

Article

Gas Injection Optimization and Shrinkage Control for Salt Cavern CO₂ Storage (SCCS) Based on Creep-Shrinkage Sensitivity Analysis

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Abstract

Salt cavern CO₂ storage (SCCS) technology represents a crucial pathway for achieving large-scale carbon sequestration. However, its long-term operation faces the challenge of cavern shrinkage due to surrounding rock creep, which directly impacts storage safety and stability. Despite its importance, there is currently a lack of research focusing on the proactive control of SCCS cavern shrinkage and its collaborative optimization with operational economy. To fill this gap, this paper first investigated the effects of the stress state (f_1), height-to-diameter ratio (f_2), symmetry factor (f_3), and cavern volume (f_4) on the volumetric shrinkage rate through numerical simulations of regular caverns and univariate sensitivity analysis. The sensitivity ranking and quantitative relationships of these factors were clarified as $f_1(2.31) > f_4(0.309) > f_2(0.166) > f_3(0)$. Subsequently, a multi-objective nonlinear optimization model was established, and the primal-dual interior-point method was adopted as the solution algorithm. Using actual cavern data as a case study for the solution, the results demonstrate that the optimization model converges stably in approximately 1.1 s. The resulting optimal gas injection allocation scheme achieves a 14.77% improvement in the comprehensive score compared to the baseline scheme. This study provides a theoretical basis and a practical tool for the rapid generation of SCCS gas injection allocation schemes.

Keywords: salt cavern CO₂ storage; creep shrinkage; sensitivity analysis; gas injection optimization; primal-dual interior-point method



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1. Introduction

Against the backdrop of global efforts to advance carbon neutrality, CO₂ capture, utilization, and storage technology has become an indispensable critical pathway for achieving deep emission reductions [1]. Among these, SCCS technology, which involves constructing artificial cavities in deep salt formations through solution mining to store high-pressure CO₂, is regarded as a crucial technical option for realizing large-scale, long-term, and safe geological sequestration [2,3]. However, during long-term storage facility operation, the surrounding rock of the cavern undergoes continuous creep driven by pressure differentials, leading to a slow shrinkage of the cavern volume, which directly impacts the long-term stability of the cavern [4]. Therefore, revealing the primary controlling factors of cavern

creep shrinkage and, based on this, establishing scientific gas injection operation strategies to achieve collaborative optimization of shrinkage control and operational economy represents a core scientific problem that must be solved for SCCS technology to advance towards large-scale and intelligent application [5].

Substantial research has been conducted by scholars worldwide on the creep and stability issues of salt caverns, primarily focusing on three core aspects. First, concerning creep mechanisms and constitutive models, studies have revealed the rheological properties of salt rock under the influence of stress, temperature, and time. Constitutive models have evolved from classical empirical models like Norton's law to complex models considering damage, hardening, and multi-field coupling, providing a theoretical foundation for numerical calculations [6,7]. Second, regarding the influencing factors and prediction of cavern shrinkage, several key factors have been identified. For instance, Zhang et al. [8] found through numerical simulation that, besides the inherent creep properties of salt rock, the operational modes of adjacent caverns, the proportion of low-pressure operation time within a single cycle, and the cavern height-to-diameter ratio are sensitive factors. Third, in the field of injection–production optimization for underground energy storage, multi-objective optimization models have been developed that comprehensively consider economics, safety, and equipment capacity. Good results in energy saving and pressure equalization have been achieved through optimization algorithms [9,10].

Despite the progress made in existing research, significant limitations remain concerning the active control of creep shrinkage and collaborative operational optimization for SCCS. Firstly, the identification of factors influencing cavern shrinkage in current studies is not comprehensive [8]. Secondly, there is a disconnect between mechanistic research and operational control; most sensitivity analyses merely reveal patterns without translating the sensitivity of shrinkage rates to various factors into control indicators and optimization objectives that can be embedded into daily scheduling [11]. Furthermore, traditional optimization models treat shrinkage merely as a passive constraint rather than a core objective to be actively minimized, making it difficult for research outcomes to achieve targeted control applications [10,12].

To address these limitations, this study first conducts systematic numerical simulations and sensitivity analyses to investigate the influence of key parameters—such as the in situ stress-to-internal pressure ratio, cavern shape, and cavern volume—on the long-term volumetric shrinkage rate of SCCS caverns. It determines their sensitivity ranking and quantitative functional relationships. Subsequently, using this quantitative function as the core control element, a multi-objective nonlinear optimization model is constructed, incorporating gas injection energy consumption. The decision variables are the daily gas injection volumes for each cavern over the upcoming month, and the model is solved efficiently using the primal-dual interior-point method. Finally, through a case study of an actual cavern group, the effectiveness of the proposed method in collaboratively optimizing shrinkage control, operational economy, and scheduling flexibility is verified, providing theoretical support for the intelligent and refined operation of SCCS facilities.

2. Influencing Factors and Sensitivity Analysis of Cavern Shrinkage

2.1. Key Influencing Factors of Creep Shrinkage

During the operation of a salt cavern CO₂ storage facility, the gas pressure inside the cavern is typically lower than the vertical and horizontal stresses of the surrounding rock mass, leading to an unbalanced stress state in the salt rock. This stress state drives the salt rock to creep towards the cavern interior, macroscopically manifested as a continuous shrinkage of the cavern volume [13]. Due to the influence of solution mining techniques and salt layer heterogeneity, actual cavern geometries are complex and irregular, making

it difficult to reveal universal laws through direct studies on irregular caverns [14]. In this study, actual caverns are simplified into a rotationally symmetric pear-shaped model to isolate the influence of minor geometric details and extract the primary mechanical mechanisms. As the gas injection scheduling period is fixed at one month, this study does not consider the independent effect of time on shrinkage. The investigation focuses on the impact of three categories of factors on shrinkage behavior: stress state, cavern shape, and cavern volume.

2.1.1. Stress State of the Cavern

The driving force for cavern shrinkage originates from the difference between the internal gas pressure and the in situ stress. Existing research indicates that, under otherwise constant conditions, the lower the internal pressure of the cavern, the greater the volumetric shrinkage rate, with the shrinkage rate increasing nonlinearly as the internal pressure decreases [15]. To quantify this mechanical influence, this study adopts the ratio of in situ stress to cavern internal pressure (f_1) as the sensitivity analysis parameter reflecting the cavern's stress state.

2.1.2. Cavern Shape

The shape of a regular cavern is primarily determined by two parameters: the height-to-diameter ratio and the symmetry factor. The height-to-diameter ratio is defined as the ratio of cavern height to diameter; the symmetry factor is the ratio of the height of the upper semi-ellipsoid to that of the lower semi-ellipsoid. The cavern shape directly affects the stress distribution within the surrounding rock mass, which in turn macroscopically influences the volumetric shrinkage of the cavern. Ma et al. [16] pointed out that under conditions where the burial depth exceeds 433.2 m, salt rock can be considered to be in a hydrostatic stress state. In this stress environment, when the cavern is spherical, its mechanical state is optimal, and the volumetric shrinkage rate is minimal; the further the shape deviates from a sphere, the more significant the volumetric shrinkage trend becomes. This study incorporates the height-to-diameter ratio (f_2) and the symmetry factor (f_3) into the sensitivity analysis system to quantitatively assess their regulatory effects on shrinkage behavior.

2.1.3. Cavern Volume

Under the premise that the geometric shape remains constant, variations in cavern volume can still influence its creep shrinkage behavior. Caverns with significantly different volumes may exhibit different shrinkage trends under identical stress conditions [17]. Therefore, this study lists cavern volume (f_4) as another factor for sensitivity analysis, aiming to reveal its inherent relationship with the volumetric shrinkage rate.

2.2. Numerical Simulation Method and Benchmark Model

To quantitatively evaluate the effects of various influencing factors, this study employed Rhino 7 software to establish three-dimensional geometric models of different caverns. After meshing, these models were imported into FLAC3D (a numerical simulation software widely used for geotechnical creep analysis) for creep simulation. Tetrahedral elements were selected for the mesh, with refinement applied to the region near the cavern and a gradual transition to coarser elements outwards, balancing computational accuracy and efficiency. To conduct parametric sensitivity analysis, a benchmark state was defined to unify dimensions and serve as a basis for comparing the influence of each factor. A pear-shaped cavern with a diameter of 70 m, height of 115.5 m, top burial depth of 1200 m, internal pressure of 12 MPa, and a symmetry factor of 1.9 was selected as the benchmark model. The corresponding parameters of the model are taken from a typical cavern in the

Jintan gas storage facility [13]. The model comprised 48,359 tetrahedral elements, and its three-dimensional geomechanical model is shown in Figure 1.

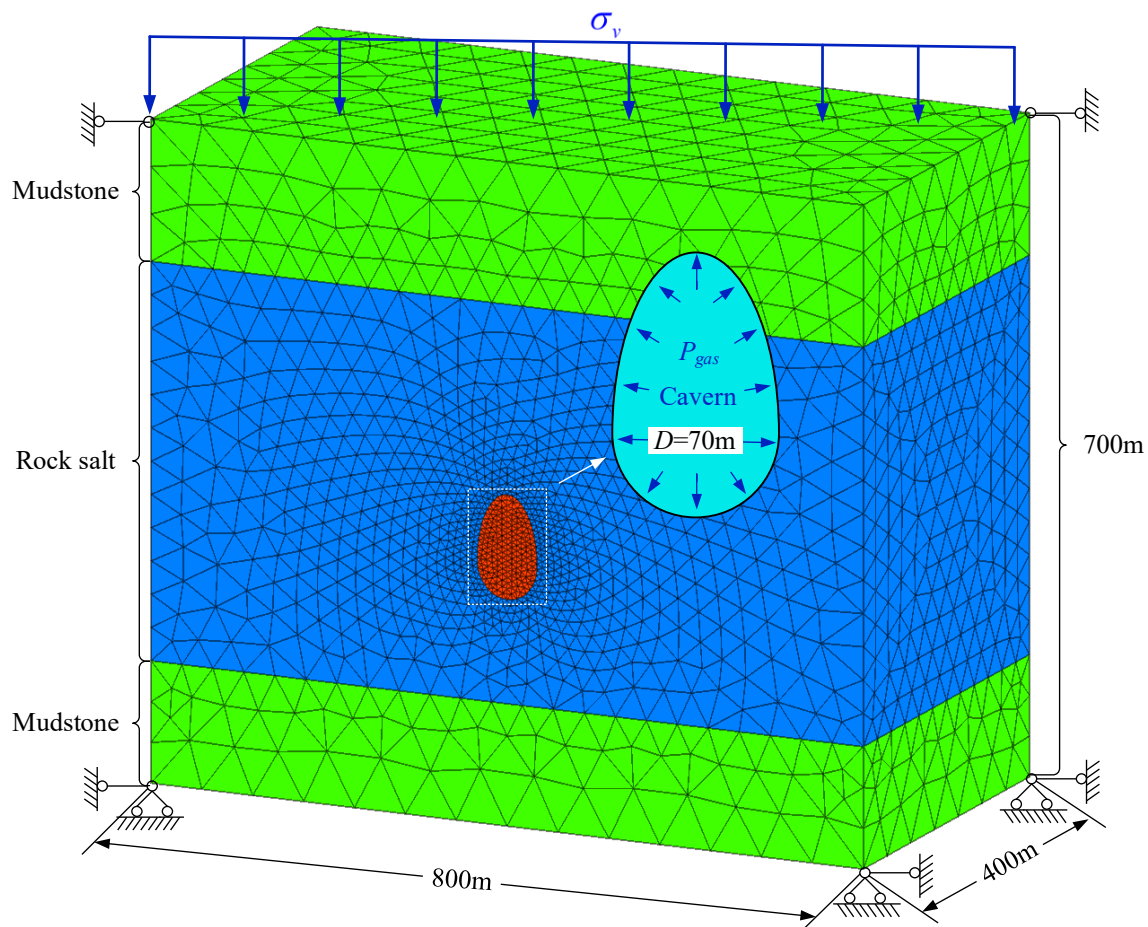


Figure 1. Three-dimensional geological model of the benchmark cavern.

The in situ stress field of the model was set as a triaxial equal-compressive, self-weight stress field, consistent with the hydrostatic pressure assumption for deep salt rock. Boundary conditions were configured as follows: vertical displacement was constrained at the model base, and normal displacement constraints were applied to the four lateral boundaries, effectively treating the surrounding rock mass as rigid boundaries. The model top was a free surface, uniformly subjected to the self-weight stress corresponding to the thickness of the overlying strata. The salt rock layer had a thickness of 421 m, interbedded between thick mudstone layers above and below, with the model top buried at a depth of 850 m. In the numerical simulation, the Mohr-Coulomb criterion was adopted to assess the plastic zone and damage distribution in the surrounding rock, while the Norton model was selected to describe the steady-state creep behavior of the salt rock [18]. Its constitutive relation for creep strain rate is

$$\dot{\epsilon}(t) = A\sigma^n \quad (1)$$

where $\dot{\epsilon}$ is the creep strain rate, s^{-1} ; σ is the deviatoric stress, MPa; and A and n are experimental constants for the salt rock material. The mechanical parameters used in the model for mudstone and salt rock were referenced from actual engineering test data from the Jintan gas storage facility, with the specific values being as follows: for salt rock, density is 2.25 g/cm^3 and cohesion is 1.0 MPa; for mudstone, density is 2.76 g/cm^3 and cohesion is 1.0 MPa, along with other parameters listed in Table 1 [19]. All comparative models were constructed and calculated following the same methodology.

Table 1. Elastoplastic and creep calculation parameters for the surrounding rock [19].

Lithology	Elastic Modulus (GPa)	Poisson's Ratio	Internal Friction Angle (°)	Tensile Strength (MPa)	Creep Parameters	
					$A/(\text{MPa}^{-5} \cdot \text{d}^{-1})$	n
Mudstone	10	0.27	30	1.0	4×10^{-7}	3.5
Salt Rock	13	0.30	35	1.0	4×10^{-6}	3.5

2.3. Univariate Sensitivity Analysis

The volumetric shrinkage rate of the cavern after 100 years of operation is defined as V_{lost} . According to the analysis in Section 2.1, it is primarily controlled by four factors, namely the ratio of in situ stress to internal pressure, i.e., $V_{lost} = F(f_1, f_2, f_3, f_4)$. The parameters of the benchmark model are set as $f^* = \{f_1^*, f_2^*, f_3^*, f_4^*\}$, corresponding to the benchmark shrinkage rate V_{lost}^* . By varying a single factor, f_i , within its reasonable range while keeping the other parameters at their benchmark values, the degree and trend of deviation of V_{lost} from V_{lost}^* can be systematically analyzed, thereby assessing the sensitivity of each factor.

For the height-to-diameter ratio f_2 , this study directly adopts the linear fitting relationship obtained from existing research [20,21]. Equation (2); therefore, data for f_2 is not included in the table, and its sensitivity coefficient obtained from existing studies is 0.166.

$$V_{lost}/V_{lost}^* = 0.166(f_2/f_2^*) + 0.833 \quad (2)$$

Based on this principle, the univariate comparative simulation scheme designed for this study is shown in Table 2. The table presents the ratio of each influencing factor f_i relative to the benchmark value f_i^* . Taking f_1 as an example, its variation is achieved by adjusting the cavern's top burial depth and internal pressure: Comparative Models 1 and 2 share the same burial depth (1200 m) but have different internal pressures (8 MPa and 10 MPa, respectively); Comparative Models 3 and 4 share the same internal pressure (8 MPa) but have different burial depths (600 m and 1000 m, respectively). A value such as 0.5 in the table indicates that the ratio of the parameter f_i of the comparison model to the corresponding parameter f_i^* of the baseline model is 0.5.

Table 2. Design of comparative schemes for each influencing factor.

Model	f_1	f_2	f_3	f_4
Comparative Model 1	0.500	—	0.474	0.356
Comparative Model 2	0.833	—	0.737	0.624
Comparative Model 3	1.200	—	1.263	1.254
Comparative Model 4	1.500	—	1.526	1.502

The cavern volumetric shrinkage rates after 100 years of simulated operation for each scheme are listed in Table 3.

Table 3. Calculated volumetric shrinkage rates for different schemes.

Model	Relative Volumetric Shrinkage Rate			
	f_1	f_2	f_3	f_4
Comparative Model 1	2.238	—	1.007	0.727
Comparative Model 2	1.510	—	1.028	0.839
Comparative Model 3	0.399	—	1.049	1.031
Comparative Model 4	0.017	—	1.021	1.077

Based on the data in Table 3, scatter plots of the relative volumetric shrinkage rate against each parameter were generated, as shown in Figure 2, and function fitting was performed. The slope α_i of the curve represents the sensitivity coefficient of that factor on the volumetric shrinkage rate. A positive slope value indicates that the shrinkage rate varies in the same direction as the factor, while a negative value indicates an inverse relationship.

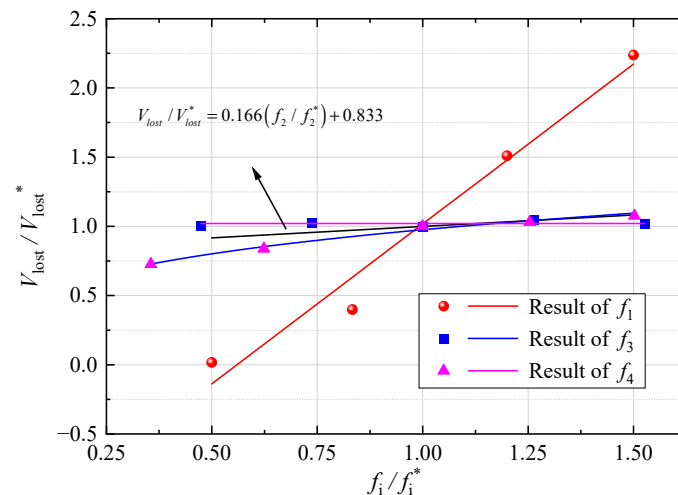


Figure 2. Fitting curves between each factor and the relative volumetric shrinkage rate.

According to the fitting results, the ratio of in situ stress to internal pressure f_1 exhibits a significant linear positive correlation with the relative shrinkage rate. The fitted straight line has a slope of 2.31, and the expression is

$$V_{lost}/V_{lost}^* = 2.31(f_1/f_1^*) - 1.29 \quad (R^2 = 0.973) \quad (3)$$

The relationship between the symmetry factor f_3 and the relative shrinkage rate is approximately constant, with a sensitivity coefficient near 0. This indicates that, within the parameter range studied, the variation in the ratio of the upper to lower parts of the cavern has a negligible impact on the total volumetric shrinkage.

The cavern volume f_4 shows a nonlinear relationship with the relative shrinkage rate, best fitted by a power function:

$$V_{lost}/V_{lost}^* = 0.98(f_4/f_4^*)^{0.28} \quad (R^2 = 0.986) \quad (4)$$

Its sensitivity (curve slope) varies with the relative volume. Within the actual engineering parameter range, the curve is relatively flat. For ease of comparison, the average of the maximum and minimum curve slopes within this interval was taken as its equivalent sensitivity coefficient, calculated to be 0.309.

Based on the comprehensive analysis above, the ranking of the influencing factors on cavern volumetric shrinkage rate is $f_1(2.31) > f_4(0.309) > f_2(0.166) > f_3(0)$. This indicates that the ratio of in situ stress to internal pressure f_1 is the most critical factor controlling the long-term volumetric shrinkage of salt caverns, with its influence significantly greater than other geometric factors. In the actual operation of gas storage facilities, maintaining a reasonable internal pressure level is the primary measure for controlling cavern shrinkage and ensuring storage stability. Cavern volume f_4 and the height-to-diameter ratio f_2 also have certain influences, while the symmetry factor f_3 has the least impact under the conditions of this study.

3. Development and Application of the Gas Injection Optimization Model for Salt Cavern CO₂ Storage

3.1. Optimization Model Formulation

3.1.1. Injection Optimization Model Decision Variables

To achieve the intelligent allocation of gas injection volumes and the intelligent control of shrinkage in SCCS, this study constructs a gas injection optimization model. Aiming to maximize the system's comprehensive score, this model automatically allocates the gas injection volume for each cavern based on the current system state while strictly satisfying various safety and engineering constraints. The model employs a one-dimensional decision vector X as the decision variable, whose element $Q_{t,j}$ is defined as the gas injection volume for the j -th cavern on the t -th day (Nm³). The expression for X is as follows:

$$X = [Q_{1,1}, Q_{2,1}, \dots, Q_{t,1}, Q_{1,2}, \dots, Q_{t,2}, \dots, Q_{1,j}, \dots, Q_{t,j}] \quad (5)$$

Considering that a monthly cycle is a common period for inventory assessment, market settlement, and maintenance scheduling in energy storage systems, this paper selects $t = 30$ days as the total number of gas injection days and conducts the optimization study using a storage facility comprising 8 caverns as an example.

3.1.2. Multi-Objective Function Design

(1) Cavern Shrinkage Control Term

To mitigate cavern creep shrinkage and ensure long-term stability, this paper incorporates the cavern shrinkage rate into the optimization objective based on the sensitivity analysis results. SCCS projects involve multiple caverns with service lives spanning up to a hundred years. The safety control objective is to minimize the comprehensive volumetric shrinkage rate of the storage system over the 100-year sequestration period. According to the research in Section 2, the primary factor controlling the volumetric shrinkage rate of a single cavern is its internal pressure state, which has a sensitivity coefficient 7.5 to 13.92 times higher than other factors. To simplify the calculation, this paper neglects the influence of other factors and utilizes the fitted Equation (3) to calculate the volumetric shrinkage rate of a single cavern over its 100-year service life.

Based on Equation (3), the volumetric shrinkage rate V_{lost} for a single cavern over 100 years of operation is

$$V_{lost} = 2.31V_{lost}^* (f_1/f_1^*) - 1.29V_{lost}^* \quad (6)$$

where f_1 is the ratio of in situ stress to internal pressure.

The gas injection period for SCCS typically lasts only a few months. During the CO₂ compression and injection phase, influenced by gas compression work and the throttling effect, significant transient thermal effects occur within the cavern, leading to short-term local temperature fluctuations. However, the time span of this transient thermal process is extremely short, accounting for less than 1% of the service cycle, which is negligible compared to the century-long service life. After injection, during the long-term storage period, the CO₂ within the cavern undergoes sufficient heat exchange with the massive surrounding salt rock formation, and the early transient thermodynamic disturbances gradually dissipate. The gas temperature ultimately stabilizes at the formation temperature and maintains long-term stability. Moreover, the amplitude of pressure fluctuations caused by transient thermodynamic disturbances is relatively small, usually within 10% of the stable operating pressure of the cavern. Therefore, when evaluating the volumetric shrinkage rate on a century scale, short-term transient thermal effects can be reasonably ignored. The

cavern pressure is mainly determined by the total injection volume $\sum Q_{t,j}$. At this stage, the stabilized gas pressure inside the cavern P_j^{100} is calculated as

$$P_j^{100} = P_j^0 + \frac{Z(P, T)nRT}{V_j} = P_j^0 + \frac{Z(P, T)RT}{0.0224V_j} \sum Q_{t,j} \quad (7)$$

where V_j is the cavern volume, n is the moles of gas, $Q_{t,j}$ is the gas volume in standard states (Nm^3), $R = 8.314 \times 10^{-6} \text{ MPa} \cdot \text{m}^3 / (\text{mol} \cdot \text{K})$ is the molar gas constant, and T is the gas temperature. The burial depth of the Jintan gas storage caverns ranges from 975 m to 1200 m; this study adopts the formation temperature corresponding to the median depth of 1100 m. Measured data show that the salt cavern temperature at this depth is 53°C , so $T = 53^\circ\text{C}$ is used. The gas compressibility factor Z is determined by the Peng–Robinson equation. In SCCS projects, cavern pressure is generally maintained near the upper limit. For the Jintan gas storage facility, the upper pressure limit is between 16 MPa and 19 MPa, corresponding to Z values between 0.48 and 0.54. This study adopts the median value, $Z = 0.51$.

Although the construction of Equation (7) neglects secondary factors and may introduce certain errors, the core of the optimization model lies in comparing the relative impact of different injection allocation schemes on the shrinkage trend. This simplification does not affect the effective pursuit of the optimal solution within the feasible region while significantly reducing computational complexity. In this study, this simplified equation is used only for the relative comparison of volumetric shrinkage between different injection schemes and not for the precise calculation of absolute shrinkage.

Each cavern in an SCCS project differs in shape, volume, and burial depth; therefore, the same injection volume produces different shrinkage control effects in different caverns. To quantify the contribution of each cavern to the overall system shrinkage, this study defines a weight coefficient w_j for each cavern j based on the sensitivity coefficients of each influencing factor:

$$w_j = \alpha_1 \cdot \frac{f_{1,j}}{f_1^*} + \alpha_2 \cdot \frac{f_{2,j}}{f_2^*} + \alpha_3 \cdot \frac{f_{3,j}}{f_3^*} + \alpha_4 \cdot \frac{f_{4,j}}{f_4^*} \quad (8)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, and $f_{i,j}$ is the initial state value of cavern j for the i -th factor.

To standardize the dimension within the objective function and provide a quantifiable reference for engineering decisions, the relative shrinkage rate of the cavern is normalized into a score (0–100) through linear transformation. The calculation formula is as follows:

$$S_{v,j} = 100 \times \left(1 - \frac{F_{\text{actual}} - F_{\min}}{F_{\max} - F_{\min}} \right) \quad (9)$$

where $F_{\max}$ and $F_{\min}$ are the theoretical maximum and minimum relative volumetric shrinkage rates for the cavern, respectively; $F_{\text{actual}, j} = V_{\text{lost}, j} / V_{\text{lost}}^*$ is the actual relative volumetric shrinkage rate, with $F_{\text{actual}} = 1$ indicating that the cavern shrinkage rate equals that of the benchmark cavern. In this study, $F_{\max} = 2.5$ and $F_{\min} = 0.25$ are adopted, corresponding to the two boundary scenarios where the shrinkage rate increases to four times the benchmark value or decreases to one-quarter of it. This range covers the majority of shrinkage variations that may occur in actual operation [8]. The total system score is the weighted average of the scores for all caverns on each day. The objective of shrinkage control is to minimize this total score, i.e.,

$$\min Z = \sum_{j=1}^8 [w_j \cdot S_{v,j}] \quad (10)$$

(2) Cost Control Term

To control gas injection energy consumption, this paper incorporates an energy consumption control term. The core energy consumption of a salt cavern energy storage facility is concentrated in the gas injection process. This process requires high-power compressors to pressurize the gas for injection into the underground caverns and is a key aspect of cost control [22]. The energy consumption per unit of gas injection is closely related to the injection rate, cavern pressure, and compressor performance. The analysis process is relatively complex; therefore, this paper adopts a simplified treatment. Based on the principle that, given other parameters are identical, a higher initial cavern pressure results in more work required to inject a unit volume of CO₂, the total energy consumption for gas injection—given the known pressures $P_{i,1}$ and $P_{i,100}$ before and after injection—is approximately estimated using the following expression:

$$W = \sum_{j=1}^8 \sum_{t=1}^{30} \left[Q_{t,j} \cdot RT \times \frac{P_{i,0}}{P_{i,100}} \right] \quad (11)$$

This simplified model is only used for the comparison of energy consumption between different injection schemes and does not serve as a precise calculation of absolute energy consumption. To facilitate engineering decision-making, unify dimensions, and coordinate with other terms in the objective function, the gas injection cost is also normalized to a range of 0 to 100 using the linear transformation method shown in Equation (9).

Furthermore, to improve operational efficiency, this paper incorporates a time cost control term. The time cost T_X is defined as the total time required to complete the gas injection task, which is also normalized to the 0–100 range using the linear transformation method in Equation (9).

(3) Comprehensive Objective Function

The cavern shrinkage control term and cost control terms are respectively normalized and then summed with weights to form the following comprehensive objective function:

$$\min Z = 0.8 \cdot \sum_{j=1}^8 (w_j \cdot S_{v,j}) + 0.15 S_w + 0.05 S_{T_X} \quad (12)$$

where S_w is the normalized energy consumption control term, and S_{T_X} is the normalized time cost control term. The weights 0.8, 0.15 and 0.05 in the objective function are the coefficients for the different objectives. The values of these weight coefficients are determined based on engineering experience. This allocation reflects an optimization orientation that prioritizes the long-term stability of the caverns as the core while also considering operational economy.

3.1.3. Constraints

To ensure the engineering feasibility and safety of the gas injection operation optimization scheme, the model must satisfy the following three categories of constraints:

(1) Pressure and Storage Capacity Physical Constraints

Let the minimum safe pressure of the storage facility be $P_{\min}^j$, corresponding to the minimum working gas volume $V_{\min}^j$, and the maximum safe pressure be $P_{\max}^j$ corresponding to the maximum working gas volume $V_{\max}^j$. The initial working gas volume is set as V_0^j . The working gas volume of the cavern must always remain within the permissible safe range, with the specific expression as follows:

Let the upper pressure limit of the storage cavern be $P_{\max}^j$, corresponding to the maximum storage capacity $V_{\max}^j$. If the initial injected gas volume is V_0^j (Nm^3), then the newly injected gas volume must always remain within the permissible safe range, with the specific expression as follows:

$$V_0^j + \sum_{t=1}^T q_t^j \leq V_{\max}^j \quad (13)$$

(2) Equipment Capacity Constraints

The total daily gas injection volume of the storage system is constrained by the delivery capacity of surface facilities and pipelines. Additionally, to ensure the safety of the wellbore and injection string, the daily injection volume for each individual cavern must also be limited. Let C_{inj} be the maximum total daily injection capacity, and C_{per} be the maximum daily injection capacity for each cavern. The constraints are

$$\sum_{j=1}^N q_t^j \leq C_{inj}, q_t^j \leq C_{per} \quad (14)$$

(3) Task Completion Constraint

At the end of the optimization period, the cumulative gas injection volume for each cavern must meet the predetermined production task requirement. The corresponding constraint is

$$\sum_{j=1}^N \sum_{t=1}^T q_t^j = V_T \quad (15)$$

where V_T is the gas injection task for all caverns over the designed optimization period (T days), Nm^3 .

3.1.4. Analytical Proof of the Strict Convexity of the Comprehensive Objective Function

Based on the volumetric shrinkage control term and the cost control term in the comprehensive objective function, given the initial cavern pressure $P_{i,0}$, the single-cavern volumetric shrinkage rate and the injection energy consumption are mainly influenced by the final pressure $P_{i,100}$, both of which are non-linear functions of the total injection volume Q_i . Meanwhile, the time cost is a linear function of the injection time. According to Equation (7), the relationship between the volumetric shrinkage rate $V_{lost,i}$ (and the injection energy consumption) of cavern i and the total injection volume Q_i can be abstracted into the following function form F :

$$F(Q_i) = \frac{C_1}{P_i^0 + k_i Q_i} - C_2 \quad (16)$$

where C_1 , C_2 , P_i^0 , and k_i are all non-negative real constants determined by the physical properties of the cavern.

Taking the first and second derivatives of the function F with respect to Q_i yields

$$\frac{\partial F(Q_i)}{\partial Q_i} = -\frac{C_1 \cdot k_i}{(P_i^0 + k_i Q_i)^2} \quad (17)$$

$$\frac{\partial^2 F(Q_i)}{\partial Q_i^2} = \frac{2C_1 \cdot k_i^2}{(P_i^0 + k_i Q_i)^3} \quad (18)$$

Since both the constant terms and the variable domains are positive, the condition $\partial^2 F(Q_i)/\partial Q_i^2$ always holds. Therefore, F is a strictly convex function with respect to the injection volume Q_i .

Furthermore, the normalization of the relative shrinkage rate in Equation (9) represents a linear transformation with positive coefficients. Because the strict convexity of a convex function remains unchanged after such a linear transformation. At the same time, the time cost is a linear function of the injection time, and any linear function is both convex and concave. In summary, because the comprehensive objective function is a combination of a strictly convex function and a linear function, the gas injection allocation optimization model established in this study constitutes a strictly convex optimization problem.

3.2. Model Standardization and Solution Method

To effectively solve the injection optimization model for the salt cavern CO₂ storage facility, it is necessary to transform it into a standard form suitable for numerical computation. This study first completes the model standardization and subsequently employs the primal-dual interior-point method as the optimization algorithm for the solution.

3.2.1. Model Standardization

The optimization model comprises a nonlinearized objective function equation and linear constraint equation. The decision variables are the daily gas injection volumes for each storage cavern, formulated as a one-dimensional vector with a dimension of $30 \times 8 = 240$. To adapt to the input specifications of the interior-point method solver, the objective function and constraint conditions are organized according to the standard form of nonlinear programming, as shown below:

$$\min f(x), \text{ s.t. } \begin{cases} g_i(x) \leq 0, & i = 1, \dots, m \\ h_j(x) = 0, & j = 1, \dots, p \end{cases} \quad (19)$$

where $g_i(x) \leq 0$ represents inequality constraints and $h_j(x) = 0$ represents equality constraints. This model comprises a total of 510 inequality constraints and 1 equality constraint, with a constraint matrix size of approximately 511×240 , representing a typical medium-scale strictly convex nonlinear optimization problem.

3.2.2. Solution Method and Principles

For the nonlinear convex optimization model, this paper adopts the primal-dual interior-point method for the solution. Since Karmarkar's pioneering work in 1984, this method has undergone considerable development. It has now been successfully extended from solving linear programming to the field of nonlinear programming and has become one of the most mainstream and efficient algorithms for solving nonlinear programming (NLP) problems with complex inequality constraints [23]. Its advantages include: ① high computational efficiency, where solution time does not increase explosively with problem size, making it suitable for gas injection allocation involving multiple caverns and long periods in actual salt cavern CO₂ storage systems; ② high numerical stability, as the algorithm reliably converges even when the constraint matrix exhibits significant order-of-magnitude disparities, ensuring the accuracy of the optimization results; and ③ low requirement for the initial point, as it can rapidly converge to the optimal solution even when starting from an infeasible point. These characteristics make it particularly suitable for handling multi-cavern, multi-constraint gas injection optimization problems, capable of meeting the engineering demands for rapidly generating gas injection allocation schemes while ensuring solution accuracy.

The primal-dual interior-point method relaxes the strict optimality conditions by introducing a barrier parameter, thereby constructing a central path located inside the feasible region and converging rapidly to the optimal solution along this path [24]. Its solution process is illustrated in Figure 3. The method first transforms the inequality constraints of the original problem into equality constraints by adding non-negative slack variables and constructs the Lagrangian function accordingly. Based on this function, the Karush-Kuhn-Tucker (KKT) system, encompassing primal feasibility, dual feasibility, and complementary slackness, is derived [25]. To overcome the difficulty of solving this system directly, the algorithm introduces a positive perturbation parameter (i.e., the barrier parameter) into the complementary slackness condition, forming a series of perturbed KKT systems that approximate the original conditions [24]. In each iteration step, Newton's method is employed to solve the linear approximation of this nonlinear system, obtaining update directions for the primal and dual variables. To ensure the iteration points remain strictly within the feasible region, a backtracking line search determines an appropriate step size, guiding the solution along the central path towards the optimal vertex [26]. During the iterative process, the barrier parameter is gradually reduced. When the duality gap falls below a preset threshold, the algorithm is deemed to have converged, yielding a high-precision solution within tens of iterations.

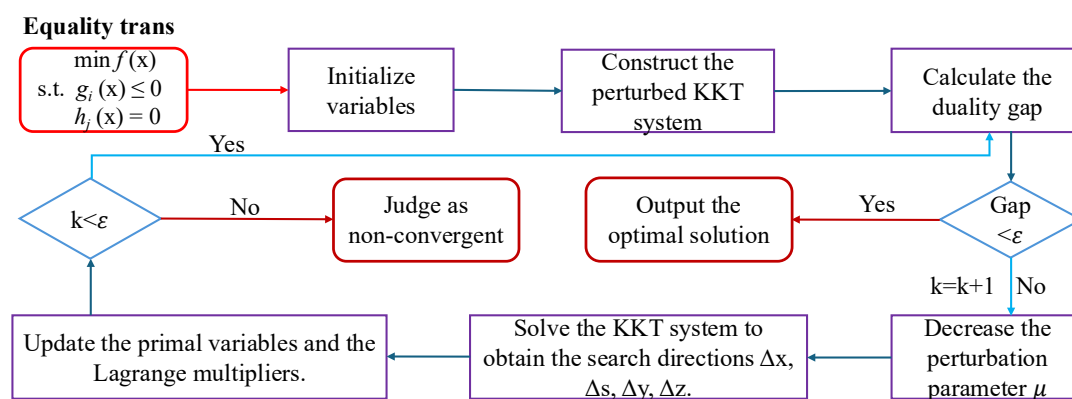


Figure 3. Solution flowchart of the primal-dual interior-point method.

3.2.3. Solution Implementation

The numerical computations were performed in the MATLAB R2023a environment. The CVX toolbox was used to solve the convex optimization problem. This toolbox internally invokes the SDPT3 solver, which employs the primal-dual interior-point method for solving convex optimization problems. During the solution process, the convergence tolerance was set to the default value of 10^{-6} , the maximum number of iterations was set to 10^3 , and other parameters were kept at their default settings.

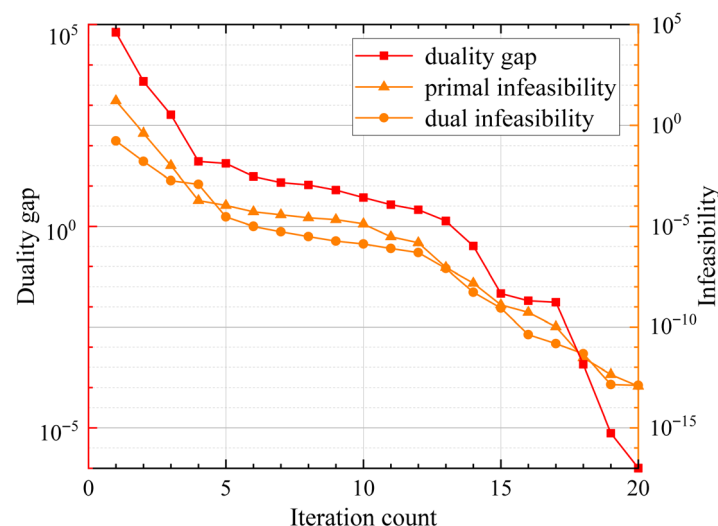
3.3. Application in an Actual Engineering Case

To validate the engineering applicability of the established optimization model and the primal-dual interior-point method solution strategy, this study selected eight typical caverns from a salt cavern storage facility in East China as the research objects. An optimization scheduling study for CO₂ injection was conducted over a scheduling period of one month. The key parameters for each cavern, derived from geological surveys and sonar cavity measurements, are detailed in Table 4.

Table 4. Operational parameters of typical caverns.

Cavern	$P_{\max}$ (MPa)	P_0 (Mpa)	Cavern Volume (m^3)	Cushion Gas (Nm^3)	Burial Depth (m)	Cavern Height (m)	Radius (m)
1	14.5	13.5	543,770	62,533,603	945.3	86.5	54.8
2	15.5	9	330,457	34,202,363	931	93	41.2
3	17.5	12.8	231,342	15,962,612	997.5	87.2	35.6
4	17.5	14.8	206,174	26,081,126	1015	85.2	40
5	17.5	13.6	247,602	17,084,594	999.8	86.4	37
6	17.5	12.5	296,004	20,424,321	996.2	87.5	40.2
7	17.5	11.2	301,898	20,831,020	996.8	88.8	40.3
8	17.5	10.8	165,819	11,441,549	999.6	89.2	29.8

The total amount of CO_2 to be injected into the storage system was set at $4 \times 10^7 \text{ Nm}^3$. The maximum daily injection capacity of the surface system was $4 \times 10^6 \text{ Nm}^3/\text{d}$, and the maximum daily injection volume for each individual cavern was $1 \times 10^6 \text{ Nm}^3$. These engineering parameters were substituted into the established nonlinear programming model and solved using MATLAB R2023a on a computing environment with an Intel Core i5-10400F processor and 24 GB of RAM. The iterative optimization process of the primal-dual interior-point method is illustrated in Figure 4.

**Figure 4.** Iteration process of the primal-dual interior-point method.

As can be observed from the figure, with the increase in iteration number, the duality gap, primal infeasibility, and dual infeasibility all exhibit a monotonically decreasing trend. After approximately 20 iterations, the primal infeasibility and dual infeasibility decreased by about 11 to 12 orders of magnitude, and the duality gap was reduced from 10^5 to 10^{-6} , meeting the preset tolerance requirement (duality gap $< 10^{-6}$). In the initial iterations, all metrics decreased rapidly, indicating the algorithm's ability to quickly enter the feasible region. In the later stages, the curves tended to flatten, reflecting the favorable local convergence characteristics of the interior-point method near the optimal solution. The entire solution process took approximately 1.1 s, which can meet the rapid decision-making requirements of engineering, and no oscillation or divergence occurred during the solution process, demonstrating the good robustness of the model.

Figure 5a illustrates the gas injection volume allocation scheme for each cavern. As shown in the figure, there are significant differences in the injection volumes among caverns: Caverns 2 and 7 received the highest amount, reaching $20.0 \times 10^6 \text{ Nm}^3$; Cavern 8 followed

with $14.0 \times 10^6 \text{ Nm}^3$; and Cavern 4 received the lowest, at only $4.0 \times 10^6 \text{ Nm}^3$. Combined with the data in the table, under similar burial depths, the allocation of injection volume is negatively correlated with the initial pressure of the cavern, and the model tends to allocate more injection volume to caverns with lower initial pressure. The internal mechanism is that caverns with lower initial pressure experience a larger pressure change after injecting the same amount of gas, which provides better control over the volumetric shrinkage rate and can more quickly reduce the overall system shrinkage rate. For caverns with higher safety, the injection intensity is moderately controlled. Under the global optimization framework, the model automatically achieves differentiated allocation by weighing the safety benefits of each cavern, reflecting an effective response to cavern heterogeneity.

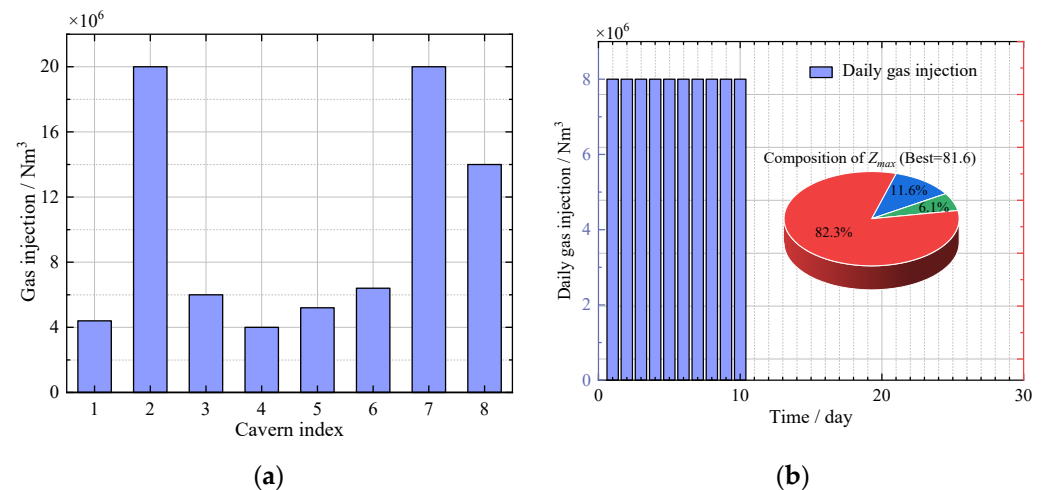


Figure 5. Gas injection scheme for each cavern within (a) gas injection volume allocation scheme and (b) variation in daily gas injection volume over time.

The daily gas injection volume of the storage system and the optimal value of the objective function during the injection period are shown in Figure 5b. The figure shows that the normalized optimal objective function score was 81.6. The optimized scheme involved continuous full-capacity injection into the storage system at the maximum daily injection rate of $4 \times 10^6 \text{ Nm}^3/\text{d}$ until the injection task was completed, achieving the minimum time cost.

To validate the effectiveness of the proposed model, the optimal solution generated in this study was compared against a baseline strategy. In the baseline scenario, the total gas injection volume is equally distributed across all cavities, with the injection task scheduled for completion within a 10-day timeframe. The performance comparison between the optimal and baseline solutions is summarized in the Table 5 below. As indicated by the results, the composite score of the optimal solution is 14.77% higher than that of the baseline. Specifically, the volume shrinkage score improved by 6.1%, and the energy consumption score increased by 219.8%, while the time cost score remained identical to the baseline. These findings demonstrate that the proposed model is capable of generating reliable optimal solutions that enhance operational efficiency.

Table 5. Performance comparison between the optimal solution and baseline strategy.

Scheme	Volume Shrinkage Score	Energy Consumption Score	Time Cost Score	Composite Score
Optimal Solution	84	63	100	81.6
Baseline Strategy	78.8	19.7	100	71.1

3.4. Model Applicability and Directions for Improvement

The analysis above indicates that this optimization model can effectively address the multi-cavern gas injection allocation and volumetric shrinkage control problem of SCCS facilities, generating safe and reliable allocation schemes while ensuring solution efficiency.

It should be clarified that the objective of the optimization of this model focuses on the century-long control of the cavern volumetric shrinkage rate and economic balance; therefore, the impact of short-term transient thermal effects is ignored during the modeling process. Considering the practical application scenarios of salt cavern energy storage projects, the core feature of salt cavern CO₂ sequestration is that CO₂ is not extracted after being injected into the cavern. The gas pressure inside the cavern remains basically stable throughout its life cycle, which highly aligns with the long-term optimization assumption of the model. Therefore, this model can be effectively applied to the long-term operational scenarios of such projects, providing a scientific reference for their gas injection allocation. However, for scenarios with high-frequency injection and production processes, such as salt cavern gas storage or compressed air energy storage projects, the internal gas pressure of the cavern fluctuates significantly, and transient thermal effects will significantly impact the flow allocation and safety of each cavern. In this case, if the daily flow allocation scheme generated by this model is directly adopted, significant deviations will occur due to the neglect of pressure fluctuations and thermal-mechanical coupling effects, making it unable to meet the accuracy requirements of daily scheduling for flow decision-making. To address this limitation, future work needs to further expand the applicable scope of the model, optimize the model assumptions, and improve its adaptability to high-frequency injection–production scenarios and daily scheduling. In addition, the model still has the following limitations:

First, the description of physical mechanisms within the objective function is not yet complete. The current adoption of a volumetric shrinkage rate expression for regular caverns, rather than an analytical expression for actual irregular caverns, is primarily constrained by the complexity of the actual cavern geometry, making it difficult to accurately describe its shrinkage characteristics with a simple formula. Furthermore, the model employs simplified treatments for both the volumetric shrinkage rate and energy consumption; while this does not affect the relative comparison and selection between different schemes, it prevents the precise calculation of absolute values. Subsequent work could combine numerical simulations or machine-learning-based surrogate models to construct an objective function that more closely aligns with physical reality.

Second, the safety evaluation index is relatively singular. Although the volumetric shrinkage rate is an important indicator for assessing cavern safety, cavern stability is also influenced by factors such as roof integrity and interlayer sealing. Future studies could introduce a multi-objective optimization framework, incorporating multiple safety indicators into the objective function or constraints to achieve collaborative optimization of safety and economy.

4. Conclusions

This paper addresses the research gap in the proactive control of SCCS cavern shrinkage and its synergistic optimization with economic performance, conducting sensitivity analysis of influencing factors and optimization scheduling research. The main conclusions are as follows:

1. The primary controlling factors for the volumetric shrinkage rate of SCCS caverns and their sensitivity ranking were clarified. The research reveals that the ratio of in situ stress to internal pressure (f_1) is the core factor affecting the long-term shrinkage of the cavern, with its sensitivity significantly higher than that of cavern volume

(f_4) and the height-to-diameter ratio (f_2), while the influence of the symmetry factor (f_3) is negligible. The quantitative ranking of the sensitivity of each factor is $f_1(2.31) > f_4(0.309) > f_2(0.166) > f_3(0)$.

2. A multi-objective gas injection optimization model was constructed, integrating proactive shrinkage control and operational economy. The model comprehensively considers energy consumption, volumetric shrinkage rate, and time cost; utilizes sensitivity coefficients to weight heterogeneous caverns; and integrates engineering constraints, such as pressure safety, equipment capacity, and gas injection tasks. The model can effectively support the comparison and selection of different gas injection schemes; however, due to simplification, it cannot accurately calculate energy consumption and volumetric shrinkage rates and is only applicable to long-term volumetric shrinkage control.
3. The efficiency and robustness of the primal-dual interior-point method for solving the model were verified. The solution of the actual case study demonstrated that the algorithm could converge stably to the optimal solution within 20 iterations and approximately 1.1 s, with the duality gap decreasing by over 11 orders of magnitude, meeting the real-time requirements of engineering scheduling.
4. The significant role of the optimization model in improving the reliability and economy of the scheduling scheme was confirmed. Optimization results show that the model tends to allocate more gas volume to caverns with lower initial pressure and completes the task in the shortest possible time. The comprehensive score of the optimal scheme is 14.77% higher than that of the average allocation baseline scheme, indicating that the model can fully respond to cavern heterogeneity and generate scientific and reliable scheduling schemes.

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