

Article

Incorporating Carbon-Related Costs into Production Decisions in the Fastener Industry Under the Net-Zero Transition

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Abstract

The implementation of global net-zero emission policies and carbon pricing mechanisms, including the Carbon Border Adjustment Mechanism (CBAM), has increased the need for export-oriented manufacturers to make carbon-related costs visible in production decisions. Taiwan's fastener industry is particularly exposed because of its multi-stage production structure, intensive use of steel-based inputs, and reliance on international markets. Traditional single-base cost allocation can obscure differences in resource consumption across products and processes. To enhance the decision-making value of cost information, this study develops an Activity-Based Costing (ABC) framework that assigns modeled resource consumption to activities and products while separately internalizing carbon-related costs in a mixed-integer production-planning model. Theory of Constraints (TOC) is used as a bottleneck-focused managerial interpretation of finite production capacities rather than as a separate objective function. Using Taiwan's fastener manufacturing industry as the research context, the model is demonstrated through an illustrative numerical example parameterized to reflect representative production and resource consumption relationships. The reported findings are therefore numerical results of the model rather than statistical empirical validation of a sampled population or a disclosed company dataset. Under the specified baseline parameters, Product 4 is selected because its revenue, purchasing-tier, activity driver, capacity, and carbon-related cost coefficients jointly dominate the alternatives. Product exclusion scenarios further show how the preferred product changes when individual products are exogenously removed while carbon price parameters are held fixed. The framework provides a transparent decision support structure for jointly examining activity costs, constrained resources, and carbon-related charges; it should not be interpreted as a legal replication of CBAM certificate obligations. Industry 5.0 extensions are positioned as future research.



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1. Introduction

1.1. Research Background

Global climate change has become a critical issue, with countries progressively strengthening environmental regulations and carbon management policies, presenting new challenges for enterprises regarding carbon reduction and sustainable operations. Uncontrolled greenhouse gas emissions will severely exacerbate climate change, emphasizing

the importance of immediate carbon reduction actions [1]. Against the backdrop of rapid global economic growth, all countries must confront the severe challenges of global warming, greenhouse effects, and environmental pollution. This situation requires governments and enterprises to adopt comprehensive approaches to reduce environmental footprints, including formulating strategies to reduce carbon emissions, carbon rights, carbon taxes, and carbon neutrality, while strengthening environmental responsibility to avoid serious reputational damage [2].

To meet the carbon emission reduction commitments established under the 1997 Kyoto Protocol, the European Union introduced the EU Emissions Trading System (EU ETS), becoming the first jurisdiction to implement a mandatory carbon trading market. The EU ETS operates under a cap-and-trade framework that caps aggregate emissions and uses tradable allowances to encourage cost-effective reductions. Since its launch in 2005, carbon pricing instruments have expanded across regions. According to the World Bank's State and Trends of Carbon Pricing 2026, direct carbon pricing instruments now cover nearly 30% of global greenhouse gas emissions [3]. Firms exposed to these mechanisms therefore increasingly need carbon cost information for operational and strategic decision-making. This development strengthens the rationale for integrating carbon-related costs into production planning.

Although Industry 5.0 emphasizes human centricity, sustainability, and resilience, these dimensions are not modeled in the present optimization framework. The current study therefore focuses on the narrower operational problem of linking activity-level cost information, finite production capacities, and carbon-related costs. Industry 5.0 is retained only as a future research direction.

1.2. Research Motivation

In July 2021, the European Union adopted the proposal for the Carbon Border Adjustment Mechanism (CBAM). Following the transitional phase from October 2023 through December 2025, the EU CBAM entered its definitive regime on 1 January 2026. Under the definitive regime, the legal obligation is borne by EU importers or their indirect customs representatives acting as authorized CBAM declarants. They must report embedded emissions and surrender CBAM certificates, whose price is linked to EU ETS allowance prices. For iron and steel goods listed in Annex II of Regulation (EU) 2023/956, only direct embedded emissions are currently taken into account for CBAM purposes [4,5]. Consequently, the carbon cost function used in this study is treated as a generic/internal carbon pricing proxy for production planning and should not be interpreted as a legal replication of CBAM certificate obligations. Taiwan's fastener industry therefore faces increasing pressure to make carbon information more traceable and to incorporate carbon-related costs into export-oriented production decisions.

Taiwan is one of the world's leading fastener production and export centers. The official Taiwan International Fastener Show platform identifies Taiwan as the world's third-largest fastener exporter and emphasizes its highly integrated supply chain and established industrial clusters. Because steel-based inputs and multi-stage downstream processing transmit carbon-related costs through the production system, carbon cost transparency is particularly relevant to export-oriented fastener production planning.

Many fastener manufacturers are small and medium enterprises and therefore face practical constraints in capital, manpower, data integration, and carbon management capabilities. These limitations increase the importance of a decision framework that uses available operational data to identify resource-intensive activities, potential bottlenecks, and the product-level effects of carbon-related charges.

Accordingly, this study develops a mathematical programming framework that integrates Activity-Based Costing (ABC) with bottleneck-focused Theory of Constraints (TOC) logic and a modeled carbon-related cost term. The framework is demonstrated through an illustrative numerical example parameterized to represent fastener production relationships. The application is used to examine product mix and resource use decisions under the specified cost and capacity structure rather than to claim statistical empirical validation of a disclosed company dataset.

1.3. Research Purpose

This study has the following four specific objectives:

1. This study analyzes the impacts and implementation challenges of the Carbon Border Adjustment Mechanism (CBAM) for Taiwan's fastener industry.
2. This study explores the carbon emission characteristics, inventory status, and supply chain management issues of the fastener industry.
3. This study applies Activity-Based Costing (ABC) and the Theory of Constraints (TOC) to production planning in the fastener manufacturing industry and develops a carbon inventory process framework along with corresponding improvement strategies.
4. This study provides recommendations for carbon reduction and low-carbon transformation that comply with international standards for the fastener industry.

1.4. Paper Architecture

This study examines carbon-related production decision-making in the fastener industry and is organized into five sections.

Section 1 provides the introduction, outlining the research background, motivation, and objectives. Section 2 reviews the fastener industry, CBAM, carbon pricing, ABC, and TOC. Section 3 presents the methodology, Section 4 reports and discusses the numerical results, and Section 5 concludes the study.

2. Literature Review

2.1. Overview of the Carbon Border Adjustment Mechanism (CBAM)

The EU Carbon Border Adjustment Mechanism (CBAM) emerged from the European Union's broader climate governance framework. The EU Emissions Trading System (EU ETS) has undergone successive tightening, and the current 2030 cap trajectory is designed to reduce emissions from covered sectors by 62% relative to 2005 levels [4]. CBAM was introduced as a complementary instrument addressing carbon leakage concerns as free allocation is phased down. This linkage makes the interaction between carbon pricing, embedded emissions, and industrial competitiveness increasingly important for export-oriented production decisions.

While the EU ETS can effectively internalize carbon costs, it also raises production costs within the EU, potentially causing enterprises to relocate capacity to countries with looser regulations, leading to "carbon leakage," which actually undermines global reduction effectiveness [6,7]. The EU also defines carbon leakage as the phenomenon where EU producers' increased costs due to the EU ETS lead to the transfer of trade and production to third countries, resulting in increased emissions in those countries [8]. To simultaneously prevent carbon leakage and maintain the competitiveness of EU products, the EU further imposes additional carbon costs on high-carbon goods from outside the EU to level the carbon cost differences between domestic and foreign goods and increase incentives for other countries to reduce emissions; this is the prototype of the CBAM system.

In the legislative process, the European Commission officially published the CBAM legislative proposal on 14 July 2021, stipulating that carbon-intensive products imported

into the EU must declare embedded carbon emissions and purchase CBAM certificates, with prices linked to EU ETS carbon prices. This ensures that non-EU producers bear carbon costs comparable to EU enterprises; hence, it is also called a “carbon tariff” [8]. The initial proposal covered high-carbon industries, including cement, electricity, fertilizers, steel, and aluminum, planning for a three-year transition period with formal collection starting 1 January 2026, and the gradual phase-out of EU ETS free allowances expected by 2036.

Subsequently, the European Parliament passed the proposal in 2022, shortening the transition period and expanding coverage to organic chemicals, plastics, and some downstream products of steel and aluminum, while including indirect emissions, demonstrating the legislature’s intent to enhance system completeness and environmental rigor. In December 2022, the European Parliament and the Council of the European Union reached a provisional political agreement on the Carbon Border Adjustment Mechanism (CBAM). The agreement confirmed CBAM as a complementary instrument to the EU Emissions Trading System (EU ETS), aligned with the phase-out schedule of free allowances under the EU ETS. Together, these measures establish an integrated framework combining carbon pricing, emissions trading, and border adjustment.

In summary, the EU Carbon Border Adjustment Mechanism (CBAM) can be understood as a policy instrument developed within the framework of the European Green Deal and the Fit for 55 agenda. It is designed to address carbon leakage concerns arising from the EU Emissions Trading System (EU ETS) by imposing carbon-related costs on imports of carbon-intensive products in line with EU ETS pricing. In doing so, CBAM serves multiple policy objectives, including mitigating carbon leakage risks, safeguarding industrial competitiveness, and extending carbon pricing principles beyond the EU’s borders.

2.2. Summary of Taiwan’s Fastener Industry

Taiwan’s fastener industry emerged after World War II, initially relying on manual labor and Japanese-imported machinery and primarily supplying the domestic market with products such as non-heat-treated machine screws and wood screws. In 1949, the “Chun Yu” screw and nut factory was established in Gangshan, Kaohsiung. During the Vietnam War, it became a designated procurement center for the U.S. military, marking the beginning of Taiwan’s reputation as the “Screw Kingdom.” Between 1968 and 1980, driven by the Ten Major Construction Projects and the establishment of China Steel Corporation, large quantities of high-quality domestic wire rod became available. Additionally, successful independent development of screw-forming machines occurred, and the industry entered a phase of mass production and export expansion of standard parts, establishing complete industrial clusters in Gangshan, Luzhu, and other areas. Subsequently, driven by global trade liberalization and export orders, Taiwan’s fastener industry gradually established a complete supply chain from wire rod forging, processing, and electroplating to packaging, earning international recognition as the “Screw Kingdom” and consistently ranking among the world’s top three exporters.

Taiwan’s fastener sector has developed a highly integrated supply chain spanning wire rod, forming, processing, heat treatment, electroplating, and packaging. The Taiwan International Fastener Show identifies Taiwan as the world’s third-largest fastener exporter and highlights the concentration of specialized suppliers within established industrial clusters. For the present study, these characteristics are important because multi-stage activity structures and steel-based inputs create heterogeneous resource consumption and carbon-related cost exposure across products.

International competition and evolving carbon regulations increase the need for traceable cost and emissions information in export-oriented fastener production. Under the

definitive CBAM regime, covered imports are linked to embedded emissions reporting and certificate obligations on EU declarants [4,5]. Although the model in this study does not reproduce the legal CBAM certificate calculation, this regulatory environment motivates the use of an internal carbon-related cost signal when evaluating product mix decisions.

The operational challenge is also shaped by the resource limitations of small and medium enterprises. Limited data integration, carbon accounting capability, and investment capacity can make sophisticated environmental management difficult to implement. A production planning framework therefore needs to remain transparent enough to connect available activity and capacity data with product-level cost consequences.

In summary, Taiwan's fastener industry faces simultaneous pressures from market competition, carbon costs, resource constraints, and changing regulatory requirements. For the purpose of the present model, the following review is therefore narrowed to the three mechanisms that enter the decision framework directly: carbon pricing signals, activity-based resource and cost allocation, and finite production capacities/bottlenecks. Broader issues such as workforce transition and Industry 5.0 are discussed only as contextual or future-research considerations. Section 2.3 next reviews the carbon pricing structures used to motivate the modeled carbon cost term.

2.3. Carbon Tax

Carbon taxes are an important policy instrument designed to reduce carbon emissions by pricing fuels according to their carbon content. In the green building industry, effective management of carbon emissions in construction projects can improve the accuracy of cost information while mitigating environmental impacts. However, the implementation of carbon taxes also raises construction costs, thereby increasing cost pressures on construction firms. As awareness of corporate social responsibility (CSR) grows, firms are increasingly required to incorporate carbon emission costs into cost estimation and decision-making processes in order to more accurately forecast project costs and mitigate overall environmental impacts [9]. Therefore, carbon taxes are levied based on a specified price per ton of carbon emissions. In this way, carbon taxes serve as an important environmental policy instrument that incentivizes firms to pursue more efficient environmental management in response to carbon pricing signals.

Due to heterogeneity in environmental policies across countries, carbon tax regimes differ substantially, requiring firms to incorporate carbon tax costs into production and pricing decisions. This study examines how alternative carbon tax collection mechanisms affect product mix decisions. In particular, it focuses on progressive and stepped incremental carbon tax schemes, which differ in their incentive structures for emissions reduction. Under a progressive tax regime, tax rates increase gradually with higher emission levels, thereby encouraging continuous improvements in emission reduction performance. By contrast, a stepped incremental tax regime imposes higher tax rates once predefined emission thresholds are exceeded, creating strong financial incentives for firms to keep emissions below these limits. Together, these tax structures provide insights into how different carbon pricing designs influence firms' strategic decisions and financial outcomes in managing carbon emissions.

Recent research further emphasizes that carbon pricing affects operational decisions through the interaction of emitters, regulators, and economic incentives. Maevsky et al. [10] modeled greenhouse gas regulation through a game-theoretic framework linking emission sources and regulatory actions, while Mezatio et al. [11] integrated a carbon tax directly into mathematical supply chain planning. These studies illustrate the importance of connecting policy signals with operational choices. Relative to this stream, the present

model focuses on activity-level cost allocation and bottleneck-constrained product mix decisions in fastener manufacturing.

Figure 1 illustrates the carbon tax threshold (i.e., no tax is collected when emissions remain below the threshold), with a fixed tax rate applied once the threshold is exceeded. Figure 2 presents a taxation method in which carbon taxes are collected at specific rates without a threshold, that is, a fully progressive tax rate system wherein each emission stage carries a corresponding progressive tax rate. While a progressive tax rate system is generally effective in reducing carbon emissions, a stepped incremental tax rate system is more economically reasonable, as it applies differentiated tax rates across discrete emission intervals and generates stronger marginal incentives for emission reduction.

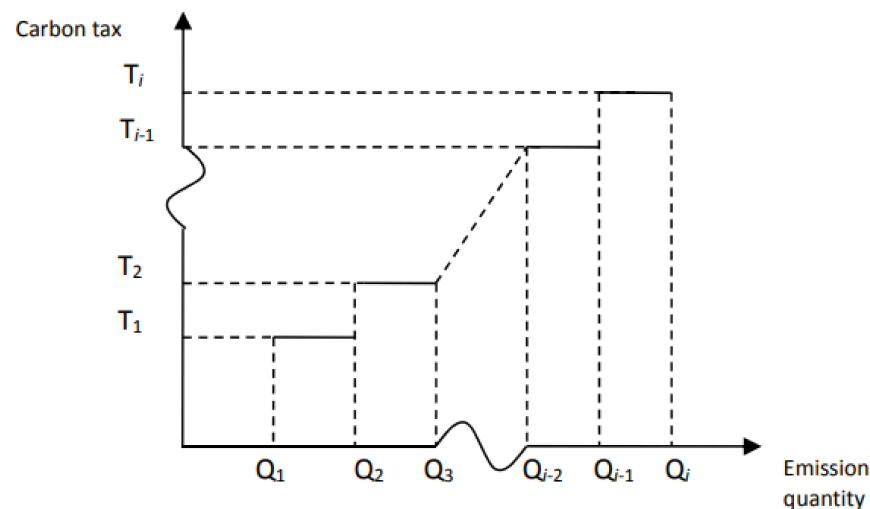


Figure 1. The fixed carbon tax rate schedule.

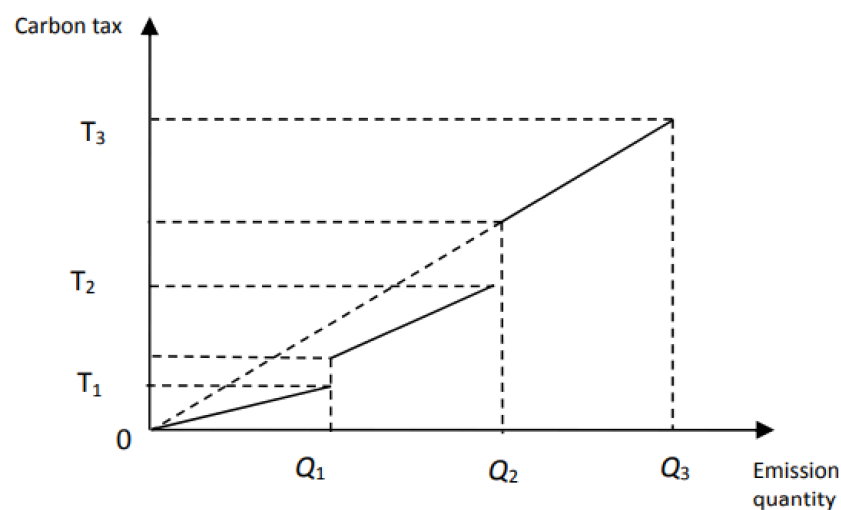


Figure 2. The full progressive carbon tax rate.

2.4. ABC and TOC

Activity-Based Costing (ABC) was first proposed by Kaplan and Cooper (1988) [12] to address cost distortion problems in traditional cost systems that used only quantity as the allocation basis. ABC employs two-stage cost allocation: first, resource costs are allocated to activity centers, and then these costs are allocated to cost objects based on activity drivers (such as unit, batch, and product levels). Its advantage lies in strengthening enterprises' ability to identify "value-added/non-value-added activities" as a basis for product design, pricing, and product mix decisions [12]. However, ABC requires setting numerous drivers

for complex processes, has high estimation subjectivity, and has difficulty reflecting actual capacity utilization, which may lead to inconsistent decisions [13].

Activity-based costing is not the only framework available for cost and environmental burden allocation. Thermoeconomic and exergoeconomic approaches combine thermodynamic/exergy information with economic cost formation and have also been extended to carbon footprint allocation in multi-energy systems [14,15]. These approaches are particularly valuable when detailed thermodynamic state and exergy flow information is available. The present study retains ABC because its decision objects are manufacturing activities and products and its input structure is expressed through activity/resource drivers rather than a complete thermodynamic network. Accordingly, the contribution claimed here is a more disaggregated and traceable representation relative to a single-base conventional costing representation within the modeled activity system, not universal superiority over alternative allocation methodologies.

To address the limitations of traditional Activity-Based Costing (ABC), Kaplan and Anderson [16,17] proposed Time-Driven Activity-Based Costing (TDABC), which assigns costs based on unit time costs and the time required for activities. This approach enhances the objectivity of cost drivers and enables time equations to capture operational complexity [13]. In contrast, the Theory of Constraints (TOC), introduced by Goldratt [18], focuses on identifying production bottlenecks and improving their utilization efficiency in order to maximize overall contribution margins. It is particularly well suited for short-term decision-making under resource constraints. TOC assumes that, except for direct materials, all other costs are treated as fixed, thus not accounting for idle capacity costs [13]. Plenert (1993) [19] and Fredendall and Lea (1997) [20] emphasized that in multi-bottleneck situations, integer linear programming (ILP) is needed to obtain optimal solutions.

Based on sustainable production requirements, the present study combines ABC resource-cost information, finite production capacities, and a carbon-related cost term in a single-objective profit-maximization model. The integrated structure is intended to make activity costs and potentially binding resources visible in the same product mix decision rather than to propose a new general theory of ABC or TOC.

However, prior studies, including the authors' earlier green product mix research, have already combined activity-based costing, capacity constraints, and mathematical programming [13]. Accordingly, the incremental contribution of this study is not the generic integration of ABC and TOC. Instead, it adapts an activity-based, bottleneck-constrained product mix formulation to the multi-stage fastener setting and explicitly introduces process-level carbon-related costs into production decisions relevant to export-oriented manufacturers. The framework is therefore positioned as a context-specific decision support extension under the net-zero transition.

In the fastener setting, products pass through multiple resource and activity categories, including material preparation, labor, forging, machine/heat treatment capacity, electroplating, maintenance, and pre-coating. Different products consume these resources at different rates; the activity-based structure is therefore used to trace modeled resource consumption and its cost consequences at the product level.

Capacity limits at labor, machine/heat treatment, forging, electroplating, pre-coating, and maintenance stages can restrict the feasible product mix. In the present framework, such finite and potentially binding resources are interpreted through TOC bottleneck logic for managerial attention. Mathematically, however, they remain capacity constraints within the optimization model, and no separate TOC objective function is solved. Table 1 summarizes the complementary roles of ABC and TOC in the fastener-industry decision context and the integrated managerial benefits considered in this study.

Table 1. Comparison of ABC and TOC.

Fastener Industry Problem	ABC Function	TOC Function	Integrated Benefits
Multi-variety, multi-stage process costs difficult to clarify	Transparent activity costs and carbon emission information	Understand bottleneck load costs	Help eliminate high-carbon, high-cost products
Aging equipment, fluctuating scheduling efficiency	Accurately calculate equipment resource consumption	Improve bottleneck capacity utilization	Reduce energy waste and carbon emissions
Carbon tariff pressure under CBAM	Incorporate carbon costs into product profitability analysis	Maximize “low-carbon contribution margin”	Enhance export competitiveness
High customization demand, large opportunity cost loss	Improve product mix decision quality	Strengthen order acceptance assessment	Reduce high-carbon, low-profit order risks

This study focuses on Taiwan’s fastener industry as the research context, which is characterized by multi-variety production, a complex cost structure, and dependence on carbon-intensive upstream steel inputs. Prior ABC-TOC and green product mix studies provide the methodological foundation for the present work; the incremental contribution is the adaptation of that foundation to the multi-stage fastener setting while internalizing process-level carbon-related costs in the same finite-capacity product mix decision.

Within this activity-based setting, the model provides more disaggregated and traceable visibility of resource consumption than a single-allocation-base conventional costing representation. ABC supplies activity-level cost information, while TOC contributes the managerial logic of focusing attention on finite and potentially binding resources. The combined framework is evaluated as one integrated decision model and does not claim that standalone ABC, TOC, TDABC, thermoeconomic, or heuristic approaches are universally inferior.

3. Research Methodology

3.1. Research Framework

This study examines the joint product nature of multi-variety production in the fastener industry by integrating Activity-Based Costing (ABC) with bottleneck-focused Theory of Constraints (TOC) logic and incorporating a generic carbon-related cost term into a mixed-integer programming (MIP) model to identify the profit-maximizing product mix. The carbon-related cost term is a production planning proxy and is not intended to reproduce legal CBAM certificate obligations. The research framework process is shown in Figure 3.

Within the integrated model, ABC supplies the activity cost rates and resource–driver relationships used to calculate product-level resource consumption. TOC is operationalized as a managerial interpretation of finite production resources: labor, machine/heat treatment, forging, electroplating, pre-coating, and maintenance capacities constrain the feasible product mix, and capacity limits that become binding are interpreted as bottlenecks requiring managerial attention. Mathematically, these remain conventional capacity constraints; the TOC contribution is the bottleneck-focused decision logic applied to their utilization and improvement. The study does not solve a separate TOC objective function and therefore does not claim a standalone TOC profit level that exceeds ABC. The combined framework thus links activity-level cost visibility with constrained-resource product mix planning.

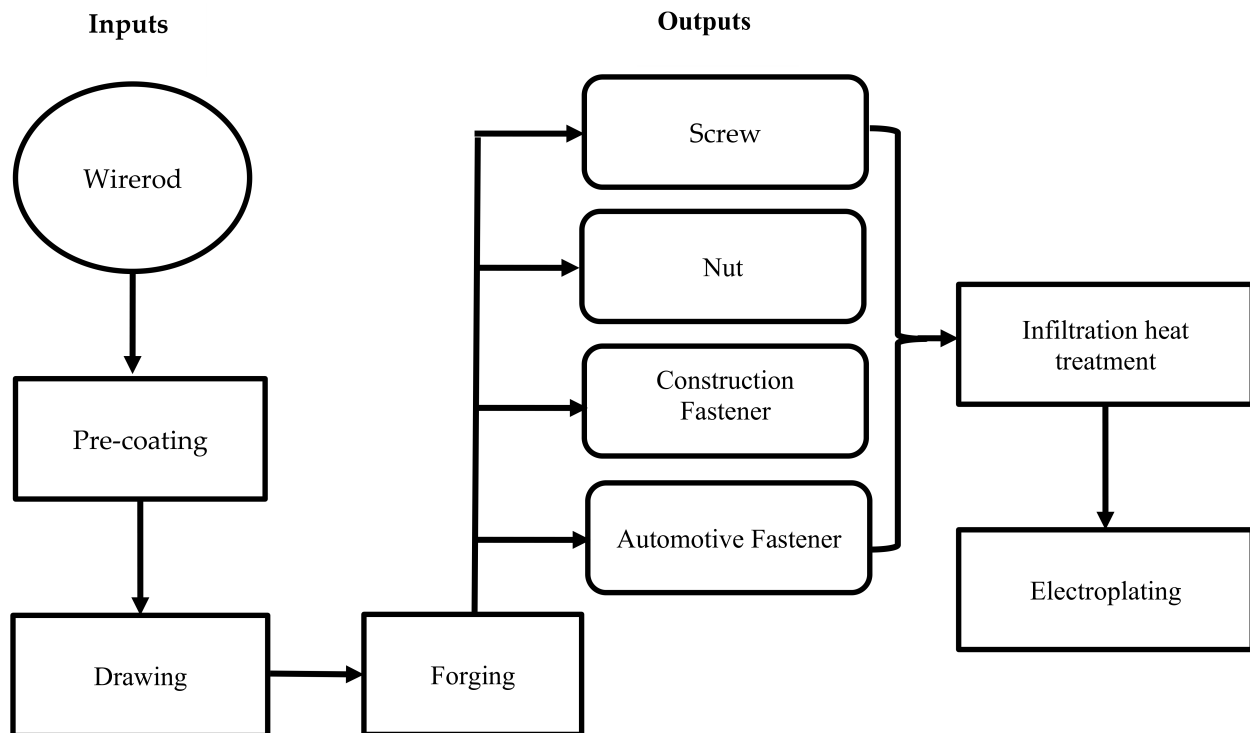


Figure 3. Production planning and control for the fastener industry.

3.2. Integrating Cost Models

This study formulates a single-objective mixed-integer programming model whose objective is to maximize profit (π). No second objective is optimized in the reported analysis. The mathematical structure allocates material, labor, activity, capacity, and carbon-related costs subject to the stated constraints. The objective function is expressed as follows:

$$\begin{aligned} \text{Maximize } \pi = & (\text{Total Revenue}) - (\text{Direct Material Cost}) - (\text{Direct Labor Cost}) \\ & - (\text{Material Handling Cost}) - (\text{Machine Cost}) - (\text{Carbon Tax Cost}) - (\text{Fixed Cost}) \end{aligned}$$

$$\begin{aligned} \text{MAX } \pi = & \sum_{i=1}^n p_i x_i - \sum_{r=1}^s (L_r M_r + L_{d_r} M_{d_r} + L_{dd_r} M_{dd_r}) \\ & - [WC_1 + (WC_2 - WC_1)\alpha_1 + (WC_3 - WC_1)\alpha_2] - \sum_{i=1}^n (t_{oi} + t_{ci})K_o O_i \\ & - \sum_{i=1}^n (t_{\delta i} \delta_i + t_{pi} q_i) k_{\delta} - \sum_{i=1}^n (t_{\tau i} k_{\tau} \tau_i) \\ & - (WA_1 \beta_1 + WA_2 \beta_2 + WA_3 \beta_3) - [W_1 \gamma_1 + (X_1 + X_2 + X_3 + X_4) \gamma_5] \\ & - \sum_{k=0}^t MC_k \theta_k - FC \end{aligned} \quad (1)$$

The current implementation is therefore a single-objective profit-maximization model. The subsequent constraints determine feasible material procurement, labor use, activity consumption, capacity utilization, and the modeled carbon-related charge.

The proposed model comprises several constraint categories. Specifically, the Direct Material Quantity Constraint Equations (2)–(10) govern raw material procurement and consumption while accommodating specific quantity discount mechanisms. Figure 4 illustrates the associated cost structure.

$$X_r \leq Q_r \quad r = 1, 2, \dots, s, \quad r \neq D \quad (2)$$

$$\sum_{i=1}^n d_{ir} q_i \leq X_r \quad r = 1, 2, \dots, s, \quad r \neq D \quad (3)$$

$$Md_r \geq TD_r SD_r \quad r = 1, 2, \dots, s, \quad r \in D \quad (4)$$

$$Md_r \leq W_r SD_r \quad r = 1, 2, \dots, s, \quad r \in D \quad (5)$$

$$M_r \leq TD_r ND_r \quad r = 1, 2, \dots, s, \quad r \in D \quad (6)$$

$$Mdd_r \geq W_r OD_r \quad r = 1, 2, \dots, s, \quad r \in D \quad (7)$$

$$Mdd_r \leq X_r OD_r \quad r = 1, 2, \dots, s, \quad r \in D \quad (8)$$

$$ND_r + SD_r + OD_r = 1 \quad r = 1, 2, \dots, s, \quad r \in D \quad (9)$$

$$M_r \geq 0 \quad r = 1, 2, \dots, s, \quad (10)$$

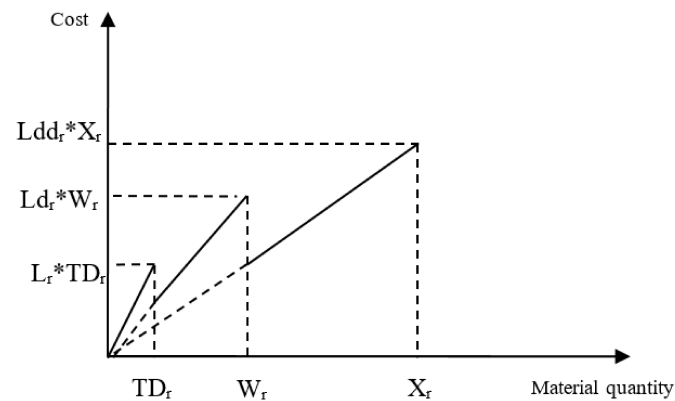


Figure 4. Direct Material Cost.

Direct Labor Hour Constraint Equations (11)–(16): These constraints define the availability of labor hours, accounting for regular time, overtime, and extended shifts. The corresponding cost structure for direct labor is depicted in Figure 5.

$$TWH \leq WH_1 + (WH_2 - WH_1)\alpha_1 + (WH_3 - WH_1)\alpha_2 \quad (11)$$

$$\alpha_0 - \eta_1 \leq 0 \quad (12)$$

$$\alpha_1 - \eta_1 - \eta_2 \leq 0 \quad (13)$$

$$\alpha_2 - \eta_2 \leq 0 \quad (14)$$

$$\alpha_0 + \alpha_1 + \alpha_2 = 1 \quad (15)$$

$$\eta_1 + \eta_2 = 1 \quad (16)$$

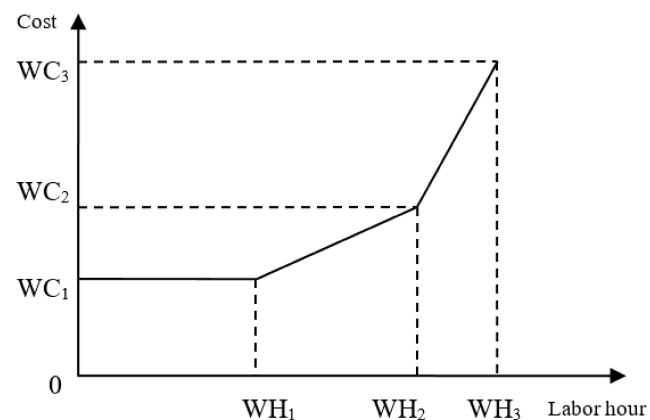


Figure 5. Direct labor cost.

Electroplating Constraint Equation (17): This constraint captures the specific technical requirements and operational limits inherent in the electroplating process.

$$\sum_{i=1}^n (t_{oi}\beta_i + t_{ci}\beta_i) \leq \sigma_o \quad (17)$$

Furthermore, the Maintenance Constraint Equations (18) and (19) characterize mandatory equipment maintenance, ensuring the seamless incorporation of scheduled downtime into the operational model.

$$q_i = s_i\delta_i \quad i = 1, 2, \dots, n, \quad (18)$$

$$\sum_{i=1}^n (t_{\delta i}\delta_i + t_{pi}q_i) \leq \sigma_{\delta} \quad i = 1, 2, \dots, n, \quad (19)$$

Pre-coating Constraint Equations (20) and (21): These constraints characterize the technical limitations inherent in the pre-treatment coating process, ensuring that the operational parameters remain within feasible bounds.

$$\sum_{i=1}^n (t_{\tau i}\tau_i) \leq \sigma_{\tau} \quad i = 1, 2, \dots, n, \quad (20)$$

$$0 \leq q_i \leq R_i\tau_i \quad i = 1, 2, \dots, n, \quad (21)$$

Stepwise Machine Hour Constraint Equations (22) and (23): These constraints characterize the stepwise nature of machine capacity and the associated cost structures. The discrete, process-level activities and their incremental transitions are further illustrated in Figure 6.

$$\sum_{k=0}^t MA_k\theta_k \leq 0 \quad (22)$$

$$\sum_{k=0}^t \theta_k = 1 \quad (23)$$

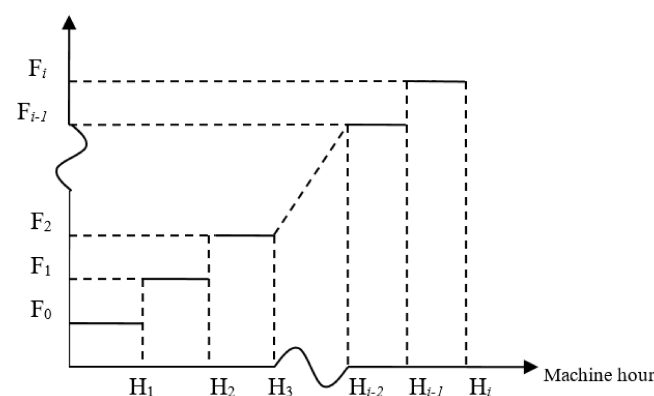


Figure 6. Stepwise process-level activity.

Forging Quantity Constraint Equations (24)–(30): These constraints regulate the forging throughput and the associated waste disposal requirements. The economic impact of waste management is further detailed in Figure 7, which illustrates the waste disposal cost structure.

$$WAQ = WQ_1\beta_1 + WQ_2\beta_2 + WQ_3\beta_3 \quad (24)$$

$$\beta_0 - \mu_1 \leq 0 \quad (25)$$

$$\beta_1 - \mu_1 - \mu_2 \leq 0 \quad (26)$$

$$\beta_2 - \mu_2 - \mu_3 \leq 0 \quad (27)$$

$$\beta_3 - \mu_3 \leq 0 \quad (28)$$

$$\beta_0 + \beta_1 + \beta_2 + \beta_3 = 1 \quad (29)$$

$$\mu_1 + \mu_2 + \mu_3 = 1 \quad (30)$$

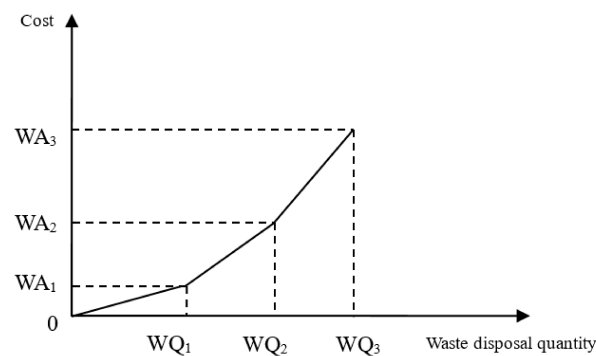


Figure 7. Waste disposal cost.

Carbon Emission Tax Constraint Equation (31): This constraint internalizes a modeled carbon-related cost within the mathematical model to reflect a carbon pricing signal. In the numerical example, Equation (31) applies the stepped carbon cost structure defined by the tier rates T1–T3 and the corresponding emission thresholds reported in Table 2. The fully progressive structure discussed in Section 2.3 is presented as a conceptual alternative and is not used in the reported numerical solution. Figure 8 illustrates the taxation structure associated with the modeled carbon cost constraint.

$$\begin{aligned} 2X_1 + 3X_2 + 3X_3 &\leq G_0 + \omega_1(G_1 - G_0) + \omega_2(G_2 - G_0) \\ \omega_0 - \varphi_1 &\leq 0 \\ \omega_1 - \varphi_1 - \varphi_2 &\leq 0 \\ \omega_2 - \varphi_2 &\leq 0 \\ \omega_0 + \omega_1 + \omega_2 &\leq 1 & \omega_1, \omega_2, \omega_3 &\leq 1 \\ \varphi_1 + \varphi_2 &= 1 & \varphi_1, \varphi_2 &= 0, 1 \end{aligned} \quad (31)$$

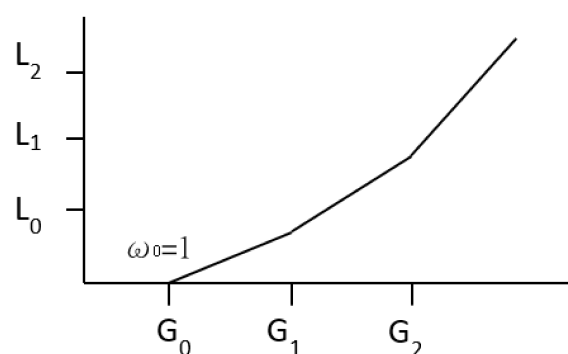


Figure 8. Stepped carbon-related cost structure.

Implementation and solver settings. The mathematical model was implemented in LINGO 20.0 (LINDO Systems, Inc., Chicago, IL, USA) and solved as a deterministic mixed-integer programming model. The default LINGO MIP solver settings were used; no user-defined optimality gap tolerance, branching rule, cut generation setting, or heuristic parameter was imposed. The model was solved to the default optimality criterion. Equations (1)–(31) and Tables 2 and 3 provide the algebraic formulation and numerical parameterization used to reproduce the reported illustrative solution.

Table 2. Data for the numerical example.

Panel A: Production information				Product ×1	Product ×2	Product ×3	Product ×4	Available Capacity
Maximum demand				70,000	40,000	50,000	6000	
Selling price				\$4000	\$5000	\$6000	\$6500	
Quantity of batch	b_{i0}			10	4	4	4	
Quantity of shipment	s_{i0}			5	2	2	2	
Direct material constraint								
Cost/unit	Lr1 = \$50/unit	Ldr1 = \$40/unit	Lddr1 = \$30/unit	6	8	10	10	Xd1 = 30,000
	Lr2 = \$40/unit			2.5	3	4	4	Xd2 = 10,000
Material quantities	Td1 = 10,000	Wd1 = 20,000						
Direct labor constraint								
Cost	WC1 = \$600,000	WC2 = \$1,100,000	WC3 = \$2,400,000					
Labor hours	WH1 = 3000	WH2 = 5000	WH3 = 8000	2	2	3.5	3.5	WH3 = 8000
Wage rate	A1 = \$200/h	A2 = \$220/h	A3 = \$300/h					
Forging constraint								
Cost	WC1 = \$400,000	WC2 = \$900,000	WC2 = \$2,000,000					
Forging quantity	WH1 = 2000	WH2 = 3000	WH2 = 5000	2	3	3	3	WH2 = 5000
Forging rate	A1 = \$200/h	A1 = \$300/h	A1 = \$400/h					
Time driver								
Pre-coating hours	$t_{\tau i0}$			150	300	250	250	1000
Drawing hours				2	2	3	3	
Carbon emission constraint								
Cost/unit	T1 = \$40/unit	T2 = \$50/unit	T3 = \$60/unit					
Carbon emission quantities	o1 = 10,000	o2 = 12,000	o3 = 15,000	20	40	50	20	20,000
Process-level activity: Infiltration heat treatment				Machine 1	Machine 2	Machine 3		
Current capacity								
Cost				F10 = \$200,000	F20 = \$300,000	F30 = \$720,000	F40 = \$800,000	
Machine hours				H10 = 4000	H20 = 6000	H30 = 14,400	H40 = 16,000	
Capacity expansion 1								
Cost				F11 = \$450,000	F21 = \$600,000	F31 = \$1,200,000	F41 = \$1,500,000	
Machine hours				H11 = 56,250	H21 = 75,000	H31 = 150,000	H41 = 187,500	
Capacity expansion 2								
Cost				F12 = \$1,000,000	F22 = \$1,200,000	F32 = \$1,800,000	F42 = \$2,000,000	
Machine hours				H12 = 4000	H22 = 4000	H32 = 7200	H42 = 8000	
Panel B: Resources consumed								
	Electroplating	Maintenance	Pre-coating					
Resources	\$100,000	\$1230,000	\$702,500					
Capacity (hours)	$\sigma_o = 250$	$\sigma_\delta = 4100$	$\sigma_\tau = 1000$					
Per hour/rate	$k_o = \$400/h$	$k_\delta = \$300/h$	$k_\tau = \$702.5/h$					

Table 3. Optimal product mix decision with capacity expansions after adding carbon-related cost constraints.

$ \begin{aligned} \text{Max } \pi = & 4000 \times q_1 + 5000 \times q_2 + 6000 \times q_3 + 6500 \times q_4 - 50 \times M_{r1} - 40 \times Md_{r1} - 30 \times Mdd_{r1} - 40 \times M_{r2} - 600,000 \times \alpha_0 \\ & - 1,100,000 \times \alpha_1 - 2,400,000 \times \alpha_2 - 200,000 \times \theta_{10} - 300,000 \times \theta_{20} - 720,000 \times \theta_{30} - 800,000 \times \theta_{40} - 450,000 \times \theta_{11} \\ & - 600,000 \times \theta_{21} - 1,200,000 \times \theta_{31} - 1,500,000 \times \theta_{41} - 1,000,000 \times \theta_{12} - 1,200,000 \times \theta_{22} - 1,800,000 \times \theta_{32} \\ & - 2,000,000 \times \theta_{42} - 400,000 \times \beta_1 - 900,000 \times \beta_2 + 200,000 \times \beta_3 - (200 \times BA_1 + 220 \times BA_2 + 300 \times BA_3) \div 60 \\ & - (200 \times BA_1 + 300 \times BA_2 + 400 \times BA_3) \div 60 - 400 \times (50 \times \delta_1 + 50 \times \delta_2 + 50 \times \delta_3 + 50 \times \delta_4) \div 60 \\ & - 300 \times (8 \times q_1 + 8 \times q_2 + 8 \times q_3 + 8 \times q_4) \div 60 - 702.5 \times (250 \times \tau_1 + 300 \times \tau_2 + 250 \times \tau_3 + 250 \times \tau_4) \\ & - 40 \times DT - 50 \times ET - 60 \times FT - 200,000 \end{aligned} $	
Subject to-Direct material $6 \times q_1 + 8 \times q_2 + 10 \times q_3 - Md_{r1} - Md_{r1} - Mdd_{r1} \leq 0$ $2.5 \times q_1 + 3 \times q_2 + 4 \times q_3 - M_{r2} \leq 0$ $ND + SD + OD = 1 \quad M_{r2} \leq 10,000 \quad M_{r2} \geq 0$ $M_{r1} \leq 10,000 \times ND \quad Md_{r1} \leq 20,000 \times SD \quad Mdd_{r1} \leq 30,000 \times OD$ $Md_{r1} \geq 10,000 \times SD \quad Mdd_{r1} \geq 20,000 \times OD$	Subject to-Direct labor $2 \times q_1 + 2 \times q_2 + 3.5 \times q_3 + 3.5 \times q_4 - 3000 \times \alpha_0$ $- 5000 \times \alpha_1 - 8000 \times \alpha_2 \leq 0$ $\alpha_0 - \eta_1 \leq 0 \quad \alpha_1 - \eta_1 - \eta_2 \leq 0 \quad \alpha_2 - \eta_2 \leq 0$ $\eta_1 + \eta_2 = 1 \quad \alpha_0 + \alpha_1 + \alpha_2 = 1$ Subject to-Electroplating $15 \times BA_1 + 15 \times BA_2 + 15 \times BA_3 + 50 \times BA_1$ $+ 55 \times BA_2 + 60 \times BA_3 \leq 250 \times 60$ $q_1 = BA_1 \times 10 \quad q_2 = BA_2 \times 4 \quad q_3 = BA_3 \times 4$ $q_4 = BA_4 \times 4$
Subject to-Pre-coating $150 \times \tau_1 + 300 \times \tau_2 + 250 \times \tau_3 + 250 \times \tau_4 \leq 1000$ $q_1 \leq 7000 \times \tau_1 \quad q_2 \leq 4000 \times \tau_2 \quad q_3 \leq 5000 \times \tau_3 \quad q_4 \leq 6000 \times \tau_4$ Subject to-Forging $2 \times q_1 + 3 \times q_2 + 3 \times q_3 + 3 \times q_4 - 2000\beta_1 - 3000 \times \beta_2 - 5000 \times \beta_3 \leq 0$ $\beta_0 - \mu_1 \leq 0 \quad \beta_1 - \mu_1 - \mu_2 \leq 0 \quad \beta_2 - \mu_2 - \mu_3 \leq 0 \quad \beta_3 - \mu_3 \leq 0$ $\beta_0 + \beta_1 + \beta_2 + \beta_3 = 1 \quad \mu_1 + \mu_2 + \mu_3 = 1$ Subject to-Maintenance $50\delta_1 + 50\delta_2 + 50\delta_3 + 8q_1 + 8q_2 + 8q_3 \leq 4100 \times 60$ $q_1 = 5 \times \delta_1 \quad q_2 = 2 \times \delta_2 \quad q_3 = 2 \times \delta_3 \quad q_4 = 2 \times \delta_4$	Subject to-Stepwise machine hour $2 \times q_1 - 2000 \times \theta_{10} - 3000 \times \theta_{11} - 5000 \times \theta_{12} \leq 0$ $\theta_{10} + \theta_{11} + \theta_{12} \leq 1$ $2 \times q_2 - 3000 \times \theta_{20} - 4000 \times \theta_{21} - 6000 \times \theta_{22} \leq 0$ $\theta_{20} + \theta_{21} + \theta_{22} \leq 1$ $3 \times q_3 - 6000 \times \theta_{30} - 8000 \times \theta_{31} - 9000 \times \theta_{32} \leq 0$ $\theta_{30} + \theta_{31} + \theta_{32} \leq 1$ $3 \times q_4 - 8000 \times \theta_{40} - 10,000 \times \theta_{41} - 10,000 \times \theta_{42} \leq 0$ $\theta_{40} + \theta_{41} + \theta_{42} \leq 1$ Subject to-Carbon emission tax $20 \times q_1 + 40 \times q_2 + 50 \times q_3 \leq CT + DT + ET + FT$ $CT \geq 0 \quad CT \leq G_1 \times 10,000$ $DT \geq G_2 \times 10,000 \quad DT \leq G_2 \times 12,000$ $ET \geq G_3 \times 12,000 \quad ET \leq G_3 \times 15,000$ $FT \geq G_4 \times 15,000 \quad G_1 + G_2 + G_3 + G_4 = 1$

3.3. Numerical Application and Parameter Mapping

The application uses four representative product families, P1–P4, as analytical labels rather than named commercial SKUs. All four are modeled through the same resource categories—direct materials, direct labor, forging, machine/heat treatment capacity, electroplating, maintenance, and pre-coating—but with different activity driver coefficients, as reported in Table 2. Selling price and maximum demand determine the revenue and demand terms. Material coefficients and quantity discount thresholds map to Equations (2)–(10); labor coefficients and labor capacity tiers map to Equations (11)–(16); electroplating maps to Equation (17); maintenance maps to Equations (18) and (19); pre-coating maps to Equations (20) and (21); machine capacity maps to Equations (22) and (23); forging and related production constraints map to Equations (24)–(30); and the stepped carbon cost parameters map to Equation (31). The values in Table 2 are treated as an illustrative parameter set used to demonstrate the model rather than as a disclosed statistical sample of firms. Section 4 then reports the product mix solution generated from this mapped parameter set.

Because the submitted material does not document a sampling frame, collection period, external validation dataset, or auditable source for every parameter, the numerical application is not presented as statistical empirical validation. It is designed to demonstrate the behavior of the prescriptive optimization framework under a specified parameter set, not to estimate predictive fitting accuracy. Future validation can compare model recom-

mentations with historical multi-period production decisions when auditable company data are available.

4. Results and Discussion

4.1. Results

Using the illustrative production activity data shown in Table 2 together with the related input parameters, this study expands the ABC objective function and constraints into the numerical form shown in Table 3. Carbon-related costs are incorporated as a production planning proxy for the broader carbon pricing environment associated with the net-zero transition; the term is not presented as a direct calculation of legal CBAM certificate obligations. The resulting mixed-integer model is then solved for the optimal product mix under the specified resource capacities and cost parameters.

After incorporating the modeled carbon-related cost, the model uses wire processing as the joint input and treats available process capacities as constraints. The results show (Table 4) that the optimal product mix consists solely of Product 4 under the specified numerical parameters, requiring 1428 units of wire rod input. The first material reaches its purchasing discount tier, decreasing from \$50 to \$40 per unit, and requires 14,280 units; the second material is \$40 per unit and requires 5712 units. Labor costs total \$1,100,000, machine operating costs are \$800,000, forging costs are \$200,000, electroplating requires 357 batches, maintenance requires 714 batches, pre-coating requires 250 h, and the modeled carbon-related charge is \$28,560.

Table 4. Original program.

Variable	Value	Variable	Value	Variable	Value	Variable	Value
Profit	2,970,673	W ₃₀	0	BA ₂	0	SD	1
Q ₁	0	W ₄₀	1	BA ₃	0	OD	0
Q ₂	0	W ₁₁	0	S ₁	0	r ₁	0
Q ₃	0	W ₂₁	0	S ₂	0	r ₂	1
Q ₄	1428	W ₃₁	0	S ₃	0	B ₀	0
M ₁	0	W ₄₁	0	S ₄	714	U ₁	0
MD ₁	14,280	W ₁₂	0	Z ₁	0	U ₂	0
MDD ₁	0	W ₂₂	0	Z ₂	0	U ₃	1
M ₂	5712	W ₃₂	0	Z ₃	0	BA ₄	357
A ₀	0	W ₄₂	0	Z ₄	1	CT	10,000
A ₁	1	B ₁	0	DT	0	G ₁	1
A ₂	0	B ₂	0	ET	0	G ₂	0
W ₁₀	0	B ₃	1	FT	18,560	G ₃	0
W ₂₀	0	BA ₁	0	ND	0	G ₄	0

Combining ABC cost and the modeled carbon-related cost, the maximum profit is \$2,970,673 under the specified conditions. Product 4 dominates the baseline because its unit selling price (\$6500) is the highest among the four products, while its modeled carbon emission coefficient (20 units) is equal to Product 1 and lower than Products 2 and 3 (40 and 50 units, respectively). The baseline procurement quantity also reaches the material discount tier. Although Product 4 is more labor and process intensive than Product 1, its revenue contribution and purchasing discount outweigh the additional activity and carbon-related costs under the specified capacities and rates. The result should therefore be interpreted as a consequence of the joint revenue, activity driver, purchasing, capacity, and carbon cost coefficients rather than of carbon intensity alone.

Within the assumptions of this numerical example, the activity-based formulation provides a more disaggregated and traceable representation of modeled resource consump-

tion than a single-allocation-base conventional costing approach. This statement is not intended as evidence that ABC, TDABC, TOC, thermoeconomic, or heuristic approaches are universally inferior. The main decision value of the model is the joint visibility of activity costs, carbon-related costs, and finite resource capacities.

4.2. Product Exclusion Scenario Analysis

The following analyses examine alternative product exclusion scenarios under the same carbon price rates, tax thresholds, emission coefficients, and other baseline parameters. They are intended to show how the preferred product changes when products are exogenously removed from the feasible set.

Assuming Product 4 is temporarily excluded from production, Q4 is set to 0 and the single-objective profit-maximization model is re-solved while the remaining parameters are held unchanged.

The results show (Table 5) that the optimal product mix consists solely of Product 1, requiring 1500 units of wire rod input. The first material reaches the purchasing discount tier and is priced at \$40 per unit, requiring 10,000 units; the second material is \$40 per unit and requires 3750 units. Labor costs are \$600,000, machine operating costs are \$450,000, forging costs are \$900,000, electroplating requires 150 batches, maintenance requires 300 batches, pre-coating requires 250 h, and the modeled carbon-related charge is \$30,000.

Table 5. Product exclusion scenario: Q4 = 0.

Variable	Value	Variable	Value	Variable	Value	Variable	Value
Profit	1,936,073	W ₃₀	0	BA ₂	0	SD	1
Q ₁	1500	W ₄₀	0	BA ₃	0	OD	0
Q ₂	0	W ₁₁	1	S ₁	300	r ₁	1
Q ₃	0	W ₂₁	0	S ₂	0	r ₂	0
Q ₄	0	W ₃₁	0	S ₃	0	B ₀	0
M ₁	0	W ₄₁	0	S ₄	0	U ₁	0
MD ₁	10,000	W ₁₂	0	Z ₁	1	U ₂	0
MDD ₁	0	W ₂₂	0	Z ₂	0	U ₃	1
M ₂	3750	W ₃₂	0	Z ₃	0	BA ₄	0
A ₀	1	W ₄₂	0	Z ₄	0	CT	10,000
A ₁	0	B ₁	0	DT	0	G ₁	1
A ₂	0	B ₂	1	ET	0	G ₂	0
W ₁₀	0	B ₃	0	FT	20,000	G ₃	0
W ₂₀	0	BA ₁	150	ND	0	G ₄	0

Combining ABC cost and the modeled carbon-related cost, the maximum profit is \$1,936,073 under this product-exclusion condition.

Next, Products 4 and 1 are both excluded. Q4 and Q1 are set to 0 and the same single-objective profit-maximization model is re-solved with all other parameters unchanged.

The results show (Table 6) that the optimal product mix consists solely of Product 2, requiring 664 units of wire rod input. Because procurement does not reach the discount threshold, the first material is priced at \$50 per unit and requires 5312 units; the second material is \$40 per unit and requires 1992 units. Labor costs are \$600,000, machine operating costs are \$300,000, forging costs are \$400,000, electroplating requires 166 batches, maintenance requires 332 batches, pre-coating requires 300 h, and the modeled carbon-related charge is \$26,560.

Table 6. Product exclusion scenario: $Q_4 = 0$, $Q_1 = 0$.

Variable	Value	Variable	Value	Variable	Value	Variable	Value
Profit	338,942.2	W_{30}	0	BA_2	166	SD	0
Q_1	0	W_{40}	0	BA_3	0	OD	0
Q_2	664	W_{11}	0	S_1	0	r_1	1
Q_3	0	W_{21}	0	S_2	332	r_2	0
Q_4	0	W_{31}	0	S_3	0	B_0	0
M_1	5312	W_{41}	0	S_4	0	U_1	0
MD_1	0	W_{12}	0	Z_1	0	U_2	1
MDD_1	0	W_{22}	0	Z_2	1	U_3	0
M_2	1992	W_{32}	0	Z_3	0	BA_4	0
A_0	1	W_{42}	0	Z_4	0	CT	10,000
A_1	0	B_1	1	DT	0	G_1	1
A_2	0	B_2	0	ET	0	G_2	0
W_{10}	0	B_3	0	FT	16,560	G_3	0
W_{20}	1	BA_1	0	ND	1	G_4	0

Combining ABC cost and the modeled carbon-related cost, the maximum profit is \$338,942.2 under this product-exclusion condition.

Finally, Products 4, 1, and 2 are excluded. Q_4 , Q_1 , and Q_2 are set to 0 and the same single-objective profit-maximization model is re-solved with the remaining parameters unchanged.

The results show (Table 7) that the optimal product mix consists solely of Product 3, requiring 664 units of wire rod input. Because procurement does not reach the discount threshold, the first material is priced at \$50 per unit and requires 6640 units; the second material is \$40 per unit and requires 2656 units. Labor costs are \$600,000, machine operating costs are \$720,000, forging costs are \$400,000, electroplating requires 166 batches, maintenance requires 332 batches, pre-coating requires 250 h, and the modeled carbon-related charge is \$33,200.

Table 7. Product exclusion scenario: $Q_4 = 0$, $Q_1 = 0$, $Q_2 = 0$.

Variable	Value	Variable	Value	Variable	Value	Variable	Value
Profit	91,669.58	W_{30}	1	BA_2	0	SD	0
Q_1	0	W_{40}	0	BA_3	166	OD	0
Q_2	0	W_{11}	0	S_1	0	r_1	1
Q_3	664	W_{21}	0	S_2	0	r_2	0
Q_4	0	W_{31}	0	S_3	332	B_0	0
M_1	6640	W_{41}	0	S_4	0	U_1	0
MD_1	0	W_{12}	0	Z_1	0	U_2	1
MDD_1	0	W_{22}	0	Z_2	0	U_3	0
M_2	2656	W_{32}	0	Z_3	1	BA_4	0
A_0	1	W_{42}	0	Z_4	0	CT	10,000
A_1	0	B_1	1	DT	0	G_1	1
A_2	0	B_2	0	ET	0	G_2	0
W_{10}	0	B_3	0	FT	23,200	G_3	0
W_{20}	0	BA_1	0	ND	1	G_4	0

Combining ABC cost and the modeled carbon-related cost, the maximum profit is \$91,669.58 under this product exclusion condition.

The analyses in Tables 5–7 are product exclusion scenarios. Carbon price rates, tax thresholds, and emission coefficients are held fixed; therefore, these results should not be interpreted as a parametric sensitivity analysis of carbon tax characteristics. They show how the preferred product changes when products are exogenously excluded while the remaining model parameters are unchanged. A systematic sensitivity analysis of carbon

prices, emission factors, demand, material prices, and capacities is left for future work. Within this scope, the scenarios provide an alternative product comparison rather than evidence of robustness to changing regulatory parameters.

The scenario results can support managerial interpretation of alternative product choices under a fixed parameter structure, but they do not quantify the effect of changing carbon prices, renewable energy incentives, or other policy instruments. Such broader policy comparisons should therefore be treated as future analytical extensions rather than outcomes established by the present numerical example.

4.3. Discussion

In this study, ABC supplies activity-level cost information, while TOC is represented by the managerial emphasis on constrained resource utilization embedded in the model's finite capacity constraints. Because the study did not solve separate standalone ABC and TOC models, no claim is made that TOC yields a higher profit than ABC. The integrated interpretation is used to identify how activity costs and constrained resources jointly shape the product mix solution.

For practical use, firms would need consistent data on bill-of-material quantities, process routings and activity drivers, labor and machine capacities, energy/emission factors, and carbon price parameters. These inputs can be extracted from ERP/MES and energy management records, transferred to the optimization model, and then used to return an optimized product mix recommendation to production planning. The model is therefore best viewed as a decision support layer that depends on the quality and governance of the underlying operational data.

The proposed decision model was implemented and solved using LINGO 20.0 (LINDO Systems, Inc.) with the default MIP solver settings. No customized optimality gap tolerance, branching, cut generation, or heuristic settings were imposed. The reported implementation is a single-objective profit-maximization model; no separate multi-objective or standalone TOC solution is reported. The numerical results are conditional on the specified input values and constraints, and the model should be interpreted as a prescriptive optimization tool rather than a predictive forecasting model.

4.4. Future Research Toward Industry 5.0

Industry 5.0 is not modeled in the current optimization framework. Accordingly, human centricity, resilience, collaborative robots, AI, digital twins, real-time IoT sensing, and human-machine collaboration are positioned as future extensions rather than as results demonstrated in this study. Future work may also examine green activity-based costing, renewable energy, and circular economy variables after the present carbon-cost-and-capacity framework is validated with richer operational data.

5. Conclusions

Against the backdrop of increasingly stringent climate policies and expanding carbon pricing mechanisms, firms need decision tools that make both resource consumption and carbon-related costs visible. Focusing on Taiwan's fastener industry as an illustrative manufacturing context, this study combines Activity-Based Costing (ABC), bottleneck-focused Theory of Constraints (TOC) logic, and mathematical programming to analyze how product characteristics, resource capacities, and a modeled carbon-related cost influence product mix decisions.

The numerical example shows that the integrated formulation can identify a profit-maximizing product mix while making activity costs, finite capacities, and carbon-related charges visible in the same decision. Under the specified baseline coefficients, Product 4

is selected because its selling price, material purchasing tier, activity requirements, capacity use, and carbon emission coefficient jointly produce the highest modeled profit. The reported results are conditional numerical outcomes and should not be interpreted as statistical empirical validation or as a direct legal calculation of CBAM certificate obligations.

The study contributes in three narrower ways. First, it adapts an established ABC-capacity mathematical programming foundation to a multi-stage fastener production setting and explicitly internalizes process-level carbon-related costs. Second, it provides a disaggregated and traceable activity-based representation of modeled resource consumption while using finite capacity constraints to identify potentially binding resources for bottleneck-focused managerial attention. Third, the product exclusion scenarios demonstrate how the preferred product changes when alternatives are removed from the feasible set. Because carbon price rates and emission coefficients are not varied, these scenarios are not presented as evidence of sensitivity or robustness to changing carbon tax characteristics.

The numerical analysis directly supports conclusions about the modeled cost, capacity, carbon cost, and product exclusion conditions. Broader implications concerning carbon trading, renewable energy incentives, resilience, and international competitiveness are therefore treated as managerial considerations or future research directions rather than outcomes directly demonstrated by the present numerical example.

From a managerial perspective, the framework can be implemented as a decision support layer linked to operational data on bills of material, process routings, activity drivers, capacities, energy/emission factors, and carbon price parameters. Its usefulness therefore depends on the quality and governance of the underlying ERP/MES and energy management data. The model can assist managers in examining how activity-level costs and finite resources jointly affect product mix choices under a carbon pricing signal.

This study is subject to several limitations. First, the model assumes deterministic demand and production conditions, whereas actual fastener markets are affected by demand volatility, price fluctuations, and supply chain disruptions. Accordingly, the numerical results should be interpreted as conditional on the specified deterministic parameter values rather than as forecasts of dynamic export market behavior. Future research can incorporate stochastic, scenario-based, or robust optimization to represent demand, carbon price, material price, and capacity uncertainty. Second, the model assumes static competitor responses and fixed carbon cost structures. Third, because the numerical application is illustrative and industry specific, its coefficient values should not be generalized directly to other firms, countries, or industries. Future studies should validate the framework using auditable multi-period company data and examine external validity across production systems and regulatory environments.

Future research may extend the framework toward Industry 5.0 by incorporating human centrality, resilience, collaborative robots, AI-assisted decision-making, digital twins, real-time IoT data, renewable energy, and circular economy variables. These elements are proposed as future extensions and are not claimed as modeled contributions of the present study.

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Nomenclature

Symbol	Definition
i, n	i = product index; n = number of products
r, s	r = direct material index; s = number of direct materials
D	Set of materials subject to the quantity discount structure in Equations (4)–(9).
k, t	k = machine capacity segment index; t = highest segment index used in the stepwise machine capacity representation
j	Generic tier/segment index used for piecewise forging/waste and carbon-related cost terms
π	Maximum corporate profit; objective value in Equation (1)
p_i	Unit selling price of product i
x_i	Production quantity of product i in the revenue/material/capacity formulation
q_i	Activity-linked quantity of product i used in the electroplating, maintenance, and pre-coating constraints; in the numerical application, it is mapped to the same product output through the corresponding batch/lot relationship
FC	Fixed cost included in the objective function
L_r	Unit cost of material r without a purchase discount
Ld_r	Unit cost of material r under the first purchase discount tier
Ldd_r	Unit cost of material r under the second purchase discount tier
M_r	Procurement quantity of material r purchased without a discount
Md_r	Procurement quantity of material r purchased under the first discount tier
Mdd_r	Procurement quantity of material r purchased under the second discount tier
d_{ir}	Units of material r required per unit of product i
X_r	Material- r availability/allocated quantity used to bound total material consumption in Equation (3)
Q_r	Upper material supply/procurement bound associated with material r in Equation (2)
TD_r	Lower purchase quantity threshold for entering the first discount tier for material r
W_r	Purchase quantity breakpoint separating the first and second discount tiers for material r
ND_r	Binary variable equal to 1 when material r is purchased in the non-discount region
SD_r	Binary variable equal to 1 when material r is purchased in the first discount region
OD_r	Binary variable equal to 1 when material r is purchased in the second discount region
WC_1, WC_2, WC_3	Total direct labor cost at the three labor capacity breakpoints
TWH	Total direct labor hours required by the selected product mix
WH_1, WH_2, WH_3	Direct labor hour breakpoints defining regular time, overtime, and extended-shift capacity
$\alpha_0, \alpha_1, \alpha_2$	Binary/segment selection variables for the piecewise direct labor cost and capacity structure in Equations (11)–(16)
η_1, η_2	Auxiliary binary variables linking adjacent direct labor segments in Equations (12)–(16)
t_{oi}	Fixed/setup electroplating time coefficient associated with product i
t_{ci}	Variable/cycle electroplating time coefficient associated with product i
β_i	Electroplating batch/activity count variable for product i in Equation (17); the product subscript distinguishes this use from the separate forging tier selectors in Equations (24)–(30)
$K_o (k_o)$	Electroplating activity cost rate per unit of resource time; the lower-case form of the same symbol is used in Table 2
O_i	Electroplating activity quantity for product i used in the objective function cost term
σ_o	Available electroplating resource capacity
s_i	Lot/shipment quantity coefficient linking product output to the maintenance activity count in Equation (18)
δ_i	Maintenance/lot activity count variable for product i

$t_{\delta i}$	Fixed maintenance time required per maintenance/lot event for product i
t_{pi}	Variable maintenance time coefficient per unit of product i
k_{δ}	Maintenance activity cost rate per unit of resource time
σ_{δ}	Available maintenance resource capacity
τ_i	Binary product activation variable for the pre-coating activity in Equations (20) and (21)
$t_{\tau i}$	Pre-coating time required when product i is activated
R_i	Upper bound/maximum feasible output of product i used in the pre-coating linking constraint
k_{τ}	Pre-coating activity cost rate per unit of resource time
σ_{τ}	Available pre-coating resource capacity
MA_k	Machine capacity expression (or net capacity load) associated with segment k in Equation (22)
MC_k	Machine/capacity cost associated with segment k in the objective function
θ_k	Binary/selection variable for machine capacity segment k ; Equation (23) enforces selection of one applicable segment
WAQ	Total forging/waste disposal quantity to which the piecewise cost structure is applied
WQ_j	Forging/waste quantity breakpoint for tier j
WA_j	Cost associated with forging/waste disposal tier j
β_j	Forging/waste tier selection variable in Equations (24)–(30)
μ_j	Auxiliary binary variable linking adjacent forging/waste tiers in Equations (25)–(30)
G_j	Carbon emission breakpoint/threshold used in the stepped carbon-related cost representation in Equation (31)
ω_j	Carbon cost segment selection variable used to activate the applicable emission interval in Equation (31)
φ_j	Auxiliary binary variable linking adjacent carbon cost segments in Equation (31)
T_1, T_2, T_3	Carbon-related unit cost rates assigned to the stepped emission intervals in the numerical application of Equation (31)
W_1	Base/threshold quantity term appearing in the carbon-related cost component of Equation (1)
γ_m	Carbon-related marginal cost coefficient(s) appearing in the piecewise carbon-cost component of Equation (1)
$X_m (m = 1, \dots, 4)$	Product-specific quantity terms entering the carbon-related cost component of Equation (1); these are numerically mapped to the four product outputs in Table 3

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