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A Bilevel Optimization Framework for Power–Traffic Network Coordination with Incentive-Based Driver Decisions

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Abstract

Electric vehicles have strengthened the coupling between transportation systems and power distribution networks, giving rise to new challenges in the coordinated management of traffic flow and charging demand. Monetary incentives, such as tariffs and subsidies, have been widely adopted to influence drivers' route and charging decisions and to improve system-level performance. This paper proposes a user-centric incentive framework in which a system operator allocates rewards to guide drivers' behavior, thereby enabling coordinated operation of power–traffic networks. A reward scheme is developed to provide joint subscription-based and path-based incentives that account for drivers' behavioral responses through a logit choice model for scheme adoption embedded within a traffic assignment model. The resulting interaction is formulated as a bilevel optimization problem, in which a coupled power–traffic system operator determines incentive schemes to achieve system optimality within a given budget constraint, while individual drivers respond by selecting routes and charging strategies to minimize their perceived travel costs. A single-level Karush–Kuhn–Tucker (KKT) reformulation is developed, and linearization techniques are employed to compute the resulting equilibrium, yielding a tractable mixed-integer second-order cone program (MISOCP). Numerical experiments demonstrate the effectiveness of the subscription-based and path-based reward schemes in improving network performance and budget saving.

Keywords: bilevel optimization; electric vehicles; logit choice model; power–traffic network; reward scheme; system optimality

1. Introduction

With the rapid growth of electric vehicles (EVs), the demand for EV charging has risen and public charging is gaining popularity [1]. According to the International Energy Agency, EV sales exceeded 17 million units in 2024, rising by more than 25% [2]. Meanwhile, recent advances in EV energy management and control technologies have further promoted the deployment of EVs, such as energy-oriented torque-vector control frameworks for electric vehicles [3]. The large-scale integration of EVs has strengthened the coupling between transportation networks (TNs) and power distribution networks (PDNs), where traffic conditions and charging-related factors jointly influence EV routing, charging behavior, and system operations [4–6]. In this context, the coordinated operation of transportation and power distribution networks is increasingly important to address the operational challenges and safety risks in PDNs induced by large-scale EV charging, and to mitigate traffic congestion in TNs [7].



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Extensive research has investigated the interdependence between urban power networks and transportation networks (TNs). These studies primarily focus on characterizing the stable equilibrium states arising from such interactions. Such equilibrium formulations—including static [8], semi-dynamic [9], and dynamic equilibria [10]—describe the resulting distributions of traffic flows and EV charging demand under the assumption that drivers act as self-interested agents and select routes by minimizing their perceived travel costs.

Unlike the aforementioned equilibrium analysis of the power–transportation system, which focuses on the strategic interactions among drivers, transportation operators, and the power grid through locational marginal pricing (LMP) schemes and results in a user equilibrium, the power–traffic coordinated optimization problem aims to jointly design price signals to improve the operational performance of both transportation networks and power distribution networks while accounting for drivers' path choices [11–13], thereby shifting the system from user equilibrium toward system optimality.

By modeling travelers as non-cooperative decision-makers who select travel plans by minimizing individual costs, some studies primarily focus on EV charging prices at charging stations (CSs), neglecting the role of road pricing in regulating user choices and mitigating traffic congestion [14,15]. Road congestion tolls have been introduced as an efficient traffic regulation policy solely from the perspective of transportation networks [16]. Collaborative pricing schemes generally coordinate nodal electricity prices with congestion tolls to achieve system-wide performance optimization across the integrated network. Although transportation and power systems are presently managed by independent operators [17], coupled power–traffic network operators are often adopted as an analytical framework to demonstrate the benefits of coordinated operation and to highlight the suboptimality and potential reliability concerns associated with independently operated systems.

Many existing studies attempt to induce the system optimum (SO) from the user equilibrium (UE) through price-based mechanisms. The SO can be interpreted as a modified UE in which travelers respond to marginal costs. Under the tariff-based scheme, a marginal-cost pricing approach based on first-best tolls and locational marginal prices [18] is designed to minimize the total social cost, measured by the sum of travel time and energy generation costs. The coupled power–transportation network is analyzed under the SO and UE states, and the inverse optimization [19] is applied to the pricing paradigm for minimizing additional user charges. A privacy-preserving cooperative pricing framework [20] is developed for coupled power–traffic networks under a predefined privacy budget, which avoids direct access to true demand data while ensuring feasibility and near-optimal performance. Such high tariffs or increased travel times may undermine user acceptance, thereby limiting the practical attainability of the SO state. However, traditional marginal cost pricing often leads to excessive user charges, as travelers are required to account for congestion externalities.

Under the subsidy-based scheme, reference [21] introduces subsidy-based incentives on both the power and transportation sides using a Nash–Stackelberg–Nash game model, improving the efficiency and flexibility of price-based incentive mechanisms. Reference [22] investigates the collaborative interactions and profit allocation mechanisms between travelers and network operators, aiming to improve the regulatory effectiveness of monetary incentives and enhancing social welfare in coupled power–traffic networks. A path-based reward credit mechanism [23] is developed to incentivize commuters, aiming to mitigate carbon emissions while minimizing the overall system travel time. However, the power network is not taken into account in this scheme.

The coupled power–traffic optimization problem aims to jointly design price signals to minimize the combined operational costs of both networks or the overall social cost, accounting for low-carbon [24,25] and economic objectives [26] as well as drivers' individual

costs. However, achieving these goals depends not only on monetary incentives but also on the strategic interactions among multiple entities, including drivers, network operators, and the power system. To account for these interactions and evaluate their impact on system performance, coordinated modeling frameworks have been developed. Existing studies on coupled power–traffic networks can be broadly grouped into several types. Some works model the PDN operator and drivers as separate decision-making entities [27]. Others extend this setting by introducing additional participants, such as charging network operators, under non-cooperative pricing schemes [15] or trilevel frameworks [28]. A further stream of research considers cooperative interactions among network operators by explicitly accounting for the role of the TN operator [29]. This paper focuses on a bilevel framework in which power and transportation operators act collaboratively as leaders by offering subscription-based and path-based incentives, while EV/GV drivers respond by deciding whether to participate in the incentive scheme and by choosing routes and charging strategies.

Table 1 provides a clear comparison between this study and the existing literature. Most existing studies fail to account for drivers’ endogenous participation behavior in response to incentive schemes, as well as the need for differentiated incentive design across heterogeneous vehicle energy types. For example, some studies, such as [22], assume a fixed proportion of collaborative and independent travelers. To address this research gap, this paper presents a reward-based method for the coordinated operation of coupled power–traffic networks aimed at improving system-level performance. Specifically, a subscription-based and path-based reward scheme is introduced to incentivize drivers to select paths that enhance social welfare within a given budget constraint. A logit choice model based on drivers’ utility functions is developed to capture their decisions on whether to participate in the incentive scheme. The resulting bilevel problem is transformed into a single-level optimization problem through the application of Karush–Kuhn–Tucker (KKT) conditions, along with linearization techniques. Specifically, this paper makes the following contributions:

- (1) A bilevel framework is developed to model the interactions among the transportation network (TN) operator, power distribution network (PDN) operator, and electric vehicle (EV)/gasoline vehicle (GV) drivers. The leader problem corresponds to the reward allocation by cooperative TN and PDN operators to guide drivers’ behavior under a given budget constraint. The follower problem captures drivers’ endogenous decisions on whether to subscribe to the reward scheme, as well as their collective route and charging choices, which are modeled as a Wardrop user equilibrium accounting for generalized costs.
- (2) A logit choice model is designed to represent drivers’ decisions on whether to participate in the reward program. In practice, drivers have the flexibility to voluntarily join or opt out of the scheme at the beginning of each operating period.
- (3) Karush–Kuhn–Tucker (KKT) reformulation, together with linearization techniques, is applied to solve the coupled power–traffic optimization problem. Simulations conducted on a test system demonstrate the effectiveness of the proposed approach.

The remainder of this paper is organized as follows. Section 2 introduces the overall framework, the drivers’ logit choice model, and the operational models for each entity. Section 3 details the solution methodology for the proposed bilevel problem. Section 4 presents the experimental results. Section 5 analyzes the experimental results while Section 6 concludes the paper.

Table 1. Comparison of the existing literature and this paper.

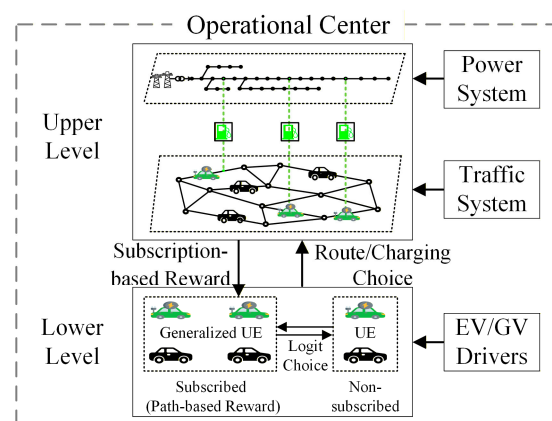
Ref.	Scheme	Incentive		Solution Methodology
		PDN	TN	
[15]	Noncooperative	Tariff (charging pricing)	-	Fixed point theory
[18]	Cooperative	Tariff (LMP)	Tariff (road toll)	Optimality condition decomposition
[19]	Inverse optimization-based	Tariff (charging pricing)	-	Decentralized bilevel decomposition
[20]	Cooperative	Tariff (charging pricing)	-	Privacy-preserving cooperative optimization
[21]	Noncooperative	Subsidy (charging pricing)	Subsidy (link-based)	KKT-based bilevel reformulation with Gauss–Seidel iteration
[22]	Cooperative	Subsidy	Subsidy (path-based)	Asymmetric Nash bargaining
[23]	Cooperative	-	Subsidy (subscription and path-based)	KKT-based bilevel reformulation with B&B algorithm
This paper	Cooperative	Subsidy	Subsidy (subscription and path-based)	KKT-based bilevel reformulation with linearization techniques

2. Model Formulation

In this section, a bilevel framework is first introduced in Section 2.1. The operational models for drivers and the TN/PDN operators are then presented in Sections 2.2 and 2.3, respectively. Finally, the bilevel optimization formulation is summarized in Section 2.4.

2.1. Overall Framework

Figure 1 depicts the bilevel structure for power–traffic network coordination, where an operational center is introduced to maximize social welfare. At the upper level, both network operators aim to improve the performance of the coupled networks within a given budget constraint by offering rewards to drivers who subscribe to the incentive program. The proposed reward scheme consists of two components: a subscription-based reward, in which drivers receive a fixed amount of rewards upon joining the program, and a path-based reward for choosing system-optimal routes with lower generalized travel costs. At the lower level, drivers have the flexibility to voluntarily participate in or opt out of the scheme at the beginning of each operating period. Each electric vehicle/gasoline vehicle (EV/GV) driver independently makes route and charging choices according to the generalized cost. The resulting traffic flow distribution reaches a user equilibrium (UE) state, in which no driver can reduce their generalized travel cost by unilaterally changing their route or charging choices under the reward scheme. Accordingly, the interactions between TN/PDN operators and EV/GV drivers can be formulated as a bilevel optimization problem.

**Figure 1.** System framework.

2.2. Operation Model of EV/GV Drivers (Follower Problem)

Given that GVs still account for a large portion of the vehicle fleet, both GVs and EVs are explicitly considered in the proposed modeling framework. To maintain consistency in traffic flow representation, EVs that do not require charging are treated as GVs in the model.

Since drivers have the flexibility to voluntarily participate or opt out of the reward at the beginning of every operating period, we denote the variable demand of GVs and EVs who choose to participate for OD pair r - s by $q_{rs}^{g,I}$ and $q_{rs}^{e,I}$, respectively. Similarly, the variable demand of GVs and EVs who choose not to participate is denoted by $q_{rs}^{g,N}$ and $q_{rs}^{e,N}$ for the same OD pair. Accordingly, the path flow variables $f_{k,rs}^{g,N}$, $f_{k,rs}^{g,I}$, $f_{k,rs}^{e,N}$ and $f_{k,rs}^{e,I}$ on path k for OD pair r - s must be nonnegative and satisfy the corresponding demand constraints:

$$\sum_{k \in K_{rs}} f_{k,rs}^{g,I} = q_{rs}^{g,I}, \forall r \in R, s \in S \quad (1)$$

$$\sum_{k \in K_{rs}} f_{k,rs}^{g,N} = q_{rs}^{g,N}, \forall r \in R, s \in S \quad (2)$$

$$\sum_{k \in K_{rs}} f_{k,rs}^{e,I} = q_{rs}^{e,I}, \forall r \in R, s \in S \quad (3)$$

$$\sum_{k \in K_{rs}} f_{k,rs}^{e,N} = q_{rs}^{e,N}, \forall r \in R, s \in S \quad (4)$$

$$q_{rs}^{g,N} + q_{rs}^{g,I} = q_{rs}^g, \forall r \in R, s \in S \quad (5)$$

$$q_{rs}^{e,N} + q_{rs}^{e,I} = q_{rs}^e, \forall r \in R, s \in S \quad (6)$$

$$f_{k,rs}^{g,I} \geq 0, f_{k,rs}^{g,N} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (7)$$

$$f_{k,rs}^{e,I} \geq 0, f_{k,rs}^{e,N} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (8)$$

where q_{rs}^g and q_{rs}^e are assumed to be fixed, denoting the fixed demand of GVs and EVs on OD pair r - s .

We use a logit choice model to capture the stochasticity in the proportions of GV and EV drivers who choose to participate in the reward scheme. The utilities of GV and EV drivers, for both participating and non-participating cases, denoted by $U_{rs}^{g,I}$, $U_{rs}^{g,N}$, $U_{rs}^{e,I}$ and $U_{rs}^{e,N}$, are defined as follows:

$$U_{rs}^{g,I} = -\theta u_{rs}^{g,I} + \theta h_{rs}^g, \forall r \in R, s \in S \quad (9)$$

$$U_{rs}^{g,N} = -\theta u_{rs}^{g,N}, \forall r \in R, s \in S \quad (10)$$

$$U_{rs}^{e,I} = -\theta u_{rs}^{e,I} + \theta h_{rs}^e, \forall r \in R, s \in S \quad (11)$$

$$U_{rs}^{e,N} = -\theta u_{rs}^{e,N}, \forall r \in R, s \in S \quad (12)$$

where θ is an empirical parameter representing drivers' sensitivity to the utility difference between subscribing and not subscribing, capturing their responsiveness to monetary incentives and travel cost variations. $u_{rs}^{g,I}$ and $u_{rs}^{g,N}$ denote the minimum generalized travel costs of s subscribing and non-subscribing GVs for OD pair r - s , respectively, while $u_{rs}^{e,I}$ and $u_{rs}^{e,N}$ denote the corresponding costs for EVs. h_{rs}^g and h_{rs}^e denote the subscription-based reward of GVs and EVs, respectively.

The utility of subscribing GV drivers is based on their path travel time cost and the subscription-based reward, whereas the utility of non-subscribing GV drivers depends only on their perceived travel time cost. Similarly, the utility of subscribing EV drivers accounts for path travel time cost, charging price, and the subscription-based reward, while the utility of non-subscribing EV drivers is determined solely by travel time cost and charging

price. Accordingly, the minimum generalized costs for subscribing and non-subscribing GV/EV drivers, denoted by $u_{rs}^{g,I}$, $u_{rs}^{g,N}$, $u_{rs}^{e,I}$ and $u_{rs}^{e,N}$ are defined as below:

$$u_{rs}^{g,I} = \min_{k \in K_{rs}} \left(\sum_{a \in A_{rg}} \eta t_a \delta_{ak,rs}^g - m_{k,rs}^g \right) \quad (13)$$

$$u_{rs}^{g,N} = \min_{k \in K_{rs}} \left(\sum_{a \in A_{rg}} \eta t_a \delta_{ak,rs}^g \right) \quad (14)$$

$$u_{rs}^{e,I} = \min_{k \in K_{rs}} \left(\sum_{a \in A_{rg}} \eta t_a \delta_{ak,rs}^e + \sum_{a \in A_{ch}} (\eta t_a + \lambda_a E^e) \delta_{ak,rs}^e - m_{k,rs}^e \right) \quad (15)$$

$$u_{rs}^{e,N} = \min_{k \in K_{rs}} \left(\sum_{a \in A_{rg}} \eta t_a \delta_{ak,rs}^e + \sum_{a \in A_{ch}} (\eta t_a + \lambda_a E^e) \delta_{ak,rs}^e \right) \quad (16)$$

where $m_{k,rs}^g$ and $m_{k,rs}^e$ denote the path-based reward on path k for OD pair r - s . K_{rs} denotes the feasible path set for OD pair r - s . η denotes the monetary cost associated with traveling time and t_a denotes the link travel time on link a . The indicator $\delta_{ak,rs}^g$ and $\delta_{ak,rs}^e$ equals 1 if path k between origin–destination (O–D) pair r - s contains link a and 0 otherwise. λ_a is the charging price at charging stations (CSs) located on link a and E^e denotes the energy demand of each EV.

Accordingly, the traffic demand of drivers subscribing to the reward scheme for OD pair r - s is given by:

$$q_{rs}^{g,I} = q_{rs}^g \frac{\exp(-\theta u_{rs}^{g,I} + \theta h_{rs}^g)}{\exp(-\theta u_{rs}^{g,N}) + \exp(-\theta u_{rs}^{g,I} + \theta h_{rs}^g)}, \forall r \in R, s \in S \quad (17)$$

$$q_{rs}^{e,I} = q_{rs}^e \frac{\exp(-\theta u_{rs}^{e,I} + \theta h_{rs}^e)}{\exp(-\theta u_{rs}^{e,N}) + \exp(-\theta u_{rs}^{e,I} + \theta h_{rs}^e)}, \forall r \in R, s \in S \quad (18)$$

Travel time on each link is mainly affected by the associated traffic flow. To represent both GVs and EVs consistently, the traffic network is modeled with three types of links: regular, charging, and bypass links, as shown in Figure 2. Regular links correspond to physical road segments, while charging and bypass links are virtual links representing charging/queuing and bypassing behaviors at charging stations (CSs), respectively. For example, a route traversing a charging station may follow the sequence $T1 \rightarrow I \rightarrow O \rightarrow T2$ via a charging link to represent EV charging, or follow the same sequence via a bypass link to indicate that GVs skip charging at the station. Based on this, the detailed model for GVs and EVs is described as follows:

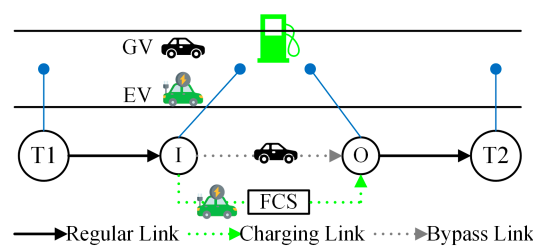


Figure 2. Unified topology of transportation network.

Cons-Flow:

$$x_a^{rg} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} \left(f_{k,rs}^{g,N} + f_{k,rs}^{g,I} \right) \delta_{ak,rs}^g + \left(f_{k,rs}^{e,N} + f_{k,rs}^{e,I} \right) \delta_{ak,rs}^e, \forall a \in A_{rg} \quad (19)$$

$$x_a^{ch} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} \left(f_{k,rs}^{e,N} + f_{k,rs}^{e,I} \right) \delta_{ak,rs}^e, \forall a \in A_{ch} \quad (20)$$

Cons-Time:

$$t_a(x_a^{rg}) = t_a^0 \left[1 + 0.15 \left(\frac{x_a^{rg}}{C_a^{rg}} \right)^4 \right], \forall a \in A_{rg} \quad (21)$$

$$t_a(x_a^{ch}) = \frac{E^e}{p^e} + t_a^{ch,max} \left(\frac{x_a^{ch}}{C_a^{ch}} \right)^3, \forall a \in A_{ch} \quad (22)$$

where x_a^{rg} and x_a^{ch} denote aggregated flow on link a . The parameter t_a^0 denotes the free-flow travel time on a regular link $a \in A_{rg}$ and C_a^{rg} denotes the capacity of the corresponding road segment. For charging stations (CSs), p^e denotes average service rate, $t_a^{ch,max}$ denotes maximum waiting time and C_a^{ch} denotes the station capacity. Finally, A_{rg} and A_{ch} denote the set of regular links and charging links in the traffic network, respectively.

The constraint set Cons-Flow enforces flow conservation in the TN. Specifically, constraints (19)–(20) describe the relationship between path flows and link flows. The constraint set Cons-Time specifies the travel time formulations for different link types. For a regular link $a \in A_{rg}$, constraint (21) models the link travel time using the Bureau of Public Roads (BPR) function as a function of x_a^{rg} . For a charging link $a \in A_{ch}$, constraint (22) captures the queuing behavior at the charging station based on an $M/M/c/K$ queueing model, which is commonly used to represent systems with c servers and a maximum system capacity of K customers. In constraint (22), the first term corresponds to the charging time, while the second term represents the expected queuing delay.

The above-mentioned UE model, which accounts for the proportion of users adopting subscription-based and path-based rewards, is formulated as follows:

$$\begin{aligned} \min C_{UE} = & \sum_{a \in A_{rg}} \int_0^{x_a^{rg}} \eta t_a(\omega) d\omega + \sum_{a \in A_{ch}} \int_0^{x_a^{ch}} [\eta t_a(\omega) + \lambda_a E^e] d\omega \\ & + \frac{1}{\theta} \sum_{r \in R} \sum_{s \in S} (q_{rs}^{g,I} \ln q_{rs}^{g,I} + q_{rs}^{g,N} \ln q_{rs}^{g,N} + q_{rs}^{e,I} \ln q_{rs}^{e,I} + q_{rs}^{e,N} \ln q_{rs}^{e,N}) \\ & - \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} (f_{k,rs}^{g,I} m_{k,rs}^g + f_{k,rs}^{e,I} m_{k,rs}^e) - \sum_{r \in R} \sum_{s \in S} (q_{rs}^{g,I} h_{rs}^g + q_{rs}^{e,I} h_{rs}^e) \end{aligned} \quad (23)$$

subject to: (1)–(8), (13)–(22)

2.3. Operation Model for Power–Traffic Operator (Leader Problem)

The coupled network operators aim to promote social welfare, which comprises the travel time cost in the TN as given in (30) and the total economic cost in the PDN as given in (31). The travel time cost includes the time spent on regular links, as well as the waiting and charging times at CSs, which are converted into monetary cost using η .

$$C = C_{TN} + C_{PDN} \quad (24)$$

To prevent drivers from engaging in socially inefficient behaviors, such as making unnecessary detours or cyclic trips solely to obtain rewards, the model imposes that the utility gained from rewards on any path does not exceed the minimum generalized travel cost.

$$u_{rs}^{g,I} \geq 0, u_{rs}^{e,I} \geq 0, \forall r \in R, s \in S \quad (25)$$

$$m_{k,rs}^g \geq 0, m_{k,rs}^e \geq 0, \forall r \in R, s \in S \quad (26)$$

In the subscription-based reward scheme, $\bar{h}_{rs}^g$ and $\bar{h}_{rs}^e$ are defined as upper bounds denoting the maximum reward granted to subscribers on OD pair r - s .

$$0 \leq h_{rs}^g \leq \bar{h}_{rs}^g \quad (27)$$

$$0 \leq h_{rs}^e \leq \bar{h}_{rs}^e \quad (28)$$

Let B denote the total reward budget. Accordingly, the budget constraint is imposed to ensure that the total reward does not exceed budget.

$$C_{re} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} (f_{k,rs}^{g,I} m_{k,rs}^g + f_{k,rs}^{e,I} m_{k,rs}^e) + \sum_{r \in R} \sum_{s \in S} (q_{rs}^{g,I} h_{rs}^g + q_{rs}^{e,I} h_{rs}^e) \leq B \quad (29)$$

The total traffic delay cost can be expressed as follows:

$$C_{TN} = \eta \left(\sum_{a \in A_{rg}} t_a(x_a^{rg}) \cdot x_a^{rg} + \sum_{a \in A_{ch}} t_a(x_a^{ch}) \cdot x_a^{ch} \right) \quad (30)$$

Consider a radial distribution network consisting of a set of buses and branches. The slack bus is indexed by 0, and each branch is denoted by the pair (i, j) , indicating a directed connection from buses i to bus j . The ACOPF problem aims to minimize the total operating cost of the PDN, which includes local generation costs and the cost of electricity procured from the main grid, and is formulated as follows:

$$C_{PDN} = \sum_j [a_m (P_j^g)^2 + b_m P_j^g] + \lambda_0 \sum_k P_{0k} \quad (31)$$

where the first term denotes the generation cost of local distributed generators, and the second term denotes the cost of electricity purchased from the main grid. a_m and b_m denote the quadratic and linear cost coefficients of generator m . P_{0k} denotes the active power purchased from the main grid and λ_0 is the electricity price from the main grid.

The relationship between line flows and bus voltages in the PDN is given as follows:

$$P_j^g + P_{ij} - r_{ij} \bar{I}_{ij} = \sum_{k \in \Omega(j)} P_{jk} + P_{L,j} + P_{FCS,j}, \forall j \in N^+ \quad (32)$$

$$Q_j^g + Q_{ij} - x_{ij} \bar{I}_{ij} = \sum_{k \in \Omega(j)} Q_{jk} + Q_{L,j}, \forall j \in N^+ \quad (33)$$

$$\bar{U}_i - \bar{U}_j = 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) - (r_{ij}^2 + x_{ij}^2) \bar{I}_{ij}, \forall (i, j) \in L \quad (34)$$

$$\bar{U}_i \bar{I}_{ij} \geq (P_{ij})^2 + (Q_{ij})^2, \forall (i, j) \in L \quad (35)$$

$$0 \leq P_j^g \leq P_j^{\max}, \forall j \in N^+ \quad (36)$$

$$Q_j^{\min} \leq Q_j^g \leq Q_j^{\max}, \forall j \in N^+ \quad (37)$$

$$U_{\min}^2 \leq \bar{U}_i \leq U_{\max}^2, \forall j \in N^+ \quad (38)$$

$$0 \leq \bar{I}_{ij} \leq I_{\max}^2, \forall (i, j) \in L \quad (39)$$

$$P_{ij}^{\min} \leq P_{ij} \leq P_{ij}^{\max}, \forall (i, j) \in L \quad (40)$$

$$Q_{ij}^{\min} \leq Q_{ij} \leq Q_{ij}^{\max}, \forall (i, j) \in L \quad (41)$$

where P_j^g and Q_j^g denote active and reactive power outputs of distributed generators (DGs), respectively. P_{ij} and Q_{ij} denote the active and reactive power flows on branch ij . r_{ij} and x_{ij} denote the resistance and reactance of branch ij , respectively. $\bar{I}_{ij}$ and $\bar{U}_i$ denote the squared current amplitude on branch ij and the squared voltage amplitude at bus i , respectively. $P_{L,j}$ and $Q_{L,j}$ denote the regular active and reactive load demand at bus j . $P_{FCS,j}$ denote the charging load demand of the fast-charging station supplied by bus j , which is determined through its interaction with the transportation network. $P_j^{\max}$ and $Q_j^{\max}$ are the upper limits of the active and reactive power outputs of DGs, respectively, while $Q_j^{\min}$ denotes the lower limit of the reactive power output. $U_{\max}$ and $U_{\min}$ denote the upper and lower voltage limits, respectively, and $I_{\max}$ denotes upper limits of the branch current. $P_{ij}^{\max}$, $P_{ij}^{\min}$, $Q_{ij}^{\max}$ and $Q_{ij}^{\min}$ denote the upper and lower limits of the active and reactive power flows on branch ij . N^+ denotes the set of buses, and L denotes the set of branches. If a branch is directed from bus k to bus j , $k \in \Omega(j)$ denotes the set of all child buses of bus j .

The apparent power constraint for each line in its original form is given by:

$$\bar{U}_i \bar{I}_{ij} = (P_{ij})^2 + (Q_{ij})^2, \forall (i, j) \in L \quad (42)$$

here the inequality in (35) is used in place of (42), and its SOC form is given as follows:

$$\left\| \begin{bmatrix} 2P_{ij} \\ 2Q_{ij} \\ \bar{I}_{ij} - \bar{U}_i \end{bmatrix} \right\|_2 \leq \bar{I}_{ij} + \bar{U}_i, \forall (i, j) \in L \quad (43)$$

This relaxation has been shown to be exact, and a detailed proof of the equivalence between (35) and (42) is provided in [30].

2.4. Summary

The bilevel optimization formulation is presented as follows:

$$\min_{\mathbf{h}^g, \mathbf{h}^e, \mathbf{m}^g, \mathbf{m}^e, \mathbf{f}^{g,I}, \mathbf{f}^{g,N}, \mathbf{f}^{e,I}, \mathbf{f}^{e,N}, \mathbf{q}^{g,I}, \mathbf{q}^{e,I}} C \quad (44)$$

subject to: (24)–(34), (36)–(41), (43).

$$\left(\mathbf{f}^{g,I}, \mathbf{f}^{g,N}, \mathbf{f}^{e,I}, \mathbf{f}^{e,N}, \mathbf{q}^{g,I}, \mathbf{q}^{e,I} \right) \in \operatorname{argmin}_{C_{UE}} \quad (45)$$

subject to: (1)–(8), (13)–(22).

Where variables in bold denote vectors. The leader's main decisions are the subscription-based and path-based reward vectors $\mathbf{h}^g$, $\mathbf{h}^e$, $\mathbf{m}^g$ and $\mathbf{m}^e$ for GV and EVs, respectively. The follower problem solves a UE parameterized by $\mathbf{m}^g$ and $\mathbf{m}^e$, incorporating a discrete choice model with parameters $\mathbf{h}^g$ and $\mathbf{h}^e$. Anticipating the followers' optimal responses, the leader can then select the strategy that best aligns with its own objective.

3. Solution Methodology

We first derive a single-level reformulation of the problem in Section 3.1 based on the Karush–Kuhn–Tucker (KKT) conditions. To enable tractable computation, the nonlinear and bilinear terms in the reformulated model are then linearized in Section 3.2, resulting in a tractable mixed-integer second-order cone (SOC) programming (MISOCP) problem.

3.1. Single-Level KKT Reformulation

As shown in Figure 3, at the upper level, the operational center designs a budget-constrained incentive scheme, including subscription-based and path-based rewards, to

improve social welfare. At the lower level, drivers voluntarily decide whether to participate in the scheme and choose their routes and charging behaviors to minimize individual generalized travel costs under the given incentives. Since the lower-level problem is convex, it can be equivalently reformulated using its KKT conditions.

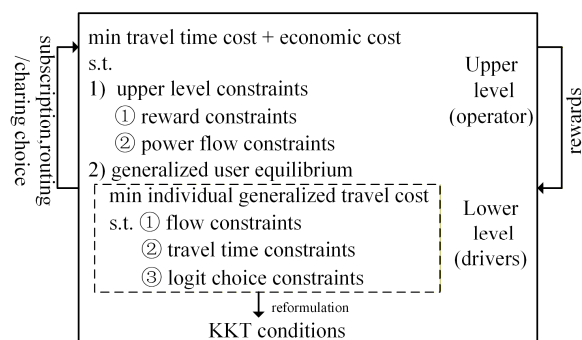


Figure 3. Architecture for the interaction mechanism between drivers and operators.

The Lagrangian multipliers associated with the flow conservation constraints (1)–(4) are denoted by $\lambda_{rs}^{g,I}$, $\lambda_{rs}^{g,N}$, $\lambda_{rs}^{e,I}$ and $\lambda_{rs}^{e,N}$, with bold face symbols indicating vector-valued multipliers. The Lagrangian function of the follower problem is then given by:

$$L(\mathbf{f}^{g,I}, \mathbf{f}^{g,N}, \mathbf{f}^{e,I}, \mathbf{f}^{e,N}, \mathbf{q}^{g,I}, \mathbf{q}^{e,I}, \lambda_{rs}^{g,I}, \lambda_{rs}^{g,N}, \lambda_{rs}^{e,I}, \lambda_{rs}^{e,N}) = C_{UE} - \sum_{r \in R} \sum_{s \in S} \lambda_{rs}^{g,I} \left(\sum_{k \in K_{rs}} f_{k,rs}^{g,I} - q_{rs}^{g,I} \right) - \sum_{r \in R} \sum_{s \in S} \lambda_{rs}^{g,N} \left(\sum_{k \in K_{rs}} f_{k,rs}^{g,N} - q_{rs}^{g,N} + q_{rs}^{g,I} \right) - \sum_{r \in R} \sum_{s \in S} \lambda_{rs}^{e,I} \left(\sum_{k \in K_{rs}} f_{k,rs}^{e,I} - q_{rs}^{e,I} \right) - \sum_{r \in R} \sum_{s \in S} \lambda_{rs}^{e,N} \left(\sum_{k \in K_{rs}} f_{k,rs}^{e,N} - q_{rs}^{e,N} + q_{rs}^{e,I} \right) \quad (46)$$

The convex follower problem can be replaced by its KKT conditions, thereby reducing the bilevel model to a single-level problem. The following conditions must then hold:

$$f_{k,rs}^{g,I} \frac{\partial L}{\partial f_{k,rs}^{g,I}} = 0, \frac{\partial L}{\partial f_{k,rs}^{g,I}} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (47)$$

$$f_{k,rs}^{g,N} \frac{\partial L}{\partial f_{k,rs}^{g,N}} = 0, \frac{\partial L}{\partial f_{k,rs}^{g,N}} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (48)$$

$$f_{k,rs}^{e,I} \frac{\partial L}{\partial f_{k,rs}^{e,I}} = 0, \frac{\partial L}{\partial f_{k,rs}^{e,I}} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (49)$$

$$f_{k,rs}^{e,N} \frac{\partial L}{\partial f_{k,rs}^{e,N}} = 0, \frac{\partial L}{\partial f_{k,rs}^{e,N}} \geq 0, \forall r \in R, s \in S, k \in K_{rs} \quad (50)$$

$$q_{rs}^{g,I} \frac{\partial L}{\partial q_{rs}^{g,I}} = 0, \frac{\partial L}{\partial q_{rs}^{g,I}} \geq 0, \forall r \in R, s \in S \quad (51)$$

$$q_{rs}^{e,I} \frac{\partial L}{\partial q_{rs}^{e,I}} = 0, \frac{\partial L}{\partial q_{rs}^{e,I}} \geq 0, \forall r \in R, s \in S \quad (52)$$

$$\frac{\partial L}{\partial \lambda_{rs}^{g,I}} = 0, \frac{\partial L}{\partial \lambda_{rs}^{g,N}} = 0, \forall r \in R, s \in S \quad (53)$$

$$\frac{\partial L}{\partial \lambda_{rs}^{e,I}} = 0, \frac{\partial L}{\partial \lambda_{rs}^{e,N}} = 0, \forall r \in R, s \in S \quad (54)$$

The partial derivative of the Lagrangian function with respect to $f_{k,rs}^{g,N}$ is given by:

$$\begin{aligned} \frac{\partial L}{\partial f_{k,rs}^{g,I}} &= \frac{\partial C_{UE}}{\partial x_a} \frac{\partial x_a}{\partial f_{k,rs}^{g,I}} - \lambda_{rs}^{g,I} = \left(\sum_{a \in A_{rg}} \eta t_a(x_a) \right) \delta_{ak}^{rs} - m_{k,rs}^g - \lambda_{rs}^{g,I} \\ &= c_{k,rs}^g - m_{k,rs}^g - \lambda_{rs}^{g,I}, \forall r \in R, s \in S, k \in K_{rs} \end{aligned} \quad (55)$$

Following a similar derivation, the following expressions can be obtained:

$$\frac{\partial L}{\partial f_{k,rs}^{g,N}} = c_{k,rs}^g - \lambda_{rs}^{g,N}, \forall r \in R, s \in S, k \in K_{rs} \quad (56)$$

$$\frac{\partial L}{\partial f_{k,rs}^{e,I}} = c_{k,rs}^e - m_{k,rs}^e - \lambda_{rs}^{e,I}, \forall r \in R, s \in S, k \in K_{rs} \quad (57)$$

$$\frac{\partial L}{\partial f_{k,rs}^{e,N}} = c_{k,rs}^e - \lambda_{rs}^{e,N}, \forall r \in R, s \in S, k \in K_{rs} \quad (58)$$

$$\frac{\partial L}{\partial q_{rs}^{g,I}} = \frac{1}{\theta} \ln \frac{q_{rs}^{g,I}}{q_{rs}^g - q_{rs}^{g,I}} - h_{rs}^g + \lambda_{rs}^{g,I} - \lambda_{rs}^{g,N}, \forall r \in R, s \in S \quad (59)$$

$$\frac{\partial L}{\partial q_{rs}^{e,I}} = \frac{1}{\theta} \ln \frac{q_{rs}^{e,I}}{q_{rs}^e - q_{rs}^{e,I}} - h_{rs}^e + \lambda_{rs}^{e,I} - \lambda_{rs}^{e,N}, \forall r \in R, s \in S \quad (60)$$

where $c_{k,rs}^g$ and $c_{k,rs}^e$ denote the travel costs of GVs and EVs on path k for OD pair $r-s$, respectively.

Given that $q_{rs}^{g,I}$ and $q_{rs}^{e,I}$ are nonzero, (51)–(52) can be rewritten as:

$$\frac{\partial L}{\partial q_{rs}^{g,I}} = 0, \forall r \in R, s \in S \quad (61)$$

$$\frac{\partial L}{\partial q_{rs}^{e,I}} = 0, \forall r \in R, s \in S \quad (62)$$

The single-level KKT reformulation of bilevel optimization formulation is summarized as follows:

$$\min_{\mathbf{h}^g, \mathbf{h}^e, \mathbf{m}^g, \mathbf{m}^e, \mathbf{f}^{g,I}, \mathbf{f}^{g,N}, \mathbf{f}^{e,I}, \mathbf{f}^{e,N}, \mathbf{q}^{g,I}, \mathbf{q}^{e,I}} C \quad (63)$$

subject to:

$$\begin{aligned} &(1)–(8), (13)–(16) \\ &(24)–(34), (36)–(41), (43) \\ &(47)–(50), (61)–(62) \end{aligned}$$

3.2. Model Linearization Techniques

C_{TN} contains nonlinearities arising from the BPR function on regular links and from the queuing model used to characterize charging and waiting times. To handle these nonlinearities, a piecewise linear approximation based on a special ordered set of type 2 (SOS2) is employed. This approach has been widely adopted in traffic assignment studies and is consistent with the methodologies in [31,32]. Another type of nonlinearity in (61)–(62) involves logarithmic functions, which can be linearized using the same approach. The number of segments is determined offline by numerical optimization to achieve a trade-off between computational efficiency and approximation accuracy.

The complementarity conditions in constraints (47)–(50) and (7)–(8) can be converted into an equivalent mixed-integer linear formulation via the Big-M method, with the inclusion of binary variables:

$$0 \leq f_{k,rs}^{g,I} \leq M_1(1 - v_{k,rs}^{g,I}), \forall r \in R, s \in S, k \in K_{rs} \quad (64)$$

$$0 \leq c_{k,rs}^g - m_{k,rs}^g - \lambda_{rs}^{g,I} \leq M_2 v_{k,rs}^{g,I}, \forall r \in R, s \in S, k \in K_{rs} \quad (65)$$

where a binary variable $v_{k,rs}^{g,I}$ is introduced to indicate whether a given path is selected. The Big-M constants M_1 and M_2 are chosen to be sufficiently large, corresponding to the OD demand and the maximum perceived path cost, respectively. The remaining cases, including subscription-based GV drivers and non-subscription GV and EV drivers, can be handled in an analogous manner.

The bilinear terms in (29) are linearized using a binary expansion approach, where the reward variables are discretized into multiple equal intervals represented by binary variables, mitigating the relaxation errors associated with the McCormick envelope method. As an illustrative example, consider the bilinear term xy :

$$x = x_{\min} + \sum_{n=1}^N 2^{N-n} u_n \Delta\tau, u_n \in \{0, 1\} \quad (66)$$

where $\Delta\tau = (\tau_{\max} - \tau_{\min}) / (2^N - 1)$ denotes the minimum interval. The bilinear term can then be formulated as:

$$xy = x_{\min}y + \sum_{n=1}^N 2^{N-n} u_n y \Delta\tau = x_{\min}y + \sum_{n=1}^N 2^{N-n} v_n \Delta\tau \quad (67)$$

where auxiliary variables $v_n = u_n y$ are introduced and linearized as follows:

$$\begin{cases} y_{\min} u_n \leq v_n \leq y_{\max} u_n \\ y_{\min}(1 - u_n) \leq y - v_n \leq y_{\max}(1 - u_n) \end{cases} \quad (68)$$

With the above transformations, the original bilevel program is reformulated as a mixed-integer second-order cone program (MISOCP), which can be efficiently solved using off-the-shelf solvers such as Gurobi [33].

4. Case Studies

4.1. Example System

In this section, system consisting of a 20-link traffic network [34] and a modified IEEE 33-bus power distribution network [35] is employed to validate the effectiveness of the proposed scheme, as illustrated in Figure 4. The base values of traffic flow in the TN and power demand in the PDN are set to 10 vehicles/hour and 10 MVA, respectively. The lower and upper voltage limits are 0.9 p.u. and 1.1 p.u. The cost coefficients a_m and b_m associated with local distributed generators are set to 0.3 \$/(MW)² and 200 \$/MW, respectively. The average charging rate per EV is assumed to be $p^e = 50$ kW. The electricity price from the main grid is set to 200 \$/MW. The parameters $P_j^{\max}$, $Q_j^{\max}$ and $Q_j^{\min}$ are set to 5 MW, 2.5 MVar and −2.5 MVar, respectively. The charging price at charging stations is set to 220 \$/MW. In the TN, 11 O–D pairs are considered, all departing from the northwest and traveling toward the southeast, with the corresponding traffic demands listed in Table 2. Eight CSs are connected to buses 4, 7, 10, 16, 19, 25, 27 and 31 in the PDN. The monetary cost of travel time is set to 10 \$/h.

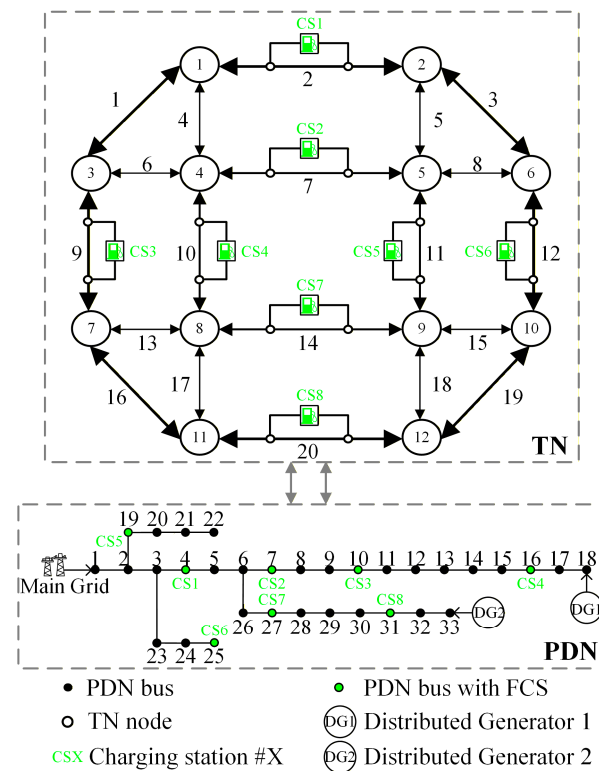


Figure 4. Topology of the coupled power-traffic system.

Table 2. Traffic demands (P.U.).

OD Pair	EV Demand	GV Demand
1–6	10	30
1–10	5	60
1–11	10	40
1–12	5	40
4–9	10	50
4–12	10	40
3–6	10	30
3–10	5	50
3–11	10	40
3–12	5	50
4–10	10	40

The optimization problem is solved using Gurobi 12.0 via Yalmip platform [36] in MATLAB 2022b, running on a computer equipped with an Intel i9-13900H CPU (2.6 GHz) and 32 GB memory.

4.2. Result Analyses

Figures 5 and 6 illustrate the path flows and the corresponding travel costs for the selected OD pair 4–12 of gasoline vehicles (GVs) and electric vehicles (EVs). The figures compare the actual travel costs and the perceived generalized travel costs using line plots. Specifically, the red curves represent the path travel costs, while the blue bars denote the actual path flows and the corresponding travel cost budgets. The simulation results show that, for all paths with nonzero flows, the generalized travel cost reaches the minimum among all feasible paths.

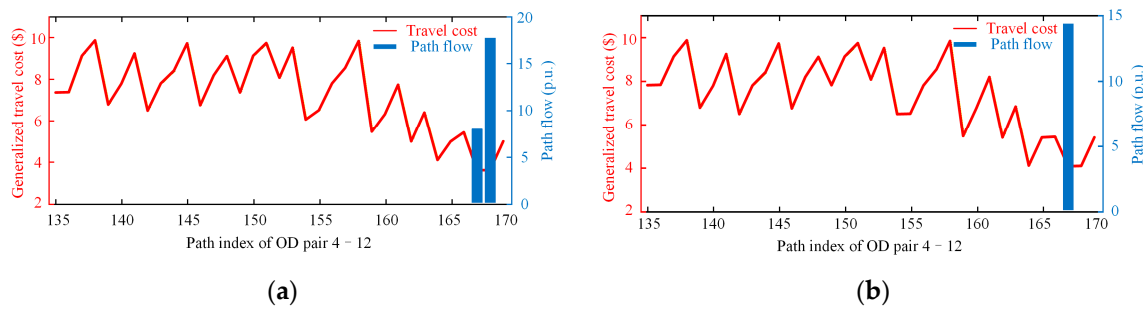


Figure 5. Path flows and travel costs of GV for OD pair 4–12. (a) Incentivized gasoline vehicles on OD pair 4–12; (b) Non-incentivized gasoline vehicles on OD pair 4–12.

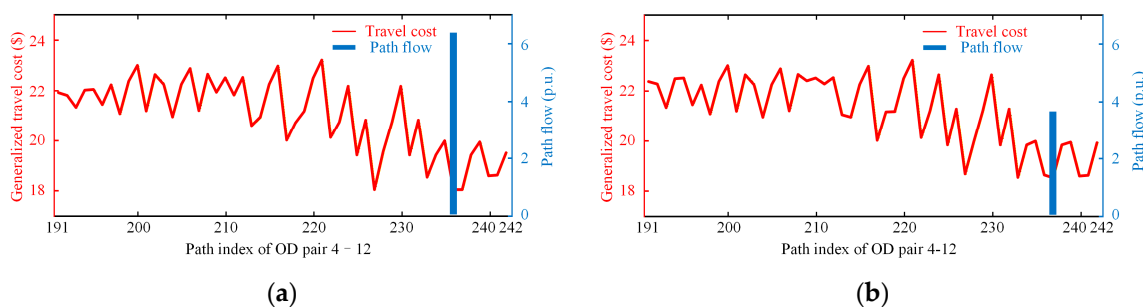


Figure 6. Path flows and travel costs of EVs for OD pair 4–12. (a) Incentivized electric vehicles on OD pair 4–12; (b) Non-incentivized electric vehicles on OD pair 4–12.

Drivers first decide whether to participate in the incentive scheme and then select routes based on the resulting generalized travel costs. According to the logit choice model, approximately 64% of GV drivers on OD pair 4–12 are incentivized. These drivers receive a fixed subscription reward of \$0.12 and path-based rewards of \$0.46 and \$0.48 on paths 167 and 168, respectively. Similarly, about 64% of EV drivers are incentivized on path 236, receiving a subscription reward of \$0.08 and a path-based reward of \$0.59.

After the subscription-based rewards are provided, drivers may either retain their original routes or switch to alternative routes based on the updated generalized travel costs. For example, before the rewards are offered, drivers predominantly choose the path 4→8→11→12 due to its lowest travel cost. After the subscription reward of \$0.12 is introduced, approximately 64% of drivers voluntarily participate in the incentive scheme, reflecting an endogenous participation decision under the logit model. Among the participating drivers, around 30% continue to select the path 4→8→11→12 and receive a path reward of \$0.46, while the remaining 70% switch to the path 4→8→9→12 and obtain a higher reward of \$0.48.

5. Discussions

In this section, the operational results are compared with the user equilibrium in Section 5.1. A sensitivity analysis is performed in Section 5.2, and the feasibility and effectiveness of the proposed model are demonstrated through four incentive scheme models in Section 5.3.

5.1. Comparison of Operational Results Against the Baseline UE

We first compare the network conditions under uncoordinated and coordinated operations. In the uncoordinated scenario, drivers independently determine their routes and charging decisions by minimizing individual travel costs, without any intervention from the coupled power–traffic network operator, which results in a user equilibrium (UE) flow. Under coordinated operation, the coupled operator provides incentive rewards to drivers

with the objective of maximizing social welfare. Figure 7 illustrates the unregulated UE flow and the traffic flow patterns under the proposed coordinated reward scheme in the TN. By offering rewards, the operator encourages drivers to divert from congested inner links to peripheral routes, thereby alleviating congestion on links in the central district while maintaining the secure and economic operation of the coupled networks. The standard deviation of charging flows in the TN is reduced from 1.89 p.u. to 1.35 p.u., indicating that the charging demand among different charging stations is more evenly distributed with the incorporation of the incentive scheme.

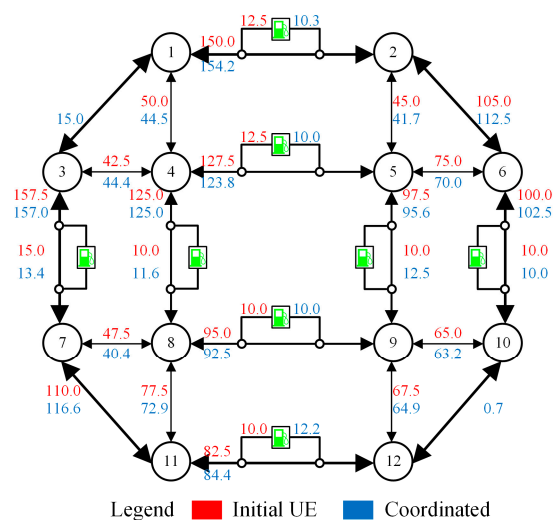


Figure 7. Distribution of traffic flow under different coordination schemes.

By comparing the flow distributions in the UE and coordinated operation scenarios, it is evident that the coordinated operation allocates rewards to influence route choice. This mechanism encourages users to distribute traffic over a larger set of routes, thereby mitigating congestion. As a result, the flow distribution under the coordinated operation is more balanced than in the UE scenario, where traffic is concentrated on a limited number of paths. Specifically, the total number of paths utilized by EVs increases from 16 in the UE scenario to 21 under the coordinated operation. Figure 8 presents the congestions levels on each link under the mixed-equilibrium scenario and the corresponding congestion levels in the TN. The congestion level is quantified as the link flow that causes the travel time to reach 1.5 times the free-flow travel time, representing a threshold for severe congestion. As shown, four links are congested under the initial UE state, and congestion is alleviated on three of these links after incorporating incentive scheme.

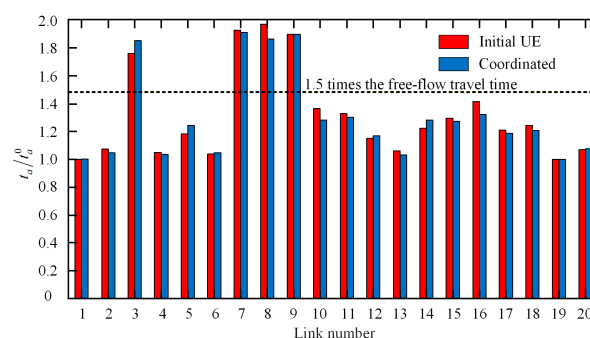


Figure 8. Congestion levels on each link in the TN.

To ensure the operational security of the PDN, the voltage magnitude at each bus is analyzed, as illustrated in Figure 9. This figure presents the voltage magnitudes of all

33 buses both before and after implementing the incentivization scheme, with distributed generators installed at buses 18 and 33. As expected, the voltages at buses 18 and 33 are higher than those at the other buses, primarily due to the local energy injection from the distributed generators in combination with the continuous energy support from the main grid. Despite these variations, all bus voltages remain within the acceptable operating range of 0.95–1.01 p.u., reflecting satisfactory voltage regulation throughout the network. Furthermore, the coordinated operation reduces voltage fluctuations across all buses, resulting in a more stable and uniform voltage profile, which highlights the effectiveness and potential benefits of the coordinated reward-based approach for maintaining PDN operational stability.

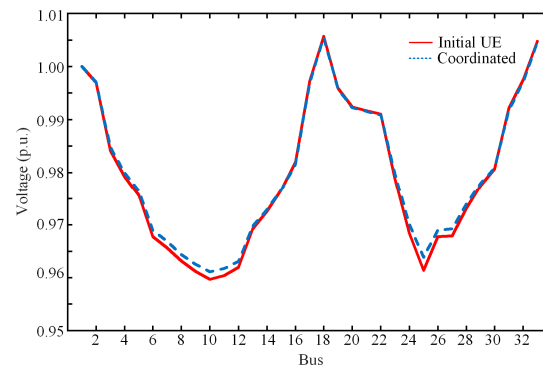


Figure 9. Bus voltage in different scenarios in PDN.

To illustrate the distribution of benefits across OD pairs, Figure 10 compares the travel costs of GV and EV users under different scenarios. As shown in the figure, all participating gasoline and electric vehicle users experience cost reductions after the implementation of the incentivization scheme, although the magnitude of the benefits varies across OD pairs. For non-participating gasoline and electric vehicle users, travel costs on a few OD pairs (32%) increase slightly due to traffic reallocation effects; however, the majority of OD pairs still exhibit cost decreases. Overall, these results indicate that the proposed incentivization scheme generates positive user-level impacts and distributes benefits broadly across the network.

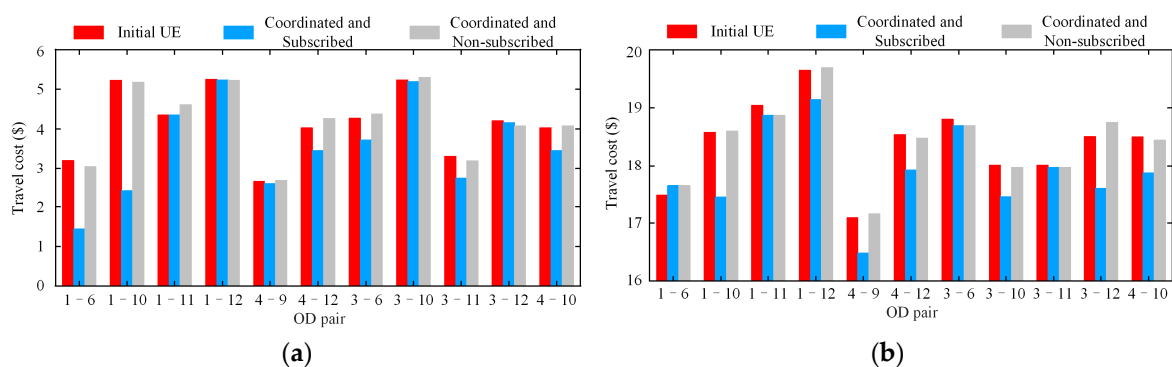


Figure 10. Comparison of travel costs across OD pairs under different scenarios. (a) Gasoline vehicles (GVs); (b) Electric vehicles (EVs).

5.2. Sensitivity Analysis

The objective of this section is to evaluate the effects of the utility sensitivity parameter θ in the logit choice model and the drivers' value of time η on model performance. In the utility and travel cost formulations adopted in this study, these parameters characterize drivers' willingness to participate in the reward scheme and the sensitivity of travel cost,

respectively. The sensitivity coefficient θ is set to a value around 1, which is commonly adopted in the literature to represent a moderate level of user rationality in subscription decisions, consistent with [23]. The parameter η converts time units into monetary units. A relatively low value is adopted in this study, consistent with empirically observed estimates of travel time [37]. To investigate their impacts on the model outcomes, a series of simulations are conducted under different parameter settings by solving the mixed-integer second-order cone programming (MISOCP) model.

Figure 11 illustrates the variation in the proportion of incentivized drivers under different parameter settings. The results indicate that a higher utility sensitivity coefficient (θ) and a lower value of time (η) lead to an increased proportion of incentivized drivers, with the proportion being more sensitive to θ . Specifically, within the range of $\theta = 0.8$ – 1.1 , the proportion of incentivized drivers increases approximately linearly with θ . When θ exceeds 1.1, the growth rate gradually slows, because although drivers become more responsive to rewards, the path-based subsidies are constrained by the available budget, limiting further increases in the proportion of incentivized drivers.

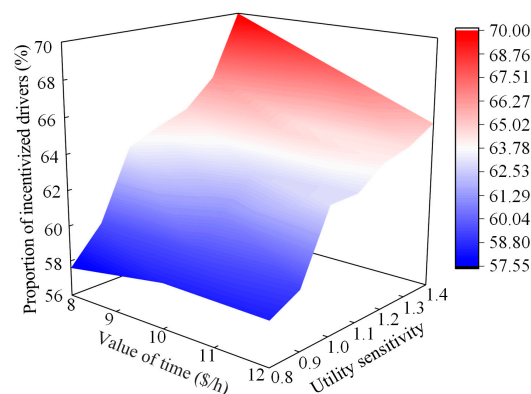


Figure 11. Proportion of incentivized drivers under different parameter settings.

In contrast, the proportion of incentivized drivers is less sensitive to variations in η . Within the range of $\theta = 0.8$ – 1.4 , the proportion decreases as η increases. This behavior can be explained as follows: as η rises, drivers' perceived travel costs increase, leading to larger cost differentials across alternative paths. Consequently, the leader must offer higher path-based subsidies to induce route shifts. However, under a limited budget, the subsidies cannot be sufficiently increased, which reduces their effectiveness in motivating drivers to change paths.

5.3. Comparison of Different Incentive Scheme Models

Four different incentive scheme models are presented to illustrate the feasibility and effectiveness of the proposed model.

Case 1: The choice model is relaxed, allowing the coupled power–traffic operator to fully control the proportion of incentivized drivers.

Case 2: The link-based incentive strategy, consistent with reference [21] is adopted as a benchmark to represent existing incentive schemes.

Case 3: Only path-based rewards are provided to regulate the network and guide drivers, while the subscription-based rewards are set to zero.

Case 4: Both subscription-based rewards and path-based rewards are considered in this paper.

The travel and charging/waiting time in the TN and the operating cost in the PDN together with the total subscription-based cost, total path-based cost, and overall cost to maximize social welfare are presented in Table 3.

Table 3. Experiment results for the four models.

Cases	Travel Time in TN (Min)	Operating Cost in PDN (\$)	Total Path-Based Costs (\$)	Total Subscription-Based Costs (\$)	Total Costs (\$)
Case 1	15,476.00	1682.43	246.0	0	246.0
Case 2	15,477.08	1682.11	523.3	0	523.3
Case 3	15,475.83	1682.50	270.9	0	270.9
Case 4	15,476.08	1682.48	136.3	88.4	224.7

Case 1 represents the scenario in which drivers are insensitive to their choices, and the coupled power–traffic system operator is assumed to directly control the proportion of incentivized drivers. Such an assumption may be less realistic in practice, as it neglects the endogenous decision-making process of drivers regarding whether to participate in the incentive scheme.

From a global perspective, the network performance improvement achieved in Case 2 is inferior to that obtained in Case 4. This is mainly because Case 2 adopts a link-based incentive strategy, which restricts reward allocation to individual links rather than entire paths. As a result, Case 2 lacks the flexibility to guide drivers' route and charging decisions in a differentiated manner across alternative paths. Instead, driver behavior is influenced in a relatively coarse and generalized way at the link level. This structural limitation reduces the variability of reward allocation, thereby leading to suboptimal network performance and higher incentive costs compared with the path-based strategies. The comparison between Case 4 and Case 2 further confirms that path-based incentive schemes are generally more effective in achieving better budget efficiency.

Case 3 demonstrates the effectiveness of path-based rewards and shows that the discrete choice model constraints can be satisfied even in the absence of subscription-based rewards. However, this advantage comes at the expense of a relatively high incentive cost. Although subscription-based rewards are not required in Case 3, the reliance on path-based rewards alone necessitates higher reward levels to induce the desired behavioral responses, which significantly increases the overall incentive expenditure. In contrast, Case 4 exhibits superior performance relative to Case 3. While the introduction of subscription-based rewards incurs additional costs, it effectively reduces the dependence on high path-based rewards, leading to a lower total reward cost. This result highlights the benefits of jointly implementing subscription-based and path-based incentive mechanisms, which enables a more balanced trade-off between system performance improvement and budget efficiency.

5.4. Computational Performance

For larger networks, the enumeration of all possible alternative paths is computationally infeasible. Therefore, a representative path set is generated offline, where each path contains a maximum of 12 links based on the corresponding OD pair. During path set generation, repeated node visits are avoided as a data preprocessing step. It is assumed that electric vehicles must pass through at least one charging-related link based on their remaining energy, while gasoline vehicles have no charging-related links along their paths. In this study, 306 paths are generated for gasoline vehicles and 496 paths for electric vehicles.

The resulting optimization problem contains 1868 continuous decision variables, 1392 binary variables, and 3024 constraints, including SOC and SOS2 constraints for power network and traffic-flow modeling. To solve this MISOCP problem efficiently, the Gurobi solver was employed with the following settings: MIPFocus = 1 to balance optimality and feasible solution search, Cuts = 2 to enhance cut generation, Heuristics = 0.3 to enable moderate heuristic search, Presolve = 2 for preprocessing, and NumericFocus = 1 to improve numerical stability. With these settings, the computational time for solving the full-scale

problem is 929 s. Since the traffic assignment model is static in this paper, the computation time is acceptable for practical purposes.

6. Conclusions

This paper designed reward schemes to guide drivers from the perspective of a coupled power–traffic operator, aiming to maximize social welfare for the coordinated operation of coupled power–traffic networks. By designing subscription-based and path-based reward schemes and explicitly modeling travelers’ behavioral responses through a logit-based adoption model embedded within a traffic assignment framework, the complex interactions between system operators and individual drivers were captured. Numerical results demonstrate that the proposed joint subscription-based and path-based incentive schemes significantly improve the operational efficiency of coupled power and transportation networks, outperforming traditional price-based mechanisms. By accounting for drivers’ responses and budget constraints, the proposed bilevel framework provides a practical and scalable approach for system operators to achieve coordinated system optimality and economic dispatch. The proposed framework also offers valuable insights into the design of incentive-based coordination mechanisms in integrated energy and transportation systems.

This study provides a basis for multiple future extensions. With the emergence of coupled power–traffic network operators and platforms, practical implementation becomes increasingly feasible, enabling system operators to deploy the proposed reward mechanisms for coordinated operation of power–traffic networks. In this paper, the reward mechanisms are implemented under the assumption of voluntary driver participation, in which drivers report their chosen paths. As a path-dependent road management approach, the practical implementation of such mechanisms presents operational challenges, including monitoring users’ path selections while protecting privacy. Future studies should include the analysis of driver heterogeneity, applications to large-scale networks with empirical behavioral data to improve more realistic power–traffic systems, and focus on additional operational objectives, such as reducing peak–valley differences in PDN–TN demands and carbon emissions. In addition, the modeling framework can be further extended to a multi-period setting to capture both spatial and temporal benefits of the proposed reward schemes.

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