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Asymmetrical Interleaved DC/DC Switching Converters for Photovoltaic and Fuel Cell Applications—Part 2: Control-Oriented Models

Eliana Arango ^{1,*}, Carlos Andres Ramos-Paja ¹, Javier Calvente ², Roberto Giral ²
and Sergio Ignacio Serna-Garces ³

¹ Universidad Nacional de Colombia, Medellin 05001000, Colombia; E-Mail: caramosp@unal.edu.co

² Universitat Rovira i Virgili, Tarragona 43007, Spain; E-Mails: javier.calvente@urv.cat (J.C.); roberto.giral@urv.cat (R.G.)

³ Instituto Tecnológico Metropolitano, Medellin 05001000, Colombia; E-Mail: sergioserna@itm.edu.co

* Author to whom correspondence should be addressed; E-Mail: eiarangoz@unal.edu.co;
Tel.: +57-4-4255344; Fax: +57-4-2341002.

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Abstract: A previous article has presented the members of the asymmetrical interleaved dc/dc switching converters family as very appropriate candidates to interface between photovoltaic or fuel cell generators and their loads because of their reduced ripple and increased current processing capabilities. After a review of the main modeling methods suitable for high-order converters operating, as the asymmetrical interleaved converters (AIC) ones, in discontinuous current conduction mode a full-order averaged model has been adapted and improved to describe the dynamic behavior of AIC. The excellent agreement between the mathematical model predictions, the switched simulations and the experimental results has allowed for satisfactory design of a linear-quadratic regulator (LQR) in a fuel-cell application example, which demonstrates the usefulness of the improved control-oriented modeling approach when the switching converters operate in discontinuous conduction mode.

Keywords: DC/DC switching converters; photovoltaic; fuel cell; interleaved topologies; asymmetrical interleaving; discontinuous conduction mode (DCM); control-oriented model

1. Introduction

DC-to-DC switching converters are frequently used to adapt the voltage and current levels provided by power generators, such as fuel cells or photovoltaic modules, to the ones required by electrical loads. Large switching ripples at the converters' input current reduce the efficiency of the photovoltaic generation and shorten the life expectancy of fuel cells. Adding input filters to the switching stages [1] or using conventionally interleaved converters [2] are some well known procedures to reduce the detrimental ripple amplitudes of the currents requested by switching converters from power generators. Conventional interleaving is the parallel connection of N identical switching stages whose control signals and subsequent current ripple waveforms are uniformly phase-shifted over the switching period. The resultant harmonic cancellation lowers ripple amplitudes by raising the effective switching frequency without increasing switching losses or device stresses [2]. In addition to these interleaving benefits, since the aggregated current is evenly shared among the paralleled stages, high amounts of power can be processed. However, specific current control strategies are required in conventional interleaving to ensure an even sharing of the current among the paralleled power modules [3–5]. Without these sharing control strategies [6], small inequalities between converter parameters in addition to a bidirectional current implementation of the switches result in large circulating currents associated with high losses and stresses in power inductors and semiconductor devices. The use of unidirectional current switches without current sharing control strategies could also have severe drawbacks such as the appearance of discontinuous conduction modes (DCM) [1]. In this case, one of the modules frequently operates in continuous conduction mode (CCM) and processes the largest amount of power, while the rest of the paralleled modules, being in DCM, transfer very small amounts of power from the generator to the load.

The asymmetrical interleaved converters (AIC) presented in [7] are a particular case of interleaving in which the inputs of two non-identical unidirectional converters are connected in parallel and an open-loop straightforward control strategy consisting in the complementary activation of the converter's switches is enforced. The breaking of the symmetry results in one of the stages to operate in CCM and the other in DCM but the specific form in which the non-identical converter's outputs are connected causes that a significant fraction of the input current is processed by the DCM phase. The simplicity of the open-loop control strategy required to obtain a partial input current sharing in addition with a significant reduction in the amplitudes of input and output current ripples are some of the advantages of the recently reported new high-order family of converters. Since discontinuous conduction modes are inherent to the AIC family, closing a regulation loop requires using specific modeling techniques that are reviewed in Section 2 from which the revised averaging method [8] has been selected as the most suitable one. The application of the revised averaging method to the boost derived asymmetrical interleaved dual boost (AIDB) converter is detailed in Section 3. However, as it will be seen in Section 4, the high-order of the converter and the simultaneous coexistence of continuous and discontinuous conduction have required an improved modeling approach that provides better agreement between the frequency responses of the model and the simulated switching converter. The results of applying the new modeling approach to the remaining members of the AIC family [asymmetrical interleaved dual buck-boost (AIDBB) and asymmetrical interleaved dual flyback (AIDF)] are provided in Section 5. Section 6 shows an example

of multivariable control design that uses the model previously obtained in Section 4. Finally, conclusions and some proposals for future studies are discussed in Section 7.

2. DCM Modeling Review

Power converter's usual modeling produces detailed continuous-time nonlinear time-varying models in state-space form. Since reasonable approximations such as neglecting dynamics above the switching frequency are usually made, these models could have rather low orders [9], but they could still be too detailed and difficult to work with. Therefore, the obtention of simplified and preferably time-invariant approximated models has been object of intensive research. The two main systematic ways to obtain the desired simplifications are averaging and sampled-data modeling. By excluding detailed switching intricacies in case of averaging approaches or by focusing on cycle-to-cycle behavior in case of sampled-data modeling, both methods can produce time-invariant but still nonlinear models. In comparison with converters in CCM, converters in DCM are characterized by the presence of internally controlled switching instants which results in additional topologies and complicate their modeling. Since most of the time the modeling methods applied to converters operating in DCM are a specialization of the CCM methods, DCM modeling methods can also be classified in reduced-order and full-order models. Focusing in DCM that, like in AIC converters, affect an inductor current, the main contributions in these two categories are discussed in next subsections. Analysis of less-common DCM can be found in [10,11].

2.1. Reduced-Order Models

The most referenced reduced-order model is [12] in which the authors view DCM modeling as a particular case of the state-space averaging CCM modeling described in [13]. The space-state averaging method is applied considering DCM converters as having three states with additional constraints that take into account the inductor current particular behavior. In the averaged three state system of equations, since the inductor current becomes always zero at the DCM subinterval, the time-derivative of the inductor current is considered to be null and the corresponding static equation is removed. This method considers that the inductor current is no longer a true state variable in DCM. In [14], this method was used to modeling the small signal behavior of the pulsewidth modulation (PWM) controlled Cuk converter operating in DCM. Since the analyzed Cuk converter had two inductors and two capacitors, two constraints were considered, one for each inductor current. A similar procedure applied to the single-ended primary-inductor converter (SEPIC) provided also third order small-signal transfer functions in [15]. More recently, some inconsistencies in the use of variables when using the state-space averaging method [12] have been revised in [8,16]. Since the corrections in averaging and duty-cycle constraints can yield also full order models, the revised averaging approach will be discussed with more detail in Subsection 2.2.

In the slow-fast variables approach [17,18], the converter state variables are partitioned into slow and fast ones, from which the averaged model is obtained in three steps. In DCM, the inductor current is considered a fast variable. This method provides the same reduced-order models obtained by using the classical averaging method but without the inconsistencies in the inductor current averaging calculation.

This method is not applicable to all converters because of the need for a period-one analytical solution of the relaxed fast subsystem. Non-periodic solutions of the fast subsystem exhibiting components at frequencies lower than the switching one significantly affect the slow subsystem.

Circuit averaging has also been proposed for modeling DCM converters in [19], where the resulting averaged model is the same reduced-order method provided by the previous methods. However, as it is shown in [8], reduced-order models predict responses that, in comparison with those obtained from switched simulations, exhibit significant discrepancies at high frequencies that are specially large in the phase plots.

2.2. Full-Order Models

Full order methods improve the model approximation to the converter real behavior at high frequencies. The method reported in [10] is based on the definition of an equivalent duty ratio m as a function of the actual duty ratio d . This equivalent duty ratio m is a load dependent function that is used instead of d in the CCM averaged model from where a small-signal ac model is derived. Therefore, the order of the model is the same as the order of the CCM state-space averaged model. Also, the zeroes of the control-to-output transfer function, including right-half plane (RHP) zeros if there are any, are the same in DCM than in CCM, exhibiting the same dependence on the circuit parameters and DC conversion ratio.

The approach proposed in [20] to obtain a full order model is to modify a previous CCM model of the PWM switch [20] by adding small-signal damping resistors. A debate on the correctness of this model attempting to replace the switching part of PWM converters without including information on the voltage across the DCM inductor was published in [21]. The switch model in [20] is referred to as c-PWM model in [22,23], where the alternate p-PWM and a-PWM models are proposed to facilitate the analysis of those converters whose PWM-switch-cell common terminal corresponds to the model. Interestingly in [23], converters in CCM can be analyzed by using particular simplified cases of the DCM switch models. Similarly, the unified switched inductor model (USIM) [24] considers that CCM is a limit or special case of DCM. Applying a geometric approach to obtain the relationships between the average inductor current and its peak value, a basic equivalent circuit of the switched inductor independent of the topology is constructed. It is claimed in [24] that when tested in large and small signal simulations, the model automatically follows CCM and DCM operation with fewer convergence problems in comparison with previous simulation models. A small-signal circuit model made of controlled current sources, an independent voltage source and resistors, where the principle of energy conservation was used to take into account transistor and diode parasitic resistances and the diode threshold voltage, was presented in [25]. This DCM circuit model that includes parasitics was applied to the analysis of a DCM boost converter whose control-to-output transfer function, input-to-output voltage transfer function, input and output impedances were derived. Unlike average models that use current and voltage dependent sources, reference [1] presents an equivalent model for the PWM DCM switch cell based on a dependent power source and a loss-free resistor (LFR).

Sampled data models constitute another important approach to modeling CCM and DCM converters. In [26], a zero order hold equivalent discrete-time model of the boost converter, which takes into

account non-ideal conductive losses in both CCM and DCM, is obtained, and the derived small-signal frequency response and closed loop behavior are verified experimentally. A comparative study between multi-frequency averaging and sampled-data models has been reported in [27]. The main conclusion of the comparative study is that sampled-data models can provide all the results of higher-order multi-frequency previously reported but with less computational effort. A sampled data approach has been adopted to find a discrete-time full order state-space model of a Cuk converter in [28]. In [29], a nonlinear discrete recurrence relating the state vector at the beginning of a cycle to the state vector at the end of the same cycle is used to obtain a large signal modeling of a boost converter operating in CCM and DCM. This method has some similarities with the current injected approach that is used in [30] to derive the transfer functions of the three basic converters operating in DCM at constant frequency with duty ratio control. A similar result using the current injected equivalent circuit approach is achieved in [31].

Starting with a review of the previous models of converters operating in DCM, the authors of [8] present a physically based three-step modeling procedure for PWM converters which serves as a general framework to compare different models, along with new full-order averaged models. The revised averaged models are provided in both analytical and circuital forms and are claimed to overcome the problems of existing models. The frequency response of a boost converter obtained with the revised averaging approach is compared with detailed switched simulations and it is found to be correctly predicted up to a third of the switching frequency where the phase starts to diverge from the detailed switched simulations. In comparison with the reduced [12] and full-order analytical methods [10,20], the frequency responses start to diverge at frequencies above one tenth of the switching frequency, and discrepancies are also detected in the prediction of poles and RHP zero.

3. Modeling the AIDB Converter Using the Revised Averaging Method

The discontinuous conduction mode, in which the AIDB converter works, limits the modeling possibilities of methods that are traditionally applied to CCM. Among the DCM modeling methods, the one that offers the best approximation is *the revised averaging method (RAM)* proposed in [8] due to its improved approximation of the high-frequency poles. This section presents the modeling of the AIDB converter using the RAM to illustrate its limitations in the reproduction of the converter dynamic behavior. The electrical circuit of the AIDB converter is depicted in Figure 1a, where the asymmetrical interleaved structure is observed. Moreover, Figure 1b shows the inductor current waveforms where the DCM condition is put in evidence at $d_3 \cdot T$. In addition, such a figure shows the three operation intervals that define the switching period: $d_1 \cdot T$, $d_2 \cdot T$ and $d_3 \cdot T$, which duty cycles are related as in Equations (1) and (2). It must be pointed out that the modeling procedures presented in this section are based on the circuital analyses given in [7] for the AIDB converter:

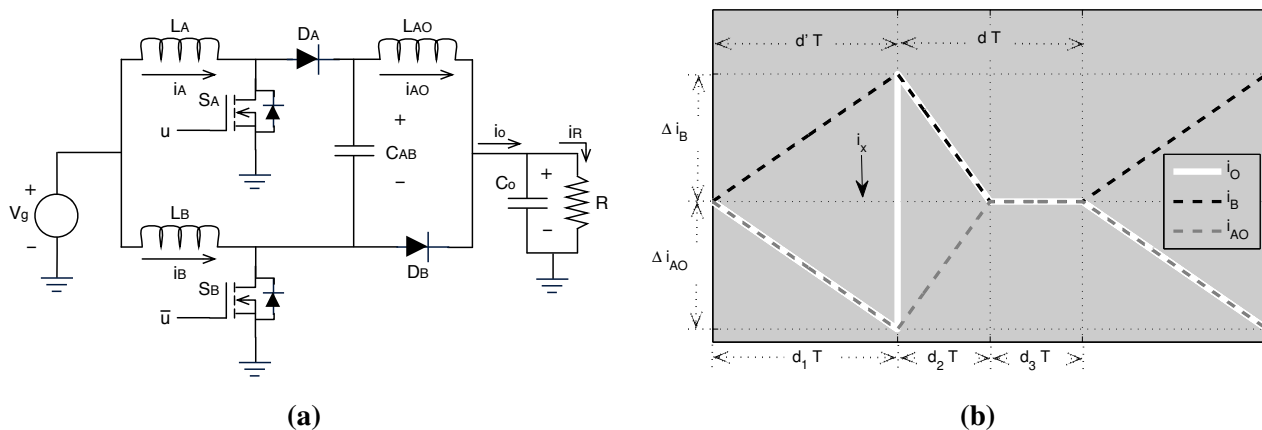
$$d_1 + d_2 + d_3 = 1 \quad (1)$$

$$d_1 = d'; d_2 + d_3 = d \quad (2)$$

The RAM consists of three steps: averaging, correction, and duty-cycle constraint definition [8]. The first step consist in the conventional state variables averaging as given in [1], while the second one consists of dividing all the inductor currents in the converter by the factor $(d_1 + d_2)$ since these are

the two duty ratios in which the inductors are conducting. Such a correction must be made because, in DCM, the inductor currents (or the discontinuous variables) have three intervals instead of the two intervals present in CCM. Figure 1b shows that such a DCM condition exists in the inductor currents i_B and i_{AO} of the AIDB converter.

Figure 1. Asymmetrical interleaved dual boost (AIDB) converter circuit and waveforms: (a) electrical scheme; and (b) discontinuous currents.

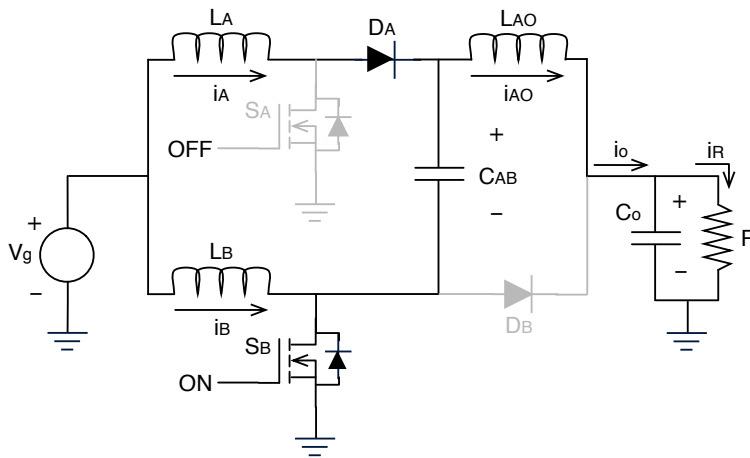


To perform the averaging procedure, the AIDB topologies and equations obtained in [7] must be analyzed. In addition, reference [7] was demonstrated that the AIC converters must be operated at duty cycles within $d > 38\%$, in which low ripple conditions are ensured. Therefore, the modeling approaches consider the three topologies exhibited by the AIDB for the duty cycles of interest:

- Topology 1 (S_B and D_A ON, S_A and D_B OFF): it is active in the first interval ($0 \leq t \leq d_1 \cdot T$, $d' = d_1$) during $d_1 \cdot T$, it begins when S_B is turned ON and S_A is turned OFF, and it ends when S_B is turned OFF and S_A is turned ON (Topology 2). In this topology i_B increases while i_A and i_{AO} decrease;
- Topology 2 (S_A and D_B ON, S_B and D_A OFF): it is active in the second interval [$d_1 \cdot T < t \leq (d_1 + d_2) \cdot T$] during $d_2 \cdot T$, it begins when S_B is turned OFF and S_A is turned ON, and it ends when D_B is automatically turned OFF (Topology 3). In this topology i_B decreases while i_A and i_{AO} increase;
- Topology 3 (S_A ON, S_B , D_A and D_B OFF): it is active in the third interval [$(d_1 + d_2) \cdot T \leq t < T$] during $d_3 \cdot T$, it begins when D_B is automatically turned OFF by the DCM condition generated when $i_B = i_{AO}$ (null current in D_B), and it ends when S_B is turned ON and S_A is turned OFF (Topology 1). In this topology i_B and i_{AO} are constant and equal while i_A still increases.

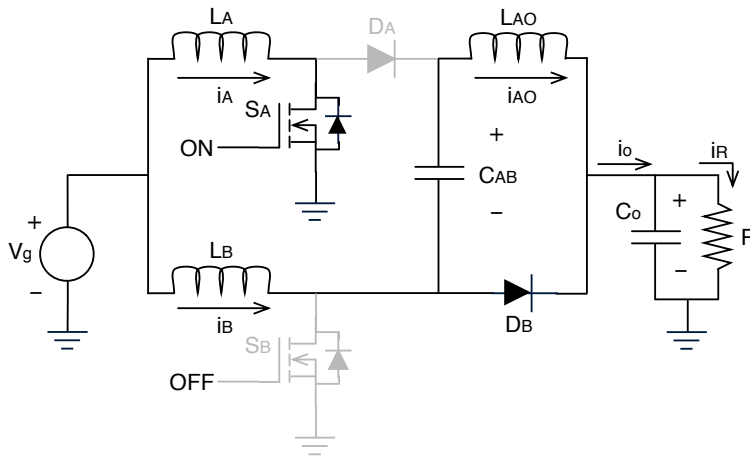
Figures 2–4 show the topologies and their equations, where v_{AB} and v_o are the voltages of capacitors C_{AB} and C_o , respectively. In Topology 3, the equation marked with an asterisk (*) reflects the reduction of order that occurs in DCM, which makes difficult to perform the averaging calculations.

Figure 2. Topology 1.



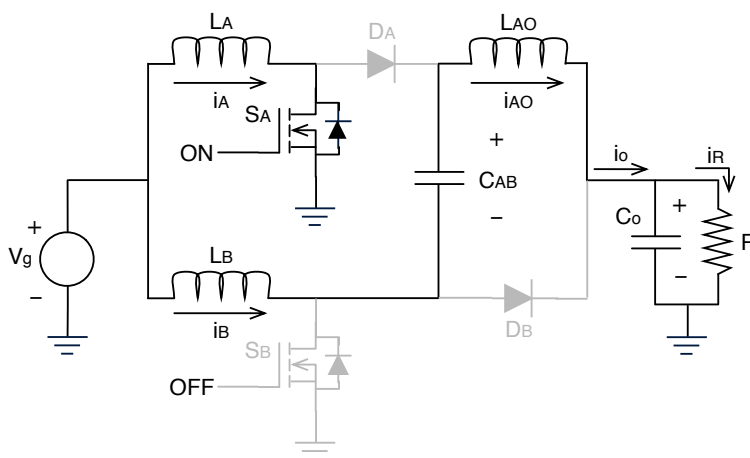
$$\begin{aligned}
 L_A \frac{di_A}{dt} &= v_g - v_{AB} \\
 L_B \frac{di_B}{dt} &= v_g \\
 L_{AO} \frac{di_{AO}}{dt} &= v_{AB} - v_o \\
 C_{AB} \frac{dv_{AB}}{dt} &= i_A - i_{AO} \\
 C_o \frac{dv_o}{dt} &= i_{AO} - \frac{v_o}{R}
 \end{aligned} \quad (3)$$

Figure 3. Topology 2.



$$\begin{aligned}
 L_A \frac{di_A}{dt} &= v_g \\
 L_B \frac{di_B}{dt} &= v_g - v_o \\
 L_{AO} \frac{di_{AO}}{dt} &= v_{AB} \\
 C_{AB} \frac{dv_{AB}}{dt} &= -i_{AO} \\
 C_o \frac{dv_o}{dt} &= i_B - \frac{v_o}{R}
 \end{aligned} \quad (4)$$

Figure 4. Topology 3.



$$\begin{aligned}
 L_A \frac{di_A}{dt} &= v_g \\
 (L_B + L_{AO}) \frac{di_B}{dt} &= v_g + v_{AB} - v_o \\
 i_{AO} &= i_B * \\
 C_{AB} \frac{dv_{AB}}{dt} &= -i_{AO} \\
 C_o \frac{dv_o}{dt} &= i_{AO} - \frac{v_o}{R}
 \end{aligned} \quad (5)$$

The averaged equations are obtained using the conventional method of multiplying each equation by the corresponding intervals and dividing it by the period [1]. Then, the state space averaged system (for $d > 38\%$) is:

$$\begin{aligned}
 \frac{d\bar{i}_A}{dt} &= \frac{v_g - \bar{v}_{AB}d_1}{L_A} \\
 \frac{d\bar{i}_B}{dt} &= \frac{v_g}{L_B}(d_1 + d_2) - \frac{\bar{v}_o}{L_B}d_2 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{i}_{AO}}{dt} &= \frac{\bar{v}_{AB}}{L_{AO}}(d_1 + d_2) - \frac{\bar{v}_o}{L_{AO}}d_1 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_Ad_1 - \bar{i}_{AO}}{C_{AB}} \\
 \frac{d\bar{v}_o}{dt} &= \frac{\bar{i}_{AO}}{C_o} - \frac{\bar{v}_o}{RC_o}
 \end{aligned} \tag{6}$$

In the correction step proposed in [8], all the equations are affected. But from the equations on Figures 2–4, it is noted that i_{AO} is always continuous in the AIDB, while i_B and i_{AO} are discontinuous in Topology 3. Therefore, in the AIDB it is only necessary to apply the correction to the equations in which i_B and i_{AO} appear since the CCM averaging in i_A is correct. Consequently, the averaged equations of i_B and i_{AO} must be divided by the factor $(d_1 + d_2)$, as described in [8], to obtain the corrected expressions:

$$\begin{aligned}
 \frac{d\bar{i}_A}{dt} &= \frac{v_g - \bar{v}_{AB}d_1}{L_A} \\
 \frac{d\bar{i}_B}{dt} &= \frac{1}{d_1 + d_2} \left[\frac{v_g}{L_B}(d_1 + d_2) - \frac{\bar{v}_o}{L_B}d_2 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \right] \\
 \frac{d\bar{i}_{AO}}{dt} &= \frac{1}{d_1 + d_2} \left[\frac{\bar{v}_{AB}}{L_{AO}}(d_1 + d_2) - \frac{\bar{v}_o}{L_{AO}}d_1 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \right] \\
 \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_Ad_1 - \bar{i}_{AO}}{C_{AB}} \\
 \frac{d\bar{v}_o}{dt} &= \frac{\bar{i}_{AO}}{C_o} - \frac{\bar{v}_o}{RC_o}
 \end{aligned} \tag{7}$$

To obtain the dynamic model, the duty cycle d_2 must be replaced in Equation (7) by an expression depending on other variables. Equation (8) gives the expression obtained in [8] for d_2 , where $\bar{i}_L$ is the average value of the discontinuous current and v_{on} is the voltage across the inductor when the switch is ON:

$$d_2 = \frac{2L\bar{i}_L}{d_1Tv_{on}} - d_1 \tag{8}$$

Since there are two discontinuous currents in the AIDB, it is possible to define d_2 from i_B as in Equation (9) or from i_{AO} as in Equation (10):

$$d_{2,i_B} = \frac{2L_B\bar{i}_B}{d_1Tv_g} - d_1 \tag{9}$$

$$d_{2,i_{AO}} = \frac{2L_{AO}\bar{i}_{AO}}{d_1T(\bar{v}_o - \bar{v}_{AB})} - d_1 \tag{10}$$

But replacing Equation (9) in Equation (7) produces an operating point that does not match the simulation results of [7], predicting a i_B value close to zero and predicting RHP poles that make this model unstable, it behaving totally differently from the converter. Similarly, by replacing Equation (10) in Equation (7), the prediction is worst since the variable i_B does not appear in the equations, which makes impossible to obtain a solution for the system. Therefore, it became necessary to explore another way to calculate d_2 to apply the RAM approach described in [8].

3.1. New Calculation Procedure for Duty Cycle d_2

The average converter currents are calculated using the areas of the triangles in the waveforms of Figure 1b, where the average value of i_B and i_{AO} currents are given in Equations (11) and (12), respectively. In such expressions, the peak currents Δi_B and Δi_{AO} are calculated from Equation (5), i.e., Topology 3, as in Equation (13). Then, the expression for d_2 given in Equation (14) is obtained by subtracting Equations (11) and (12):

$$\bar{i}_B = i_x + \frac{\Delta i_B}{2} (d_1 + d_2) \quad (11)$$

$$\bar{i}_{AO} = i_x - \frac{\Delta i_{AO}}{2} (d_1 + d_2) \quad (12)$$

$$\Delta i_B = \frac{v_g}{L_B} d_1 T; \quad \Delta i_{AO} = \frac{\bar{v}_o - \bar{v}_{AB}}{L_{AO}} d_1 T \quad (13)$$

$$d_2 = \frac{2(\bar{i}_B - \bar{i}_{AO})}{\left(\frac{v_g}{L_B} + \frac{\bar{v}_o - \bar{v}_{AB}}{L_{AO}}\right) d_1 T} - d_1 \quad (14)$$

3.2. RAM Applied to the AIDB Using the New d_2 Value

Replacing Equations (1) and (14) in Equation (7) leads to the dynamic model of the AIDB (for $D > 38\%$) based on the approach proposed in [8]:

$$\begin{aligned} \frac{d\bar{i}_A}{dt} &= \frac{v_g - \bar{v}_{AB} d_1}{L_A} \\ \frac{d\bar{i}_B}{dt} &= \frac{2\bar{i}_B L_B L_{AO}^2 (a_1) + 2\bar{i}_{AO} L_B L_{AO}^2 (a_2) + \bar{v}_{AB} d_1 T L_B^2 (a_3) + \bar{v}_o d_1 T L_B L_{AO} (a_4) + d_1 T V_g (a_5)}{2L_{AO} L_B^2 (L_{AO} + L_B) (-\bar{i}_B + \bar{i}_{AO})} \\ \frac{d\bar{i}_{AO}}{dt} &= \frac{2(\bar{i}_{AO} - \bar{i}_B) L_B L_{AO} (a_6) + d_1 T V_g L_{AO}^2 (a_7) + \bar{v}_o d_1^2 T L_B (a_8) + d_1 T L_{AO} L_B (a_9)}{2L_{AO}^2 L_B (L_{AO} + L_B) (-\bar{i}_B + \bar{i}_{AO})} \\ \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_A d_1 - \bar{i}_{AO}}{C_{AB}} \\ \frac{d\bar{v}_o}{dt} &= \frac{\bar{i}_{AO}}{C_o} - \frac{\bar{v}_o}{RC_o} \end{aligned} \quad (15)$$

where:

$$\begin{aligned}
 a_1 &= -V_g + \bar{v}_o + \bar{v}_{AB} & a_2 &= V_g - \bar{v}_o - \bar{v}_{AB} & a_3 &= V_g + \bar{v}_{AB} + \bar{v}_o d_1 - 2\bar{v}_o \\
 a_4 &= V_g - d_1 V_g + d_1 \bar{v}_{AB} - \bar{v}_o & a_5 &= (-V_g - \bar{v}_{AB}) L_B L_{AO} - \bar{v}_o L_B^2 - \bar{v}_o d_1 L_{AO}^2 \\
 a_6 &= (-V_g + \bar{v}_o) L_{AO} + \bar{v}_{AB} L_B & a_7 &= -V_g + \bar{v}_o (d_1 + 1) - \bar{v}_{AB} & a_8 &= -\bar{v}_{AB} + \bar{v}_o \\
 a_9 &= d_1 \bar{v}_o (V_g - \bar{v}_{AB}) + V_g (\bar{v}_{AB} - \bar{v}_o) - 2\bar{v}_{AB} \bar{v}_o + \bar{v}_{AB}^2 + \bar{v}_o^2
 \end{aligned} \tag{16}$$

The input and state variables of the model are represented in terms of steady-state and small signal components as in Equations (17) and (18), where $x = [i_A \ i_B \ i_{AO} \ v_{AB} \ v_o]^T$. Moreover, $\bar{x}$ are the averaged states, X , V_g and D' are the steady-state values while $\hat{x}$, $\hat{v}_g$ and $\hat{d}_1$ are small-signal quantities:

$$\bar{x} = X + \hat{x}; \quad v_g = V_g + \hat{v}_g \tag{17}$$

$$d_1 = D' + \hat{d}_1 \tag{18}$$

Then, the steady-state condition given in Equation (19) is obtained by solving Equation (15) with all the derivatives equal to zero, where the capital letters indicate steady-state values for the states in X :

$$\left\{ \begin{aligned} I_A &= \frac{1}{D'} \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} \\ I_B &= \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} + \frac{V_g T (L_{AO} + L_B)}{2 L_{AO} L_B} D'^2 (1 + D') \\ I_{AO} &= \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} \\ V_{AB} &= \frac{V_g}{D'} \\ V_o &= \left(1 + \frac{1}{D'} \right) V_g \end{aligned} \right. \tag{19}$$

The prediction provided by such steady-state expressions has been contrasted with numerical values obtained from electrical simulations (using PSIM software) of the AIDB converter with $L_A = 246 \mu\text{H}$, $L_B = 222 \mu\text{H}$, $L_{AO} = 217 \mu\text{H}$, $C_{AB} = 50 \mu\text{F}$, $C_O = 23.5 \mu\text{F}$, $R = 10 \Omega$, and $T = 20 \mu\text{s}$ (switching period) and duty cycles from 40% to 90% each 10%. In such conditions, expressions from Equation (19) exhibit a Root Mean Squared Error (RMSE) equal to 0.3%.

Similarly, the small-signal model, given in Equation (20), of the state space averaged system, *i.e.*, Equation (15), is obtained by linearizing, around the operating point, the states and inputs Jacobians A and B as in Equations (21) and (22), respectively:

$$\hat{\dot{x}} = A\hat{x} + B\hat{d}_1 \tag{20}$$

$$A = \left. \frac{\partial f(\bar{x}, d_1)}{\partial \bar{x}} \right|_{\bar{x}=X, d_1=D'} \tag{21}$$

$$B = \left. \frac{\partial f(\bar{x}, d_1)}{\partial d_1} \right|_{\bar{x}=X, d_1=D'} \tag{22}$$

To test the model accuracy in the reproduction of the converter dynamic behavior, a numerical small-signal model was obtained for the converter parameters previously described. Moreover, the AIDB converter was simulated in PSIM to obtain the circuit frequency response, which was contrasted with the small-signal model predictions. Figures 5 and 6 show the Bode diagrams of the states $x = [i_A \ i_B \ i_{AO} \ v_{AB} \ v_o]^T$ obtained with both the model and the circuit, where the model approximation is not accurate enough: at low frequencies the model reproduces with very small differences the circuit behavior, but at high frequencies the model exhibits large errors. Hence, the model obtained following the approach given in [8] is not suitable for control purposes, or even for dynamic analyses, due to its large errors. Therefore, a new approach must be adopted to find a more accurate dynamic model suitable for control design.

Figure 5. AIDB Bode diagrams from circuit simulation and revised averaging method (RAM) simulation: (a) i_A ; (b) i_B ; (c) i_{AO} ; and (d) v_{AB} .

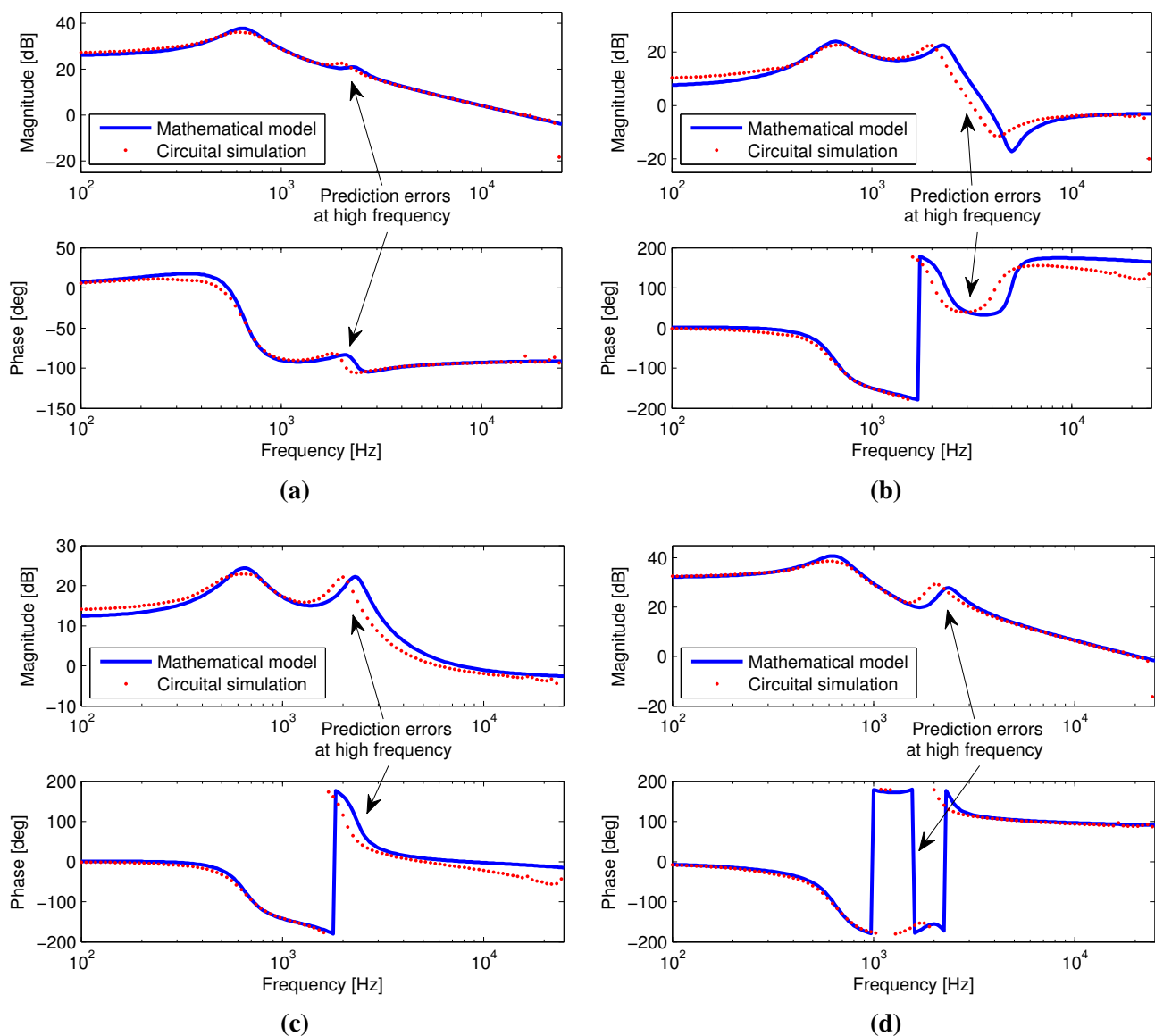
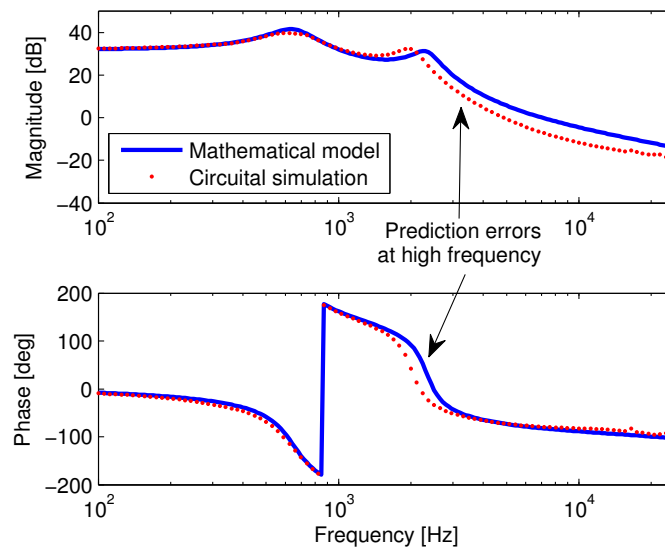


Figure 6. AIDB Bode diagrams from circuit simulation and RAM simulation: output voltage v_o .



4. Improved Method for Modeling High-Order Converters in DCM

This method has the aim of improving the prediction of dynamical behaviors in converters operating in DCM, it focused in the AIC family despite it is applicable to any converter. The design of this method was necessary due to the simultaneous appearance of discontinuous currents (with three intervals) and continuous currents (with two intervals) in the AIDB converter, as in the AIC family also. In theory, if both continuous and discontinuous currents were not concurrent, the RAM would provide an accurate approach. In such a way, this new method averages the state variables of the converter in a conventional way but includes two changes to improve the approximation of the model: the definition of an additional variable to describe the behavior of discontinuous variables and a novel procedure to calculate the duty cycle d_2 .

The steps of the new *improved averaging method*, named IAM, are the following:

1. Identify new variables that affect the converter dynamics using circuit analyses;
2. Include in the topologies equations the new variable;
3. Obtain the averaged equations from the modified topologies equations;
4. Calculate the correct expression for the duty cycle d_2 ;
5. Calculate the new variable in terms of the circuit states and inputs;
6. Replace both the new variable and d_2 in the averaged equations;
7. Calculate the steady-state and small-signal expressions.

In the following subsections, the IAM is illustrated by modeling the AIDB converter. In addition, such a model is contrasted with circuit simulations and experimentally tested to validate the IAM.

4.1. Modeling the AIDB Using the IAM

To illustrate the IAM technique, each step is described considering an AIDB converter:

- **Step 1:** From the AIDB scheme in Figure 1a, it is noted that the output current I_O is obtained by adding two currents: the current in the inductor L_{AO} and the current in the diode D_B . Therefore, to provide an accurate prediction of the AIDB output current, it is necessary to include the current in diode D_B in the analytical model as the new variable i_{DB} . In this way, the error generated in RAM [8] by neglecting this current is avoided.
- **Step 2:** From the circuital topologies (Figure 3) it is noted that diode D_B conducts in Topology 2. Therefore, the expressions in Equation (4) describing this topology are modified to include the current i_{DB} as follows:

$$\begin{aligned}
 L_A \frac{di_A}{dt} &= v_g \\
 L_B \frac{di_B}{dt} &= v_g - v_o \\
 L_{AO} \frac{di_{AO}}{dt} &= v_{AB} \\
 C_{AB} \frac{dv_{AB}}{dt} &= -i_{AO} \\
 C_o \frac{dv_o}{dt} &= i_{AO} + i_{DB} - \frac{v_o}{R}
 \end{aligned} \tag{23}$$

- **Step 3:** The new state space averaged system ($D > 38\%$) including the new variable i_{DB} is given in Equation (24), where $\bar{i}_{DB}$ represents the averaged value of i_{DB} :

$$\left\{ \begin{aligned}
 \frac{d\bar{i}_A}{dt} &= \frac{v_g - \bar{v}_{AB}d_1}{L_A} \\
 \frac{d\bar{i}_B}{dt} &= \frac{v_g}{L_B} (d_1 + d_2) - \frac{\bar{v}_o}{L_B}d_2 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{i}_{AO}}{dt} &= \frac{\bar{v}_{AB}}{L_{AO}} (d_1 + d_2) - \frac{\bar{v}_o}{L_{AO}}d_1 + \frac{v_g - \bar{v}_o + \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_A d_1 - \bar{i}_{AO}}{C_{AB}} \\
 \frac{d\bar{v}_o}{dt} &= \frac{\bar{i}_{AO} + \bar{i}_{DB}}{C_o} - \frac{\bar{v}_o}{RC_o}
 \end{aligned} \right. \tag{24}$$

- **Step 4:** The procedure to calculate the correct duty cycle d_2 , for the AIDB converter, was described in Subsection 3.1 leading to Equation (14). Moreover, the duty cycle d_3 is calculated from the fundamental relation given in Equation (1).
- **Step 5:** Taking into account that diode D_B only conducts in Topology 2 with $i_{DB} = i_B - i_{AO} > 0$, it describes a triangular waveform in i_{DB} with a peak current equal to $\Delta i_B + \Delta i_{AO}$: since $di_B/dt < 0$ and $di_{AO}/dt > 0$ in Topology 2, the maximum current in D_B (peak current) is achieved at the beginning of the interval $[d_1 \cdot T, (d_1 + d_2) \cdot T]$ when both i_B and i_{AO} exhibit their peak currents,

i.e., Δi_B and Δi_{AO} , respectively. Then, $\bar{i}_{DB}$ is calculated as in Equation (25), where substituting the expressions for Δi_B and Δi_{AO} obtained in Equation (13) and the expression of d_2 given in Equation (14), the $\bar{i}_{DB}$ expression given in Equation (26) is obtained:

$$\bar{i}_{DB} = \frac{\Delta i_B + \Delta i_{AO}}{2} d_2 \quad (25)$$

$$\bar{i}_{DB} = \left(\frac{v_g}{L_B} + \frac{\bar{v}_o - \bar{v}_{AB}}{L_{AO}} \right) \frac{d_1 T}{2} \left(\frac{2(\bar{i}_B - \bar{i}_{AO})}{\left(\frac{v_g}{L_B} + \frac{\bar{v}_o - \bar{v}_{AB}}{L_{AO}} \right) d_1 T} - d_1 \right) \quad (26)$$

- **Step 6:** Replacing Equations (1), (14) and (26) in Equation (24), the new averaged equations in terms of the duty cycle $d_1 = d'$ are:

$$\begin{aligned} \frac{d\bar{i}_A}{dt} &= \frac{v_g - \bar{v}_{AB}d_1}{L_A} \\ \frac{d\bar{i}_B}{dt} &= \frac{2\bar{i}_B L_B L_{AO}^2 (a_1) + 2\bar{i}_{AO} L_B L_{AO}^2 (a_2) + \bar{v}_{AB} d_1 T L_B^2 (a_3) + \bar{v}_o d_1 T L_B L_{AO} (a_4) + d_1 T V_g (a_5)}{(-V_g L_{AO} - L_B \bar{v}_o + L_B v_{AB}) L_B (L_{AO} + L_B) d_1 T} \\ \frac{d\bar{i}_{AO}}{dt} &= \frac{2(\bar{i}_{AO} - \bar{i}_B) L_B L_{AO} (a_6) + d_1 T V_g L_{AO}^2 (a_7) + \bar{v}_o d_1^2 T L_B (a_8) + d_1 T L_{AO} L_B (a_9)}{(-V_g L_{AO} - L_B \bar{v}_o + L_B v_{AB}) L_{AO} (L_{AO} + L_B) d_1 T} \\ \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_A d_1 - \bar{i}_{AO}}{C_{AB}} \\ \frac{d\bar{v}_o}{dt} &= \frac{R_L [2\bar{i}_B L_B L_{AO} - d_1^2 T (V_g L_{AO} + \bar{v}_o L_B - \bar{v}_{AB} L_B)] - 2\bar{v}_o L_B L_{AO}}{C_o R_L L_B L_{AO}} \end{aligned} \quad (27)$$

where:

$$\begin{aligned} a_1 &= -V_g + \bar{v}_o + \bar{v}_{AB} & a_2 &= V_g - \bar{v}_o - \bar{v}_{AB} & a_3 &= V_g + \bar{v}_{AB} + \bar{v}_o d_1 - 2\bar{v}_o \\ a_4 &= V_g - d_1 V_g + d_1 \bar{v}_{AB} - d_1 \bar{v}_o & a_5 &= (-V_g - \bar{v}_{AB}) L_B L_{AO} - \bar{v}_o L_B^2 - \bar{v}_o d_1 L_{AO}^2 \\ a_6 &= (-V_g + \bar{v}_o) L_{AO} + \bar{v}_{AB} L_B & a_7 &= -V_g + \bar{v}_o (d_1 + 1) - \bar{v}_{AB} & a_8 &= -\bar{v}_{AB} + \bar{v}_o \\ a_9 &= d_1 \bar{v}_o (V_g - \bar{v}_{AB}) + V_g (\bar{v}_{AB} - \bar{v}_o) - 2\bar{v}_{AB} \bar{v}_o + \bar{v}_{AB}^2 + \bar{v}_o^2 \end{aligned} \quad (28)$$

- **Step 7:** The calculation of the expressions for both the steady-state and small-signal model are performed following the procedures given in Subsection 3.2, where the states vector is defined as $x = [i_A \ i_B \ i_{AO} \ v_{AB} \ v_o]^T$. Again, the input and state variables of the model are represented in terms of steady-state and small signal components as in Equations (17) and (18).

The small-signal model, i.e., $\hat{x} = A\hat{x} + B\hat{d}_1$, of the state space averaged system, i.e., Equation (27), is obtained by linearizing, around the steady-state operating point, the states and inputs Jacobians A and B as in Equations (21) and (22), respectively.

Then, the steady-state condition given in Equation (29) is obtained by solving Equation (27) with all the derivatives equal to zero, where the capital letters indicate steady-state values. Expressions in Equation (29) include modifications in the equations of the three currents (I_A , I_B and I_{AO}), with respect to Equation (19), which are the result of the correction introduced by i_{DB} current. Such modifications

improve the prediction of the steady-state conditions: the prediction of those steady-state expressions exhibit a RMSE equal to 0.01% with respect to the electrical simulations performed with the same circuit parameters reported in Subsection 3.2. Therefore, the proposed model improves the accuracy of the steady-state calculations in one order of magnitude. But the most important improvement of the new model concerns its high accuracy in dynamic conditions, which will be illustrated in the following subsection by means of simulation and experimental results:

$$\left\{ \begin{array}{l} I_A = \frac{1}{D'} \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} - \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}} \right) D'^2 T \\ I_B = \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} + \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}} \right) D'^2 T \\ I_{AO} = \left(1 + \frac{1}{D'} \right) \frac{V_g}{R} - \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}} \right) D'^3 T \\ V_{AB} = \frac{V_g}{D'} \\ V_o = \left(1 + \frac{1}{D'} \right) V_g \end{array} \right. \quad (29)$$

4.2. Simulation and Experimental Validation

To verify the accuracy of the dynamic model for the AIDB converter obtained with IAM, *i.e.*, Equation (27), the IAM model was parameterized with the values reported in Subsection 3.2. Both the IAM model and PSIM circuit (from Subsection 3.2) were simulated to obtain the IAM and circuitual frequency responses presented in Figures 7 and 8: the high accuracy of IAM over RAM at high frequency is observed for all the states, but also improvements at low frequency are obtained with this new modeling technique for the prediction of the currents. Such simulation results put in evidence the satisfactory prediction of the AIDB dynamics provided by the IAM, which makes such a model useful for control purposes.

In addition to the simulations, an experimental prototype with the same circuit parameters was used to obtain the frequency response of a practical AIDB converter: Figure 9a shows the block diagram of the experimental setup, where a frequency response analyzer (FRA) excites the converter by means of an error amplifier (for isolation) and a PWM (to drive the switches) with multiple frequencies. Then, the signals at the PWM input and converter output are measured to calculate the experimental frequency response of the output voltage. Figure 9b shows the test bench, where the FRA and electrical circuits used in the experiments are observed.

Figure 10 shows the Bode diagrams obtained by means of the new model (IAM) and the one measured experimentally (FRA) for the output voltage. The figure shows the satisfactory prediction provided by the model at both high and low frequency conditions with some small errors in the phase. This result validates the IAM approach, demonstrating its accurate prediction of the converter dynamic behavior, which in turn is useful for control purposes.

Figure 7. AIDB Bode diagrams from circuit simulation and improved averaging method (IAM) simulation: (a) i_A ; (b) i_B ; (c) i_{AO} ; and (d) v_{AB} .

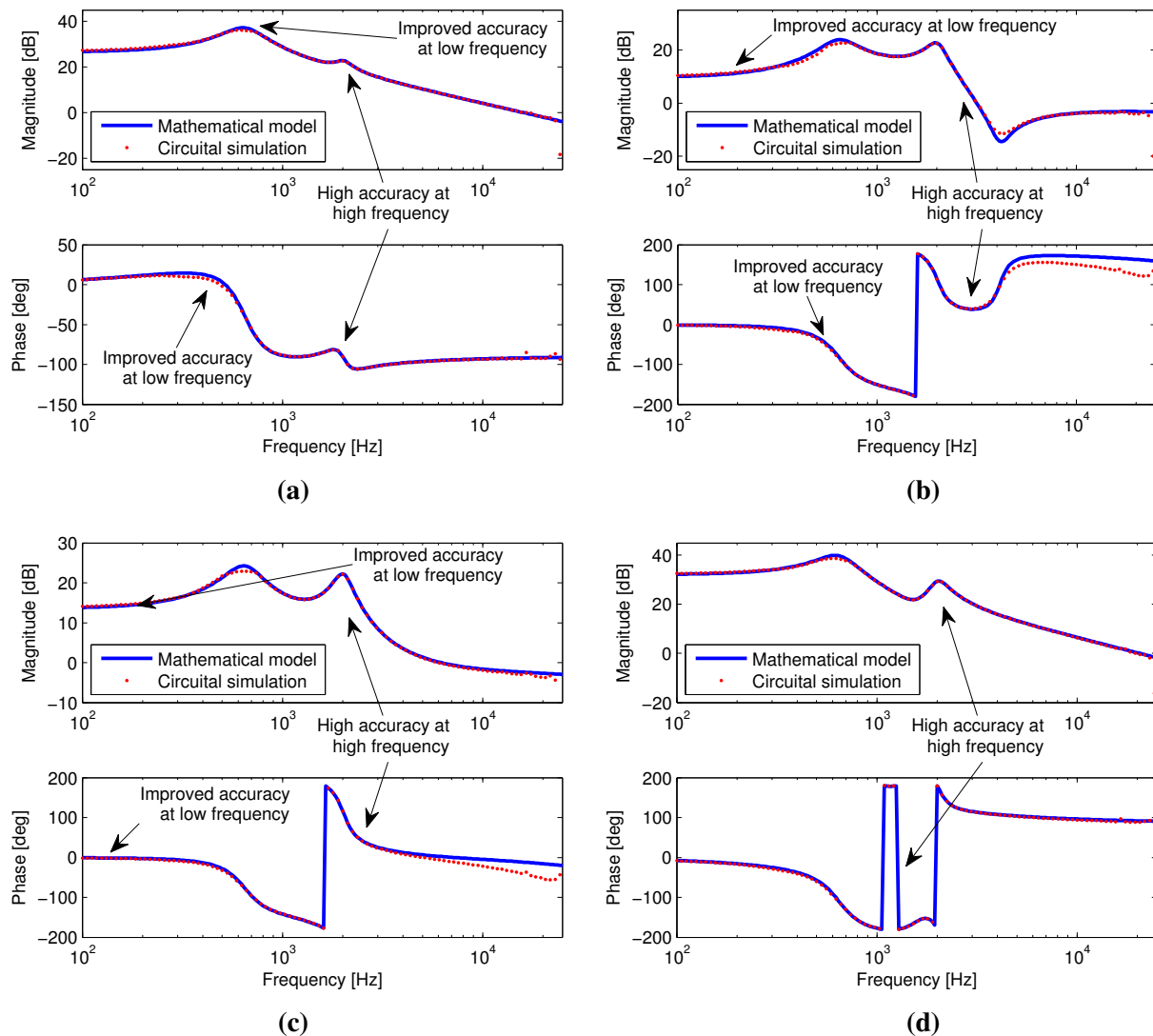


Figure 8. AIDB Bode diagrams from circuit simulation and IAM simulation: output voltage v_o .

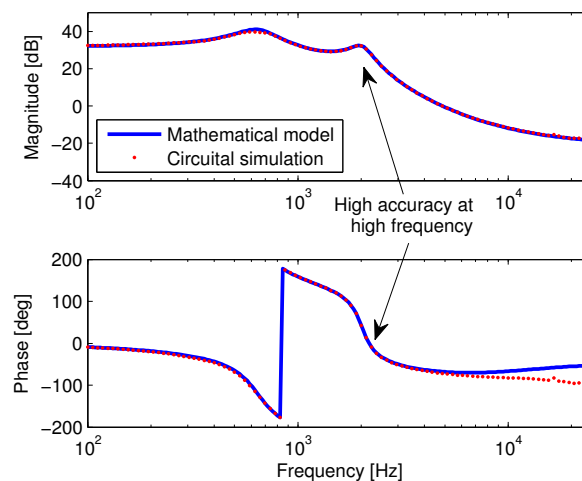
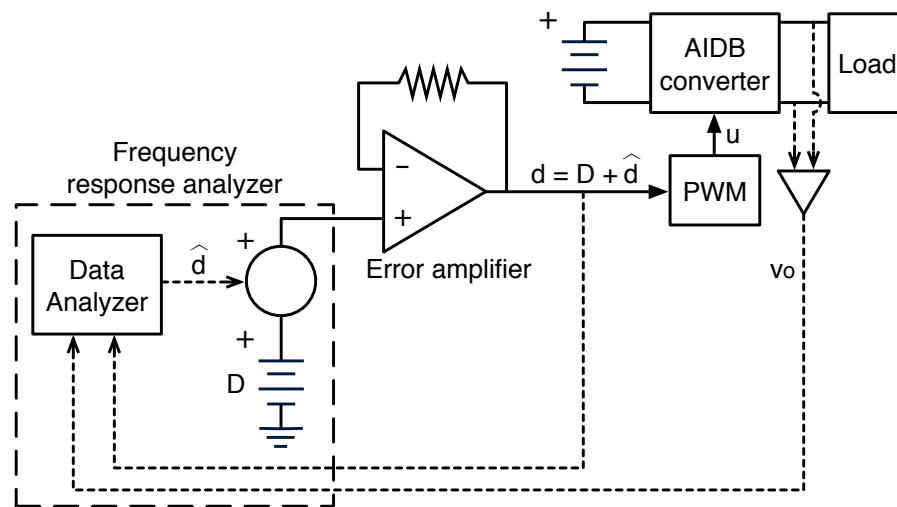
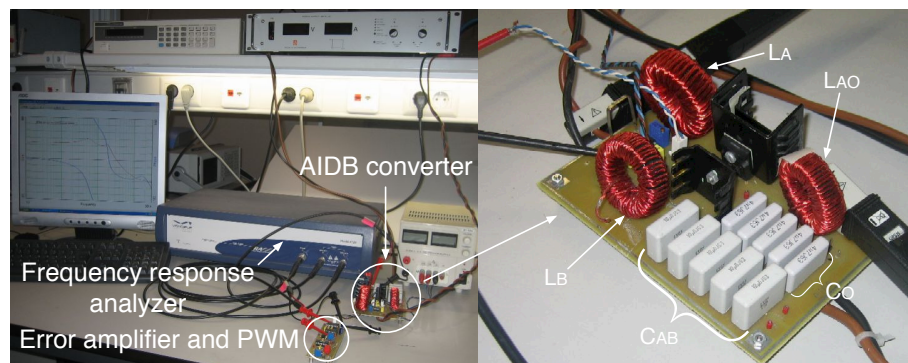


Figure 9. Experimental setup used to obtain the AIDB frequency response: (a) block diagram; and (b) test bench.

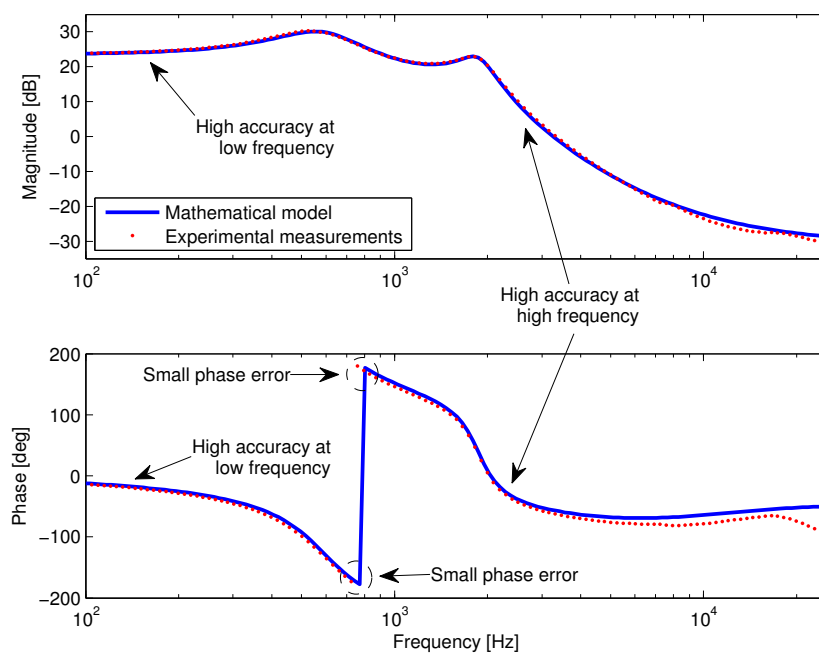


(a)



(b)

Figure 10. AIDB Bode diagrams from experiments and IAM simulation.



5. Other AIC Family Models Based on IAM

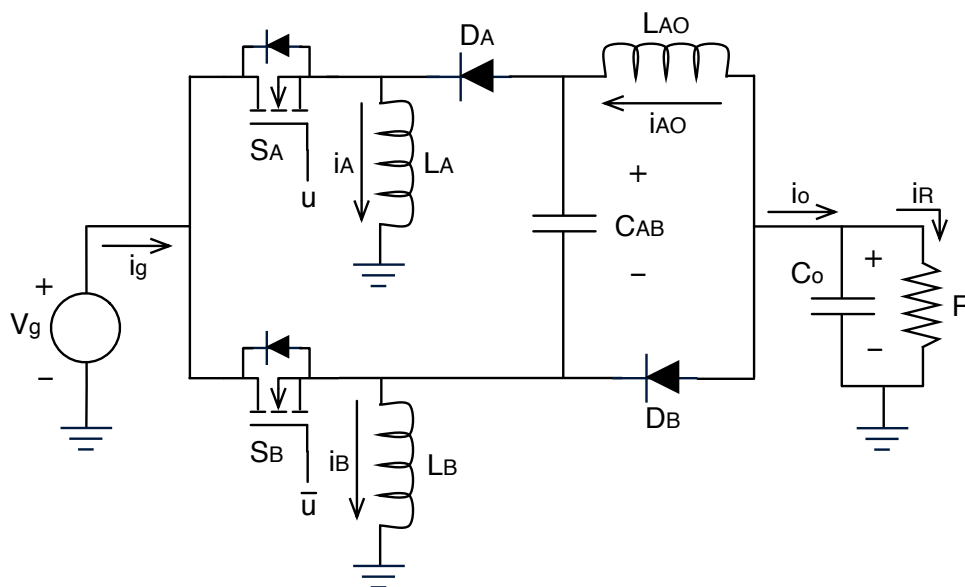
This section presents the dynamic models, obtained with the IAM technique, of other converters of the AIC family [7]: the AIDBB and the AIDF group. In sake of shortness, the corrected average equations, the corrected duty cycles d_2 and the expressions of the new variables are presented without describing each step of the IAM. In any case, the step-by-step procedure previously illustrated in Section 4 can be easily reproduced for all the converters in the AIC family.

5.1. The AIDBB Converter

The circuital scheme of the AIDBB converter is presented in Figure 11, where again the current in the diode D_B must be taken into account to improve the prediction of the output current, similar to the situation in the AIDB. Following the procedure presented in the previous section, the corrected averaged equations (for $D > 38\%$) shown in Equation (30) were obtained. Such expressions are in terms of the diode D_B current i_{DB} and the duty cycle d_2 :

$$\begin{aligned}
 \frac{d\bar{i}_A}{dt} &= \frac{v_g + \bar{v}_{AB}d_1}{L_A} \\
 \frac{d\bar{i}_B}{dt} &= \frac{v_g}{L_B}d_1 + \frac{\bar{v}_o}{L_B}d_2 + \frac{\bar{v}_o - \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{i}_{AO}}{dt} &= \frac{\bar{v}_o - v_g}{L_{AO}}d_1 - \frac{\bar{v}_{AB}}{L_{AO}}(d_1 + d_2) + \frac{\bar{v}_o - \bar{v}_{AB}}{L_B + L_{AO}}d_3 \\
 \frac{d\bar{v}_{AB}}{dt} &= \frac{-\bar{i}_A d_1 + \bar{i}_{AO}}{C_{AB}} \\
 \frac{d\bar{v}_o}{dt} &= \frac{-\bar{i}_{AO} - \bar{i}_{DB}}{C_o} - \frac{\bar{v}_o}{RC_o}
 \end{aligned} \tag{30}$$

Figure 11. Electrical circuit of the asymmetrical interleaved dual buck-boost (AIDBB) converter.



Following the IAM technique, the corrected duty cycle d_2 and the averaged new variable $\bar{i}_{DB}$ are calculated as in Equations (31) and (32), respectively:

$$d_2 = \frac{2(\bar{i}_B - \bar{i}_{AO})}{\left(\frac{v_g}{L_B} + \frac{v_g + \bar{v}_{AB} - \bar{v}_o}{L_{AO}}\right) d_1 T} - d_1 \quad (31)$$

$$\bar{i}_{DB} = \left(\frac{v_g}{L_B} + \frac{v_g + \bar{v}_{AB} - \bar{v}_o}{L_{AO}}\right) \frac{d_1 T}{2} \left(\frac{2(\bar{i}_B - \bar{i}_{AO})}{\left(\frac{v_g}{L_B} + \frac{v_g + \bar{v}_{AB} - \bar{v}_o}{L_{AO}}\right) d_1 T} - d_1 \right) \quad (32)$$

Then, the corrected averaged system in terms of the duty cycle d_1 is obtained by replacing Equations (1), (31) and (32) in Equation (30). Moreover, solving the corrected averaged system for derivatives equal to zero, the expressions given in Equation (33) are obtained for steady-state conditions. Finally, the small-signal model is obtained as in Equations (20–22) around a given operating point:

$$\left\{ \begin{array}{l} I_A = \left(\frac{1}{D'^2}\right) \frac{V_g}{R} - \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}}\right) D'^2 T \\ I_B = \left(\frac{1}{D'}\right) \frac{V_g}{R} + \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}}\right) D'^2 T \\ I_{AO} = \left(\frac{1}{D'}\right) \frac{V_g}{R} - \frac{V_g}{2} \left(\frac{L_B + L_{AO}}{L_B L_{AO}}\right) D'^3 T \\ V_{AB} = -\frac{V_g}{D'} \\ V_o = -\frac{V_g}{D'} \end{array} \right. \quad (33)$$

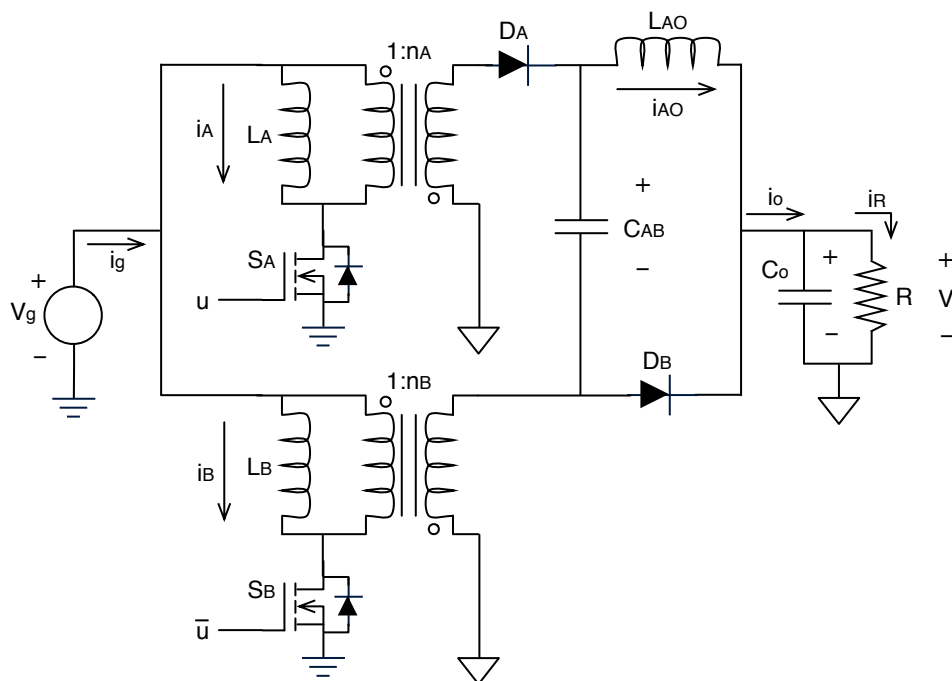
5.2. The AIDF Group

The AIDF group is composed by four converters: the isolated inverting AIDF, the isolated non-inverting AIDF, the non-isolated inverting AIDF and the non-isolated non-inverting AIDF. As described in [7], the isolated versions provide galvanic isolation between the source (photovoltaic array or fuel cell) and the load, while the non-isolated versions provide larger voltage conversion ratios. Among such topologies, the isolated non-inverting AIDF converter (I-NI-AIDF), depicted in Figure 12, was selected to illustrate IAM expressions. However, the step-by-step procedure previously illustrated in Section 4 can be easily applied to the other members of the AIDF-group.

The AIDF-group shares the same condition present in the AIDB and AIDBB: the prediction of the output current is improved by taking into account the dynamic behavior of the diode D_B current i_{DB} . Therefore, as in the AIDB and AIDBB, the new variable introduced in the model is i_{DB} . From such a consideration, the corrected averaged equations ($D > 38\%$) for the I-NI-AIDF are given in Equation (34). Again, i_{DB} and the corrected duty cycle d_2 are needed:

$$\begin{aligned}
 \frac{d\bar{i}_A}{dt} &= \frac{v_g (n_B d_1 + n_A (1 - d_1)) - \bar{v}_{AB} d_1}{n_A L_A} \\
 \frac{d\bar{i}_B}{dt} &= \frac{v_g}{L_B} d_1 - \frac{\bar{v}_o}{n_B L_B} d_2 + \frac{\bar{v}_{AB} - \bar{v}_o}{n_B^2 L_B + L_{AO}} n_B d_3 \\
 \frac{d\bar{i}_{AO}}{dt} &= \frac{-n_B \bar{v}_g - \bar{v}_o}{L_{AO}} d_1 + \frac{\bar{v}_{AB}}{L_{AO}} (d_1 + d_2) + \frac{\bar{v}_{AB} - \bar{v}_o}{n_B^2 L_B + L_{AO}} d_3 \\
 \frac{d\bar{v}_{AB}}{dt} &= \frac{\bar{i}_A d_1 - \bar{i}_{AO}}{C_{AB}} \\
 \frac{d\bar{v}_o}{dt} &= \frac{\bar{i}_{AO} + \bar{i}_{DB}}{C_o} - \frac{\bar{v}_o}{RC_o}
 \end{aligned} \tag{34}$$

Figure 12. Electrical circuit of the isolated non-inverting asymmetrical interleaved dual flyback (AIDF) converter.



In such a way, the expression for the corrected d_2 of the I-NI-AIDF converter is given in Equation (35), while its averaged current $\bar{i}_{DB}$ is given in Equation (36):

$$d_2 = \frac{2 \left(\frac{\bar{i}_B}{n_B} - \bar{i}_{AO} \right)}{\left(\frac{v_g}{n_B L_B} + \frac{n_B v_g - \bar{v}_{AB} + \bar{v}_o}{L_{AO}} \right) d_1 T} - d_1 \tag{35}$$

$$\bar{i}_{DB} = \left(\frac{v_g}{n_B L_B} + \frac{n_B v_g - \bar{v}_{AB} + \bar{v}_o}{L_{AO}} \right) \frac{d_1 T}{2} \left(\frac{2 \left(\frac{\bar{i}_B}{n_B} - \bar{i}_{AO} \right)}{\left(\frac{v_g}{n_B L_B} + \frac{n_B v_g - \bar{v}_{AB} + \bar{v}_o}{L_{AO}} \right) d_1 T} - d_1 \right) \tag{36}$$

As in the previous case, the corrected averaged system for the I-NI-AIDF in terms of the duty cycle d_1 is obtained by replacing Equations (1), (35) and (36) in Equation (34), while the steady-state conditions are given by Equation (37). Again, the small-signal model is obtained as in Equations (20–22) around a given operating point:

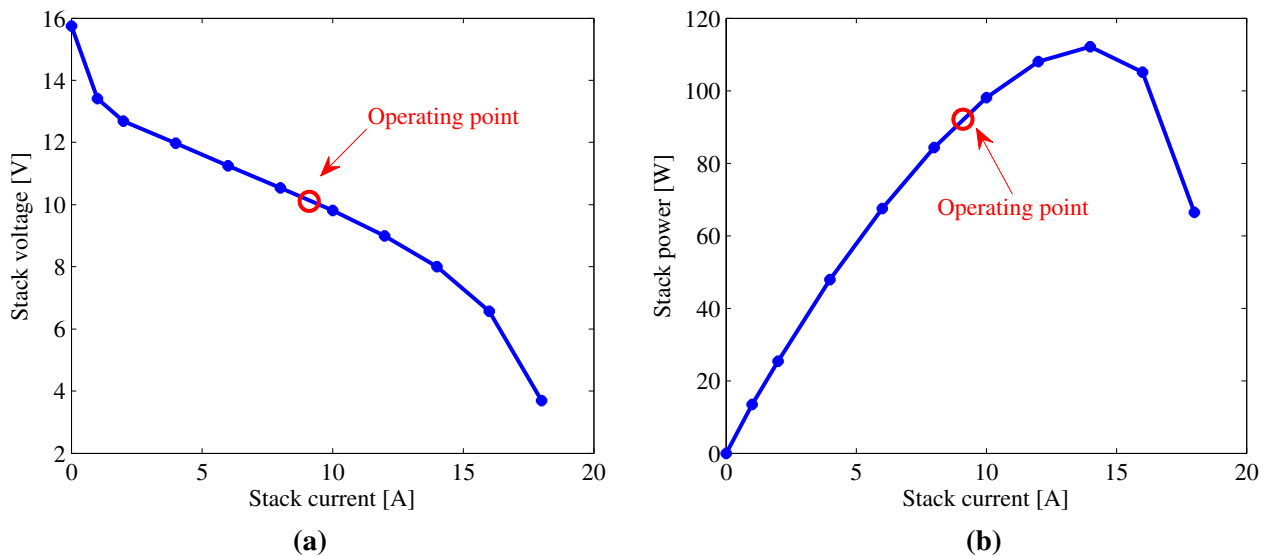
$$\left\{ \begin{array}{l} I_A = \left(\frac{n_A (D' n_B + (1 - D') n_A)}{D'^2} \right) \frac{V_g}{R} - \frac{V_g}{2} \left(\frac{n_A (n_B^2 L_B + L_{AO})}{L_B L_{AO} (D' n_B + (1 - D') n_A)} \right) D'^2 T \\ I_B = \left(\frac{n_B (D' n_B + (1 - D') n_A)}{D'} \right) \frac{V_g}{R} + \frac{V_g}{2} \left(\frac{n_B^2 L_B + L_{AO}}{L_B L_{AO}} \right) D'^2 T \\ I_{AO} = \frac{I_A D'}{n_A} \\ V_{AB} = V_g \left(\frac{D' n_B + (1 - D') n_A}{D'} \right) \\ V_o = V_g \left(\frac{D' n_B + (1 - D') n_A}{D'} \right) \end{array} \right. \quad (37)$$

Finally, the IAM models for the AIDB, AIDBB and AIDF converters are useful to design control strategies aimed at improving the behavior of power systems.

6. Application Example: AIDB Control Design

This section illustrates the usefulness of the proposed IAM technique in practical applications: the AIDB converter is controlled to supply a regulated voltage to a resistive load, where the electrical power is provided by a fuel cell. As clearly described in [7], the AIC family provides the useful conditions to interface fuel cells: low current ripple and high voltage conversion ratio. But, since the fuel cell voltage changes with the requested power, the AIDB output voltage must be regulated to compensate both load and fuel cell perturbations.

This example considers the AIDB parameters given in Subsection 3.2 operating with a fuel cell stack from Pragma Industries (named FCPc) composed by 2 parallel strings of 18 cells in series each. The polarization and power curves of such a FCPs stack are depicted in Figure 13a and 13b, respectively. Moreover, the application considers a load with a nominal impedance of 10 Ω that requires a fixed voltage of 30 V to operate. In such a condition the fuel cell must to provide 90 W, disregarding losses, which imposes stack current and voltage near to 9 A and 10 V, respectively. Such an operating point is highlighted in Figure 13a and 13b. Moreover, taking into account that the load voltage must be regulated, such figures put in evidence that changes on the load impedance generate changes in the fuel cell voltage, which imposes the voltage at the input of the AIDB converter. Therefore, a suitable controller must be designed to compensate both input and output perturbations in the converter.

Figure 13. Fuel cell electrical characteristics: (a) polarization curve; and (b) power curve.

Then, the improved model based on the IAM approach, and presented in Subsection 4.1, is used to design a linear-quadratic regulator (LQR) to provide the required compensation. Such a control technique was adopted since it is based on the states feedback and not only on the output feedback as Proportional-Integral (PI) or Proportional-Integral-Derivative (PID) controllers. Therefore, the design of a LQR controller helps to illustrate, in a better way, the usefulness of the modeling approach proposed in this paper.

6.1. The LQR Controller

The quadratic optimal control theory is used to design stable control systems by choosing the control vector $u(t)$ such that a given performance index is minimized. Therefore, considering the system representation given in Equation (38), the designing problem is reduced to determine the matrix K of the optimal control vector [32], as in Equation (39), to minimize the complex quadratic performance index given in Equation (40):

$$\dot{x} = Ax + Bu \quad (38)$$

$$u(t) = -Kx(t) \quad (39)$$

$$J = \int_0^{\infty} (x^* Q x + u^* R u) \quad (40)$$

In Equation (40), Q is a positive-definite (or positive-semidefinite) Hermitian or real symmetric matrix and R is a positive-definite Hermitian or real symmetric matrix. The matrices Q and R determine the relative importance of the error and the expenditure of the energy in the system.

The procedure followed to find the matrix K begins with the definition of the parameter-optimizing problem, then using the second method of Liapunov the performance index J is evaluated and minimized, and the result is the reduced-matrix Riccati equation which solution provides the optimal matrix K . The details of such a procedures are given in [32].

Moreover, to ensure null steady-state error, a sixth state variable was introduced to the system: the integral of the error between the output voltage and the reference. Such additional state equation is given in Equation (41), where $V_{ref} = 30$ V is the reference voltage:

$$\frac{d\bar{e}}{dt} = \bar{v}_o - V_{ref} \quad (41)$$

The feedback vector gains K Equation (39) can be obtained using the MATLAB command $[K, S, e] = lqr(A, B, Q, R)$, which provides the solution to the Riccati equation in continuous time (matrix S), determines the optimal feedback matrix gains (vector K) and the closed loop poles location (vector e). The matrices Q and R are usually selected by the designer and adjusted by trial and error. In this example, such matrices have been defined taking into account energy considerations and the relative weight of the output voltage and its error: considering the energy stored in the capacitors (E_C) and inductors (E_L) given in Equation (42), and taking into account that Q matrix is related to energy expenditure, the coefficients of such a matrix were defined as in Equation (43). In such a matrix, the coefficients related to the currents have the same weight since in the adopted converter all the inductors have very similar values (see circuit parameters given in Subsection 3.2); but the coefficients related to the voltages are weighted in agreement with the capacitor values since they are very different. In addition, the coefficient related to the error was selected large enough to give priority to the null steady-state error condition. Finally, the Q matrix was normalized with respect to the output voltage coefficient to avoid numerical values too small:

$$E_C = \frac{1}{2}CV^2, E_L = \frac{1}{2}LI^2 \quad (42)$$

$$Q = \frac{1}{V_o^2 \times 23.5} \begin{bmatrix} I_A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_B^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{AO}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{AB}^2 \times 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_o^2 \times 23.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \times 10^{10} \end{bmatrix} \quad (43)$$

Then, matrix R must be small enough to provide a fast correction of the perturbations as given in Equation (44). Finally, using Q and R matrices given in Equations (43) and (44), the IAM model parameterized as in Subsection 3.2, and both the FCPc fuel cell stack and load characteristics previously described, the numerical K vector given in Equation (45) was calculated:

$$R = [1] \quad (44)$$

$$K = \begin{bmatrix} -3.79 & 0.12 & 0.10 & -0.40 & 0.31 & -745 \end{bmatrix} \quad (45)$$

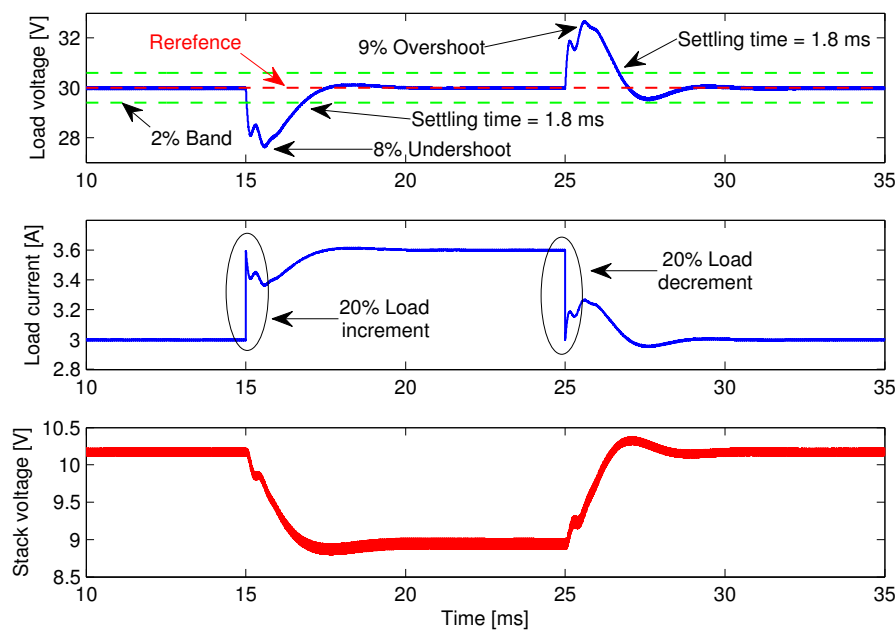
6.2. Simulation Results

The designed LQR controller was tested by means of circuital simulations in PSIM. The simulation considers the fuel cell modeled by a Thevenin equivalent, while the load perturbations were introduced

by changing the load current. In such a way, 20% step-up and step-down load perturbations were introduced to evaluate the controller performance.

Figure 14 shows the simulation results: the controller compensates both step-up and step-down load perturbations by providing settling times equal to 1.8 ms in both cases.

Figure 14. Dynamic simulation in PSIM.



The figure also shows the change on the stack voltage caused by the increment in the load power. In any case, both perturbations are satisfactorily attenuated. Moreover, the operating point defined by the interaction between the converter and the fuel cell slightly diverges from the theoretical values (10 V, 9 A) due to the stack electrical characteristic (Figure 13a and 13b), but such a difference is compensated by the controller to provide the desired conditions to the load.

Finally, this example put in evidence the usefulness of the proposed IAM modeling approach in the design of control strategies for practical applications.

7. Conclusions

The challenge of accurately modeling the members of the AIC family has been initially addressed by selecting the revised averaging method [8], which consists of three steps: averaging, correction and duty-cycle constraint definition. Being the AIC family a particular case of high-order dc/dc switching converters operating in discontinuous conduction mode in which two of the inductor currents are in discontinuous conduction while another one is not, the application of the revised averaging method is not straightforward. A new calculation procedure for the duty cycle d_2 in the duty-cycle constraint definition step has been illustrated in the modeling of the AIDB converter. However, the resulting full-order continuous non-linear model of the AIDB converter is not suitable for advanced control design because it exhibits large errors at high frequencies.

An improved averaging method that overcomes the high frequency modeling deficiencies of the previous revised averaging method has been carefully detailed and validated by means of both PSIM

simulations and experimental measurements of an AIDB stage operating at duty cycles above 38%. The key factor in the improved modeling is the explicit inclusion in the equations of the current flowing through the diode that operates in discontinuous conduction mode.

The improved method has also been successfully applied to the remaining members of the AIC family. Examples of modeling the AIDBB and the I-NI-AIF converters are provided in the paper. The modeling of other high-order converters operating in DCM such as Cuk, sepic or zeta will be addressed in future works. Similarly, a simplified version of the proposed technique, aimed for field engineers, is an open problem to be addressed.

Finally, the excellent accuracy in the frequency response provided by the improved averaging method has allowed to design a voltage regulator for a fuel cell application in which an AIDB converter is successfully controlled by means of a linear quadratic regulation technique.

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Conflicts of Interest

The authors declare no conflict of interest.

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