



A Convex Optimization Algorithm for Electricity Pricing of Charging Stations

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Abstract: The problem of electricity pricing for charging stations is a multi-objective mixed integer nonlinear programming. Existing algorithms have low efficiency in solving this problem. In this paper, a convex optimization algorithm is proposed to get the optimal solution quickly. Firstly, the model is transformed into a convex optimization problem by second-order conic relaxation and Karush–Kuhn–Tucker optimality conditions. Secondly, a polyhedral approximation method is applied to construct a mixed integer linear programming, which can be solved quickly by branch and bound method. Finally, the model is solved many times to obtain the Pareto front according to the scalarization basic theorem. Based on an IEEE 33-bus distribution network model, simulation results show that the proposed algorithm can obtain an exact global optimal solution quickly compared with the heuristic method.

Keywords: charging station; convex optimization; multi-objective programming; polyhedral approximation; scaling method

1. Introduction

With the popularization of traffic electrification, the number of electric vehicles (EVs) is increasing year by year. It is estimated that the number of EVs will reach 11 million in 2025 and 30 million in 2030 [1]. In order to satisfy the charging demand of EVs, a large number of charging stations (CSs) have been built in the distribution network. Considering the charging behavior of EV closely related to people's daily work and rest, EV users usually charge after work in the evening. In addition, the operation of the distribution network will be severely impacted by a large number of charging loads [2]. Therefore, the electric energy plan of CSs must be optimized to ensure the orderly charging of EVs [3]. Electric vehicle users are guided to change charging time by CSs setting charging prices [4]. The relationship among electric vehicle users, charging station operators, and distribution network operators (DSO), needs to be taken into consideration. However, existing algorithms are inefficient in solving this problem [5]. Therefore, it is very meaningful to develop the optimization algorithm for electric power planning of charging stations.

For the time being, the research on charging price of EVs mainly focuses on time-of-use price [6]. In [7–9], the distribution locational marginal pricing (DLMP) method is applied to optimize the charging price of EVs to reduce the impact of charging load on distribution network. In [10–12], the electricity demand and travel characteristics of EV users are considered, and charging price is optimized by the



price demand elasticity coefficient. In [13], a Stackelberg leadership game model is applied to optimize the electricity prices for encouraging EV users to participate in the electricity market, improving their own interests, and promoting the development of smart energy systems. However, the problem

of power planning for charging stations is a multi-objective mixed integer nonlinear programming (MINLP) and some constraints are nonconvex. Heuristic algorithms are usually used to solve this problem such as genetic algorithm (GA) in [14] and particle swarm optimization (PSO) in [15], which take a long time to solve and easily fall into local optimum.

Aiming at getting the global optimal solution quickly, the main contribution of this paper is to propose a convex optimization algorithm for electricity pricing of charging stations. Firstly, the model is transformed into a convex optimization problem by second-order conic relaxation and Karush–Kuhn–Tucker optimality conditions. Secondly, a polyhedral approximation method is applied to construct a mixed integer linear programming, which can be solved quickly by branch and bound method. Finally, the model is solved iteratively to obtain the Pareto front according to the scalarization basic theorem.

On the one hand, the optimal solution can be obtained in the strict sense provided with a convex optimization problem. The conic problem is approximated to a simple type based on polyhedral approximation in order to improve the speed of solution. In addition, the tripartite game problem is decomposed into a Stackelberg leadership game and a cooperative game. Besides, the multi-solution property of the multi-objective optimization problem is preserved under the scaling method.

The remainder of the paper is organized as follows. Section 2 introduces the model of charging station electric power planning. Section 3 proposes the convex optimization algorithm based on second order conic relaxation (SOCR) and polyhedral approximation method. Simulations and analysis are carried out in Section 4. Finally, a conclusion is drawn in Section 5.

2. The Model of Electricity Pricing of Charging Stations

The benefits of DSO and EVs should be taken into account when CSs set electricity prices. The relationship among DSO, CSs, and EVs is shown in Figure 1. Usually, optimizing the power plan of CSs can not only improve its own revenue, but also reduce the peak-valley difference of the distribution network. DSO and CSs can achieve a win-win situation, which constitutes a cooperative relationship [11]. In order to adjust the load curve, DSO guides CSs through marginal price to formulate an appropriate power plan. Charging fee is a cost for EVs but a revenue for CSs. Therefore, there is a conflict of benefits between EVs and CSs, which can be considered as a non-cooperative game relationship [16,17]. A more accurate description should be the Stackelberg game between EVs and CSs because there is a sequence of decisions between the two sides [18]. After charging station operators set charging service price, electric vehicle users decide charging mode according to the price. The goal of EVs is to reduce charging cost and thus EVs will charge when the charging price is low. When CSs formulate the power plan, guiding EVs to charge orderly by setting an appropriate charging price is necessary, and the revenue of CSs should also be maximized.



Figure 1. The relationship among distribution network operators (DSO), charging stations (CSs), and electric vehicles (EVs).

The electricity pricing problem of charging stations is a classical problem in power systems [2–11], which can be decomposed into the security checking problem of DSO, charging pricing problem of CSs, and charging mode optimization problem of EVs. In the security checking problem of DSO, the objective function of DSO is shown in Equation (1) and the constraints are shown in Equation (2), including power demand equation constraints, bus power balance constraints, bus voltage safety constraints, and line power transmission capacity constraints. The variables in this problem are $\{P_{G,t}, P_{c,t}^i, V_{i,t}, \theta_{i,t}\}$ and the purpose of this problem is to ensure the security of the power system [11].

$$\min_{\substack{P_{c,t}^{i}}} f_{DSO} = \|P_{G,t} - \overline{P}_{G}\|_{2},\tag{1}$$

s.t
$$P_{G,t} = \sum_{i \in I} (P_{L,t}^{i} + P_{c,t}^{i}) + \sum_{ij \in L} G_{ij} (V_{i,t}^{2} + V_{j,t}^{2} - 2V_{i,t}V_{j,t}\cos\theta_{ij,t}), \forall t \in T$$
$$\overline{P}_{G} = \frac{1}{T} \sum_{t \in T} P_{G,t}$$
$$P_{L,t}^{i} + P_{c,t}^{i} = G_{ij}V_{i,t}^{2} - V_{i,t}V_{j,t}(G_{ij}\cos\theta_{ij,t} + B_{ij}\sin\theta_{ij,t}), \forall t \in T, \forall i \in I$$
$$Q_{L,t}^{i} = -B_{ij}V_{i,t}^{2} + V_{i,t}V_{j,t}(B_{ij}\cos\theta_{ij,t} - G_{ij}\sin\theta_{ij,t}), \forall t \in T, \forall i \in I$$
$$V \leq V_{i,t} \leq \widetilde{V}, \forall t \in T, \forall i \in I$$
$$(G_{ij}^{2} + B_{ij}^{2})(V_{i,t}^{2} + V_{j,t}^{2} - 2V_{i,t}V_{j,t}\cos\theta_{ij,t} + 2V_{i,t}V_{j,t}\sin\theta_{ij,t}) \leq \widetilde{I}_{ij}^{2}, \forall t \in T, \forall i \in L$$

where f_{DSO} is the deviation of load in distribution network; $P_{G,t}$ is the total active power of DSO in period t; \overline{P}_G is the average active power of DSO; $P_{L,t}^i$ is the basic active power at bus i in period t; $Q_{L,t}^i$ is the basic reactive power at bus i in period t; $P_{c,t}^i$ is the charging power at bus i in period t; G_{ij} is the conductance of line ij; Line ij stands for transmission line from bus i to bus j; $V_{i,t}$ is the voltage at bus iin period t; $\theta_{ij,t}$ is the power angle difference of line ij in period t; T is the set of dispatching time; B_{ij} is the susceptance of line ij; I is the bus set of distribution network; L is the set of transmission lines of distribution network; \tilde{V} and V are the boundary of voltage.

In the charging pricing problem of CSs, the objective function of CSs is shown in Equation (3) and the constraints are shown in Equation (4), including price boundary constraints and average price equality constraints in order to ensure the fairness of the market when the competition between CSs is neglected [19]. The variables in this problem are $\{\pi_{c,t}, P_{c,t'}^i, P_{b,t}\}$ and the purpose of this problem is to maximize the revenue of the CSs [18].

$$\max_{\pi_{c,t}} f_{CSs} = \sum_{t \in T} \pi_{c,t} \sum_{i \in I} P^i_{c,t} - \sum_{t \in T} \pi_{b,t} P_{b,t},$$
(3)

s.t
$$P_{b,t} = \sum_{i \in I} P_{c,t}^{i}, \forall t \in T$$
$$P_{c,t}^{i} = \sum_{n \in N(i)} p_{c,t}^{n}, \forall t \in T, \forall i \in I$$
$$\pi \leq \pi_{c,t} \leq \overline{\pi}_{c}, \forall t \in T$$
$$\frac{1}{T} \sum_{t \in T}^{c} \pi_{c,t} = \overline{\pi}_{c}$$
(4)

where f_{CSs} is the income of CSs; $\pi_{c,t}$ is the charging price at period t; $\pi_{b,t}$ is the price of electricity purchase $P_{b,t}$ at period t; N is the set of electric vehicle users; $p_{c,t}^n$ is the charging power of EV_n at period t; $\tilde{\pi}_c$ and π_{a} are the boundary of charging price; $\bar{\pi}_c$ is the average charging price.

In charging mode optimization problem of EVs, the objective function of EVs is shown in Equation (5) and the constraints are shown in Equation (6), including power demand equation constraints, maximum charging power constraints, and available charging time constraints. The variables in this problem are $\{p_{c,t}^n\}$ and the purpose of this problem is to minimize the charging cost of EVs [18].

$$\min_{p_{\mathsf{c},t}^n} f_{\mathrm{EV},n} = \sum_{t \in T} p_{\mathsf{c},t}^n \pi_{\mathsf{c},t}, \forall n \in N,$$
(5)

s.t
$$\sum_{\substack{t \in T \\ t \in T}} p_{c,t}^n = E_{ev}^n, \forall n \in N$$
$$0 \le p_{c,t}^n \le \overline{p}_{c'}, \forall t \in T, \forall n \in N$$
$$p_{c,t}^n = 0, \forall t \notin T_e(n), \forall n \in N$$
(6)

where $f_{EV,n}$ is the charging cost of EV_n ; E_{ev}^n is the power demand of EV_n ; \overline{p}_c is the maximum charging power; $T_e(n)$ is the set of rechargeable time of EV_n .

From the above formulas, there are three objective functions in the electricity pricing problem and there are correlations among variables, what lead to the complexity of the model. Hence a series of equivalent transformations must be applied to the model. According to the Stackelberg leadership game model, the game between CSs and EVs can be regarded as a bilevel optimization problem [20], and the subproblems can be expressed as Equation (7).

$$p_{\mathsf{c},t}^n = \operatorname{argmin}_{t \in T} p_{\mathsf{c},t}^n \pi_{\mathsf{c},t}.$$
(7)

In order to obtain the optimal solution of the problem, the Lagrange function is shown in Equation (8).

$$L_{n} = \sum_{t \in T} p_{c,t}^{n} \pi_{c,t} - \lambda^{n} (\sum_{t \in T} p_{c,t}^{n} - E_{ev}^{n}) - \sum_{t \in T} \mu_{lb,t}^{n} p_{c,t}^{n} - \sum_{t \in T} \mu_{ub,t}^{n} (p_{c,t}^{n} - \overline{p}_{c}) - \sum_{t \in T} \mu_{a,t}^{n} p_{c,t}^{n}, \forall n \in N,$$
(8)

where, L_n is the Lagrange function of Equations (5) and (6); λ^n is the dual variable of power demand equation constraints; $\mu_{lb,t}^n$ and $\mu_{ub,t}^n$ are dual variables of maximum charging power constraints; $\mu_{a,t}^n$ is the dual variable of available charging time constraints.

According to the linear duality theory [21], the dual expression of Equation (5) is shown in Equation (9). At the same time, the optimum condition is Equation (10). The optimization problem of electric vehicle users is transformed into a series of Karush–Kuhn–Tucker (KKT) optimality conditions.

$$\min f_{\text{EV},n} = \max(-f_{\text{EV},n}) = \max(\lambda^n E_{\text{ev}}^n + \sum_{t \in T} \mu_{\text{ub},t}^n \overline{p}_c),$$
(9)

$$\sum_{t \in T} p_{c,t}^n - E_{ev}^n = 0, \forall n \in N$$

$$0 \le p_{c,t}^n \le \overline{p}_c$$

$$\frac{\partial L}{\partial p_{c,t}^n} = \pi_{c,t} - \lambda^n - \mu_{lb,t}^n - \mu_{ub,t}^n - \mu_{a,t}^n = 0, \forall t \in T, \forall n \in N$$

$$0 \le \mu_{lb,t}^n \perp p_{c,t}^n, \forall t \in T, \forall n \in N$$

$$0 \ge \mu_{ub,t}^n \perp (p_{c,t}^n - \overline{p}_c), \forall t \in T, \forall n \in N$$
(10)

However, Equation (10) includes some complementary constraints, which leads to the nonlinearity of the model. A big-M method is applied to relax the complementary constraints into mixed integer constraints, as Equation (11). At the same time, the main problem can be expressed as Equation (12) according to duality theory.

$$0 \le \mu_{\text{lb},t}^n \le MX_t^n, \forall t \in T, \forall n \in N$$

$$0 \le p_{c,t}^n \le M(1 - X_t^n), \forall t \in T, \forall n \in N$$

$$0 \le \overline{p}_c - p_{c,t}^n \le MY_t^n, \forall t \in T, \forall n \in N$$

$$M(Y_t^n - 1) \le \mu_{\text{ub},t}^n \le 0, \forall t \in T, \forall n \in N$$
(11)

$$\max_{\pi_{c,t}} f_{CSs} = \sum_{n \in \mathbb{N}} \left(\lambda^n E_{\text{ev}}^n + \sum_{t \in T} \mu_{\text{ub},t}^n \overline{p}_c \right) - \sum_{t \in T} \pi_{b,t} P_{b,t}, \tag{12}$$

where *M* is a given big value; X_t^n and Y_t^n are binary variables for relaxing Equation (10).

So far, the charging station power planning model is a MINLP with integer variables and non-linear expressions. Additionally, the quadratic equality constraints in Equation (2) make the model non-convex. At the same time, the benefits of DSO and CSs should be considered, which lead to multiple solutions of the model. In short, the problem of power planning for charging stations is a NP-hard problem and existing methods have difficulty in solving this problem.

3. Convex Optimization Algorithm

In this section, the second order conic relaxation method is proposed to transform the model into a convex optimization problem and the scaling algorithm is proposed to obtain the Pareto front.

3.1. SOCR Method

In [22], a method of equality transformation is proposed to promotion dimension of model like Equation (13).

$$R_{ij,t} = V_{i,t}V_{j,t}\cos\theta_{ij,t}, \forall t \in T, \forall ij \in L$$

$$S_{ij,t} = V_{i,t}V_{j,t}\sin\theta_{ij,t}, \forall t \in T, \forall ij \in L$$

$$W_{i,t} = V_{i,t}^{2}, \forall t \in T, \forall i \in I$$

$$R_{ij,t}^{2} + S_{ij,t}^{2} = W_{i,t}W_{j,t}, \forall t \in T, \forall ij \in L$$
(13)

where $R_{ij,t}$, $S_{ij,t}$, $W_{i,t}$ are temporary variables in order to replace the original variables, which have no practical physical significance.

After this transformation, decision variables have become $\{P_{G,t}, P_{c,t}^i, W_{i,t}, R_{ij,t}, S_{ij,t}\}$ in the security checking problem of DSO. Additionally, Equation (2) becomes a linear form like Equation (14). In this process, variables are projected into a higher dimensional space. In order to ensure the accuracy of the original problem, a new set of equality constraints need to be added as Equation (13), which

constitutes a set of rotating conical spaces. In order to transform the model into convex optimization form, Equation (15) is applied to relax the feasible region to some second order cones.

$$\begin{split} P_{G,t} &= \sum_{i \in I} \left(P_{L,t}^{i} + P_{c,t}^{i} \right) + \sum_{i j \in L} G_{ij} (W_{i,t} + W_{j,t} - 2R_{ij,t}), \forall t \in T \\ P_{L,t}^{i} + P_{c,t}^{i} &= G_{ij} W_{i,t} - (G_{ij} R_{ij,t} + B_{ij} S_{ij,t}), \forall t \in T, \forall i \in I \\ Q_{L,t}^{i} &= -B_{ij} W_{i,t} + (B_{ij} R_{ij,t} - G_{ij} S_{ij,t}), \forall t \in T, \forall i \in I \\ V^{2} &\leq W_{i,t} \leq \widetilde{V}^{2}, \forall t \in T, \forall i \in I \\ \left(G_{ij}^{2} + B_{ij}^{2} \right) (W_{i,t} + W_{j,t} - 2R_{ij,t} + 2S_{ij,t}) \leq \widetilde{I}_{ij}^{2}, \forall t \in T, \forall ij \in L \\ R_{ij,t}^{2} + S_{ij,t}^{2} &\leq W_{i,t} W_{j,t}, \forall t \in T, \forall ij \in L. \end{split}$$
(15)

So far, the original problem has been relaxed into a mixed integer second order conic programming (MISOCP). The existing commercial solver can solve this convex optimization problem. However, the existing convex optimization algorithms are weak in solving this problem because the problem contains a large number of variables [21]. In order to improve the efficiency of solving this problem, a polyhedral approximation method is proposed to construct a mixed integer linear programming (MILP), which can be solved quickly by general algorithms like the branch and bound method [23]. At the same time, the literature [22] has confirmed that the polyhedral approximation method can accurately control model errors and has high efficiency in power flow problems.

At first, Equation (15) is rewritten as Equation (16). And Equation (17) is applied to relax Equation (16) to a polyhedral space.

$$\sqrt{R_{ij,t}^2 + S_{ij,t}^2 + \left(\frac{W_{i,t} - W_{j,t}}{2}\right)^2} \le \frac{W_{i,t} + W_{j,t}}{2}, \forall t \in T, \forall ij \in L,$$
(16)

$$\sqrt{R_{ij,t}^2 + S_{ij,t}^2 + \left(\frac{W_{i,t} - W_{j,t}}{2}\right)^2} \le (1 + \varepsilon)\left(\frac{W_{i,t} + W_{j,t}}{2}\right), \forall t \in T, \forall ij \in Lm$$
(17)

where ε is a relaxation scalar and $\varepsilon \in (0, 0.5]$.

Secondly, the rotating cones are equivalent to two standard cones like Equations (18) and (19).

$$E_{ij,t} \ge \sqrt{R_{ij,t}^2 + S_{ij,t}^2} \,\forall t \in T, \forall ij \in L,$$
(18)

$$\frac{W_{i,t} + W_{j,t}}{2} \ge \sqrt{\left(\frac{W_{i,t} - W_{j,t}}{2}\right)^2 + E_{ij,t'}^2} \,\forall t \in T, \forall ij \in L.$$
(19)

Finally, the smooth edges of cones are replaced with polyhedrons by Equations (21)–(23). The dimensions of a polyhedron depend on the accuracy required in Equation (20).

$$\varepsilon(v) = \frac{1}{\cos(\frac{\pi}{2^{\nu+1}})} - 1.$$
(20)

The boundary of a polyhedron is calculated by recursion. The initial boundary can be calculated by Equation (21) and the recursive expression is shown in Equation (22). These expressions form a set of linear spaces when v is given.

$$\begin{aligned}
\varsigma_{ij,t}^{0} &\geq |R_{ij,t}|, \forall t \in T, \forall ij \in L \\
\eta_{ij,t}^{0} &\geq |S_{ij,t}|, \forall t \in T, \forall ij \in L \\
\xi_{ij,t}^{0} &\geq |\frac{W_{i,t} - W_{j,t}}{2}|, \forall t \in T, \forall ij \in L \\
\sigma_{ij,t}^{0} &\geq |E_{ij,t}|, \forall t \in T, \forall ij \in L
\end{aligned}$$
(21)

$$\begin{aligned} \varsigma_{ij,t}^{k} &= \cos(\frac{\pi}{2^{k+1}})\varsigma_{ij,t}^{k-1} + \sin(\frac{\pi}{2^{k+1}})\eta_{ij,t}^{k-1}, k \in 1, \dots, v, \forall t \in T, \forall ij \in L \\ \eta_{ij,t}^{k} &\geq |-\sin(\frac{\pi}{2^{k+1}})\varsigma_{ij,t}^{k-1} + \cos(\frac{\pi}{2^{k+1}})\eta_{ij,t}^{k-1}|, k \in 1, \dots, v, \forall t \in T, \forall ij \in L \\ \xi_{ij,t}^{k} &= \cos(\frac{\pi}{2^{k+1}})\xi_{ij,t}^{k-1} + \sin(\frac{\pi}{2^{k+1}})\sigma_{ij,t}^{k-1}, k \in 1, \dots, v, \forall t \in T, \forall ij \in L \\ \sigma_{ij,t}^{k} &\geq |-\sin(\frac{\pi}{2^{k+1}})\zeta_{ij,t}^{k-1} + \cos(\frac{\pi}{2^{k+1}})\sigma_{ij,t}^{k-1}|, k \in 1, \dots, v, \forall t \in T, \forall ij \in L \end{aligned}$$
(22)

Finally, the upper bound of the polyhedron is defined by Equation (23).

$$\begin{aligned}
\varsigma_{ij,t}^{v} &\leq E_{ij,t}, \forall t \in T, \forall ij \in L \\
\eta_{ij,t}^{v} &\leq \tan(\frac{\pi}{2^{v+1}})\varsigma^{v}, \forall t \in T, \forall ij \in L \\
\xi_{ij,t}^{v} &\leq \frac{W_{i,t} + W_{j,t}}{2}, \forall t \in T, \forall ij \in L \\
\sigma_{ij,t}^{v} &\leq \tan(\frac{\pi}{2^{v+1}})\xi_{ij,t}^{v}, \forall t \in T, \forall ij \in L
\end{aligned}$$
(23)

The details of polyhedral approximation method are described in Algorithm 1.

Algorithm 1: Polyhedral approximation algorithm

1: v: = 0, obtain ε by Equation (20), construct model by Equations (11), (12), and (14) while $\varepsilon > 10^{-7}$ do 2: v++, obtain ε by Equation (20) 3: 4: end while 5: **for** *t* = 1 to *T* **do** for ij = 1 to L do 6: add Equations (21) and (23) into model 7: 8: for k = 1 to v do 9: add Equation (22) into model 10: end do end do 11: 12: end do 13: solve MILP by branch and bound method 14: return $P_{G,t}$, $P_{c,t}$, $\pi_{c,t}$, $R_{ij,t}$, $S_{ij,t}$, $W_{i,t}$, f_{DSO} , f_{CSs} and $f_{EV,n}$

The relaxation process is shown in Figure 2. The original non-convex feasible region is first relaxed to a second order cone and then to a series of polyhedrons, which can be solved quickly by simplex method.



Figure 2. Relaxation process.

The original solution variables are projected into high-dimensional space by SOCR, so it is necessary to get the original variables by returning the mapping. Firstly, the voltage can be returned by Equation (24).

$$V_{i,t} = \sqrt{W_{i,t}}, \forall t \in T, \forall i \in I.$$
(24)

Then the power angle difference of the line can be obtained by Equations (25) and (26). Based on the agreement of $\theta_{0,t} = 0$, all angles can be obtained by solving the matrix.

$$\tan(\theta_{i,t} - \theta_{j,t}) = \tan \theta_{ij,t} = \frac{S_{ij,t}}{R_{ij,t}}, \forall t \in T, \forall ij \in L,$$
(25)

$$\theta_{i,t} - \theta_{j,t} = \tan^{-1} \frac{S_{ij,t}}{R_{ij,t}}, \forall t \in T, \forall ij \in L.$$
(26)

The details of return mapping method are described in Algorithm 2.

Algorithm 2: Return mapping algorithm				
1:	$R_{ij,t}$, $S_{ij,t}$, and $W_{i,t}$: calculated by Algorithm 1			
2:	for $t = 1$ to T do			
3:	for $i = 1$ to N do			
4:	$V_{i,t}$: calculated by Equation (24)			
5:	end do			
6:	$\theta_{N0,t}$:=0			
7:	for $ij = 1$ to L do			
8:	$\theta_{i,t}$: calculated by Equation (26)			
9:	end do			
10:	end do			
11: return $V_{i,t}$ and $\theta_{i,t}$				

In order to verify the accuracy of relaxation, relaxation errors are defined as Equation (27) to check whether constraints are satisfied.

$$gap_{ij,t} = |R_{ij,t}^2 + S_{ij,t}^2 - W_{i,t}W_{j,t}|, \forall t \in T, \forall ij \in L.$$
(27)

3.2. Scaling Algorithm

DSO and CSs are cooperative game relations, but both sides have their own benefits. The problem of power planning for charging stations is a multi-objective programming, and the Pareto front can balance the benefits of DSO and CSs well. According to the Pareto front, all the cooperation results can be obtained, and then the appropriate cooperation decision can be selected [24]. Therefore, the key to the problem is how to get the Pareto front of the model. According to the scalarization basic theorem [21], the Pareto front can be obtained by transforming the objective function into a set of constraints and solving the model many times. In this paper, the objective function of DSO is transformed into a constraint like Equation (28). By changing D_L , the model is solved many times to obtain the Pareto front.

$$\|P_{G,t} - \overline{P}_G\|_2 \le D_L. \tag{28}$$

In order to simplify the model, standard deviation as Equation (29) is used to simplify Equation (28), which can be rewritten as a set of linear constraints.

$$\frac{1}{T}\sum_{t\in T}|P_{G,t}-\overline{P}_G| \le D_L.$$
(29)

The details of scaling method are described in Algorithm 3.

Algorithm 3: Scaling algorithm

1 D_L : = + ∞ , $P_{G,t}$: obtain by Algorithm 1, f_{DSO} : obtain by Algorithm 1, f_{CSs} : obtain by Algorithm 1 2 $D_L: = \frac{1}{T} \sum_{t=T} |P_{G,t} - \overline{P}_G|, t: = 0, \text{ obj}_1[t]: = f_{DSO}, \text{ obj}_2[t]: = f_{CSs}$ 3 repeat 4 solve MILP by algorithm 1, f_{DSO} : obtain by Algorithm 1, f_{CSs} : obtain by Algorithm 1 5 if $f_{CSs} == obj_2[t]$ then break 6 7 else then 8 t: t++ 9 $obj_1[t]: = f_{DSO}, obj_2[t]: = f_{CSs}$ 10 end if 11 until the problem is infeasible return obj1 and obj2 12

4. Case Studies

4.1. Data Generation

The case uses an IEEE 33-bus system [25], and the urban area is divided into residential area, commercial area, and industry area [26]. The total load is 3715 kVA and there are three types of EVs in the distribution network. The structure of the distribution network and the location of the charging station are shown in Figure 3 and the electricity price of DSO is shown in Figure 4. The data of electric vehicles can be obtained in [20]. The battery capacity of EVs is assumed to be 42 kWh, and the initial state of charge (SOC) is 30%. There are 60 charging piles in the CS and the maximum charging power of each charging pile is 30 kW. In order to simplify the problem, this paper assumes that there are three types of EVs in distribution network and their parking time is shown in Figure 5. Additionally, the number of three types of electric vehicles is 200, 150, and 50 respectively. All simulations are implemented on a laptop with Intel[®] Core(TM) i5-7300HQ CPU@2.5 GHz, 8 GB memory, and the algorithms are compiled by MATLAB. YALMIP tools [23] are used to model and CPLEX [27] is called to solve the MILP model by branch and bound method. Branch and bound method is a common method, whose process only briefly described in this article can be seen in [28].



Figure 3. IEEE 33-bus system.



Figure 5. The parking time of EVs.

4.2. Discussion

The Pareto front obtained is shown in Figure 6a, and an equilibrium solution is selected to show the results. As shown in Figure 6b, charging price is higher in the peak period and lower in the valley period, so as to guide electric vehicle users to charge in the valley period. However, charging prices are the same as in many periods of time, in order to avoid users concentrating on charging at the low price period, resulting in new load peaks. Under the guidance of charging price, the charging power of different electric vehicles is shown in Figure 6c. It can be seen that charging load is mainly concentrated in the night time and some electric vehicles charge in the afternoon, due to their parking characteristics. The load curve of distribution network after orderly charging of electric vehicles is shown in Figure 6d. Under the guidance of charging price, charging load mainly concentrates on the period of low power consumption, which greatly reduces the load fluctuation of the distribution network. It can be seen that the cooperative game between DSO and CSs can achieve win-win situation and maximize social welfare.



Figure 6. Simulation results. (**a**) shows the Pareto front. (**b**) shows the charging prices. (**c**) shows the charging load of charging stations. (**d**) shows the total load of distribution system.

The voltage distribution in the distribution network is shown in Figure 7. It can be seen that the return mapping algorithm can correctly restore the original variables. At the same time, the minimum voltage of distribution network is 0.9921 pu, which occurs at 9 a.m. on bus 31. The minimum operating voltage allowed in the distribution network is 0.93 pu. In other words, the voltage security of the distribution network can be guaranteed under the scheduling strategy in this paper. As shown in Figure 8, the relaxation error of MILP is larger than that of MISOCP but both are less than 10⁻⁷. Compared with the numerical value of decision variables, this part of the error can be neglected. It can be seen that the second order cone relaxation has high accuracy. At the same time, polyhedral approximation will not greatly reduce the accuracy of the model.



Figure 7. Voltage distribution in the distribution network.



Figure 8. Relaxation error of the model.

In this paper, the proposed algorithm was compared with GA [14] and PSO [15]. As can be seen from Table 1, the performance of GA and PSO is not ideal in solving this problem. In contrast, MISOCP and MILP can obtain the optimal solution in a very short time. Besides, MISOCP and MILP get the global optimal solution according to the convex optimization theory. Moreover, MILP is more efficient than MISOCP without losing accuracy. Therefore, the polyhedral approximation algorithm proposed in this paper can effectively improve the efficiency of solving this problem.

Algorithm	f_{DSO} (kW)	<i>f_{CSs}</i> (\$)	The Sum of f_{EVs} (\$)	CPU Time (s)
GA	183.5642	133.1157	1207.11	892.4
PSO	178.7535	127.5048	1244.34	411.5
MISOCP	177.1124	135.1261	1142.67	10.843
MILP	177.1121	135.1372	1143.12	0.172

Table 1. Comparisons of calculation results.

5. Conclusions

In this paper, a convex optimization algorithm was proposed to solve the problem of power planning for charging station. Second-order conic relaxation and Karush–Kuhn–Tucker optimality conditions were proposed to transform the model into a convex optimization problem. Further, a polyhedral approximation method was applied to construct a MILP and the model was solved iteratively to obtain the Pareto front according to the scalarization basic theorem. The simulation results of IEEE 33-bus test system show that some conclusions can be drawn as follows:

- (1) The proposed second-order conic relaxation is accurate, which can transform the original model into a convex optimization model in order to obtain the global optimal solution. At the same time, the original variables can be easily restored.
- (2) The proposed polyhedral approximation method can speed up the solution of the model. Compared with other algorithms, the polyhedral approximation algorithm can get the global optimal solution with the minimum CPU time.
- (3) The proposed scaling algorithm can obtain the Pareto front by solving the problem iteratively, which can preserve the multi-solution of the model and obtain the equilibrium solution.

However, this study ignores the competitive relationship between charging stations, which may lead to an unfair market. Therefore, the game between charging stations needs further consideration, which is the focus of our future work.

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