

Article

Distributionally Robust Risk Dispatch for Integrated Electricity and Heating System Using Moment and Unimodal Information

Zeyu Yan ¹, Hua Zhuo ¹, Wentao Yan ¹, Ligong Zhang ¹, Zhanliang Dou ¹, Ling Zhou ¹, Lian Tan ^{2,*}  and Tongchui Liu ² 

¹ National Institute of Clean-and-Low-Carbon Energy, China Energy Investment Corporation, Intersection of Binhe Avenue and Yingcai North 3rd Street, Future Science City, Changping District, Beijing 102211, China

² School of Electrical Engineering, North China University of Water Resources and Electric Power, No. 36 Beihuan Road, Jinshui District, Zhengzhou 450045, China

* Correspondence: tanunited@ncwu.edu.cn

Abstract

The integration of large amounts of uncertainties in renewable energies into integrated electricity and heating systems gives rise to new challenges and threatens security. This paper presents a distributionally robust risk dispatch (DRRD) model designed to mitigate the risks arising from uncertainty, with the objective of achieving controllable risk and cost-effectiveness. Uncertainties associated with the wind power forecast error are encapsulated within an ambiguity set that encompasses moment (e.g., mean and covariance) and unimodal information to produce solutions that are more reliable and less conservative. The proposed model constrained by Conditional Value-at-Risk (CVaR) not only limits the probability of violations but also controls the magnitude of potential violations, improving the out-of-sample performance. We employ tractable reformulations and some approximation techniques to recast the resulting distributionally robust CVaR constraints as second-order cone (SOC) programs. Finally, the numerical outcomes presented utilizing the modified Barry Island system showcase the risk mitigation and economic advantages of the proposed methodology.

Keywords: distributionally robust CVaR constraint; risk dispatch; risk severity; integrated electricity and heating system; unimodality information

1. Introduction

Driven by the requirement to achieve carbon neutrality by 2060, the seamless integration of renewable energy sources is becoming a priority in energy sectors [1,2]. It is imperative to accommodate higher penetration of renewable energy being integrated into the system. The IEHS can achieve heterogeneous energy exchange and simultaneously provide electrical and thermal energy, significantly increasing the ability to accommodate higher penetration of renewable energy [3–5]. However, weather-dependent renewable sources exhibit inherent variability and uncertainty, posing great challenges and risks to the reliable operation of IEHS [6,7]. By quantifying the risk severity caused by uncertainties in wind power production, a risk dispatch associated with controllable risk and cost-effectiveness for the IEHS is implemented to realize green, clean, secure, and low-emission energy systems and enable the deep interactions between different energy systems.

In recent years, distributionally robust (DR) optimization [8,9] has emerged as a novel approach for addressing uncertainties, alongside conventional stochastic programming [10,11] and robust optimization [12,13]. DR optimization integrates the provided



Academic Editor: Alessandro Niccolai

Received: 16 September 2026

Revised: 17 October 2026

Accepted: 19 October 2026

Published: 26 October 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and

conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

probability distribution information into an ambiguity set, which effectively represents the actual probability distribution of uncertain parameters. DR approaches utilize two primary categories of ambiguity sets, namely moment-based and metric-based ambiguity sets. Moment-based ambiguity sets capture the moment characteristics of probability distributions [14,15], containing all distributions satisfying identical moment constraints, whereas metric-based ambiguity sets quantify the proximity of probability distributions to an empirical distribution using a specific statistical distance metric [16,17]. Ambiguity sets that utilize moment information, such as mean and covariance, demonstrate favorable tractability properties [18]. As a result, the moment-based distributionally robust optimization (DRO) approach has been more frequently employed in the existing literature on IEHS [19,20].

Currently, the existing investigations predominantly concentrate on the evaluation of the risk associated with IEHS. The risk source originates from two main aspects, namely the component outage and uncertainty involved. In the former, the risk caused by component outage is measured by the resulting risk indices, e.g., line overloading, node temperature-limit violation, loss of heating load, pressure-limit violation, and inadequate energy supply. Considering the component outage of IEHS, novel risk indices are proposed to provide comprehensive and quantitative metrics for risk assessment in [6,21]. A risk-averse scheduling model of IEHS against contingencies is presented in [22,23]. In the latter, the risk incurred by stochastic uncertainty involved is tackled by adopting CVaR as the risk measure to quantify the impact of uncertainties. The inclusion of the CVaR index in the objective function enhances our scheduling approach by enabling quantitative control of the risk level in [24,25]. CVaR is employed as a metric to evaluate the financial risk associated with the scheduling process of the REH-IES, considering the uncertainties of renewable sources output [26,27].

Despite vigorous endeavors, several gaps persist in the current body of research. Initially, although a considerable number of investigations have employed traditional optimization approaches to tackle stochastic uncertain production integrated into IEHS, there have been limited explorations regarding ambiguity sets with potentially various shapes and different structures. In practice, apart from moment information, we may acquire additional information regarding the structural characteristics of potential distributions concerning renewable power generation derived from historical data. Furthermore, incorporating additional information expands the ambiguity set to encompass more realistic distributions, leading to a favorable balance between the cost and risk. Secondly, the existing studies concerning the risk dispatch for the IEHS are focused on risk against contingency set incurred by the component outage and uncertainty. However, uncertain production may cause power flow overlimit, or bus voltage unbalance, water temperature issues, etc. The resulting risk has rarely been explored from the perspective of risk probability and risk severity, making it difficult for decision-makers to achieve a reasonable risk-controllable and cost-effective strategy.

In this study, we address the aforementioned gaps by providing three essential contributions.

- (1) We propose a DRRD model for the IEHS using moment and unimodal information obtained by the historical wind power forecast error. The proposed model employs DR-CVaR constraints to restrict the violation probability and violation magnitude. By measuring the risk severity, the decision-maker can manage risks of operational limit violations to achieve controllable risk and cost-effectiveness for risk dispatch.
- (2) The structured ambiguity set is constructed by incorporating moment and unimodal information of uncertain wind power forecast error, providing a more accurate char-

acterization of the true probability distribution. Thus, this results in a solution that is more reliable and less cautious.

- (3) To derive a tractable reformulation, the DR-CVaR constraints, in the form of the worst-case expectation of a non-linear function, are approximated by using two piece-wise linear supporting functions. The resulting formulation is a tractable SOC programming problem, which can be effectively resolved through the utilization of a readily available optimization package.

The remainder of this paper is organized as follows. Section 2 elaborates on the DRRD formulation for IEHS. Section 3 presents the reformulation and solution methodology for the U-DRCC framework. Section 4 presents the numerical findings derived from a case study conducted using the modified Barry Island IEHS test system. Finally, Section 5 provides a comprehensive summary of the research conducted in this paper, along with highlighting important areas for future investigation.

2. Proposed DRRD Model Formulation

In this section, we commence by formulating the DRRD model, which introduces the distributionally robust CVaR constraints, providing a guarantee on the severity of risk violations. Subsequently, we propose a structured ambiguity set, including moment and unimodality information obtained from the historical wind power forecast error.

2.1. Model Formulation

The objective function seeks to minimize the overall system cost for the DRRD, which comprises the energy and reserve cost, as well as the expected cost of adjustable energy re-dispatch and load shedding. The mathematical model of the objective function for DRRD is expressed as:

$$\min_x \sum_t \sum_e [F^c(p_{e,t}^{\text{CHP}}, H_{e,t}^{\text{CHP}}) + C^{\text{CHP,D}} R_{e,t}^{\text{CHP,D}} + C^{\text{CHP,U}} R_{e,t}^{\text{CHP,U}}] + \max_{\mathbb{P} \in \mathcal{P}} \mathcal{F}(\xi) \quad (1)$$

$$\mathcal{F}(\xi) = \min_y \mathbb{E}_{\mathbb{P}} \left[\sum_t \left(\sum_e C^{\text{CHP}} \rho_{e,t}^{\text{CHP}}(\xi) + \sum_j \left(C^{\text{Le,S}} p_{j,t}^{\text{L,S}}(\xi) + C^{\text{Lh,S}} H_{j,t}^{\text{L,S}}(\xi) \right) \right) \right] \quad (2)$$

where F^c is the fuel cost of CHP units. $\mathbb{E}_{\mathbb{P}}$ represents the expectation operator under probability distribution $\mathbb{P}$. $\max_{\mathbb{P} \in \mathcal{P}} \mathcal{F}(\xi)$ represents the expected cost under the worst-case distribution $\mathbb{P}$ within ambiguity set $\mathcal{P}$.

(1) PDN constraints modeling

The electric power flow in radial electricity network topologies is described using the linearized DistFlow branch model [19,28]. The model retains active/reactive power flows and nodal voltage magnitudes, while neglecting only the quadratic line-loss terms by dropping $I_{ij}^2 r_{ij}$ and $I_{ij}^2 x_{ij}$ in the power balance and voltage drop equations. This mathematical model can be depicted as follows:

$$P_{j,t}^G + P_{ij,t} + \rho_{e,t}^{\text{CHP}}(\xi) = \sum_{l \in S_j^{\text{B,d}}} P_{jl,t} + P_{j,t}^L - P_{j,t}^{\text{L,S}}(\xi) \quad (3)$$

$$Q_{j,t}^G + Q_{ij,t} = \sum_{l \in S_j^{\text{B,d}}} Q_{jl,t} + Q_{j,t}^L \quad (4)$$

$$V_{j,t} = V_{i,t} - (P_{ij,t} r_{ij} + Q_{ij,t} x_{ij}) / V^0 \quad (5)$$

$$\underline{V}_{j,t} \leq V_{j,t} \leq \bar{V}_{j,t} \quad (6)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [P_{ij,t}(\xi)] \leq \bar{P}_{ij,t} \quad (7)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [-P_{ij,t}(\xi)] \leq -\underline{P}_{ij,t} \quad (8)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [Q_{ij,t}(\xi)] \leq \bar{Q}_{ij,t} \quad (9)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [-Q_{ij,t}(\xi)] \leq -\underline{Q}_{ij,t} \quad (10)$$

Equations (3) and (4) enforce the nodal balance of active power and reactive power, respectively. These equations ensure the balance between the injected and consumed power at each node in the electrical network. Equation (5) captures the relationship between the magnitudes of nodal voltages and the power flows within branches. Equation (6) restricts voltage magnitudes within secure ranges. It should be noted that voltage phase angles are not included as decision variables or constraints in this formulation; they are calculated in post-processing to characterize the network operating state. Equations (7) and (8) state that the active branch power flow is restricted in the form of DR-CVaR constraints for the worst-case distribution $\mathbb{P}$ within ambiguity set $\mathcal{P}$ while the conditional expectation of $P_{ij,t}(\xi)$ is measured by $\text{CVaR}_{\mathbb{P}}^{\varepsilon} [P_{ij,t}(\xi)]$. The reactive branch power flow is enforced by (9) and (10) in the form of DR-CVaR constraints. The maximization operator $\max_{\mathbb{P} \in \mathcal{P}}$ represents the worst-case distribution $\mathbb{P}$ within ambiguity set $\mathcal{P}$. The CVaR constraints $\text{CVaR}_{\mathbb{P}}^{\varepsilon} [\cdot]$ quantifies the conditional expectation of variable ξ in its upper tail with a threshold of ε .

(2) DNS constraints modeling

The DHN comprises a series of subterranean pipe segments. Water undergoes heating through heat sources and is subsequently transferred into supply pipes. Within a designated node, the flow of hot water occurs from the supply side to the return side. The PDE can accurately describe the physical characteristics of DHN to estimate temperature dynamics. The temperature dynamics along each pipeline based on the first-order PDE can be derived from the Law of Conservation of Energy through infinitesimal analysis [29,30]. The fundamental solution approach employed in this study involves transforming the section of PDEs into AEs using the finite difference method. Under the assumption of stable hydraulic conditions in the DHN, it is expected that all branches will operate at their rated mass flow rate. The DHN model with the temperature dynamics along the pipeline can be given as:

$$\frac{\partial T_p}{\partial t} + v_p \frac{\partial T_p}{\partial x} + \frac{v_p}{\dot{m}_p c^p R_p} (T_p - T^A) = 0 \quad (11)$$

$$\sum_{S_n^{p,e}} (\dot{m}_p T_p^{\text{in}}) = T_n \sum_{S_n^{p,e}} \dot{m}_p \quad (12)$$

$$T_p^{\text{out}} = T_n \quad (13)$$

$$H_{n,t}^S = c^p m_n^S (T_{n,t}^S - T_{n,t}^R) \quad (14)$$

$$H_{n,t}^L - H_{n,t}^{L,S} = c^p m_n^L (T_{n,t}^S - T_{n,t}^R) \quad (15)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [T_{n,t}^S(\xi)] \leq \bar{T}_n^S \quad (16)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [-T_{n,t}^S(\xi)] \leq -\underline{T}_n^S \quad (17)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [T_{n,t}^R(\xi)] \leq \bar{T}_n^R \quad (18)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} \left[-T_{n,t}^R(\xi) \right] \leq -\underline{T}_n^R \quad (19)$$

Equation (11) illustrates temperature dynamics with the first-order PDE. Equation (12) describes the temperatures of the mixture at confluence nodes. Equation (13) ensures the same temperature for the mass outflows at a confluence node. Equations (14) and (15) depict the provision and utilization of thermal energy at heat nodes, correspondingly. The upper bound $\bar{T}_n^S$ and the lower bound $\underline{T}_n^S$ for the supply temperature are enforced in (16) and (17). Similarly, the return temperature is limited as constraints (18) and (19) in the form of DR-CVaR constraints.

For any given pipeline p , the temperature T_p and its partial derivatives in Equation (11) can be replaced by the following difference quotients. Substitute the temperature and its partial derivatives in Equation (11) with the discrete difference quotients listed in Equations (20)–(22).

$$\frac{\partial T_p}{\partial t} = \frac{1}{2\tau} (T_{p,b-1}^{k+1} - T_{p,b-1}^k + T_{p,b}^{k+1} - T_{p,b}^k) + O(\tau^2) \quad (20)$$

$$\frac{\partial T_p}{\partial x} = \frac{1}{2h} (T_{p,b}^k - T_{p,b-1}^k + T_{p,b}^{k+1} - T_{p,b-1}^{k+1}) + O(h^2) \quad (21)$$

$$T_p = \frac{1}{4} (T_{p,b-1}^k + T_{p,b}^k + T_{p,b-1}^{k+1} + T_{p,b}^{k+1}) + O(h^2 + \tau^2) \quad (22)$$

By performing necessary rearrangements, we can derive a discrete scheme tailored to approximate the numerical solutions of the PDEs:

$$T_{p,b}^{k+1} = \frac{1+\alpha-\beta}{1+\alpha+\beta} T_{p,b-1}^k + \frac{\alpha-1-\beta}{1+\alpha+\beta} T_{p,b-1}^{k+1} + \frac{1-\alpha-\beta}{1+\alpha+\beta} T_{p,b}^k + \frac{4\beta}{1+\alpha+\beta} T^A \quad (23)$$

where α and β are two parameters defined for simplified representation:

$$\alpha = \frac{v\tau}{h}, \beta = \frac{v\tau}{2\dot{m}cR} \quad (24)$$

$$h = \frac{L}{M}, \tau = \frac{D}{N} \quad (25)$$

(3) EH constraints modeling

Energy deliveries and conversions between the PDN and the DHN are facilitated through the utilization of EHs. Representative EHs comprise CHP units and HP, as shown in Figure 1. EHs employ CHP units to co-generate electricity and heat through the utilization of natural gas. Additionally, HPs utilize electricity consumption to produce heat energy. The extraction-turbine CHP units were chosen in this study for their inherent capacity to regulate the generation of electrical and thermal energy output [31,32]. The convex polyhedron feasible operation region can be governed by a convex combination of corner points, which is illustrated in Figure A1 of Appendix A. The operating constraints for the EH include the following:

$$P_{e,t}^{\text{CHP}} = \sum_r \mu_{e,r,t}^{\text{CP}} P_{e,r}^{\text{CP}} \quad (26)$$

$$H_{e,t}^{\text{CHP}} = \sum_r \mu_{e,r,t}^{\text{CP}} H_{e,r}^{\text{CP}} \quad (27)$$

$$0 \leq \mu_{e,r,t}^{\text{CP}} \leq 1 \quad (28)$$

$$\sum_r \mu_{e,r,t}^{\text{CP}} = 1 \quad (29)$$

$$f^c(p_{e,t}^{\text{CHP}}, H_{e,t}^{\text{CHP}}) = \sigma_0 + \sigma_1 p_{e,t}^{\text{CHP}} + \sigma_2 \cdot H_{e,t}^{\text{CHP}} + \sigma_3 p_{e,t}^{\text{CHP}^2} + \sigma_4 H_{e,t}^{\text{CHP}^2} + \sigma_5 p_{e,t}^{\text{CHP}} H_{e,t}^{\text{CHP}} \quad (30)$$

$$F^c(p_{e,t}^{\text{CHP}}, H_{e,t}^{\text{CHP}}) = \sum_r \mu_{e,r,t}^{\text{CP}} f^c(p_{e,t}^{\text{CHP}}, H_{e,t}^{\text{CHP}}) \quad (31)$$

$$p_{e,t}^{\text{CHP}} + R_{e,t}^{\text{CHP,U}} \leq \bar{p}_e^{\text{CHP}} \quad (32)$$

$$p_{e,t}^{\text{CHP}} - R_{e,t}^{\text{CHP,D}} \geq \underline{p}_e^{\text{CHP}} \quad (33)$$

$$(p_{e,t}^{\text{CHP}} + R_{e,t}^{\text{CHP,U}}) - (p_{e,t-1}^{\text{CHP}} - R_{e,t-1}^{\text{CHP,D}}) \leq RU_e^{\text{CHP}} \quad (34)$$

$$(p_{e,t-1}^{\text{CHP}} + R_{e,t-1}^{\text{CHP,U}}) - (p_{e,t}^{\text{CHP}} - R_{e,t}^{\text{CHP,D}}) \leq RD_e^{\text{CHP}} \quad (35)$$

$$\rho_{e,t}^{\text{CHP}}(\xi) + p_{e,t}^{\text{CHP}} = p_{e,t}^{\text{HP}} + p_{e,t}^{\text{EH}} \quad (36)$$

$$H_{e,t}^{\text{CHP}} + p_{e,t}^{\text{HP}} \lambda_e^{\text{HP}} = H_{e,t}^{\text{EH}} \quad (37)$$

$$\underline{p}_e^{\text{HP}} \leq p_{e,t}^{\text{HP}} \leq \bar{p}_e^{\text{HP}} \quad (38)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [\rho_{e,t}^{\text{CHP}}(\xi)] \leq R_{e,t}^{\text{CHP,U}} \quad (39)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon} [-\rho_{e,t}^{\text{CHP}}(\xi)] \leq R_{e,t}^{\text{CHP,D}} \quad (40)$$

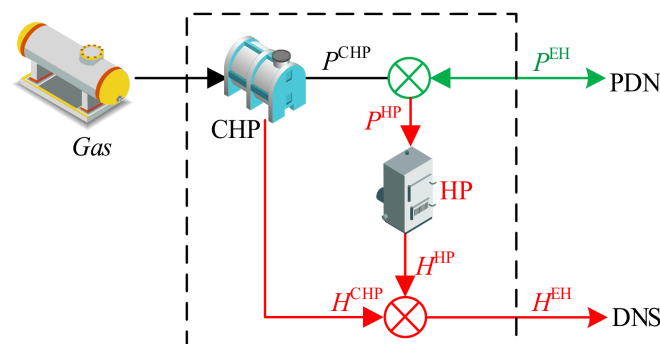


Figure 1. Schematic diagram of energy hubs. Green lines denote the direction of current flow, and red lines indicate the direction of heat flow.

Equations (26) and (27) denote the electric and thermal power outputs of extraction steam turbine CHP units. Equations (28) and (29) enforce the combination coefficient of corner point within the limit. Equations (30) and (31) state the fuel cost of CHP units, which can be approximated as a piece-wise linear function through the use of a convex combination of corner points. Constraints (32)–(35) represent the capacity and ramping constraints of CHP units. Equations (36) and (37) guarantee the preservation of the electric power balance and heat power balance, respectively. Constraint (38) states the capacity limit of HP. The adjustable capacity ρ^{CHP} of CHP units is restricted by (39) and (40) in the form of DR-CVaR constraints.

2.2. Structured Ambiguity Set Modeling

In this paper, we elucidate the employment of a moment-based ambiguity set for representing the inherent uncertainty associated with wind power forecast error. Given the observation that the empirical distributions of wind power production, as well as their corresponding forecast error distributions, frequently exhibit a unimodal nature, we

express the moment-based ambiguity set as encompassing all probability distributions that possess equivalent moments and adhere to the condition of unimodality [33,34].

$$\mathcal{P} = \mathbb{P} \in \Omega^a(\mathbb{R}^Z) : \mathbb{E}_{\mathbb{P}}[\xi] = \mu, \mathbb{E}_{\mathbb{P}}[\xi \xi^T] = \sum, Mo_{\mathbb{P}}[\xi] = \nu \quad (41)$$

where Ω^a represents the set of plausible α -unimodal distributions on $\mathbb{R}^Z$. Vectors μ in $\mathbb{R}^Z$ and matrix $\sum$ in $\mathbb{R}^{Z \times Z}$ denote the first and second moments, i.e., the mean and covariance, of wind forecast error ξ derived from historical data. $(\cdot)^T$ is the transpose operator. Without sacrificing generality, it is assumed that the forecast error mean of each wind power unit is zero, i.e., $\mu = \mathbf{0}$. Operator $Mo_{\mathbb{P}}[\cdot]$ determines the location of the mode within a unimodal distribution under the probability distribution $\mathbb{P}$. Parameter a quantifies the level of unimodality, while $\nu \in \mathbb{R}^Z$ indicates the specific location of the mode within the distribution.

3. Proposed Solution Methodology

The proposed model is intractable due to the infinite-dimensional optimization problem. Hence, tractable conservative approximations are employed to obtain the optimal solution. In this section, we initiate by introducing the proposed model in vector/matrix representations. Subsequently, we derive the simplifications of the DRRD model within the structured ambiguity set utilizing the linear decision rules. Lastly, we reformulate the DR-CVaR constraints as the SOC constraints.

3.1. Compact Form of Formulation

For notational brevity, we reformulate the DRRD model (1)–(40) using vectors and matrices as:

$$\min_x c^T x + \max_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[d^T y(\xi)] \quad (42)$$

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon}[a(y, \xi)] \leq b(y) \quad (43)$$

$$x, y \in \chi \quad (44)$$

where χ represents the feasible region of decision variables. The random function $a(y, \xi)$ is specified as a bilinear function of the design variable y and uncertainty ξ , i.e., $a(y, \xi) = a(y)\xi$.

The parameter ε is the moment ambiguity budget that controls the size of the moment-based ambiguity set. It defines the maximum allowable Mahalanobis deviation between the first- and second-order moments of any feasible distribution and the nominal empirical moments calibrated from historical training data. Physically, ε quantifies the epistemic uncertainty level regarding the true moment statistics of wind forecast errors. When $\varepsilon = 0$, the ambiguity set shrinks to distributions with exactly the nominal moments, reducing the problem to conventional stochastic programming. As ε increases, the ambiguity set expands to include distributions with larger moment deviations, covering a wider range of potential wind forecast error scenarios.

3.2. Reformulation of Objective Function

To further reduce the complexity of the proposed objective function, (1) can be approximated using linear decision rules. In this way, the functional recourse decisions $y(\xi)$ can be reformulated as $y(\xi) = y + (1^T \xi)y$. Specifically, we denote the variable $y \in \mathbb{R}$ as the participation factor in mitigating system deviations caused by the total forecast error [35]. Therefore, the three-stage objective function (1) can be written as:

$$\min_x c^T x + \max_{\mathbb{P} \in \mathcal{P}} \min_y \mathbb{E}_{\mathbb{P}}[d^T (y + (1^T \xi)y)] \quad (45)$$

Using this affine control policy for conventional units while assuming $\mathbb{E}_{\mathbb{P}}[\xi] = \mu = \mathbf{0}$, Consequently, the maximization operator $\max_{\mathbb{P} \in \mathcal{P}}$ the min–max–min objective function (45) can be eliminated [36]. This simplification results in a min–min objective function that can be expressed as a single minimization function:

$$\min_{x,y} c^T x + d^T y \quad (46)$$

3.3. Tractable Reformulation of Constraints

(1) Equation constraints reformulation

The electrical energy is provided by CHP units and wind power units. The power output of the wind farms is modeled as $W(\mu + \xi)$. Thus, (3) can be written as:

$$P_{j,t}^{\text{CHP}} + W(\mu + \xi) + P_{ij,t} + \rho_{j,t}^{\text{CHP}}(\xi) = \sum_{l \in S_j^{\text{B,d}}} P_{jl,t} + P_{j,t}^{\text{L}} - P_{j,t}^{\text{L,S}}(\xi) \quad (47)$$

By implementing linear decisions rules, (47) can be reformulated as:

$$W + \rho_{j,t}^{\text{CHP}} + P_{j,t}^{\text{L,S}} = 0 \quad (48)$$

$$P_{j,t}^{\text{CHP}} + W\mu + P_{ij,t} = \sum_{l \in S_j^{\text{B,d}}} P_{jl,t} + P_{j,t}^{\text{L,S}} \quad (49)$$

(2) Distributionally robust CVaR constraints reformulation

It is noted that the conditional expectation of $a(y, \xi)$ is measured by $\text{CVaR}_{\mathbb{P}}^{\varepsilon}[a(y, \xi)]$, which is defined as:

$$\text{CVaR}_{\mathbb{P}}^{\varepsilon}[a(y, \xi)] = \min \left\{ \theta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+ \right\} \quad (50)$$

where θ is an auxiliary variable $\mathbb{E}_{\mathbb{P}}[\cdot]^+ = \mathbb{E}_{\mathbb{P}}[\max(\cdot, 0)]^+$.

Using linear decisions rules, constraint (44) can be reformulated as:

$$\max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon}[a(y, \xi)] \leq b(y) \quad (51)$$

Then, by the definition of CVaR (50), we have

$$\begin{aligned} \max_{\mathbb{P} \in \mathcal{P}} \text{CVaR}_{\mathbb{P}}^{\varepsilon}[a(y, \xi)] &= \max_{\mathbb{P} \in \mathcal{P}} \min \left\{ \theta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+ \right\} \\ &= \min_{\theta \in \mathbb{R}} \theta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+ \end{aligned} \quad (52)$$

The switch of min and max in (50) follows from the stochastic saddle point theorem by observing that $\theta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+$ is convex in θ and concave in $\mathbb{P}$ [37]. Consequently, (50) boils down to

$$\theta + \frac{1}{\varepsilon} \max_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+ \leq b(y) \quad (53)$$

Computing $\max_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[a(y) \xi - \theta]^+$ requires calculating the worst-case expectation of a non-linear function, which is computationally prohibitive to solve [36,38]. Thus, the approach presented in References [38,39] suggests approximating the non-linear function using two piece-wise linear supporting functions, with each function composed of two linear segments. The scalar ε is the risk coefficient of the proposed DRRD model. It parameterizes the robust risk constraint and therefore calibrates the degree of risk aversion

of the dispatch. By computing the worst-case expectation of approximated functions, (53) can be recast as:

$$\left\| \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] b(\mathbf{y}) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \quad (54)$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] b(\mathbf{y}) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \quad (55)$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] b(\mathbf{y}) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) a(\mathbf{y}) \boldsymbol{\mu} \quad (56)$$

where $\Lambda = [(\frac{a+2}{a})\Sigma - (\frac{a+1}{a})^2 \boldsymbol{\mu} \boldsymbol{\mu}^T]^{1/2}$.

The tractable reformulation of DR-CVaR constraints are given in Appendix B.

4. Case Study

4.1. Test System Descriptions

The feasibility and effectiveness of the proposed approach can be validated using the Barry Island test system. The schematic diagram in Figure 2 illustrates the system composed of a 9-bus PDN and a 32-node DHN [20]. Three energy hubs are interconnected with buses 1, 9, and 6 in the PDN, as well as nodes 31, 1, and 32 in the DHN. Table 1 presents the properties of the CHP units and HP found within the three energy hubs. The load shedding cost is 500 \$/MWh [40]. In our experimental study, we generate $\boldsymbol{\mu}$ and Σ from $\mathbb{P}$ utilizing relative wind power output data for the wind farms located in southeastern Australia during the period spanning from 2012 to 2013 [41]. The dataset contains hourly day-ahead wind power forecasts and corresponding actual output data. After data cleansing to remove missing values and measurement faults, 17,384 valid hourly samples are retained. A strict chronological train-test split is adopted to eliminate data leakage and mimic the practical deployment workflow of day-ahead dispatch models: the training set from January 2012 to December 2012 includes 8682 valid hourly samples. This dataset is used exclusively to calibrate all parameters of the ambiguity set, including the nominal mean, covariance matrix, mode location, unimodality level, and the ambiguity budget via bootstrap resampling. No test data are used in any step of model construction or parameter tuning; the test set from January 2013 to December 2013 includes 8702 valid hourly samples. This independent dataset is used solely for out-of-sample performance evaluation. It is not involved in ambiguity set calibration or model parameter optimization in any form. Out-of-sample wind power scenarios are generated based on the independent test set to evaluate the actual risk control performance of the proposed model. The generation procedure preserves the statistical characteristics of real forecast errors, including both marginal distribution shape and temporal autocorrelation. The specific steps are as follows: (1) Marginal distribution fitting: The hourly wind forecast error series is extracted from the test set. A kernel density estimator (KDE) with a Gaussian kernel is applied to fit the marginal distribution of forecast errors. The kernel bandwidth is selected via Silverman's rule of thumb to balance fitting accuracy and distribution smoothness; (2) Temporal correlation preservation: A block bootstrap method is adopted to retain the diurnal autocorrelation structure of hourly forecast errors. The block length is set to 24 h, corresponding

to a complete daily dispatch cycle, to preserve the intra-day temporal dependence of wind power fluctuations. All optimization models are formulated as second-order cone programs (SOCPs). The models are implemented in Matlab 2023b using the Yalmip optimization modeling interface, and solved by the Gurobi 9.1 commercial solver. The optimality gap tolerance is set to 10^{-6} , and the primal-dual barrier algorithm is adopted for SOCP solution. All simulations are run on a standard workstation (Lenovo Group Limited, Beijing, China) equipped with an Intel Core i7 CPU (Intel Corporation, Santa Clara, California, USA) and 8 GB RAM (Samsung Electronics Co., Ltd., Suwon, Gyeonggi-do, Republic of Korea).

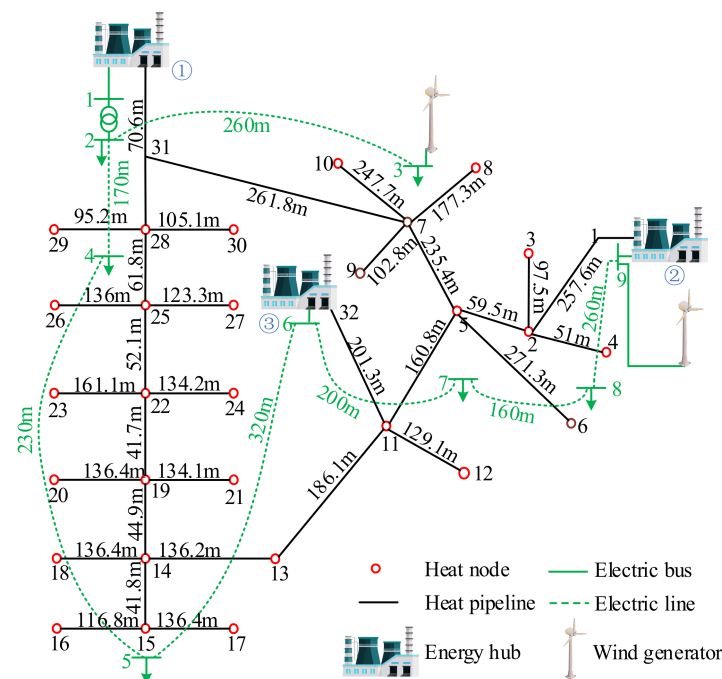


Figure 2. The topology of modified Barry Island IEHS. Green numbers denote the bus indices of the power grid, and black numbers indicate the node indices of the heating network.

Table 1. Properties of the CHP units and HP in three energy hubs.

No.	HP			CHP		
	λ^{HP}	$\bar{P/P}$ ($\bar{\text{MW}}$)	RU/RD (MW/h)	λ^{CHP}	$\bar{P/P}$ ($\bar{\text{MW}}$)	$C^{\text{CHP,U}}/C^{\text{CHP,D}}$ (\$/MWh)
1	3	2, 0	0.3, 0.3	1.3	0.6, 0	1, 1
2	3	2, 0	0.3, 0.3	1.4	0.4, 0	2, 2
3	3	2, 0	0.2, 0.2	1.2	0.5, 0	3, 3

Figure 3 illustrates the correlation of production forecast errors of wind farms w_1 and w_2 . The figure presents a matrix of plots, where the diagonal plots exhibit the histograms of production forecast errors for wind farms w_1 and w_2 . Furthermore, the off-diagonal showcases the scatter plots depicting the forecast errors that correspond to each other. The histogram plots exhibit the frequency distribution of each forecast error, while the scatter plots demonstrate the correlation between two corresponding forecast errors. From examination of the histogram plots, it is apparent that the forecast error of wind production does not adhere to a predetermined probability distribution. Moreover, it is evident that the mean of forecast errors is zero. Both marginal distributions are unimodal and right-skewed, w_1 more strongly (mode ≈ -0.15 , concentrated peak) and w_2 more mildly (mode $\approx +0.07$).

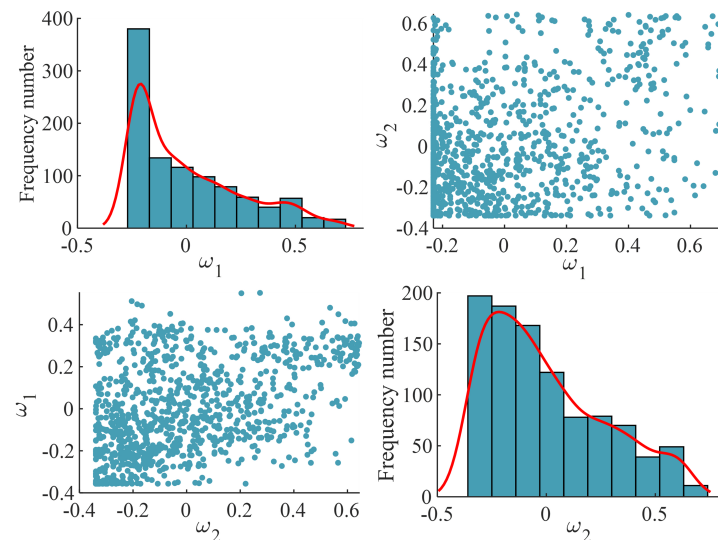


Figure 3. Production forecast errors of wind farms w_1 and w_2 .

4.2. Risk Analysis

In this section, we perform a comprehensive risk analysis to assess the level of risk caused by the uncertain wind power output. Then, the reasonable risk coefficient ε is selected by risk analysis, which can provide risk reference to balance the economic dispatch and controllable risk for the decision-maker. The distribution mode of the wind production forecast error is determined based on the underlying historical dataset. Firstly, to analyze the impact of incorporating additional unimodality information into moment-based ambiguity on risk severity, two cases are considered: unimodality information excluded (UE) and unimodality information included (UI).

Figure 4 depicts the risk cost impact on unimodal information of wind production forecast error. The dashed lines represent the mean risk cost, while the shaded area around them indicates standard deviation of the risk cost. As observed in Figure 4, incorporating unimodality information (denoted as UI) leads to a relatively reduced risk cost. Furthermore, the wind generation unit to be constructed in the case of UI is less flexible and, consequently, more cost-effective compared to those in the case of UE. The cost in the UI scenario is consistently lower than that in the UE scenario. This suggests that the inclusion of unimodality information results in less conservative risk cost solutions. Meanwhile, the approximation introduces only a 0.62% increase in dispatch cost, which is negligible for practical power system operation. In terms of risk, the approximated solution yields a slightly lower out-of-sample violation probability and CVaR than the exact benchmark, confirming the conservative property of the outer approximation. The approximation does not degrade risk control performance; on the contrary, it provides a small additional safety margin. When ε is small, the ambiguity set is narrow. Increasing ε moderately relaxes the moment constraints, allowing the model to adopt more economical dispatch strategies, which reduces the baseline operational cost. When ε exceeds 0.3, further expansion of the ambiguity set introduces more extreme distributions with heavier tails. The supremum of CVaR across the set rises rapidly, and the increasing tail risk premium dominates, causing the total risk cost to increase.

The results in Table 2 demonstrate that embedding unimodal information improves all out-of-sample risk metrics simultaneously. Compared with the UE benchmark, the UI configuration reduces the out-of-sample violation probability by 2.86 percentage points, and decreases the out-of-sample CVaR and maximum violation by 5.6% and 14.3%, respectively. This superior out-of-sample performance can be attributed to the fact that the unimodal distribution assumption is physically consistent with the statistical characteristics

of short-term wind power forecast errors. By integrating this additional valid information, the proposed DRRD model refines the description of uncertainty, avoids unnecessary conservative margins, and simultaneously enhances both economic efficiency and risk controllability in practical operation.

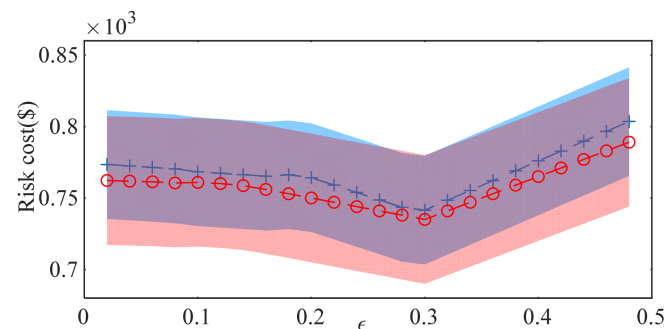


Figure 4. In-sample risk cost of the moment-only (UE) ambiguity set and the proposed moment-unimodal (UI) ambiguity set under varying uncertainty budgets ϵ (—+— UE: Mean; UE: Standard deviation; —o— UI: Mean; — UE: Standard deviation).

Table 2. Properties of the CHP units and HP in three energy hubs.

Configuration	Violation Probability (%)	Out-of-Sample CVaR (\$)	Maximum Violation (\$)
UE (moment-based ambiguity set)	7.21	806.4	1147.9
UI (moment and unimodal ambiguity set)	4.35	761.2	983.5

Figure 5 shows the load shedding influenced by the risk coefficient ϵ using two different approaches. The observation from Figure 5 is that the CVaRC model necessarily outperforms the GCC model constraints. The solution of the CVaRC model yields a robust solution compared with the GCC. The CVaRC model is more efficient since it always returns lower load shedding than the GCC model. On the contrary, load shedding with the GCC model is notably high in all cases and especially for large values of ϵ ($\epsilon < 0.3$). Note that the electrical and thermal load shedding exceed 0.2 and 0.15, respectively, when $\epsilon \geq 0.3$. Out-of-sample validation is performed to quantify the actual constraint satisfaction performance at $\epsilon = 0.3$. Both schemes meet the 5% pre-specified risk tolerance. The CVaRC scheme achieves lower violation probabilities for both branch flow and heat temperature constraints, and also significantly reduces the average violation severity, demonstrating its superior overall risk control capability.

We retain the unimodality information in the ambiguity sets and compare the performance of the chance and CVaR constraints, as shown in Figure 6. The subsequent out-of-sample analysis examines the effects of relaxing the unimodality requirement on the performance of GCC and CVaRC models. Recall that two wind farms are considered, with $a = 2$ for cases involving unimodality information. In the following analysis, the out-of-sample risk costs are given in Figure 6 for the case of $a = 2$ and the case of $a = \infty$.

As observed in Figure 6, an interesting finding is that, although the CVaRC model leads to more conservative risk dispatch decisions than the GCC model, it exhibits relatively better out-of-sample risk cost performance. This can be attributed to the CVaR-constrained model, which limits risk severity of load shedding. Moreover, as depicted in Figure 6, as a tends to infinity, the out-of-sample performance of both models converges. Specifically, the proposed GCC and CVaRC models become asymptotically equivalent when the unimodality information is excluded from the ambiguity sets. It is worth noting that the

convergence of GCC and CVaRC as $a \rightarrow \infty$ does not imply that unimodality information is irrelevant. Rather, it reflects the limiting case in which the ambiguity set degenerates to one defined solely by moment information. For moderate values of a , which correspond to the empirically observed concentration of wind power forecast errors around their mode, unimodality information plays a significant role in differentiating the two models and in improving out-of-sample performance. The parameter a , which characterizes the extent of the unimodal region, was calibrated from historical forecast error data as the radius around the empirical mode containing a prescribed fraction of the samples.

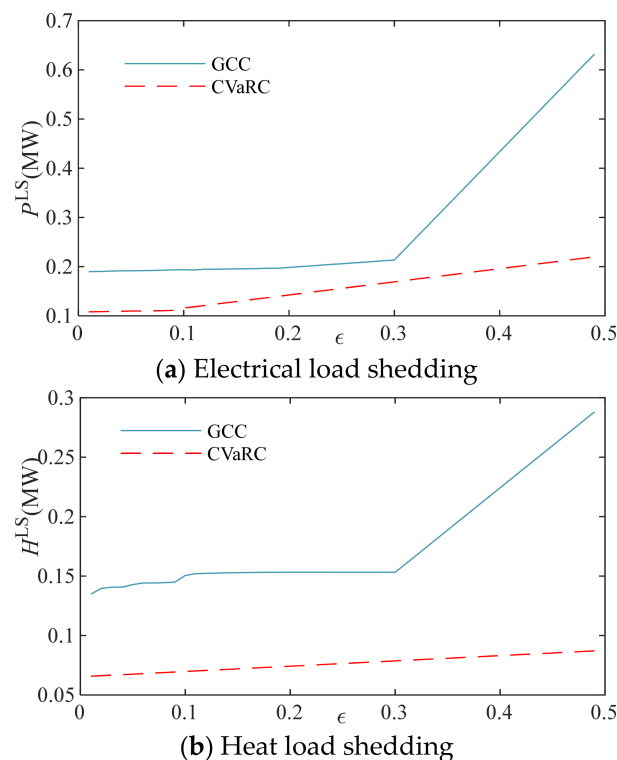


Figure 5. Electrical and heat load shedding under the GCC and DR-CVaRC schemes with varying ϵ .

Figure 7 shows that as a increases, the results of the two approaches converge, with the mode's influence on the risk cost diminishing. Additionally, Figure 7 illustrates that when the mean and mode are significantly separated and a is less than the amount of the wind farms, the mode has a pronounced effect on the worst-case distribution, resulting in a lower cost. Nevertheless, when a exceeds the amount of the wind farms, the influence of the mode decreases. This occurs because an increase in a expands the feasible region for μ , thereby reducing the impact of the same mean and mode on the solution.

4.3. DRRD Result

In this section, we select risk coefficient $\epsilon = 0.3$ according to the risk analysis. Considering moment and unimodality information obtained from the historical wind power forecast error, the bus voltage, node temperature and power output are provided from the DRRD result.

The bus voltage magnitudes and post-calculated phase angles across the PDN are more or less affected by the operational states of the DNS, as depicted in Figure 8. It is displayed in Figure 8 that bus 5 is where the maximum voltage magnitude difference across all simulation time points exceeds 0.0036 p.u. The bus voltage magnitude is an endogenous state variable constrained in the DRRD model, while the phase angle is derived from the optimal active power flow results in post-processing. It can be observed that the voltage

magnitudes experience only minor fluctuations. This is because bus voltage magnitudes are primarily influenced by reactive power flows. The phase angle variation curve for bus 5, shown in Figure 8, displays fluctuations throughout the simulation. The changes are observed at bus 5, which is expected due to the variations in active power injections at the bus, with the phase angles primarily influenced by the active power flow.

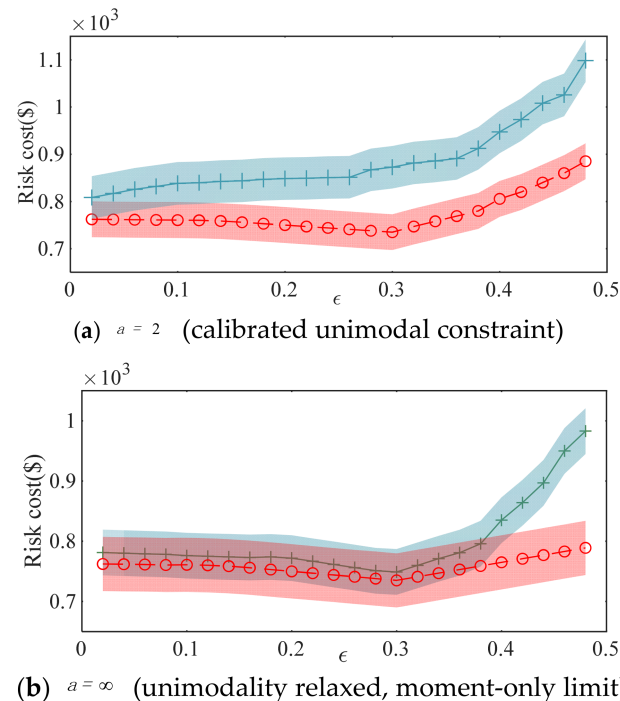


Figure 6. The risk cost of GCC and DR CVaRC modes under two unimodality ($+$: GCC Mean; $\square$: GCC Standard deviation; $-o-$: DR CVaRC Mean; $\square$: DR CVaRC Standard deviation).

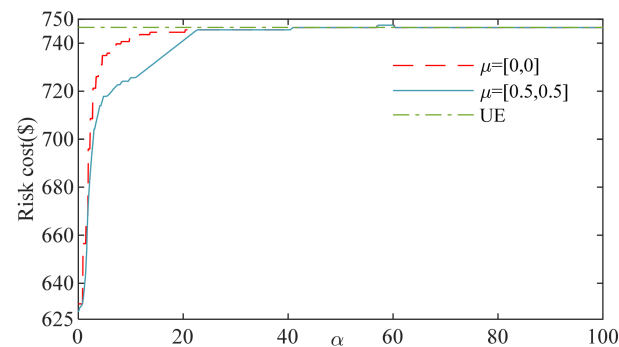


Figure 7. The risk cost with varying a .

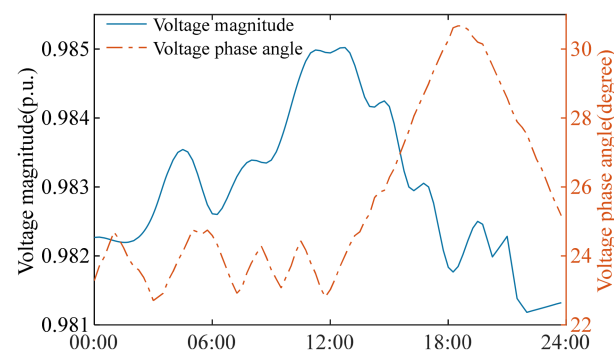


Figure 8. Diurnal profiles of voltage magnitude and post-calculated phase angle at bus 5.

The supply and return water temperatures of the DNS operate within the rated range of 95–70 °C, with a maximum deviation not exceeding 5 °C from the nominal values. The supply and return water temperatures at node 5 are shown in Figure 9.

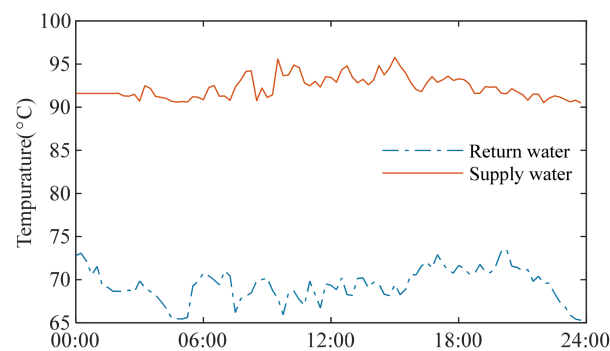


Figure 9. Supply and return water temperature profiles at node 5 of the district heating network.

Observed from Figure 9, to minimize heat loss in the DHN, the supply water temperature is maintained at a lower level. The behavior of the DNS is primarily reflected in the temperature profiles of the return water: lower temperatures in the early morning to satisfy increased heat demand, relatively higher temperatures during midday and afternoon for energy storage, and lower temperatures late at night to release energy for higher thermal loads. To quantify the actual risk control performance, out-of-sample validation is conducted with 10,000 wind scenarios generated from empirical forecast error distributions. As the DR-CVaR temperature constraints are probabilistic constraints configured at the 95% confidence level, rare violations are allowed in extreme scenarios. The statistical results show that the out-of-sample violation probabilities are 2.17% for supply water and 1.84% for return water, both well below the 5% preset risk tolerance. The worst-case violation magnitudes are 0.72 °C and 0.65 °C respectively, which are negligible for practical heating network operation.

Figure 10 shows the power output of CHP and HP in DRRD. As observed in Figure 10, since the wind farms generate a substantial amount of wind power during the early morning and at night, the power outputs of HP increase during peak wind power generation periods and decrease during off-peak hours to maximize the consumption of available wind power. In contrast, when the wind farm's power output reaches its lowest point at noon, the CHP units increase their outputs to compensate for the electrical power deficit. To quantify the cost–risk trade-off under different robustness levels and ambiguity set configurations, key metrics of typical schemes are compared in Table 3. As ε increases, the total cost rises while the out-of-sample violation risk decreases monotonically. Under the same ε , incorporating unimodal information reduces the total cost by 3.0–3.3% and achieves slightly better risk performance, demonstrating its advantage in balancing economy and robustness.

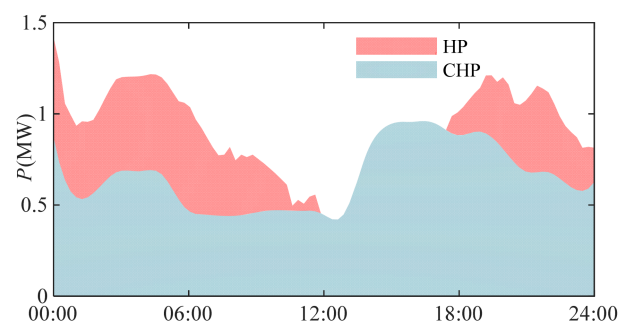


Figure 10. The optimal power output.

Table 3. Cost-risk trade-off under different dispatch configurations.

Configuration	Total Daily Cost (\$)	Violation Probability (%)	Out-of-Sample CVaR (°C)	Avg. HP Power (MW)
Deterministic	721.6	11.42	1.28	0.642
UE, $\varepsilon = 0.2$	768.3	6.15	0.74	0.671
UI, $\varepsilon = 0.2$	742.9	5.87	0.69	0.658
UI, $\varepsilon = 0.3$ (baseline)	765.1	2.17	0.38	0.686
UI, $\varepsilon = 0.5$	812.7	0.53	0.12	0.715

To quantitatively demonstrate the benefits of the proposed moment–unimodal ambiguity set, systematic comparisons are conducted with two established DRO benchmarks: the classic moment-only DRO and the state-of-the-art Wasserstein-DRO. All benchmarks share the same objective function, 5% risk tolerance, and 95% ambiguity set coverage confidence level to ensure a fair comparison. The deterministic dispatch model is also included as the economic lower bound reference. The performance metrics under the baseline configuration ($\varepsilon = 0.3$) are summarized in Table 4.

Table 4. Cost-risk trade-off under different dispatch configurations.

Formulation	Total Daily Risk Cost (\$)	Cost Premium vs. Deterministic (%)	Out-of-Sample Violation Probability (%)	Out-of-Sample CVaR (\$)
Deterministic dispatch	721.6	0.00	11.42	923.7
Moment-only DRO	789.2	9.37	3.12	802.4
Wasserstein-DRO	778.5	7.88	2.68	785.6
Proposed	765.1	6.03	2.17	761.2

Compared with the classic moment-only benchmark, the proposed formulation reduces the total risk cost by 3.1%, and simultaneously decreases the out-of-sample violation probability by 30.4% and CVaR by 5.1%. This verifies that the unimodality constraint effectively alleviates the over-conservatism of pure moment-based DRO by pruning physically implausible multimodal distributions from the ambiguity set. Compared with the data-driven Wasserstein benchmark, the proposed method reduces the out-of-sample violation probability by 19.0% and CVaR by 3.1%, while cutting the computation time by 33.0%. This demonstrates that incorporating unimodality information yields more accurate uncertainty characterization than the purely data-driven approach for wind forecast errors with well-understood distribution properties. All three robust formulations satisfy the pre-specified 5% risk tolerance. The proposed method achieves the lowest cost premium over deterministic dispatch with the best risk control performance, making it particularly suitable for day-ahead dispatch scenarios with limited historical data and clear physical distribution characteristics.

5. Conclusions

This paper presents a DRRD model for IEHS, aiming to effectively handle the risks associated with violating operational limits caused by uncertainties in wind power production. The proposed DRRD model employs a structured ambiguity set that integrates both moment-based and unimodality information. Risk severity can be measured by using the DR-CVaR constraints. Then, the proposed model problem is reformulated into SOC constraints which are tractably solved. Numerical results validate the effectiveness of

the DRRD model. The results also indicate that incorporating unimodality information into the ambiguity set leads to more precise uncertainty distribution data, yielding less conservative solutions. Furthermore, the advantages of reduced risks and operational costs, when accounting for the additional unimodality information, are also highlighted.

As potential avenues for future research, it would be valuable to investigate the implications of imprecise knowledge regarding moments and the characteristics of unimodality. We will also establish methodologies to derive combined restrictions of joint DR-CVaR incorporating unimodality, encompass additional attributes such as bounded supports and moment ambiguity, and assess the validity of the outcomes on more practical systems characterized by elevated dimensions of uncertainty and correlations, aiming to ascertain the case-specific applicability of the proposed model.

Author Contributions: Conceptualization, Z.Y. and L.T.; methodology, Z.Y., W.Y. and L.T.; software, Z.Y. and W.Y.; validation, H.Z., L.Z. (Ligong Zhang) and Z.D.; formal analysis, Z.Y., W.Y. and L.T.; investigation, L.Z. (Ligong Zhang), Z.D. and L.Z. (Ling Zhou); resources, H.Z. and T.L.; data curation, Z.D. and L.Z. (Ling Zhou); writing—original draft preparation, Z.Y.; writing—review and editing, H.Z., L.T. and T.L.; visualization, Z.Y. and L.Z. (Ling Zhou); supervision, L.T. and T.L.; project administration, H.Z. and T.L.; funding acquisition, H.Z. and T.L. All authors have read and agreed to the published version of the manuscript.

Funding: Supported by CHN Energy Investment Group (Project No. GJNY-23-175).

Data Availability Statement: The data supporting the findings of this study are not publicly available due to restrictions imposed by the data owner. The modified test system parameters and derived simulation results are available from the corresponding author upon reasonable request and subject to approval by the data owner. The Barry Island test system was adapted from Reference [20], and the wind power data used in the case study are described in Reference [41].

Conflicts of Interest: Authors Zeyu Yan, Hua Zhuo, Wentao Yan, Ligong Zhang, Zhanliang Dou, and Ling Zhou were employed by China Energy Investment Corporation during the conduct of this study. The research received funding from China Energy Investment Corporation (grant GJNY-23-175). All authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest. The funder was not involved in the study design, collection, analysis, interpretation of data, the writing of this article or the decision to submit it for publication.

Abbreviations

The following abbreviations are used in this manuscript:

Acronyms

CHP	Combined heat and power.	P^{CHP}/H^{CHP}	Electric/thermal power output of CHP unit.
PDN	Power distribution network.	P_{ij}/Q_{ij}	Active/reactive power flow of branch $i-j$.
EH	Energy hub.	V	Voltage magnitude.
HP	Heat pump.	T^S/T^R	Supply/return temperature.
DHN	District heating network.	T^{in}/T^{out}	Inlet/outlet temperature of the pipeline.
RES	Renewable energy sources.	T	Mixture temperature at confluence node.
IEHS	Integrated electricity and heating system.	ρ^{CHP}	Balancing power of CHP unit.
PDE	Partial differential equation.		
AE	Algebraic equation.	$C^{CHP,U}/C^{CHP,D}$	Upward/downward reserve cost of CHP.
DRRD	Distributionally robust risk dispatch.	$C^{L,S}$	Load shedding cost.
DRCC	Distributionally robust chance constraint.	RU^{CHP}/RD^{CHP}	Maximum ramp-up/down rate of CHP unit.
U-DRCC	DRCC with unimodal.	σ	Combination coefficient of corner point.
CVaR	Conditional Value-at-Risk.	r_{ij}/χ_{ij}	Resistance/reactance of branch $i-j$.
SOC	Second-order cone.	V^0	Voltage magnitude at slack bus.

Indices		H^L	Heat load.
i, j, l	Subscript indices of buses in PDN.	v	Flow velocity.
t	Subscript indices of time periods.	c^P	Specific heat capacity of water.
p	Subscript index of nodes in DHN.	R	Thermal resistance of the pipe segment.
n	Subscript index of pipelines in DHN.	m^S/m^L	Mass flow rate of heat source/load.
b	Subscript index of spatial difference layer.	$\dot{m}$	Mass flow rate within the pipeline.
k	Superscript index of temporal difference layer.	T^A	Ambient temperature.
e	Subscript indices of energy hubs.	λ^{HP}	Heat transfer coefficient of the HP.
r	Subscript index of corner points.	L	Length of pipeline.
Variables		D	Dispatch time horizon.
P^{CHP}/H^{CHP}	Electric/thermal power output of CHP unit.	τ	Time difference step size.
P_{ij}/Q_{ij}	Active/reactive power flow of branch $i-j$.	h	Spatial difference step size.
V	Voltage magnitude.	M	Number of the total discretized space steps.
T^S/T^R	Supply/return temperature.	N	Number of the total discretized time steps.
T^{in}/T^{out}	Inlet/outlet temperature of the pipeline.		
T	Mixture temperature at confluence node.		

Appendix A

The operation region of the extraction-turbine CHP unit is described in Figure A1.

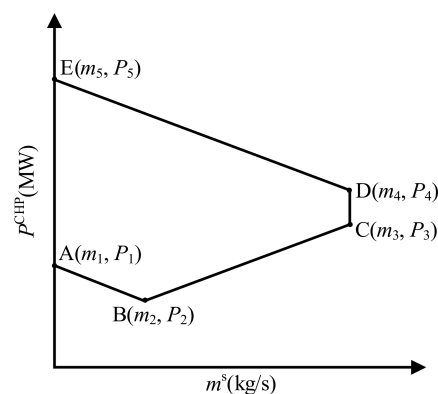


Figure A1. Operation region of the extraction steam turbine CHP units.

Appendix B

According to the reformulation explained in Section 3, probabilistic constraints (7)–(10), (16)–(19), and (39)–(40) in the form of DR-CVaR constraints can be recast as:

$$\left\| \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{P}_{ij,t} - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (A1)$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{P}_{ij,t} - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (A2)$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{P}_{ij,t} - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (A3)$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-P_{ij,t}) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (A4)$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-P_{ij,t}) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (A5)$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-P_{ij,t}) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) P_{ij,t} \mu \quad (\text{A6})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{Q}_{ij,t} - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A7})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{Q}_{ij,t} - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A8})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{Q}_{ij,t} - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A9})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-Q_{ij,t}) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A10})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-Q_{ij,t}) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A11})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-Q_{ij,t}) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) Q_{ij,t} \mu \quad (\text{A12})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^S - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A13})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^S - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A14})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^S - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A15})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^S) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A16})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^S) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A17})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^S) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^S \mu \quad (\text{A18})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^R - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A19})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^R - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A20})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] \bar{T}_n^R - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A21})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^R) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A22})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^S) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A23})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-T_n^R) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) T_{n,t}^R \mu \quad (\text{A24})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] R_{e,t}^{\text{CHP,U}} - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A25})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] R_{e,t}^{\text{CHP,U}} - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A26})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] R_{e,t}^{\text{CHP,U}} - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A27})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-R_{e,t}^{\text{CHP,D}}) - \left[\frac{2\varepsilon(a+1)}{a} - 1 \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A28})$$

$$\left\| \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-R_{e,t}^{\text{CHP,D}}) - \left[\frac{(2\varepsilon-1)(a+1)-1}{a} \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A29})$$

$$\left\| \left(\frac{a+1}{a} \right) \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \right\| \leq \left[\frac{2\varepsilon(a+1)}{a} \right] (-R_{e,t}^{\text{CHP,D}}) - \left[\frac{(2\varepsilon-1)(a+1)}{a} \right] \theta - \left(\frac{a+1}{a} \right) \rho_{e,t}^{\text{CHP}} \mu \quad (\text{A30})$$

References

1. Qiu, J.; Zhao, J.; Wen, F.; Zhao, J.; Gao, C.; Zhou, Y.; Tao, Y.; Lai, S. Challenges and pathways of low-carbon oriented energy transition and power system planning strategy: A review. *IEEE Trans. Netw. Sci. Eng.* **2023**, *11*, 5396–5416. [\[CrossRef\]](#)
2. Wang, S.; Zhai, J.; Hui, H. Optimal Energy Flow in Integrated Electricity and Gas Systems with Injection of Alternative Gas. *IEEE Trans. Sustain. Energy* **2023**, *14*, 1–18. [\[CrossRef\]](#)
3. Li, W.; Qian, T.; Zhang, Y.; Shen, Y.; Wu, C.; Tang, W. Distributionally robust chance-constrained planning for regional integrated electricity–heat systems with data centers considering wind power uncertainty. *Appl. Energy* **2023**, *336*, 120787. [\[CrossRef\]](#)
4. Tan, H.; Yan, W.; Ren, Z.; Wang, Q.; Mohamed, M.A. Distributionally robust operation for integrated rural energy systems with broiler houses. *Energy* **2022**, *254*, 124398. [\[CrossRef\]](#)
5. Wu, X.; Fang, J.; Chen, Z. Distributionally robust unit commitment of integrated electricity and heat system under bi-directional variable mass flow. *Appl. Energy* **2022**, *326*, 119788. [\[CrossRef\]](#)
6. Wang, C.; Yan, C.; Li, G.; Liu, S.; Bie, Z. Risk assessment of integrated electricity and heat system with independent energy operators based on Stackelberg game. *Energy* **2020**, *198*, 117349. [\[CrossRef\]](#)
7. Yu, J.; Guo, L.; Ma, M.; Kamel, S.; Li, W.; Song, X. Risk assessment of integrated electrical, natural gas and district heating systems considering solar thermal CHP plants and electric boilers. *Int. J. Electr. Power Energy Syst.* **2018**, *103*, 277–287. [\[CrossRef\]](#)
8. Hou, G.; Jian, X. Distributionally robust chance-constrained economic dispatch of multi-area electricity–gas–heat integrated energy systems. *Electr. Power Syst. Res.* **2023**, *217*, 109090. [\[CrossRef\]](#)
9. Lu, S.; Li, Y.; Ding, S.; Gu, W.; Xu, Y.; Song, M. Combined Electrical and Heat Load Restoration Based on Bi-Objective Distributionally Robust Optimization. *IEEE Trans. Ind. Inform.* **2023**, *19*, 9239–9252. [\[CrossRef\]](#)
10. Li, Y.; Zou, Y.; Tan, Y.; Cao, Y.; Liu, X.; Shahidepour, M.; Tian, S.; Bu, F. Optimal stochastic operation of integrated low-carbon electric power, natural gas, and heat delivery system. *IEEE Trans. Sustain. Energy* **2018**, *9*, 273–283. [\[CrossRef\]](#)
11. Arabi Nowdeh, S.; Naderipour, A.; Davoudkhani, I.F.; Guerrero, J.M. Stochastic optimization—Based economic design for a hybrid sustainable system of wind turbine, combined heat, and power generation, and electric and thermal storages considering uncertainty: A case study of Espoo, Finland. *Renew. Sustain. Energy Rev.* **2023**, *183*, 113440. [\[CrossRef\]](#)
12. Chen, L.; Tang, H.; Wu, J.; Li, C.; Wang, Y. A robust optimization framework for energy management of CCHP users with integrated demand response in electricity market. *Int. J. Electr. Power Energy Syst.* **2022**, *141*, 108181. [\[CrossRef\]](#)
13. Tan, H.; Yan, W.; Ren, Z.; Wang, Q.; Mohamed, M.A. A robust dispatch model for integrated electricity and heat networks considering price-based integrated demand response. *Energy* **2022**, *239*, 121875. [\[CrossRef\]](#)

14. Chen, C.; Wu, X.; Li, Y.; Zhu, X.; Li, Z.; Ma, J.; Qiu, W.; Liu, C.; Lin, Z.; Yang, L.; et al. Distributionally robust day-ahead scheduling of park-level integrated energy system considering generalized energy storages. *Appl. Energy* **2021**, *302*, 117493. [\[CrossRef\]](#)
15. Li, Y.; Han, M.; Shahidehpour, M.; Li, J.; Long, C. Data-driven distributionally robust scheduling of community integrated energy systems with uncertain renewable generations considering integrated demand response. *Appl. Energy* **2023**, *335*, 120749. [\[CrossRef\]](#)
16. Li, H.; Liu, H.; Ma, J.; Li, D.; Zhang, W. Distributionally robust optimal dispatching method of integrated electricity and heating system based on improved Wasserstein metric. *Int. J. Electr. Power Energy Syst.* **2023**, *151*, 109120. [\[CrossRef\]](#)
17. Skalyga, M.; Amelin, M.; Wu, Q.; Söder, L. Distributionally robust day-ahead combined heat and power plants scheduling with Wasserstein Metric. *Energy* **2023**, *269*, 126793. [\[CrossRef\]](#)
18. Zhou, Y.; Shahidehpour, M.; Wei, Z.; Sun, G.; Chen, S. Multistage robust look-ahead unit commitment with probabilistic forecasting in multi-carrier energy systems. *IEEE Trans. Sustain. Energy* **2021**, *12*, 70–82. [\[CrossRef\]](#)
19. Zhou, Y.; Shahidehpour, M.; Wei, Z.; Li, Z.; Sun, G.; Chen, S. Distributionally Robust Co-Optimization of Energy and Reserve for Combined Distribution Networks of Power and District Heating. *IEEE Trans. Power Syst.* **2020**, *35*, 2388–2398. [\[CrossRef\]](#)
20. Zhou, Y.; Shahidehpour, M.; Wei, Z.; Li, Z.; Sun, G.; Chen, S. Distributionally Robust Unit Commitment in Coordinated Electricity and District Heating Networks. *IEEE Trans. Power Syst.* **2020**, *35*, 2155–2166. [\[CrossRef\]](#)
21. Wang, C.; Bie, Z.; Wu, Q.; Li, G.; Xie, H. Coordinated post-contingency dispatch of integrated energy system with multiple participants based on distributed energy trading. *Electr. Power Energy Syst.* **2021**, *130*, 107011. [\[CrossRef\]](#)
22. Liu, T.; Pan, W.; Liu, M.; Zhu, Z. Contingencies-Based Distributionally Robust Co-Risk Operation for Combined Electricity and Heat System. *IEEE Access* **2021**, *9*, 156211–156221. [\[CrossRef\]](#)
23. Chen, Y.; Wang, J.; Bo, R.; Gu, C.; Li, Q. Risk-Averse scheduling of integrated electricity-heat systems considering multi-energy network operations for resilience enhancement against contingencies. *Int. J. Electr. Power Energy Syst.* **2023**, *153*, 109313. [\[CrossRef\]](#)
24. Xuan, A.; Shen, X.; Guo, Q.; Sun, H. A conditional value-at-risk based planning model for integrated energy system with energy storage and renewables. *Appl. Energy* **2021**, *294*, 116971. [\[CrossRef\]](#)
25. Liu, J.; Cao, X.; Xu, Z.; Guan, X.; Dong, X.; Wang, C. Resilient operation of multi-energy industrial park based on integrated hydrogen-electricity-heat microgrids. *Int. J. Hydrogen Energy* **2021**, *46*, 28855–28869. [\[CrossRef\]](#)
26. Si, F.; Wang, J.; Han, Y.; Zhao, Q. Risk-Averse Multiobjective Optimization for Integrated Electricity and Heating System: An Augment Epsilon-Constraint Approach. *IEEE Syst. J.* **2022**, *16*, 5142–5153. [\[CrossRef\]](#)
27. Zhang, X.; Zhang, Y.; Ji, X.; Ye, P.; Li, J. Unit commitment of integrated energy system considering conditional value-at-risk and P2G. *Electr. Power Syst. Res.* **2023**, *221*, 109398. [\[CrossRef\]](#)
28. Li, R.; Wei, W.; Mei, S.; Hu, Q.; Wu, Q. Participation of an Energy Hub in Electricity and Heat Distribution Markets: An MPEC Approach. *IEEE Trans. Smart Grid* **2019**, *10*, 3641–3653. [\[CrossRef\]](#)
29. Lewis, G.N.; Randall, M. *Thermodynamics*; Courier Dover Publications: Garden City, NY, USA, 2020.
30. Duquette, J.; Rowe, A.; Wild, P. Thermal performance of a steady state physical pipe model for simulating district heating grids with variable flow. *Appl. Energy* **2016**, *178*, 383–393. [\[CrossRef\]](#)
31. Zhou, Y.; Wei, Z.; Shahidehpour, M.; Chen, S. Distributionally Robust Resilient Operation of Integrated Energy Systems Using Moment and Wasserstein Metric for Contingencies. *IEEE Trans. Power Syst.* **2021**, *36*, 3574–3584. [\[CrossRef\]](#)
32. Chen, X.; Kang, C.; O'Malley, M.; Xia, Q.; Bai, J.; Liu, C.; Sun, R.; Wang, W.; Li, H. Increasing the Flexibility of Combined Heat and Power for Wind Power Integration in China: Modeling and Implications. *IEEE Trans. Power Syst.* **2015**, *30*, 1848–1857. [\[CrossRef\]](#)
33. Van Parys, B.P.G.; Goulart, P.J.; Morari, M. Distributionally robust expectation inequalities for structured distributions. *Math. Program.* **2019**, *173*, 251–280. [\[CrossRef\]](#)
34. Dharmadhikari, S.; Joag-Dev, K. *Unimodality, Convexity, and Applications*; Elsevier: Amsterdam, The Netherlands, 1988.
35. Ordoudis, C.; Nguyen, V.A.; Kuhn, D.; Pinson, P. Energy and reserve dispatch with distributionally robust joint chance constraints. *Oper. Res. Lett.* **2021**, *49*, 291–299. [\[CrossRef\]](#)
36. Xie, W.; Ahmed, S. Distributionally Robust Chance Constrained Optimal Power Flow with Renewables: A Conic Reformulation. *IEEE Trans. Power Syst.* **2018**, *33*, 1860–1867. [\[CrossRef\]](#)
37. Shapiro, A.; Kleywegt, A. Minimax analysis of stochastic problems. *SPEC.* **2010**, *17*, 523–542.
38. Pourahmadi, F.; Kazempour, J. Distributionally Robust Generation Expansion Planning with Unimodality and Risk Constraints. *IEEE Trans. Power Syst.* **2021**, *36*, 4281–4295. [\[CrossRef\]](#)
39. Li, B.; Jiang, R.; Mathieu, J.L. Ambiguous risk constraints with moment and unimodality information. *Math. Program.* **2019**, *173*, 151–192. [\[CrossRef\]](#)
40. Wei, W.; Liu, F.; Mei, S. Distributionally robust Co-optimization of energy and reserve dispatch. *IEEE Trans. Sustain. Energy* **2016**, *7*, 289–300. [\[CrossRef\]](#)
41. Dowell, J.; Pinson, P. Very-Short-Term Probabilistic Wind Power Forecasts by Sparse Vector Autoregression. *IEEE Trans. Smart Grid* **2016**, *7*, 763–770. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.