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Influence of Identification Criteria on Rolling Bearing Vibration Models Based on Geometrical and Design Parameters

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Abstract

Mathematical models describing the relationship between rolling bearing vibration levels and geometrical and design parameters are commonly identified using ordinary least squares (OLS). However, the influence of the adopted identification criterion on the properties of such models remains insufficiently investigated. This study aimed to evaluate the effects of two identification criteria, OLS and minimax, on the regression coefficients and approximation characteristics of linear bearing vibration models. The analysis included 180 single-row deep-groove ball bearings: 90 shielded (6304-2Z) and 90 open (6304). Six explanatory variables were considered: roundness and waviness deviations of the inner and outer raceways, radial clearance, and the total curvature ratio. The response variable was the vibration level in the high-frequency band (1800–10,000 Hz). Both identification criteria were applied to the same experimental dataset and model structure within each bearing group. The resulting models were compared using regression coefficients, root-mean-square error (RMSE), maximum absolute error, and the coefficient of determination (R^2). Changing the identification criterion reversed the sign of the coefficient associated with outer raceway waviness in both bearing groups and radial clearance in shielded bearings, indicating opposite directions of model-predicted vibration responses when the remaining variables were held constant. Compared with OLS, minimax reduced the maximum absolute error but increased RMSE and decreased R^2, with a particularly pronounced deterioration in overall fit for open bearings. Based on their better overall approximation of the experimental data, the OLS models were selected for subsequent diagnostic assessment and predictive validation before their potential application in bearing parameter optimization. The findings demonstrate that the identification criterion affects both the approximation characteristics and the estimated coefficients of linear bearing vibration models, which should be considered when selecting models for further engineering applications.

Keywords: rolling bearings; vibration modeling; model identification; least squares method; minimax criterion; form error measurements

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1. Introduction

Rolling bearings are fundamental components of modern mechanical systems, and their reliability has a significant impact on the durability, efficiency, and operational safety of machines and equipment [1,2]. One of the most important indicators of bearing quality and technical condition is the level of generated vibrations, whose analysis has become the subject of extensive scientific research [3–6]. To quantitatively describe the relationship between the vibration level generated by bearings and their geometrical and design parameters, mathematical models are increasingly employed, finding applications in condition monitoring, product quality assessment, and the optimization of manufacturing processes [7–9]. However, the reliability of such models depends not only on the adopted mathematical structure but also on the method used to identify their parameters.

Previous studies have shown that the vibration level of rolling bearings is influenced by numerous factors related to raceway geometry and bearing design parameters. Paper [10] presents an experimental analysis of bearing vibrations for various types of defects, including outer-race defect, inner-race defect, rolling-element defect, and combinations of these defects. Four rotational speeds and two different load levels were considered. The study confirms the effectiveness of spectral analysis in detecting bearing defects, while also highlighting the limitations arising from signal complexity and operating conditions. Work [11] presents a comprehensive analysis of the mechanism of roundness error stacking in assembled spherical roller bearings and its influence on the subcritical vibration of large rotors. It is shown that the final roundness profile of the inner raceway is the sum of the roundness error of the tapered shaft seat and the thickness-variation profile of the ring. The results indicate that low-order harmonics of the inner-ring roundness deviation are the dominant source of subcritical excitation. Their reduction through improved control of mounting errors and ring-thickness variation significantly enhances rotor dynamic performance. Article [12] presents a nonlinear dynamic model of a ball bearing that incorporates raceway waviness, rotational speed, axial load, support stiffness, and eccentricity, and the model is subsequently validated experimentally. The study emphasizes that at high rotational speeds it becomes necessary to redefine the analytical frequency bands commonly used in vibration assessment and to account for the complex effects of waviness when evaluating bearing quality. The importance of accurately representing radial clearance and inner-raceway waviness for predicting subcritical vibrations of large, flexible rotors was demonstrated in study [13]. The authors also indicated that higher-order harmonics may reach considerable amplitudes and should be taken into account already at the rotor-design stage, as they increase wear and the risk of damage and degrade the operational performance of industrial machinery. Assembly errors also affect the vibration level generated by rolling bearings [14]. The study presented in [15] introduces a theoretical and experimental model describing the influence of internal clearance on bearing vibration behavior, showing that a small clearance provides a single stable equilibrium point and leads to regular, periodic vibrations, whereas a larger clearance results in multiple equilibrium points, system instability, and a transition to chaotic vibration. These findings are confirmed by the results reported in [16], where increasing clearance was shown to significantly increase vibration amplitude. Moreover, in ball bearings, the vibration level is correlated with the thickness of the lubricant film [17]. Another design parameter influencing vibration generation is the total curvature ratio, which characterizes the geometrical conformity between the rolling element and the raceway surface [18].

An analysis of the current state of knowledge shows that, based on a limited set of parameters, numerous mathematical models have been developed to predict vibration levels and assess the significance of individual influencing factors [19,20]. In study [21], robust regression was applied to model the vibration level of rolling bearings, with

rotational speed and misalignment used as explanatory variables. The use of robust regression was aimed at reducing the impact of outliers and violations of classical regression assumptions on the parameter-estimation process. Article [22] presents a method for predicting the remaining useful life (RUL) of rolling bearings, based on dynamic regression models and a new health indicator (HI), defined as the ratio of the current RMS vibration value to a baseline value. In study [23], a novel deep-learning model, MRRAE (Memory Residual Regression Autoencoder), was proposed for detecting anomalies in rolling bearings using data from their normal operation. The model employs a memory module and an autoregressive estimator to determine the probability-density distribution of memory-residual representations, and subsequently combines reconstruction errors with surprisal values to identify abnormal states.

Previous research by Zmarzły [24] investigated relationships between selected geometrical and design parameters of rolling ball bearings and the generated vibration levels using regression models. Although that study provided a basis for assessing parameter–vibration relationships, it did not examine how the adoption of different regression identification criteria affects the estimated model coefficients and approximation characteristics.

Although regression models have been widely used to investigate relationships between bearing parameters and vibration levels, the consequences of selecting different identification criteria remain insufficiently explored. In particular, the question is not only how the adopted criterion affects approximation errors, but also whether it changes the estimated relationships between explanatory variables and vibration level. This issue is especially relevant when regression models are intended for subsequent use as surrogate objective functions in engineering optimization, since differences in the estimated coefficients may influence the directions of model-predicted responses to changes in bearing parameters. Therefore, assessing the influence of the identification criterion on model characteristics represents an important step in evaluating their suitability for further engineering applications.

The aim of this study is to investigate how the choice of identification criterion affects the coefficients and approximation characteristics of linear regression models relating selected raceway geometrical and bearing design parameters to vibration levels. Two contrasting identification criteria were considered: ordinary least squares (OLS), which minimizes the sum of squared residuals, and minimax, which minimizes the maximum absolute residual. The minimax criterion was selected to examine how prioritizing the largest absolute fitting error, rather than minimizing the sum of squared errors, affects the estimated regression coefficients and the approximation characteristics of the resulting models. Both criteria were applied to identical experimental datasets using the same model structure and explanatory variables, allowing the influence of the identification objective to be examined independently of changes in model specification. Particular attention was given to differences in the estimated coefficients and their implications for the subsequent use of regression models as surrogate objective functions in bearing parameter optimization.

The main contribution of this study is to demonstrate that changing the identification criterion from OLS to minimax, while keeping the experimental data, explanatory variables, and model structure unchanged, can alter not only the approximation errors and coefficient magnitudes but also the signs of selected regression coefficients. These sign reversals are particularly relevant when regression models are considered for use as surrogate objective functions in bearing parameter optimization, as they may imply opposite model-predicted responses to changes in the same design variables. Unlike the earlier study [24], which investigated relationships between bearing parameters and vibration levels, the present work isolates the influence of the identification criterion on

the resulting model characteristics. The findings highlight the need to consider the identification criterion when interpreting regression models and assessing their potential suitability for subsequent optimization

2. Materials and Methods

In this study, experimental data obtained during the investigations partially described in a previous publication [24] were used. In the earlier work, only shielded bearings (6304-2Z) were analyzed, and the identification of regression models was performed using the classical least-squares method. The present scope of research was extended to include an analysis of open bearings and a comparison of two identification criteria for regression models: the least-squares method and the minimax criterion. This allowed for an assessment of how the adopted identification criterion affects the properties of the regression models and their suitability for further optimization studies.

Since a detailed description of the test rigs, measurement procedures, and methods for determining geometrical parameters was provided in [24], the current article is limited to information necessary to characterize the experimental material and the dataset used.

2.1. Experimental Dataset

The experimental material comprised a total of 180 deep groove ball bearings: open 6304 bearings and shielded 6304-2Z bearings, originating from three different manufacturers. For each manufacturer, 30 open bearings and 30 shielded bearings were examined, which made it possible to form two independent datasets, each containing 90 bearings per group. The bearing components are made of AISI 52100 bearing steel.

For each bearing, a set of raceway geometrical parameters and bearing structural parameters was determined, and vibration-level measurements were performed. Vibration measurements were carried out in three frequency ranges: low (LB), medium (MB), and high (HB). The resulting data formed the basis for analyzing the relationships between the examined bearing parameters and the level of generated vibrations.

For the identification of the regression models, the vibration level in the high-frequency band (HB), expressed in Anderon units, was used as the model output variable. The selection of this band was based on the results of earlier research [24], in which the model developed for the HB range exhibited the largest number of statistically significant predictors among the models obtained for the analyzed frequency bands, as well as a good quality of fit to the experimental data. These findings provided the basis for selecting the HB band for the present analysis of the relationship between the investigated bearing parameters and vibration level.

The LB and MB bands were not included in the present analysis, as the study was intended to examine the influence of the identification criterion within a selected frequency range rather than to compare regression models across different frequency bands.

2.2. Measured Parameters

For the identification of the regression models, six input variables were used, comprising raceway geometrical parameters and bearing design parameters, along with the vibration level. The analyzed parameters were adopted as the input variables of the regression models, whereas the vibration level in the high-frequency band (HB) served as the output variable. A summary of the analyzed variables is presented in Table 1.

Table 1. Variables used for regression model identification.

Symbol	Parameter	Unit	Variable Type
x_1	Roundness deviation of the inner raceway (2–15 upr)	µm	Input
x_2	$RONt_{PZ:(2-15)}$—Roundness deviation of the outer raceway (2–15 upr)	µm	Input
x_3	$RONt_{PW:(16-50)}$—Waviness deviation of the inner raceway (16–50 upr)	µm	Input
x_4	$RONt_{PZ:(16-50)}$—Waviness deviation of the outer raceway (16–50 upr)	µm	Input
x_5	ΔR—Radial clearance	µm	Input
x_6	f_t—Total curvature ratio	-	Input
y	Vibration level (HB band)	Anderon	Output

The selection of input variables was based on the results of previous studies and on established considerations regarding the influence of raceway geometrical parameters and bearing design parameters on their dynamic behavior [24]. To ensure comparability of the identification results in both analyzed approaches, an identical set of six input variables was adopted. Thus, the only factor differentiating the models during identification was the identification criterion applied—either the least-squares criterion or the minimax criterion. This approach enabled a direct assessment of the influence of the identification criterion on the parameter values of the models and on their approximation properties.

2.3. Experimental Procedure

The experimental procedure involved determining the raceway geometrical parameters and bearing design parameters of the examined bearings, as well as measuring the level of generated vibrations. Its main stages, together with the assignment of the determined quantities to the variables used in the regression models, are presented in Figure 1.

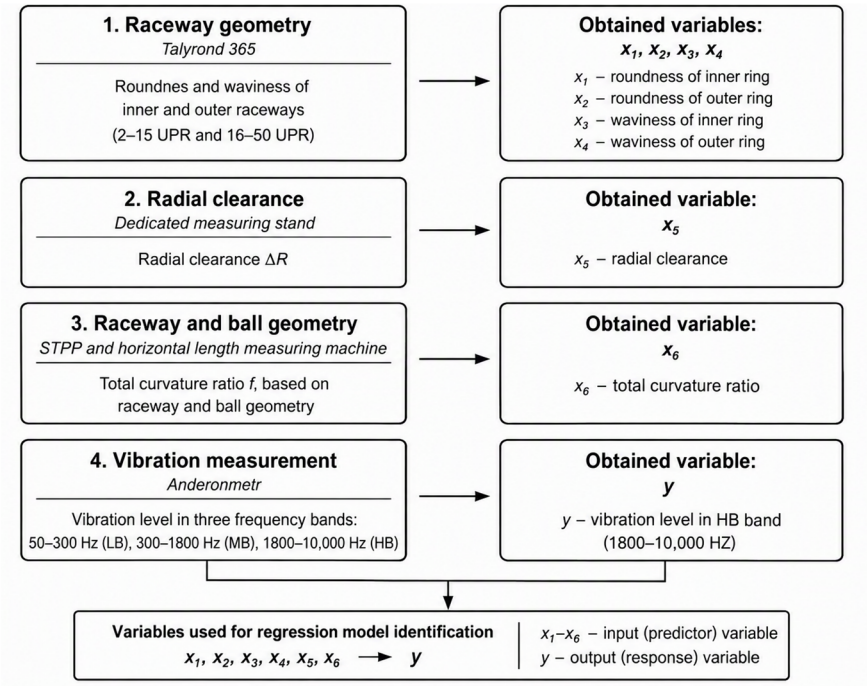


Figure 1. Experimental data acquisition scheme for regression model identification.

Roundness and waviness profiles of the inner and outer raceways were measured using a Talyrond 365 measuring system. The RONt parameter was analyzed in two ranges of undulations per revolution: 2–15 UPR, corresponding to roundness deviations, and 16-50 UPR, used for assessing raceway waviness. Based on these measurements, four geometrical parameters were determined, forming part of the explanatory variables $x_1 - x_4$.

The remaining input variables comprised bearing design parameters: the radial clearance ΔR and the total curvature ratio f_t. The radial clearance, corresponding to variable x_5, was determined using a dedicated measurement setup based on a direct measurement method. The total curvature ratio f_t, adopted as variable x_6, was calculated from the geometry of the raceways and rolling elements. Raceway radii were determined using the STPP system, while ball diameters were measured with a horizontal length measuring machine.

Vibration-level measurements were performed using an Anderonmeter system. The recorded signal was analyzed in three frequency bands: 50–300 Hz (LB), 300–1800 Hz (MB), and 1800–10,000 Hz (HB). For the identification of the regression models examined in this study, the vibration level in the HB band was used as the response variable y.

As a result of the measurement procedure, a set of six explanatory variables $x_1 - x_6$, comprising raceway geometrical parameters and bearing design parameters, together with the corresponding vibration-level value y, was obtained for each examined bearing. This dataset formed the basis for the identification of the regression models presented in the subsequent sections of the paper. A detailed description of the measurement equipment, test conditions, and the methods used to determine individual parameters is provided in [24].

The overall workflow of the conducted investigations is presented in Figure 2. Based on two experimental datasets corresponding to shielded and open bearings, independent identification of regression models describing the relationship between the analyzed bearing parameters and the vibration level in the HB band was performed. For both bearing groups, two identification criteria were applied: the least-squares method (OLS) and the minimax criterion.

The obtained models were evaluated based on the RMSE value, the maximum absolute error, and the coefficient of determination R^2. Subsequently, the properties of the models identified using the two criteria were compared separately for shielded and open bearings. The results of this comparison formed the basis for selecting the models for subsequent diagnostic assessment and predictive validation, preceding their potential application in bearing parameter optimization.

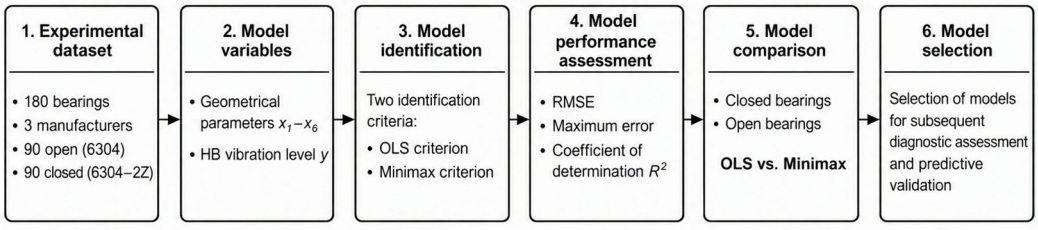


Figure 2. Workflow for regression model identification, assessment and selection.

3. Mathematical Formulation of the Parameter Identification Problem

The objective of the identification process is to determine the parameter vector of a mathematical model describing the relationship between the bearing vibration level and the analyzed bearing parameters. In this study, a linear regression model of the following form was adopted:

$$y_{m,i} = b_0 + \sum_{j=1}^{m} b_j x_{ij}, \quad i = 1, \ldots, n \tag{1}$$

where $y_{m,i}$ denotes the vibration-level value predicted by the model for the i-th observation, x_{ij} is the value of the j-th explanatory variable, b_j represents the model parameters, m is the number of explanatory variables, and n is the number of observations.

$$e_i = y_{r,i} - y_{m,i}, \quad i = 1, \ldots, n \tag{2}$$

where $y_{r,i}$ denotes the measured vibration level.

The parameter-identification problem consists of determining the values of the parameters $b_0, b_1, \ldots, b_m$, for which the adopted approximation error measure reaches its minimum. In this study, two identification criteria were compared: the classical least squares method and the minimax criterion.

3.1. Parameter Identification Using the Ordinary Least Squares (OLS) Method

The objective function for the least squares method was defined as:

$$F_{OLS}(b_0, b_1, \ldots, b_m) = \sum_{i=1}^{n} e_i^2 \tag{3}$$

By substituting expressions (1) and (2), one obtains:

$$F_{OLS}(b_0, b_1, \ldots, b_m) = \sum_{i=1}^{n} \left[y_{r,i} - \left(b_0 + \sum_{j=1}^{m} b_j x_{ij} \right) \right]^2 \tag{4}$$

The identification of the model parameters reduces to solving an optimization problem:

$$\min F_{OLS}(b_0, b_1, \ldots, b_m) \tag{5}$$

The solution of problem (5) yields the set of model parameters that minimizes the sum of squared approximation errors.

3.2. Parameter Identification Using the Minimax Criterion

The minimax criterion was selected as an alternative to ordinary least squares because it represents a fundamentally different objective in regression-model identification. Whereas OLS minimizes the sum of squared residuals, minimax minimizes the largest absolute deviation between measured and predicted vibration levels. This comparison makes it possible to examine how prioritizing the largest absolute fitting error, rather than overall squared-error performance, affects the estimated regression coefficients and the resulting model characteristics. The purpose of applying minimax was therefore not to introduce a new estimation method or to assume its superiority over OLS, but to investigate the consequences of two contrasting identification objectives for models intended for subsequent engineering optimization.

In the minimax criterion, the objective function takes the form:

$$F_{MM}(b_0, b_1, \ldots, b_m) = max|e_i|, \quad i = 1, \ldots, n. \tag{6}$$

By substituting expressions (1) and (2), one obtains:

$$F_{MM}(b_0, b_1, \dots, b_m) = max\left|y_{r,i} - \left(b_0 + \sum_{j=1}^{m} b_j x_{ij}\right)\right|, \quad i = 1, \dots, n. \tag{7}$$

The parameter-identification problem is formulated as the following optimization problem:

$$\min F_{MM}(b_0, b_1, \dots, b_m). \tag{8}$$

Due to the presence of the maximum operator and the absolute value, problem (8) cannot be solved directly using linear-programming methods. To address this, an auxiliary variable z is introduced, representing the largest absolute residual, which leads to the condition:

$$|e_i| \le z, \quad i = 1, \dots, n. \tag{9}$$

Condition (9) is equivalent to the following system of inequalities:

$$-z \le e_i \le z, \quad i = 1, \dots, n. \tag{10}$$

By substituting expressions (1) and (2), one obtains the following equivalent linear-programming problem:

$$\min z \tag{11}$$

subject to:

$$-z \le y_{r,i} - \left(b_0 + \sum_{j=1}^{m} b_j x_{ij}\right) \le z, \quad i = 1, \dots, n. \tag{12}$$

The solution of the optimization problem defined by (11)–(12) yields the set of model parameters that minimizes the maximum absolute approximation error.

3.3. Model Evaluation Criteria

To compare the models identified using the least-squares method and the minimax criterion, three approximation-quality measures were employed: the root-mean-square error (RMSE), the maximum absolute error, and the coefficient of determination R^2. These measures capture different aspects of model performance and enable a comprehensive assessment of the obtained results.

The root-mean-square error (RMSE) was defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \tag{13}$$

where e_i denotes the residual specified by expression (2).

The maximum absolute error was computed as:

$$e_{max} = max|e_i|, \quad i = 1, \dots, n. \tag{14}$$

The coefficient of determination was calculated using:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n}\left(y_{r,i} - \bar{y}_r\right)^2} \tag{15}$$

where:

$$\bar{y}_r = \frac{1}{n}\sum_{i=1}^{n} y_{r,i} \tag{16}$$

denotes the mean value of the measured vibration level.

Since the compared identification methods rely on different optimization criteria, the individual quality measures may indicate different solutions as more favorable. The least-squares method minimizes the sum of squared residuals and therefore leads to the lowest RMSE value and the highest coefficient of determination R^2 for a given dataset and model structure. In contrast, the minimax criterion minimizes the maximum absolute error, thereby limiting the largest deviation of the model-predicted value from the measured value. For this reason, all three measures were employed to compare the two identification approaches.

4. Results and Discussion

4.1. Correlation Analysis and Multicollinearity Assessment

Before estimating the model parameters, an analysis of the relationships among the explanatory variables was performed. The objective of this analysis was to assess the degree of multicollinearity among the input variables, which could affect the stability of parameter estimation as well as the interpretability of the resulting regression models. For this purpose, Pearson correlation coefficients and Variance Inflation Factors (VIF) were computed. The analyses were carried out separately for the datasets corresponding to shielded and open bearings.

The correlation matrices are presented in Tables 2 and 3, while the Variance Inflation Factor (VIF) values are summarized in Table 4. For the shielded bearings, the highest Pearson correlation coefficient (0.732) was observed between the radial clearance (ΔR) and the total curvature ratio (f_t). In the case of open bearings, the highest absolute Pearson correlation coefficient reached 0.425, indicating weaker linear relationships among the analyzed variables. In none of the examined cases were relationships identified that would indicate excessive multicollinearity among the explanatory variables.

Table 2. Pearson correlation matrix of the explanatory variables for 6304-2Z bearings.

Variable	$RONt_{PW:(2-15)}$	$RONt_{PZ:(2-15)}$	$RONt_{PW:(16-50)}$	$RONt_{PZ:(16-50)}$	ΔR	f_t
$RONt_{PW:(2-15)}$	1.000					
$RONt_{PZ:(2-15)}$	−0.075	1.000				
$RONt_{PW:(16-50)}$	−0.014	−0.376	1.000			
$RONt_{PZ:(16-50)}$	0.050	0.282	0.117	1.000		
ΔR	−0.254	0.296	0.183	0.367	1.000	
f_t	−0.175	0.540	−0.126	0.429	0.732	1.000

Table 3. Pearson correlation matrix of the explanatory variables for 6304 bearings.

Variable	$RONt_{PW:(2-15)}$	$RONt_{PZ:(2-15)}$	$RONt_{PW:(16-50)}$	$RONt_{PZ:(16-50)}$	ΔR	f_t
$RONt_{PW:(2-15)}$	1.000					
$RONt_{PZ:(2-15)}$	0.134	1.000				
$RONt_{PW:(16-50)}$	0.299	−0.079	1.000			
$RONt_{PZ:(16-50)}$	0.163	0.308	−0.147	1.000		
ΔR	−0.210	−0.174	0.047	−0.154	1.000	
f_t	0.365	0.350	−0.315	0.425	−0.433	1.000

Table 4. Variance inflation factors (VIF) of the explanatory variables.

Parameter	6304-2Z	6304
$RONt_{PW:(2-15)}$	1.10	1.48
$RONt_{PZ:(2-15)}$	1.67	1.18
$RONt_{PW:(16-50)}$	1.43	1.43
$RONt_{PZ:(16-50)}$	1.33	1.27
ΔR	2.69	1.25
f_t	3.16	2.06

All obtained VIF values were lower than the commonly accepted threshold of 5, indicating the absence of excessive multicollinearity among the explanatory variables. Slightly higher VIF values were observed for the models developed using data from shielded bearings; however, even in this case, the values did not indicate excessive multicollinearity among the explanatory variables. The results therefore indicate that none of the explanatory variables require elimination due to excessive collinearity, allowing the full set of analyzed parameters to be retained in the subsequent stages of model identification.

4.2. Comparison of OLS and Minimax Models for 6304-2Z Bearings

In the previous study [24], a linear regression model describing the relationship between the vibration level of shielded bearings and selected raceway geometrical parameters and bearing design parameters was identified using the ordinary least squares (OLS) method. In the present work, this model is used as a reference for comparison with the model obtained using the minimax identification criterion. This approach enables an assessment of the influence of the adopted identification criterion on both the regression coefficients and the approximation properties of the resulting model.

The regression coefficients obtained using both identification criteria are presented in Table 5. Differences were observed in their estimated values, with the most notable changes concerning the signs of the coefficients associated with outer raceway waviness (b_4) and radial clearance (b_5). In both cases, the sign reversal indicates opposite directions of model-predicted vibration responses to changes in the corresponding parameters, with the remaining explanatory variables held constant. These differences concern the fitted models and do not establish the underlying physical relationships.

Table 5. Regression model coefficients obtained for 6304-2Z bearings using the OLS and minimax identification criteria.

Coefficient	OLS	Minimax
b_0	−0.627	0.247
b_1	−0.075	−0.520
b_2	−0.200	−0.297
b_3	0.767	0.657
b_4	−0.509	0.969
b_5	58.533	−6.293
b_6	52.379	72.765

The quality of the obtained models was evaluated using the root-mean-square error (RMSE), the maximum absolute error, and the coefficient of determination R^2. The results are presented in Table 6.

Table 6. Approximation quality obtained for 6304-2Z bearings using the OLS and minimax identification criteria.

Criterion	RMSE	Maximum Absolute Error	R^2
OLS	0.381	1.385	0.689
Minimax	0.549	0.864	0.352

For 6304-2Z bearings, the OLS model exhibited better overall agreement with the experimental data used for identification, as indicated by a lower RMSE and a higher R^2 (Table 6). In contrast, the minimax model reduced the maximum absolute error by 37.62% relative to OLS, although this improvement was accompanied by a 44.09% increase in RMSE and a decrease in R^2. These results demonstrate a trade-off between reducing the largest individual fitting error and maintaining overall approximation accuracy. However, the observed differences characterize in-sample fitting performance and do not establish the predictive reliability of either model for previously unseen bearings.

Figure 3 presents the measured and predicted vibration levels for 6304-2Z bearings using the OLS and minimax models with trend line. The OLS predictions exhibit a more concentrated distribution around the ideal agreement line, whereas the minimax predictions show greater overall dispersion. These observations are consistent with the quantitative results presented in Table 6. The graphical comparison thus illustrates the differences in approximation characteristics resulting from the adopted identification criterion.

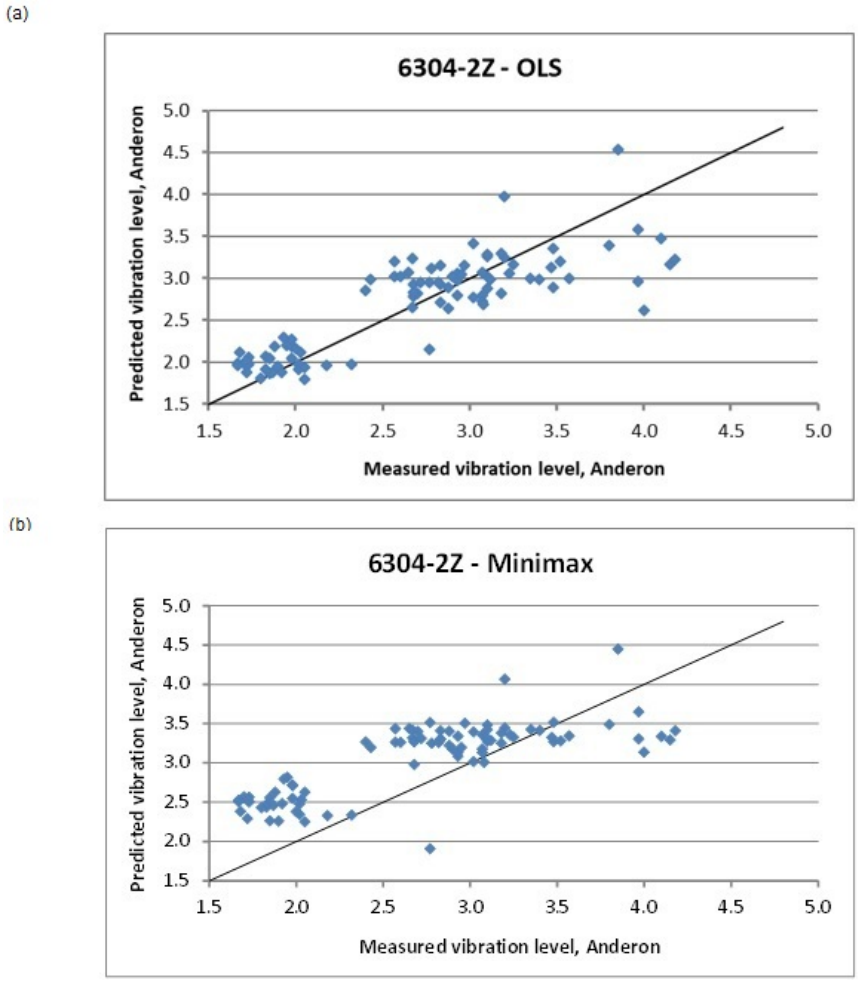


Figure 3. Measured versus predicted vibration levels for 6304-2Z bearings obtained using (**a**) the OLS criterion and (**b**) the minimax criterion.

4.3. Comparison of OLS and Minimax Models for 6304 Bearings

An analogous analysis was performed for open bearings to assess the influence of the adopted identification criterion on the regression coefficients and the quality of approximation of the experimental data. The coefficients obtained for the models identified using the least-squares method and the minimax criterion are presented in Table 7.

Table 7. Regression model coefficients obtained for 6304 bearings using the OLS and minimax identification criteria.

Coefficient	OLS	Minimax
b_0	−1.917	0.316
b_1	1.655	1.519
b_2	0.775	1.006
b_3	−1.673	−1.383
b_4	4.943	−3.647
b_5	39.397	76.082
b_6	76.725	60.561

Of particular interest is the coefficient associated with outer raceway waviness (b_4), whose sign changed from positive under OLS to negative under minimax (Table 7). This sign reversal indicates that the two models predict opposite directions of vibration-level change with increasing outer raceway waviness, assuming the remaining explanatory variables are held constant. A sign reversal was also observed for shielded bearings, although in the opposite direction.

The approximation performance of the OLS and minimax models for open 6304 bearings was evaluated using the same indicators as for shielded bearings (Table 8). The minimax model reduced the maximum absolute error by 38.15%, while RMSE increased by 79.45% relative to OLS. Notably, the coefficient of determination became negative, indicating that the minimax model produced a greater sum of squared residuals than a constant model predicting the mean vibration level. Compared with shielded bearings, the deterioration in overall approximation accuracy was substantially more pronounced, despite a comparable relative reduction in the maximum absolute error.

Table 8. Approximation quality obtained for 6304 bearings using the OLS and minimax identification criteria.

Criterion	RMSE	Maximum Absolute Error	R^2
OLS	1.027	4.676	0.369
Minimax	1.843	2.892	−1.028

Figure 4 compares the measured and predicted vibration levels for open 6304 bearings using the OLS and minimax models. The OLS predictions show better overall agreement with the experimental measurements, whereas the minimax predictions exhibit greater dispersion around the ideal agreement line. These observations are consistent with the approximation-error measures reported in Table 8. Compared with shielded bearings (Figure 3), the difference in overall fitting accuracy between the two identification approaches is more pronounced for open bearings.

The differences observed for both bearing configurations provide a basis for assessing the implications of the identification criterion for the subsequent use of regression models in bearing parameter optimization, as discussed in Section 4.4.

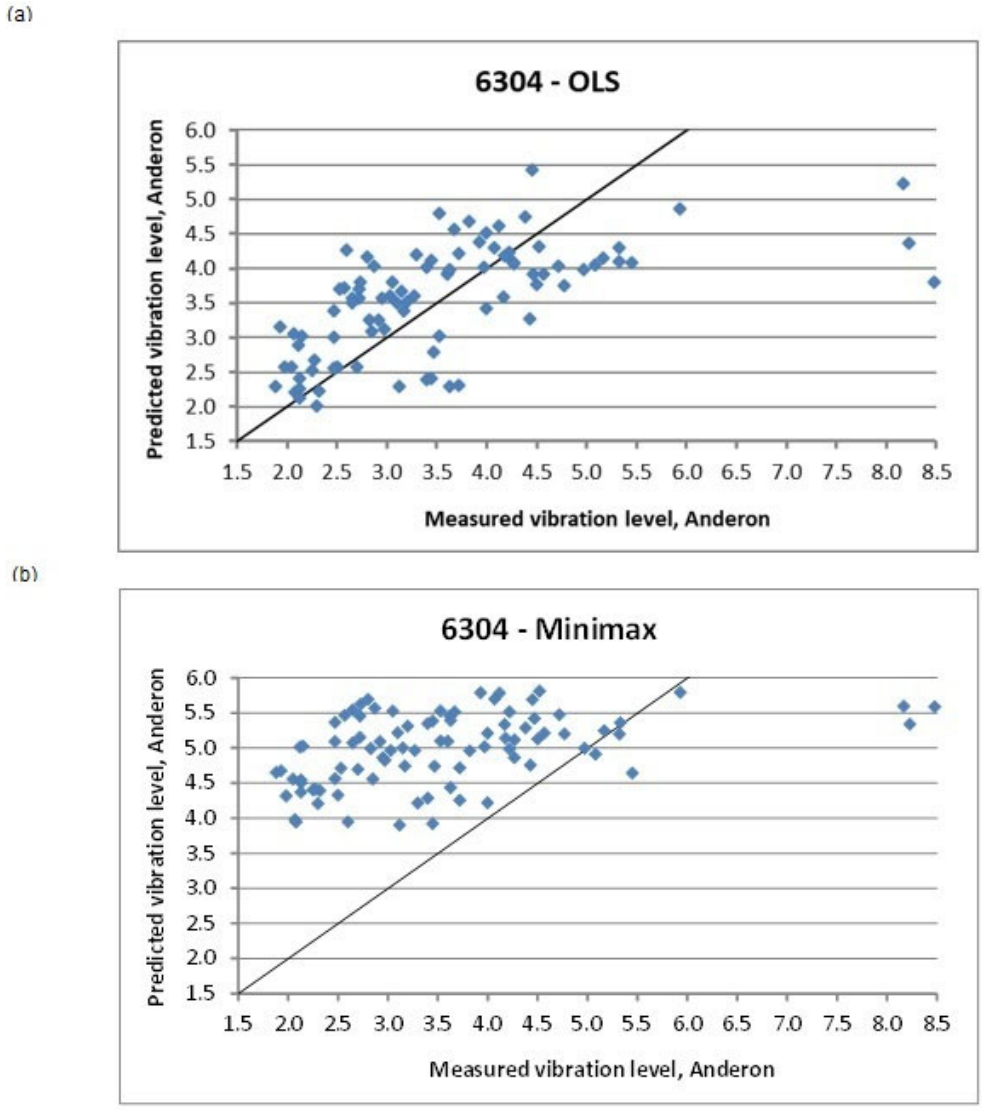


Figure 4. Measured versus predicted vibration levels for 6304 bearings obtained using (**a**) the OLS criterion and (**b**) the minimax criterion.

4.4. Comparison of Identification Criteria and Implications for Further Optimization

The comparative analysis revealed that the choice of identification criterion affects not only the approximation accuracy of rolling bearing vibration models but also the estimated relationships between the explanatory variables and vibration level. Of particular significance is the reversal of the regression coefficient associated with outer raceway waviness (b_4) between the OLS and minimax models, observed for both shielded and open bearings. An additional sign reversal was observed for the radial clearance coefficient (b_5), but only for shielded bearings. Despite using identical experimental datasets and model structures within each bearing group, the two identification criteria therefore produced models with different directional responses to changes in these parameters, which may have implications for their subsequent use in bearing parameter optimization.

Table 9 summarizes the approximation performance of the OLS and minimax models for both bearing groups. The results show that the trade-off between global fitting accuracy and maximum absolute error is considerably more pronounced for open bearings, particularly in terms of the deterioration in RMSE and R^2 under the minimax criterion. In particular, the minimax model for open bearings ($R^2 = -1.028$) exhibits insufficient overall fitting quality to be considered suitable for practical prediction or use as a surrogate objective function in optimization in its present form.

Table 9. Comparison of approximation quality obtained using the OLS and minimax identification criteria.

Bearing Type	Criterion	RMSE	Maximum Absolute Error	R^2
6304-2Z	OLS	0.381	1.385	0.689
6304-2Z	Minimax	0.549	0.864	0.352
6304	OLS	1.027	4.676	0.369
6304	Minimax	1.843	2.892	−1.028

The engineering significance of these results extends beyond the differences in approximation errors. In a linear regression model, the sign of a coefficient determines the direction of the predicted response to changes in the corresponding explanatory variable, assuming that the remaining variables are held constant. Consequently, the opposite signs of the coefficient associated with outer raceway waviness (b_4) in the OLS and minimax models imply different directions of the model-predicted vibration response for both bearing groups. A similar effect was observed for the radial clearance coefficient (b_5), but only for shielded bearings. This distinction is relevant when the regression equation is subsequently used as an objective function in bearing parameter optimization, since the identification criterion may influence the direction in which the model favors changes in these parameters.

Previous studies have highlighted the importance of raceway waviness in bearing vibration behavior. Hou et al. [12] investigated its influence using a nonlinear dynamic model, while Zmarzły [24] employed regression models to assess the relationships between selected geometrical and design parameters and vibration levels. Stoica et al. [21], applied robust linear regression to vibration-based bearing condition assessment, illustrating another approach to regression modeling in this field. Unlike these studies, the present work directly compares two identification criteria using identical experimental datasets and model structures. The observed reversals in selected regression coefficient signs demonstrate that the identification criterion can influence not only approximation accuracy but also the interpretation of model-predicted relationships between bearing parameters and vibration levels.

In the present study, the regression models are intended for subsequent use as surrogate objective functions in the optimization of bearing geometrical and design parameters. The comparison demonstrates that the identification criterion influences both approximation accuracy and the directional characteristics of the fitted models. Therefore, the selection of a model for further analysis should reflect the intended optimization objective rather than rely on a single measure of fitting accuracy. In the planned optimization study, the primary modeling objective is to represent the overall relationship between the investigated bearing parameters and vibration level. Accordingly, the OLS models were selected for subsequent diagnostic and predictive validation because they provided better global approximation of the experimental data for both bearing groups. Their suitability for use as surrogate objective functions in bearing parameter optimization remains to be assessed.

By substituting the coefficients obtained using the least-squares method into Eq. (1), the following regression model was obtained for the shielded bearings:

$$y = -0.627 - 0.075x_1 - 0.200x_2 + 0.767x_3 - 0.509x_4 + 58.533x_5 + 52.379x_6 \quad (17)$$

and for open bearings:

$$y = -1.917 + 1.655x_1 + 0.775x_2 - 1.673x_3 + 4.943x_4 + 39.397x_5 + 76.725x_6 \quad (18)$$

The same set of six explanatory variables was retained for both identification criteria within each bearing group to ensure that the observed differences in regression coefficients and approximation performance resulted from the identification criterion rather than changes in model structure. Variable selection was therefore not considered in the present study, as it would introduce an additional factor affecting the comparison.

The interpretation of the obtained results is subject to the assumptions of the adopted linear regression model. In particular, the observed differences in regression coefficients, including the reversal of the sign of b_4 in both bearing groups and b_5 in shielded bearings, describe differences between the fitted models and should not be interpreted as evidence of changes in the underlying physical relationships. Moreover, the assumed linear model structure does not necessarily capture the full complexity of the relationships between bearing geometrical and design parameters and vibration level.

The analysis was limited to vibration levels measured in the high-frequency band (HB), selected on the basis of previous studies indicating favorable regression-model performance in this frequency range. Consequently, the observed effects of the identification criterion cannot be directly generalized to other vibration frequency bands. Furthermore, the findings are specific to the investigated 6304 and 6304-2Z bearings and the ranges of geometrical and design parameters represented in the experimental data. Their applicability to other bearing types and operating conditions remains to be established.

These limitations should be taken into account in further research, particularly when considering the use of the identified models in bearing parameter optimization. The present study focuses on the influence of the identification criterion on regression coefficients and in-sample approximation characteristics, rather than on the predictive validation of the resulting models. Consequently, the reported approximation errors do not establish the predictive reliability of the models for previously unseen bearings. The selected OLS models will therefore be subjected to further diagnostic assessment and predictive validation before their suitability for optimization purposes is evaluated. Subsequent optimization should account for the experimentally investigated parameter ranges and the physical feasibility of their combinations, while the effectiveness of the resulting solutions in reducing vibration will require experimental verification.

5. Conclusions

A comparative analysis of two identification criteria for the parameters of linear regression models describing the relationship between selected raceway geometrical parameters and bearing design parameters and the generated vibration level was conducted. The study was performed for both open and shielded bearings, enabling an assessment of the influence of the adopted identification criterion on the regression coefficients and the properties of the resulting models.

The analyses showed that the identification criterion affected both the estimated regression coefficients and the approximation characteristics of the models. In both bearing groups, changing the criterion from OLS to minimax resulted in a reversal of the coefficient sign associated with outer raceway waviness, implying opposite directions of the model-predicted vibration response when the remaining variables are held constant. An additional sign reversal was observed for the radial clearance coefficient in shielded bearings. The OLS models achieved lower RMSE values and higher coefficients of determination, whereas minimax reduced the maximum absolute fitting error. The deterioration in overall fit under the minimax criterion was particularly pronounced for open bearings. Consequently, the minimax model for open bearings cannot be recommended for practical predictive or optimization applications in its present form.

The obtained results indicate that the choice of identification criterion should not be regarded solely as a numerical issue. The adopted objective function directly influences

the properties of the resulting regression model and may therefore affect its suitability for further engineering analyses. This is particularly important for surrogate models subsequently used as objective functions in optimization tasks, where accurate representation of the relationship between input variables and model response across the analyzed data domain is essential.

Based on the comparative analysis, the OLS models were selected for subsequent diagnostic assessment and predictive validation, as they provided better overall approximation of the experimental data for both bearing groups. This selection reflects the intended application of the models to representing the relationship between bearing parameters and vibration level across the investigated dataset. However, the observed fitting performance does not establish their predictive reliability or suitability for optimization purposes. These aspects require further investigation before the models can be used as surrogate objective functions in bearing parameter optimization.

The presented approach may also be applicable to other regression-model identification problems intended for optimization purposes. The results indicate that considering different identification criteria at the model-building stage may support the selection of a solution best suited to its intended application.

Further research will first focus on the diagnostic assessment and predictive validation of the selected OLS models to evaluate their reliability beyond the experimental data used for identification. Subject to satisfactory validation results, the models will subsequently be considered for use as surrogate objective functions in the optimization of bearing geometrical and design parameters aimed at minimizing vibration levels. The effectiveness of any optimized parameter combinations will require experimental verification. Further investigations may also extend the analysis to additional explanatory variables, vibration frequency bands, and bearing types.

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