum value of both relations  $s$  and  $z$ .

Union U, between the fuzzy relation matrices R (F, C), is then computed as:

$$U = \max [(F_1 \times C_1) \cup (F_2 \times C_2) \cup (F_3 \times C_3) \dots \dots \dots \cup (F_k \times C_k)] \quad (5)$$

where  $\max$  = the maximum value of the two relations.

**Step 10:** Using Equation 6, develop the fuzzy composition matrix between the fuzzy union matrices developed in Step 4 and Step 9.

$$U \circ V(x_i, z_k) = \max_{y_j} \{ \min[\mu_U(x_i, y_j), \mu_V(y_j, z_k)] \} \quad (6)$$

where  $U \circ V(x_i, z_k)$  = membership value of element  $(x_i, z_k)$  in composition matrix between U and V

$\mu_U(x_i, y_j)$  = membership value of element  $(x_i, y_j)$  in union matrix U

$\mu_V(y_j, z_k)$  = membership value of element  $(y_j, z_k)$  in union matrix V.

**Step 11:** Using Equations 7 through 9, select one row from the fuzzy composition matrix to calculate the mean and variance of the distribution.

$$P(R_I = z_k) = \frac{\mu_o(z_k)}{\sum_1^m \mu_o(z_k)} \quad (7)$$

$$\mu_I = \sum_{k=1}^m (z_k) * P(R_I = z_k) \quad (8)$$

$$\sigma^2_I = [\sum_{k=1}^m (z_k)^2 * P(R_I = z_k)] - \mu_I^2 \quad (9)$$

where  $R_I$  = risk impact

$z_k$  = element of the risk impact

$P(R_I = z_k)$  = probability of occurrence of the risk impact to be element  $z_k$

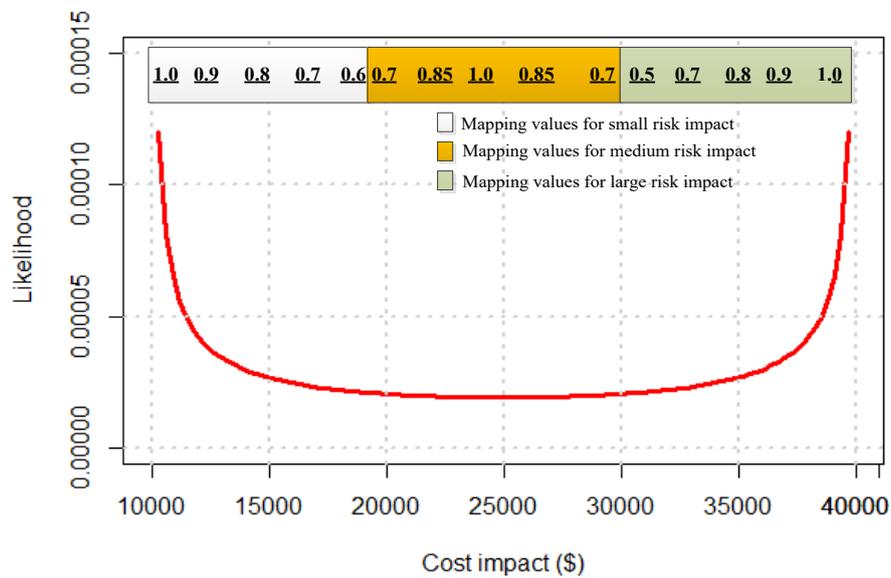
$\mu_o(z_k)$  = membership value of element  $z_k$  in subset O  
 $m$  = number of risk impact elements in subset O

**Step 12:** Using the mean and variance, fit the results to a Beta distribution, using Equations 10 and 11, to calculate the shape and scale parameters.

$$\alpha = \frac{\mu_I - A}{B - A} \left[ \frac{(\mu_I - A)(B - \mu_I)}{\sigma^2_I} - 1 \right] \quad (10)$$

$$\beta = \alpha \left[ \frac{B - \mu_I}{\mu_I - A} \right] \quad (11)$$

**Step 13:** Visualize the marginal Beta distribution of cost or schedule impact using the minimum and maximum derived in Step 5 and the scale and shape parameters derived in Step 12 (example of a resulting marginal Beta distribution is provided as follows):

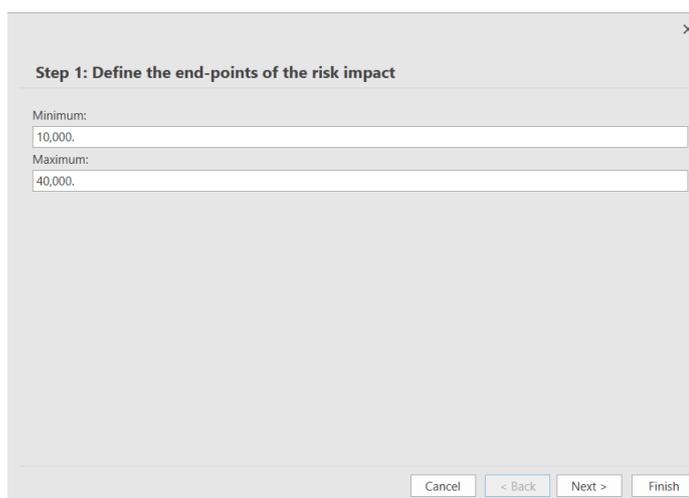


Supplement B

# Derivation of the marginal distribution using *SimphonyProject.NET*

The steps for deriving the marginal distribution in *SimphonyProject.NET*, and a graphical user interface created to make the method easy for analyst lacking expertise in fuzzy logic, are presented as follows:

Step 1:



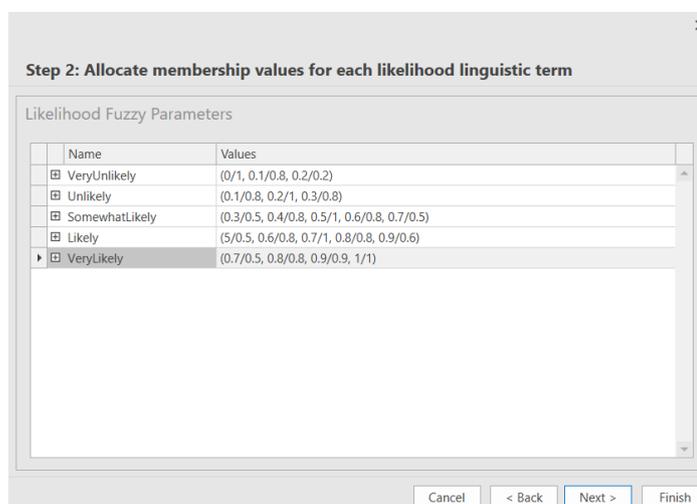
**Step 1: Define the end-points of the risk impact**

Minimum:  
10,000.

Maximum:  
40,000.

Cancel < Back Next > Finish

Step 2:



**Step 2: Allocate membership values for each likelihood linguistic term**

Likelihood Fuzzy Parameters

Name	Values
VeryUnlikely	(0/1, 0.1/0.8, 0.2/0.2)
Unlikely	(0.1/0.8, 0.2/1, 0.3/0.8)
SomewhatLikely	(0.3/0.5, 0.4/0.8, 0.5/1, 0.6/0.8, 0.7/0.5)
Likely	(5/0.5, 0.6/0.8, 0.7/1, 0.8/0.8, 0.9/0.6)
VeryLikely	(0.7/0.5, 0.8/0.8, 0.9/0.9, 1/1)

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Step 3:

**Step 3: Allocate membership values for each adverse consequence linguistic term**

Consequence Fuzzy Parameters

Name	Values
VerySmall	(0/1, 0.1/0.81, 0.2/0.25)
Small	(0/1, 0.1/0.9, 0.2/0.5)
Medium	(0.3/0.2, 0.4/0.8, 0.5/1, 0.6/0.8, 0.7/0.2)
Large	(0.8/0.5, 0.9/0.9, 1/1)
VeryLarge	(0.8/0.25, 0.9/0.81, 1/1)

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Step 4:

**Step 4: Allocate membership values for each risk impact range**

Impact Range Parameters

Name	Values
Small	(10000/0.2, 12500/0.7, 15000/1, 17500/0.6, 20000/0.4)
Medium	(20000/0.2, 22500/0.1, 25000/0, 27500/0, 30000/0)
Large	(30000/0, 32500/0, 35000/0, 37500/0, 40000/0)

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Step 5:

**Step 5: Identify and assess the root causes of a risk factor**

Root Causes

Fuzzy Parameter	Likelihood	Consequence
Construction noise is low	Likely	VerySmall
Construction noise is medium	Likely	Large
Construction noise is high	Unlikely	Large
Harm to activities is low	Unlikely	Small
Harm to activities is medium	SomewhatLikely	Large
Harm to activities is high	Unlikely	VeryLarge
Traffic disturbance is low	VeryLikely	VerySmall
Traffic disturbance is medium	SomewhatLikely	Large
Traffic disturbance is high	Unlikely	VeryLarge
Poor communication	Unlikely	Medium

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Step 6:

**Step 6: Mapping the adverse consequence of root causes to the overall risk impact**

Consequence	Impact
VerySmall	Small
Small	Small
Medium	Medium
Large	Large
VeryLarge	Large
*	

Buttons: Cancel, < Back, Next >, Finish

Step 7:

**Distribution Editor**

Probability Density Cumulative Distribution

Probability Density

Likelihood

Observed Quantile

12,811.9839 20.4% 79.6% 37,188.0161 0.0%

**Distribution Parameters**

(Type)	Simphony.Mathe...
Alpha	2.70888157894736
Beta	12.7450657894737
Low	10000
High	40000

Buttons: Fit Data..., Subjective Fit, Bayesian Update..., OK, Cancel