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Analysis of Electromechanical Swings of a Turbogenerator Based on a Fractional-Order Circuit Model

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Abstract

This paper addresses the issue of rotor swings in a high-power synchronous generator during stable operation with a stiff power grid. The analysis of electromechanical swings was conducted using a circuit model incorporating fractional-order derivatives. Assuming that variations in the load angle under small disturbances from a stable equilibrium are minor, a linearized differential equation describing the electrodynamic state of the synchronous machine was derived. Based on this linearized equation of motion and the identified parameters of the equivalent circuit, calculations were performed for a 200 MW turbogenerator. The results indicate that the electromechanical swings are characterized by a constant pulsation and a low damping factor. Calculations were also carried out using a lumped-parameter equivalent circuit model. Based on the obtained results, it can be stated that the fractional-order model provides a more accurate fit of the frequency characteristics compared with the classical model with the same number of rotor equivalent circuits. The relative approximation errors for the fractional-order model are, for the d-axis (one rotor equivalent circuit), relative magnitude error $\delta_m = 1.53\%$ and relative phase error $\delta_\varphi = 6.32\%$, and for the q-axis (two rotor equivalent circuits), $\delta_m = 3.2\%$ and $\delta_\varphi = 8.3\%$. To achieve comparable approximation accuracy for the classical model, the rotor electrical circuit must be replaced with two equivalent circuits in the d-axis and four equivalent circuits in the q-axis, yielding relative errors of $\delta_m = 2.85\%$ and $\delta_\varphi = 6.51\%$ for the d-axis, and $\delta_m = 1.86\%$ and $\delta_\varphi = 5.49\%$ for the q-axis.



Academic Editor: Branislav Hredzak

Received: 17 August 2025

Revised: 19 September 2025

Accepted: 21 September 2025

Published: 28 September 2025

Citation: Staszak, J. Analysis of Electromechanical Swings of a Turbogenerator Based on a Fractional-Order Circuit Model. *Energies* **2025**, *18*, 5170. <https://doi.org/10.3390/en18195170>

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Keywords: synchronous machine; electromechanical swings; equivalent circuit parameter identification; fractional-order derivatives

1. Introduction

During the operation of a synchronous generator connected to a stiff power grid, disturbances may arise that cause oscillatory changes in rotor speed, referred to as electromechanical swings. These speed variations also induce electromechanical swings in the generator's power angle and electromagnetic torque. A distinctive feature of these swings is their low frequency (typically in the range of 1–2 Hz) and low damping ratio. Such electromechanical swings limit the capacity for electric power transmission within the power system and, under adverse conditions, may lead to system instability [1]. Changes in the operating state of a synchronous generator may result from variations in the excitation current, voltage, and line reactance, or from sudden changes in the driving torque.

This study investigates the electromechanical swings of a turbogenerator subjected to a step change in driving torque while maintaining a constant excitation voltage. The

assumption of a sudden increase in turbine torque provides a straightforward means of illustrating transient phenomena in a synchronous generator.

The transient states of synchronous machines are described by nonlinear differential equations. To solve these equations under specified initial conditions, numerical methods are typically employed [2,3]. However, these methods have a limitation: when control and disturbance signals are known, they yield only numerical results at selected time steps. Consequently, they do not provide analytical solutions suitable for assessing the stability of a synchronous machine. By linearizing the nonlinear differential equations around a chosen operating point, a system of linear differential equations with constant coefficients is obtained. Applying operational calculus to these equations transforms them into algebraic equations, enabling analytical solutions. In this study, the linearization of the differential equations enabled the determination of the damping ratio and the frequency of electromechanical swings of a synchronous generator under small disturbances from equilibrium. The analysis of electromechanical swings is typically based on classical circuit models [4], which assume a single damper circuit in the rotor along the d-axis and another along the q-axis. Due to their low computational complexity, classical models are commonly employed in real-time simulations [5,6]. However, classical models often result in significant discrepancies between measured responses and those obtained through computer simulations. This necessitates the use of higher-order mathematical models [7–9] incorporating additional lumped-parameter RL circuits into the equivalent model of the synchronous machine. Such models more accurately reflect the skin effect occurring in the solid rotor and in conductive slot wedges. An alternative approach involves employing equivalent circuits with fractional-order operator impedances [10,11]. In the frequency domain, fractional-order operator inductance represents an impedance whose resistance and inductance vary with the frequency of eddy currents induced in the machine's solid rotor. To model eddy currents in conductive components more precisely, hybrid elements should be used [12–14]. These consist of a series connection of lumped-parameter RL elements and fractional-order operator inductance. In [15], a simulation of a three-phase short-circuit in a synchronous generator was performed, whereas in [16], the dynamic states of an induction motor with a solid rotor were analyzed. Both studies employed fractional-order differential calculus, and the nonlinear differential equations were solved using the Grünwald–Letnikov method [17]. Although fractional-order calculus is still rarely applied in the modeling of synchronous machines, its use in power system control research has been steadily increasing [18,19].

In this study, the electric circuit of a solid rotor along the d-axis is represented in the equivalent model by a single hybrid element. In the q-axis, the rotor damper circuit is modeled as a parallel connection of two branches: one containing a hybrid element and the other comprising lumped-parameter RL elements.

The parameters of the synchronous machine's equivalent circuit can be identified either through measurements [7,20] or based on design data [21]. Among the various methods for parameter identification, the SSFR (standstill frequency response) method [7] is particularly noteworthy. This method involves determining the frequency characteristics of spectral inductances, either from measurements or design data, for a stationary machine. These characteristics are then approximated to determine the parameters of the machine's equivalent circuit.

The article is structured into four chapters, organized as follows. The introduction outlines the representation of rotor electrical circuits in the equivalent model of a synchronous machine. Chapter Two presents the mathematical model of the synchronous machine. In this model, the rotor damper circuit is replaced by a single fractional-order equivalent circuit along the d-axis and two equivalent circuits along the q-axis. The corresponding

differential equations describing the electrodynamic state of the synchronous generator are also provided. Chapter Three describes the procedure for determining the parameters of the synchronous machine's equivalent circuit. The equivalent circuit parameters along the d-axis and q-axis were derived by approximating spectral inductances obtained through finite element analysis of the electromagnetic field distribution. Chapter Four presents an analysis of the electromechanical swings of the turbogenerator under small disturbances from equilibrium caused by a step change in the turbine driving torque. Through linearization of the differential equations, a fractional-order operational expression was obtained for the increments in the power angle $\Delta\delta(p)$ and angular velocity $\Delta\omega$. The damping ratio and frequency of the electromechanical swings of the high-power turbogenerator were then determined.

2. Mathematical Model of a Synchronous Machine

The analysis of transient states in synchronous machines is based on equivalent circuit models. To accurately represent the phenomena occurring within the solid rotor, the equivalent rotor electric circuit should be replaced with one or two lumped-parameter RL elements in the d-axis, and three or four RL elements in the q-axis, each characterized by different time constants (Figure 1).

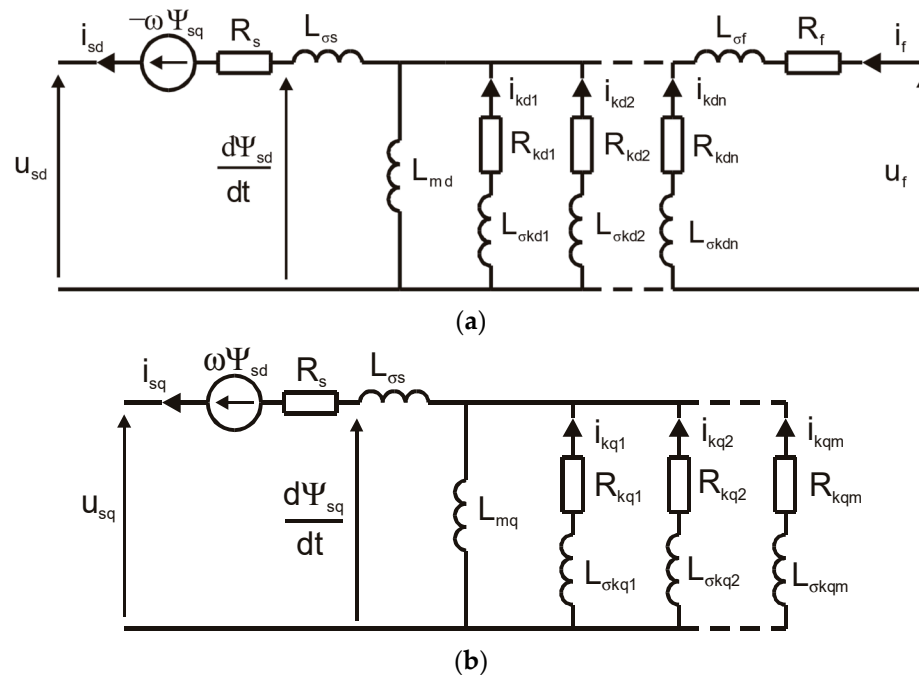


Figure 1. Equivalent circuit of a synchronous machine (a) in the d-axis and (b) in the q-axis, applying the source-oriented arrow system.

An alternative approach is to represent the damper circuit of the solid rotor using a fractional-order impedance composed of hybrid elements [16], which are series combinations of lumped integer-order RL elements and a fractional-order element (Figure 2).

In the equivalent circuits, R_s and R_f are the resistances of the armature winding and field winding, $L_{\sigma s}$ and $L_{\sigma f}$ the leakage inductances of the armature winding and field winding, L_{md} and L_{mq} the magnetizing inductances in the d- and q-axes, T_{ed} and T_{eq} the equivalent time constants of the rotor damper circuit (solid rotor, conducting wedges in the slots) in the d- and q-axes, and R_{kd} and $L_{\sigma kd}$ in the d-axis and R_{kq} and $L_{\sigma kq}$ in the q-axis are the resistance and leakage inductance with lumped parameters of the solid rotor damper circuit.

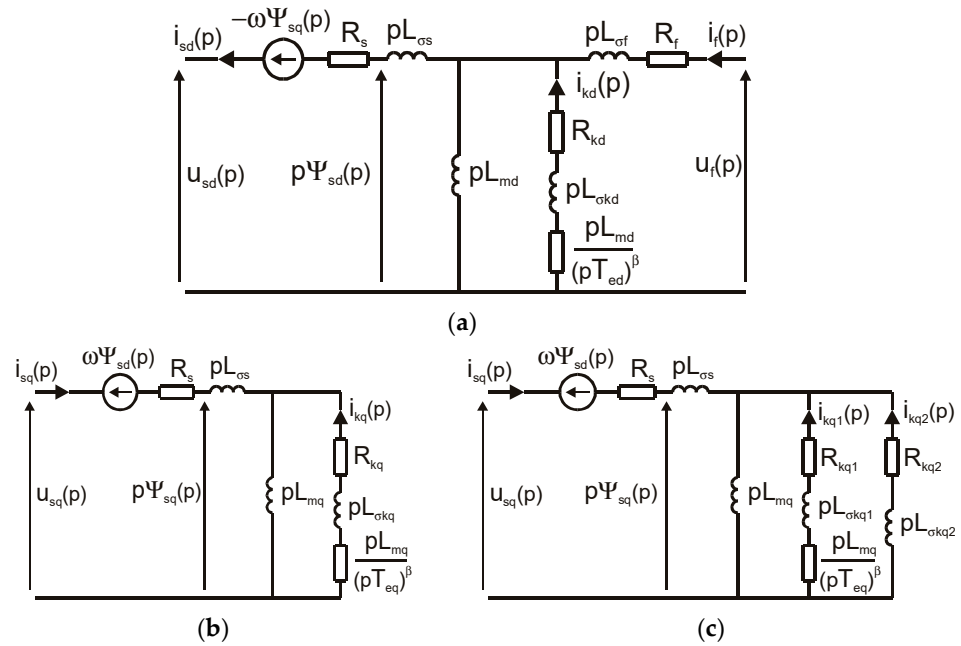


Figure 2. Equivalent circuit of a fractional-order synchronous machine: (a) in the d-axis, (b) in the quadrature q-axis with a single equivalent rotor circuit, and (c) in the q-axis with two equivalent rotor circuits, applying the source-oriented arrow system.

The analysis of electromechanical swings of a synchronous machine is conveniently carried out in Park's d-q coordinate system [22,23] in the rotor reference frame (Figure 3). In this coordinate system, the rotational voltage components present in the equivalent circuits of Figures 1 and 2 are eliminated.

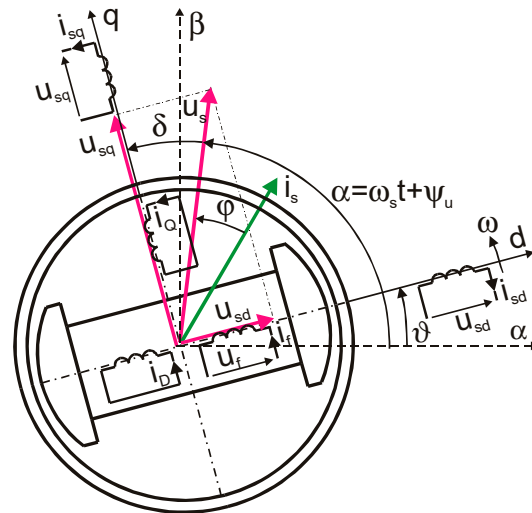


Figure 3. The d-q coordinate system of the synchronous machine in the rotor reference frame.

The electrodynamic state of a synchronous machine, where the damper circuit of the solid rotor is modeled by a fractional-order impedance, in Park's d-q coordinate system, applying the source-oriented arrow system for the stator and the load-oriented arrow system for the rotor, is described by the following equations.

The differential voltage equations for the stator in the d- and q-axes are

$$\left. \begin{aligned} u_{sd} &= -R_s i_{sd} + \frac{d\Psi_{sd}}{dt} - \omega \Psi_{sq} \\ u_{sq} &= -R_s i_{sq} + \frac{d\Psi_{sq}}{dt} + \omega \Psi_{sd} \end{aligned} \right\} \quad (1)$$

where u_{sd} and u_{sq} are the components of the armature voltage in the d- and q-axes, i_{sd} and i_{sq} are the components of the armature current in the d- and q-axes, Ψ_{sd} and Ψ_{sq} are the components of the flux linkage of the armature winding in the d- and q-axes, and ω is the electrical angular speed.

The differential voltage equation for the excitation circuit is

$$u_f = R_f i_f + \frac{d\Psi_f}{dt} \quad (2)$$

where u_f , i_f , and Ψ_f are the instantaneous values of the field voltage, field current, and flux linkage of the field winding, referred to as the armature winding side.

The differential voltage equations for the damper circuits of the solid rotor in the d- and q-axes, including fractional-order derivatives, are

$$\left. \begin{aligned} e_{md} &= \frac{d\Psi_{md}}{dt} = - \left(R_{kd} i_{kd} + L_{\sigma kd} \frac{di_{kd}}{dt} + \frac{L_m}{T_{ed}^\beta} D_\alpha(i_{kd}) \right) \\ e_{mq} &= \frac{d\Psi_{mq}}{dt} = - \left(R_{kq} i_{kq} + L_{\sigma kq} \frac{di_{kq}}{dt} + \frac{L_m}{T_{eq}^\beta} D_\alpha(i_{kq}) \right) \end{aligned} \right\} \quad (3)$$

where Ψ_{md} and Ψ_{mq} are the components of the main flux linkage in the d- and q-axes, i_{kd} and i_{kq} are the components of the damper winding current in the d- and q-axes, α is the order of the fractional derivative, and $\beta = 1 - \alpha$, where $0 < \alpha < 1$.

The fractional-order derivatives appearing in Equation (3) are defined as follows [17]:

$$D_\alpha(i_{kd}) = \frac{d^\alpha i_{kd}}{dt^\alpha} \quad D_\alpha(i_{kq}) = \frac{d^\alpha i_{kq}}{dt^\alpha}$$

The equations for the flux linkages related to the stator windings in the d- and q-axes and the excitation winding are

$$\left. \begin{aligned} \Psi_{sd} &= -L_{sd} i_{sd} + L_{md} i_f + L_{md} i_{kd} \\ \Psi_{sq} &= -L_{sq} i_{sq} + L_{mq} i_{kq} \\ \Psi_{md} &= L_{md} (-i_{sd} + i_f + i_{kd}) \\ \Psi_{mq} &= L_{mq} (-i_{sq} + i_{kq}) \\ \Psi_f &= L_{md} i_{sd} + L_{md} i_{kd} + L_f i_f \end{aligned} \right\} \quad (4)$$

The equation of motion is

$$\frac{J}{p_b} \frac{d\omega}{dt} = M_m - M_e - \frac{D}{p_b} \omega \quad (5)$$

These equations must be supplemented by the equation for the electromagnetic torque:

$$M_e = \frac{3}{2} p_b (\Psi_{sd} i_{sq} - \Psi_{sq} i_{sd}) \quad (6)$$

It is assumed that the turbogenerator operates on a symmetrical three-phase stiff grid. In this case, the voltages applied to the stator phase windings are

$$\left. \begin{aligned} u_{sa} &= U_{sm} \cos(\omega_s t + \psi_u) \\ u_{sb} &= U_{sm} \cos(\omega_s t + \psi_u - \frac{2}{3}) \\ u_{sc} &= U_{sm} \cos(\omega_s t + \psi_u - \frac{4}{3}) \end{aligned} \right\} \quad (7)$$

where u_{sa} , u_{sb} , and u_{sc} are instantaneous values of phase voltages a, b, and c, respectively; ω_s is the angular frequency of the power grid; U_{sm} is the amplitude of the phase voltage; and ψ_u is the initial phase angle of the voltage.

Applying the Park transformation [22],

$$\left. \begin{aligned} u_{sd} &= \frac{2}{3} \left(u_{sa} \cos(\vartheta) + u_{sb} \cos(\vartheta - \frac{2}{3}\pi) + u_{sc} \cos(\vartheta - \frac{4}{3}\pi) \right) \\ u_{sq} &= -\frac{2}{3} \left(u_{sa} \sin(\vartheta) + u_{sb} \sin(\vartheta - \frac{2}{3}\pi) + u_{sc} \sin(\vartheta - \frac{4}{3}\pi) \right) \end{aligned} \right\} \quad (8)$$

to the phase voltages from Equation (7), the voltage components in the stator winding d- and q-axes are obtained as follows:

$$\left. \begin{aligned} u_{sd} &= U_{sm} \sin(\omega_s t + \psi_u - \vartheta) \\ u_{sq} &= U_{sm} \cos(\omega_s t + \psi_u - \vartheta) \end{aligned} \right\} \quad (9)$$

where ϑ is the angle between the axis of phase a of the armature winding and the rotor d-axis (Figure 3).

From the vector diagram in Figure 3, it follows that

$$\left. \begin{aligned} u_{sd} &= U_{sm} \sin \delta = U_{sm} \cos(\frac{\pi}{2} - \delta) \\ u_{sq} &= U_{sm} \cos \delta = U_{sm} \sin(\frac{\pi}{2} - \delta) \end{aligned} \right\} \quad (10)$$

where δ is the load angle.

By comparing Equations (9) and (10), we obtain

$$\omega_s t + \psi_u - \vartheta = \frac{\pi}{2} - \delta$$

Thus,

$$\vartheta = \omega_s t + \psi_u + \delta - \frac{\pi}{2} \quad (11)$$

Differentiating Equation (11) with respect to time yields the following relationship for the electrical angular velocity of the turbogenerator rotor:

$$\omega = \frac{d\vartheta}{dt} = \omega_s + \frac{d\delta}{dt} \quad (12)$$

The time derivative of angular velocity is

$$\frac{d\omega}{dt} = \frac{d^2\vartheta}{dt^2} = \frac{d^2\delta}{dt^2} \quad (13)$$

The equation of motion in Equation (5) can, therefore, be expressed as

$$\frac{J}{p_b} \frac{d^2\delta}{dt^2} + \frac{D}{p_b} \frac{d\delta}{dt} + \frac{D}{p_b} \omega_s + M_e = M_m \quad (14)$$

where J and D are the moment of inertia and the viscous damping coefficient of the generator–turbine set, respectively; M_e and M_m are the electromagnetic torque and the turbine driving torque, respectively; and p_b is the number of pole pairs.

3. Identification of Equivalent Circuit Parameters

The equivalent circuit parameters in the d- and q-axes of the synchronous machine (Figure 2) were determined by approximating spectral inductances derived from the analysis of electromagnetic field distributions using the finite element method (FEM) [24], with the machine at a standstill. These inductances are determined from the relations in Equation (A21).

$$\left. \begin{aligned} L_{sd}(j\omega) &= -\frac{\Psi_{sd}(j\omega)}{I_{sd}(j\omega)} \Big|_{U_f=0} = L_{sd0}(j\omega) - \frac{pL_{fd}(j\omega)L_{df}(j\omega)}{R_f + pL_{f0}(p)} \\ G_{fd}(j\omega) &= \frac{\Psi_{sd}(j\omega)}{U_f(j\omega)} \Big|_{I_{sd}=0} = \frac{L_{df}(j\omega)}{R_f + pL_{f0}(j\omega)} \end{aligned} \right\} \quad (15)$$

$$L_{sq}(j\omega) = -\frac{\Psi_{sq}(j\omega)}{I_{sq}(j\omega)} \quad (16)$$

From relation Equation (15), it follows that the spectral transfer functions L_{sd} and G_{fd} can be determined based on the inductances L_{d0} , L_{f0} , and L_{df} , obtained with the armature winding supplied and the field winding open, as well as with the field winding supplied and the armature winding open (Appendix A). The calculations were carried out in the FEMM 4.2 [25] software, under sinusoidal current excitation of either the armature or field winding in the frequency range from 0 to 1000 Hz.

Based on the structure of the synchronous machine's equivalent circuit diagrams (Figure 2), the operator inductances $L_{sd}(p)$ in the d-axis and $L_{sq}(p)$ in the q-axis, as well as the transfer function $G_{fd}(p)$, can be represented as ratios of polynomials with real coefficients and fractional-order exponents.

$$\left. \begin{aligned} L_{sd}(p) &= L_{sd} \frac{p^2 b_{1d} + p^{\alpha+1} b_{2d} + p b_{3d} + p^{\alpha} b_{4d} + b_{5d}}{p^2 a_{1d} + p^{\alpha+1} a_{2d} + p a_{3d} + p^{\alpha} a_{4d} + a_{5d}} \\ G_{fd}(p) &= \frac{L_{md}}{R_f} \frac{p b_{1k} + p^{\alpha} b_{2k} + b_{3k}}{p^2 a_{1d} + p^{\alpha+1} a_{2d} + p a_{3d} + p^{\alpha} a_{4d} + a_{5d}} \end{aligned} \right\} \quad (17)$$

$$L_{sq}(p) = L_{sq} \frac{p b_{1q} + p^{\alpha} b_{2q} + b_{3q}}{p a_{1q} + p^{\alpha} a_{2q} + a_{3q}} \quad (18)$$

The numerator and denominator coefficients of the polynomials of the inductance $L_{sd}(p)$ and the transfer function $G_{fd}(p)$ are functions of the parameter vector λ_d of the sought parameters of the equivalent circuit in the d-axis. Similarly, the numerator and denominator coefficients of the polynomials of the armature inductance $L_{sq}(p)$ in the q-axis are functions of the vector λ_q of the sought parameters of the equivalent circuit in the q-axis.

$$\left. \begin{aligned} \lambda_d &= [L_{\sigma s} \ L_{kd\sigma} \ R_{kd} \ T_{ed}]^T \\ \lambda_q &= [L_{\sigma s} \ L_{kq\sigma} \ R_{kq} \ T_{eq}]^T \end{aligned} \right\} \quad (19)$$

The parameters of the equivalent circuit λ_d in the d-axis of the turbogenerator, and, consequently, of the approximating functions in Equation (17), are selected so that the magnitude of these functions approximates, with minimal error, the magnitude of the functions defined in Equations (15) and (16), which were obtained from electromagnetic field analysis. Similarly, the parameters of the equivalent circuit in the q-axis are determined. The sought values of the parameter vectors λ_d and λ_q are obtained by minimizing the sum of squared deviations.

For the d-axis,

$$\varepsilon_d(\lambda_d) = \sum_{i=1}^N \left\{ (|L_{sd}(f_i)| - |L_{sd}^*(f_i, \lambda_d)|)^2 + (|G_{fd}(f_i)| - |G_{fd}^*(f_i, \lambda_d)|)^2 \right\} \quad (20)$$

For the q-axis,

$$\varepsilon_q(\lambda_q) = \sum_{i=1}^N \left\{ (|L_{sq}(f_i)| - |L_{sq}^*(f_i, \lambda_q)|)^2 \right\} \quad (21)$$

where N is the number of measurement points, f is the frequency; L_{sd} , L_{sq} , and G_{fd} are the spectral quantities obtained from FEMM 4.2; and L_{sd}^* , L_{sq}^* , and G_{fd}^* are the spectral quantities from the approximating functions, determined from Equations (17) and (18) by substituting $p = j\omega$, where $\omega = 2\pi f$.

To solve the non-linear Equations (20) and (21), the Levenberg–Marquardt algorithm implemented in Matlab R2010a [26] was used.

Figures 4–6 present the frequency characteristics of the spectral inductances in the d- and q-axes of the TWW-200-2 turbogenerator, calculated using the FEM method and obtained based on the approximation with fractional-order rational functions

Equations (17) and (18). The magnitude on the frequency characteristics is expressed in p.u. (per unit, i.e., relative to the selected base values).

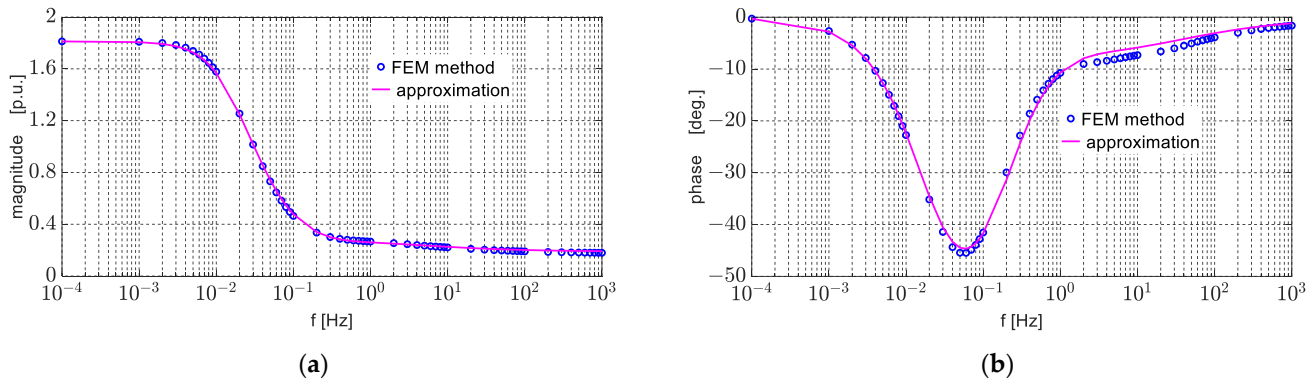


Figure 4. Frequency characteristics of the spectral inductance $L_{sd}(j\omega)$ in the d-axis: (a) magnitude; (b) phase.

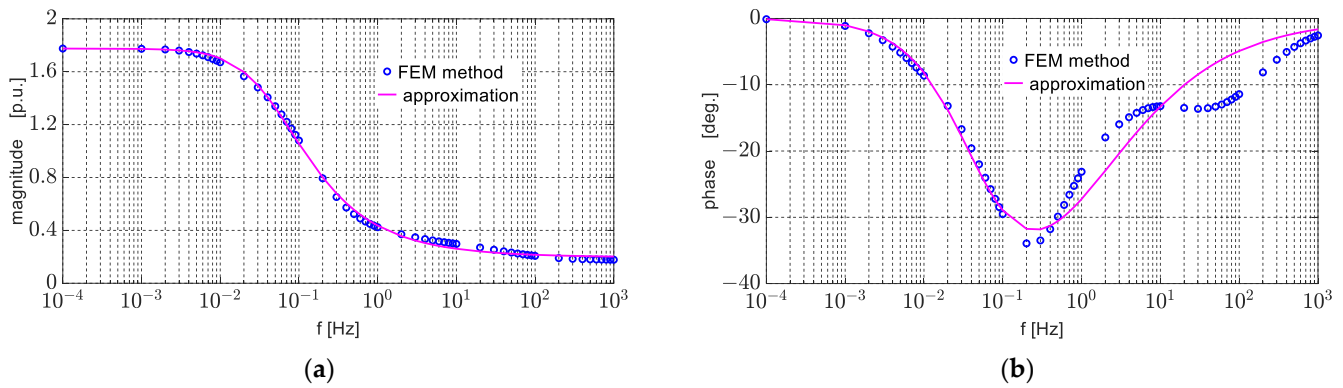


Figure 5. Frequency characteristics of the spectral inductance $L_{sq}(j\omega)$ in the q-axis: (a) magnitude; (b) phase, with one damping circuit in the q-axis of the rotor (Figure 2b).

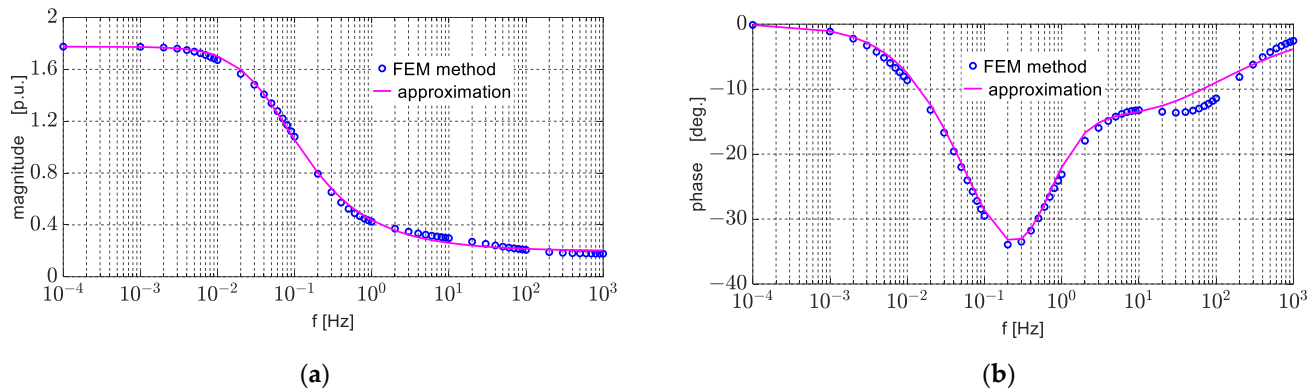


Figure 6. Frequency characteristics of the spectral inductance $L_{sq}(j\omega)$ in the q-axis: (a) magnitude; (b) phase, with two damping circuits in the q-axis of the rotor (Figure 2c).

Figures 7 and 8 show the frequency characteristics of the spectral inductances, calculated using the FEM method and obtained from the approximation for the lumped-parameter model (Figure 1). The approximating functions of the operational inductances in the d- and q-axes were assumed in the form of Equation (22)

$$\begin{aligned} L_{sd}(p) &= L_{sd} \frac{\prod_{i=1}^{n_d} (1 + pT_{di})}{\prod_{i=1}^{n_d} (1 + pT_{doi})} \\ L_{sq}(p) &= L_{sq} \frac{\prod_{i=1}^{n_q} (1 + pT_{qi})}{\prod_{i=1}^{n_q} (1 + pT_{qoi})} \end{aligned} \quad (22)$$

where L_{sd} and L_{sq} are the self-inductances of the armature winding in the d- and q-axes, respectively; T_{di} and T_{d0i} are the time constants in the d-axis; T_{qi} and T_{q0i} are the time constants in the q-axis; and n_d and n_q are the order of the operational inductance in the d- and q-axes, respectively.

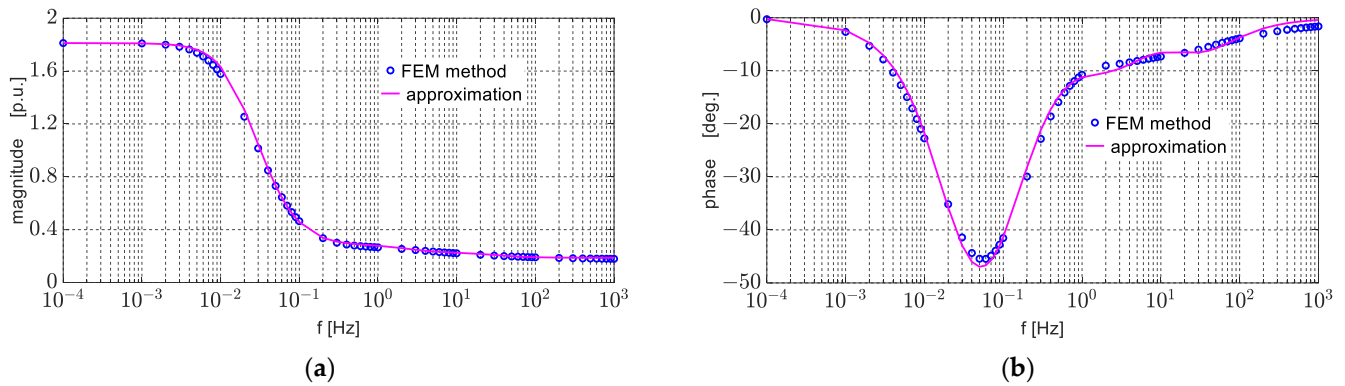


Figure 7. Frequency characteristics of the spectral inductances $L_{sd}(j\omega)$ on the d-axis for the lumped-parameter model: (a) amplitude; (b) phase.

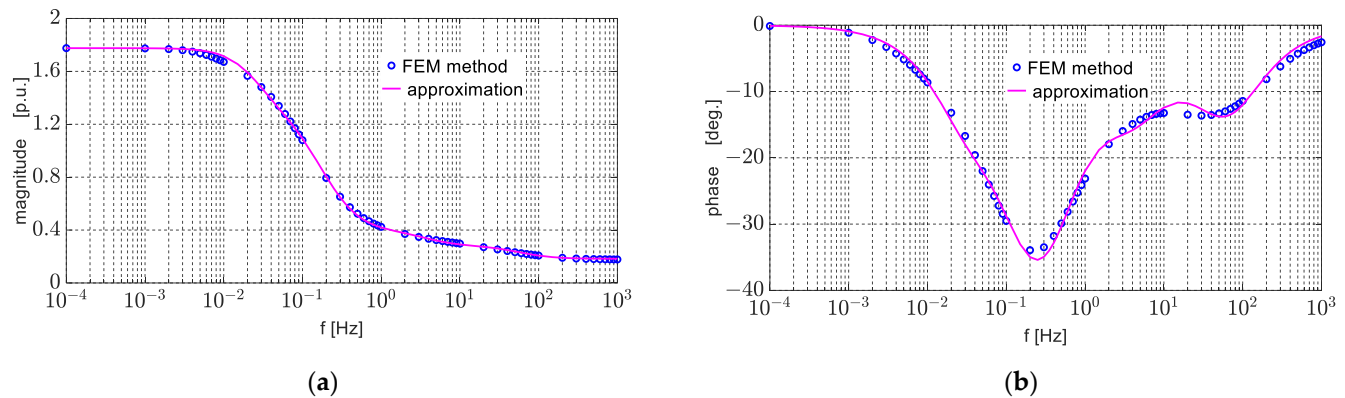


Figure 8. Frequency characteristics of the spectral inductances $L_{sq}(j\omega)$ on the q-axis for the lumped-parameter model: (a) amplitude; (b) phase.

Two equivalent circuits ($n_d = 3$) were adopted in the rotor d-axis and four equivalent circuits ($n_q = 4$) in the rotor q-axis.

To compare the amplitude and phase frequency characteristics of the spectral inductances calculated by the FEM 4.2 software with those derived from fractional-order approximations (Equations (17) and (18)), the relative error was calculated using the following definition [27]:

$$\varepsilon_y = 100 \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (Y_i - Y_i^*)^2}}{\frac{1}{n} \sum_{i=1}^n Y_i} \quad (23)$$

where Y is the actual value, and Y^* is the value obtained from approximation.

The values of the equivalent circuit parameters for the fractional-order model and the time constants for the lumped-parameter model are presented in Tables 1 and 2 (quantities marked as p.u. are expressed in per unit, i.e., relative to the selected base values). These tables also include the values of the relative errors for the magnitude (ε_m) and phase (ε_φ), as well as the coefficient of determination R^2 .

Table 1. Equivalent circuit parameters for the fractional-order model.

Parameter	d-Axis Figure 2a	Parameter	q-Axis	
			Figure 2b	Figure 2c
L_{sd} [p.u.]	1.812	L_{sq} [p.u.]	1.775	1.775
$L_{\sigma s}$ [p.u.]	0.186	$L_{\sigma s}$ [p.u.]	0.186	0.186
R_{kd} [p.u.]	0.001083	R_{kq} [p.u.]	0.001457	
$L_{\sigma kd}$ [p.u.]	0.000012	$L_{\sigma kq}$ [p.u.]	0.01029	
$L_{\sigma f}$ [p.u.]	0.1197	T_{eq} [s]	5.952	0.97
R_f [p.u.]	0.00128	R_{kq1} [p.u.]		0.001593
T_{ed} [s]	11.708	$L_{\sigma kq1}$ [p.u.]		0.000011
α	0.5	R_{kq2} [p.u.]		0.00735
ε_m [%]	1.53	$L_{\sigma kq2}$ [p.u.]		0.4722
ε_φ [%]	6.32	α	0.5	0.5
R_m^2	0.9998	ε_m [%]	2.98	3.2
R_φ^2	0.9958	ε_φ [%]	21.69	8.31
		R_m^2	0.9987	0.9985
		R_φ^2	0.8901	0.9839

Table 2. Time constants for the lumped-parameter generator model.

Parameter	d-Axis Figure 1a	Parameter	q-Axis Figure 1b
L_{sd} [p.u.]	1.812	L_{sq} [p.u.]	1.775
$L_{\sigma s}$ [p.u.]	0.186	$L_{\sigma s}$ [p.u.]	0.186
T_{d1} [p.u.]	386.560	T_{q1} [p.u.]	1362.9
T_{d2} [p.u.]	18.047	T_{q2} [p.u.]	109.288
T_{d3} [p.u.]	1.301	T_{q3} [p.u.]	10.3458
T_{d01} [p.u.]	2450.7	T_{q4} [p.u.]	0.6357
T_{d02} [p.u.]	22.872	T_{q01} [p.u.]	1912.3
T_{d03} [p.u.]	1.565	T_{q02} [p.u.]	352.0454
ε_m [%]	2.85	T_{q03} [p.u.]	14.5075
ε_φ [%]	6.51	T_{q04} [p.u.]	0.9744
R_m^2	0.9992	ε_m [%]	1.87
R_φ^2	0.9955	ε_φ [%]	5.49
		R_m^2	0.9995
		R_φ^2	0.9930

The resulting relative errors for both magnitude and phase indicate good accuracy in approximating the amplitude and phase characteristics of the spectral inductance in the d-axis. However, significant discrepancies were observed in the phase characteristics in the q-axis when the damper circuit was modeled using a single equivalent circuit (Figure 2b). To achieve a high-accuracy approximation of these characteristics, it is necessary to use two equivalent circuits in the q-axis (Figure 2c).

4. Analysis of Electromechanical Swings of a Turbogenerator Under Small Disturbances Around the Equilibrium Point

4.1. Linearization of Differential Equations

The analysis of electromechanical swings addresses small disturbances from the stable equilibrium of a synchronous generator connected to an infinite bus, caused by variations in the turbine driving torque from the steady-state condition. A variation in the driving torque ΔM_m is accompanied by variations in the voltage components Δu_{sd} and Δu_{sq} , the current components Δi_{sd} and Δi_{sq} , and the flux linkages $\Delta \Psi_{sd}$ and $\Delta \Psi_{sq}$ in the d- and q-axes, as well as variations in the electromagnetic torque ΔM_e , the angular speed $\Delta \omega$, and the load angle $\Delta \delta$. To obtain a system of linear differential equations, the nonlinear differential Equations (1) and (5) were linearized [28]. The assumption is made that, during stable operation of the generator under small disturbances, such as those that occur during normal operation of the power system [8], the electromechanical quantities are equal to the sum of the pre-disturbance values (denoted by subscript 0) and small deviations from the equilibrium point (denoted by the delta symbol). The following system of equations is then obtained:

$$\left. \begin{aligned} u_{sd} &= U_{sd0} + \Delta u_{sd} & i_{sd} &= I_{sd0} + \Delta i_{sd} \\ u_{sq} &= U_{sq0} + \Delta u_{sq} & i_{sq} &= I_{sq0} + \Delta i_{sq} \\ \Psi_{sd} &= \Psi_{sd0} + \Delta \Psi_{sd} & M_e &= M_{e0} + \Delta M_e \\ \Psi_{sq} &= \Psi_{sq0} + \Delta \Psi_{sq} & M_m &= M_{m0} + \Delta M_m \\ \omega &= \omega_s + \Delta \omega & \delta &= \delta_0 + \Delta \delta \end{aligned} \right\} \quad (24)$$

By substituting the relations in Equation (24) into Equations (1), (6), (12) and (14), and subtracting the equations for the steady states, while neglecting the armature resistance and higher-order terms, the equations in the operator form are obtained:

$$\left. \begin{aligned} \Delta u_{sd}(p) &= p \Delta \Psi_{sd}(p) - \omega_s \Delta \Psi_{sq}(p) - \Delta \omega(p) \Psi_{sq0} \\ \Delta u_{sq} &= p \Delta \Psi_{sq}(p) + \omega_s \Delta \Psi_{sd}(p) + \Delta \omega(p) \Psi_{sd0} \end{aligned} \right\} \quad (25)$$

$$\Delta \omega(p) = p \Delta \delta(p) \quad (26)$$

$$\Delta M_e(p) = \frac{3}{2} \frac{p_b}{\omega_s} [\omega_s \Psi_{sd0} \Delta i_{sq}(p) - \omega_s \Psi_{sq0} \Delta i_{sd}(p) + I_{sq0} \omega_s \Delta \Psi_{sd}(p) - I_{sd0} \omega_s \Delta \Psi_{sq}(p)] \quad (27)$$

$$\frac{J}{p_b} p^2 \Delta \delta(p) + \frac{D}{p_b} p \Delta \delta(p) + \Delta M_e(p) = \Delta M_m(p) \quad (28)$$

Taking into account the expression for the electromagnetic moment $\Delta M_e(p)$ (A31) in Equation (28), we obtain the relationship for the load angle $\Delta \delta(p)$:

$$\Delta \delta(p) = \frac{\Delta M_m(p)}{\frac{J \omega_s^2}{p_b} \left(\frac{p}{\omega_s} \right)^2 + \frac{D \omega_s}{p_b} \frac{p}{\omega_s} + \frac{p_b}{\omega_s} \left\{ Q_{e0} + \frac{3}{2} U_{sm}^2 \left(\frac{\cos^2 \delta_0}{X_{sq}(p)} + \frac{\sin^2 \delta_0}{X_{sd}(p)} \right) - \frac{p}{\omega_s} \left[P_{e0} + \frac{3}{4} U_{sm}^2 \left(\frac{1}{X_{sq}(p)} - \frac{1}{X_{sd}(p)} \right) \sin 2\delta_0 \right] \right\}} \quad (29)$$

Equation (29) allows for determining the variation in the load angle as the driving torque changes. Using polynomial calculus in Matlab, Equation (29) can be expressed in the following form:

$$\Delta \delta(p) = \Delta M_m(p) W(p) = \Delta M_m(p) \frac{N(p)}{M(p)} \quad (30)$$

Both the numerator $N(p)$ and the denominator $M(p)$ of the algebraic fraction $W(p)$ in Formula (30) are polynomials of the fractional order. This poses challenges for decomposing

the function into partial fractions and, consequently, for converting the operator form into the time domain. The order of the fractional derivative of α in the expressions for the operator inductance Equations (17) and (18) is a decimal fraction in the range $0 < \alpha < 1$. The exponent of α can be represented as a proper irreducible fraction:

$$\alpha = \frac{n}{m} \quad \alpha + 1 = \frac{n}{m} + 1 = \frac{m+n}{m} \quad (31)$$

where n and m are integers, and $n < m$.

In such a case, the exponents of the Laplace operator p of the inductances $L_{sd}(p)$ and $L_{sq}(p)$ in Equations (17) and (18) will be

$$\begin{aligned} p^2 &= \left(p^{\frac{1}{m}}\right)^{2m} \\ p^{\alpha+1} &= \left(p^{\frac{1}{m}}\right)^{m+n} \\ p^1 &= \left(p^{\frac{1}{m}}\right)^m \\ p^\alpha &= \left(p^{\frac{1}{m}}\right)^n \end{aligned} \quad (32)$$

Using the substitution

$$q = p^{\frac{1}{m}} \quad (33)$$

the operator inductances $L_{sd}(p)$ and $L_{sq}(p)$ in Equations (17) and (18) are transformed into

$$\left. \begin{aligned} L_{sd}(p) &= L_{sd} \frac{q^{2m} b_{1d} + q^{m+n} b_{2d} + q^m b_{3d} + q^n b_{4d} + b_{5d}}{q^{2m} a_{1d} + q^{m+n} a_{2d} + q^m a_{3d} + q^n a_{4d} + a_{5d}} \\ L_{sq}(p) &= L_{sq} \frac{q^m b_{1q} + q^n b_{2q} + b_{3q}}{q^m a_{1q} + q^n a_{2q} + a_{3q}} \end{aligned} \right\} \quad (34)$$

The order of the fractional derivative should be selected so that the polynomial degree of both the numerator and denominator of $L_{sd}(q)$ is as low as possible. When $\alpha = 1/2$, the resulting polynomial order is 4. When $\alpha = 2/5$ or $3/5$, the polynomial order is 10. Therefore, from the perspective of numerical computation, the most favorable order of the fractional derivative is $\alpha = 0.5$. Furthermore, for fractional-order derivatives with $\alpha = 0.5$, analytical expressions can be derived for the damping factor and the swing frequency.

4.2. Calculation of Transients

To simplify the calculations, it is convenient to use a relative units system. The following reference units are adopted:

$$\begin{aligned} I_b &= I_{smN} & U_b &= U_{smN} & S_b &= \frac{3}{2} U_{smN} I_{smN} & M_b &= \frac{p_b}{\omega_s} S_b & \omega_b &= \omega_s \\ Z_b &= \frac{U_{smN}}{I_{smN}} & L_b &= \frac{Z_b}{\omega_s} & \Psi_b &= \frac{U_b}{\omega_s} & t_b &= \frac{1}{\omega_s} \end{aligned}$$

where U_{smN} and I_{smN} are the rated maximum phase values of the armature voltage and current, ω_s is the synchronous speed, and p_b is the number of pole pairs.

Introducing the relevant notations,

$$\left. \begin{aligned} \tau_m &= \frac{J\omega_s^2}{p_b M_b} & \tau_D &= \frac{D\omega_s}{p_b M_b} & p_{e0} &= \frac{P_{e0}}{S_b} & q_{e0} &= \frac{Q_{e0}}{S_b} \\ b &= u_{sm}^2 \sin^2 \delta_0 & c &= u_{sm}^2 \cos^2 \delta_0 & d &= \frac{u_{sm}^2}{2} \sin 2\delta_0 \end{aligned} \right\} \quad (35)$$

and using a relative units system, Equation (29) can be rewritten in the following form:

$$\Delta\delta(p) = \frac{\Delta m_m(p)}{\left(\frac{p}{\omega_s}\right)^2 \tau_m + \frac{p}{\omega_s} \tau_D + q_{e0} + \frac{c}{x_{sq}(p)} + \frac{b}{x_{sd}(p)} - \frac{p}{\omega_s} \left(p_{e0} + \frac{d}{x_{sq}(p)} - \frac{d}{x_{sd}(p)}\right)} \quad (36)$$

where τ_m and τ_D are the starting time constant and the mechanical time constant of the turbogenerator set, respectively; $x_{sd}(p)$ and $x_{sq}(p)$ are the d- and q-axis operator reactance in relative units; and Δm_m is the mechanical torque in relative units.

Considering relationships Equations (32)–(34), Equation (36) can be represented as a rational algebraic function of the variable q , with the numerator and denominator being polynomials with real coefficients of the integer order:

$$\Delta\delta(p) = \Delta m_m(p) \frac{d_1 q^{r-4} + d_2 q^{r-5} + \dots + d_{r-4} q + d_{r-3}}{c_1 q^r + c_2 q^{r-1} + \dots + c_r q + c_{r+1}} = \Delta m_m(p) \frac{N(q)}{M(q)} \quad (37)$$

For an equivalent circuit comprising two rotor circuits in the d-axis and one rotor circuit in the q-axis, the polynomial degree is given by $r = 3m + 4$, where m is the denominator of coefficient α in the relationship in Equation (31). Thus, for $\alpha = 1/2$, $r = 10$, and for $\alpha = 2/5$, $r = 19$. For two rotor equivalent circuits in the d- and q-axes, the order of the polynomial is $r = 4m + 4$, and for $\alpha = 1/2$, $r = 12$, and for $\alpha = 2/5$, $r = 24$.

The transition from operator form to time form requires determining the roots of the denominator polynomial in Equation (37). Since the coefficients of the denominator polynomial $M(p)$ of function Equation (37) are real, it has r roots, which may be real or occur as complex conjugate pairs. Some roots may have positive real parts. For $\alpha = 0.5$ and two equivalent rotor circuits in the d- and q-axes (order of polynomial $r = 12$), the roots are presented in Table 3.

Table 3. Roots of the denominator polynomial of the load angle transfer function.

i	Fractional-Order Model	Lumped-Parameter Model
	Figure 2c	Figure 1
	q_i	q_i
1	$-1.1904 \cdot 10^0$	$-1.5723 \cdot 10^0$
2	$-5.3160 \cdot 10^{-1}$	$-7.6868 \cdot 10^{-1}$
3	$-1.2412 \cdot 10^{-1}$	$-9.3459 \cdot 10^{-2}$
4	$-6.5951 \cdot 10^{-4}$	$-5.3735 \cdot 10^{-2}$
5	$-9.9400 \cdot 10^{-2} + j1.2725 \cdot 10^{-1}$	$-6.4067 \cdot 10^{-3}$
6	$-9.9400 \cdot 10^{-2} - j1.2725 \cdot 10^{-1}$	$-1.4487 \cdot 10^{-3}$
7	$-1.9889 \cdot 10^{-2} + j4.550 \cdot 10^{-2}$	$-6.2823 \cdot 10^{-4}$
8	$-1.9889 \cdot 10^{-2} - j4.550 \cdot 10^{-2}$	$-4.6353 \cdot 10^{-3} + j4.0803 \cdot 10^{-2}$
9	$-8.2872 \cdot 10^{-3} + j9.3595 \cdot 10^{-2}$	$-4.6353 \cdot 10^{-3} - j4.0803 \cdot 10^{-2}$
10	$-8.2872 \cdot 10^{-3} - j9.3595 \cdot 10^{-2}$	
11	$1.3273 \cdot 10^{-1} + 1.4535 \cdot 10^{-1}$	
12	$1.3273 \cdot 10^{-1} + 1.4535 \cdot 10^{-1}$	

Thus, there are three negative real roots, six complex conjugate roots with negative real parts, and two complex conjugate roots with positive real parts, where the imaginary component exceeds the real component.

By decomposing Equation (37) into partial fractions, the operator equation for the increment in the power angle $\Delta\delta(p)$ can be written as

$$\Delta\delta(p) = \Delta m_m(p) \sum_{i=1}^r \frac{Q_i}{p^{\frac{1}{m}} - q_i} = \Delta m_m(p) \sum_{i=1}^r \frac{Q_i}{p^\lambda + b_i} \quad (38)$$

where q_i represents the roots of the denominator polynomial in expression (37), $b_i = -q_i$, Q_i represents the coefficients from the partial fraction decomposition, and $\lambda = 1/m$, where for $\alpha = 1/2$ $m = 2$, $\lambda = 0.5$.

For a step change in the mechanical torque, $\Delta m_m(t) = \Delta m_m 1(t)$, and $\Delta m_m(p) = \Delta m_m/p$, and the variation in the load angle is determined by

$$\Delta\delta(p) = \frac{\Delta m_m}{p} \sum_{i=1}^r \frac{Q_i}{p^\lambda + b_i} = \Delta m_m \sum_{i=1}^r \frac{Q_i}{p(p^\lambda + b_i)} \quad (39)$$

From the relationship in Equation (26), it follows that the increment in the rotor angular velocity $\Delta\omega(p)$, with a step change in the mechanical torque, is given by

$$\Delta\omega(p) = p\Delta\delta(p) = \Delta m_m \sum_{i=1}^r \frac{Q_i}{p^\lambda + b_i} \quad (40)$$

Applying the inverse Laplace transform (A33) yields the time form of the functions in Equations (39) and (40):

$$\left. \begin{aligned} \Delta\delta(t) &= \Delta m_m \sum_{i=1}^r \frac{Q_i}{b_i} [1 - E_\lambda(-b_i t^\lambda)] \\ \Delta\omega(t) &= \Delta m_m \sum_{i=1}^r Q_i t^{\lambda-1} E_{\lambda,\lambda}(-b_i t^\lambda) \end{aligned} \right\} \quad (41)$$

where $E_{\alpha,\beta}(z)$ is the two-parameter Mittag-Leffler function in Equation (A34).

Further analysis of the electromechanical swings of the synchronous generator is carried out for fractional-order derivatives, where $\alpha = 0.5$. In this case, the order of the denominator polynomial in Equation (37) is minimal, which improves the numerical stability of the partial fraction decomposition.

From the relationships in Equations (A35) and (A36), the load angle $\Delta\delta$ can be expressed as the sum of three components: the steady-state component $\Delta\delta_s$, the aperiodic component $\Delta\delta_a$, and the periodic component $\Delta\delta_p$.

$$\Delta\delta(t) = \Delta\delta_s(t) + \Delta\delta_a(t) + \Delta\delta_p(t) \quad (42)$$

The corresponding equations describe each component:

$$\left. \begin{aligned} \Delta\delta_s(t) &= \Delta m_m \left(\sum_{i=1}^{r-2} \frac{Q_i}{b_i} - \sum_{i=r-1}^r \frac{Q_i}{b_i} \right) \\ \Delta\delta_a(t) &= -\Delta m_m \sum_{i=1}^r \frac{Q_i}{b_i} e^{b_i^2 t} \operatorname{erfc}(b_i \sqrt{t}) \\ \Delta\delta_p(t) &= 2\Delta m_m \sum_{i=r-1}^r \frac{Q_i}{b_i} e^{b_i^2 t} \end{aligned} \right\} \quad (43)$$

When the fractional-order derivative in Equation (40) is of the order $\lambda = 0.5$, the angular velocity response $\Delta\omega$ is given by

$$\Delta\omega(t) = \Delta m_m \frac{1}{\sqrt{\pi t}} \sum_{i=1}^r Q_i - \Delta m_m \sum_{i=1}^r Q_i b_i e^{b_i^2 t} \operatorname{erfc}(b_i \sqrt{t}) + 2\Delta m_m \sum_{i=1}^r Q_i b_i e^{b_i^2 t} \quad (44)$$

From the partial fraction decomposition [2], if the degree of the numerator polynomial $N(q)$ in Equation (37) is at least two orders lower than that of the denominator $M(q)$, the sum $\sum Q_i$ in Equation (44) equals zero. Thus, Equation (44) becomes

$$\Delta\omega(t) = -\Delta m_m \sum_{i=1}^r Q_i b_i e^{b_i^2 t} \operatorname{erfc}(b_i \sqrt{t}) + 2\Delta m_m \sum_{i=r-1}^r Q_i b_i e^{b_i^2 t} \quad (45)$$

The first term of Equation (45) represents the sum of the aperiodic components of the angular velocity:

$$\Delta\omega_a = -\Delta m_m \sum_{i=1}^r Q_i b_i e^{b_i^2 t} \operatorname{erfc}(b_i \sqrt{t}) \quad (46)$$

Meanwhile, the second term of Equation (45) represents the periodic component of the angular velocity:

$$\Delta\omega_p(t) = 2\Delta m_m \sum_{i=r-1}^r Q_i b_i e^{b_i^2 t} \quad (47)$$

If $b_r = x + jy$, then $\exp(b_r^2 t)$ in Formulas (43) and (47) can be written as

$$e^{b_r^2 t} = e^{(x^2 - y^2)t} e^{j2xyt} = e^{\alpha_h t} (\cos\omega_h t + j\sin\omega_h t) \quad (48)$$

where α_h is the damping factor of the electromechanical swings, and ω_h is the angular frequency of the electromechanical swings, defined as

$$\left. \begin{aligned} \alpha_h &= x^2 - y^2 \\ \omega_h &= 2xy \end{aligned} \right\} \quad (49)$$

Given the relationship in Equation (48) and considering that $b_{r-1} = b_r^*$ and $Q_{r-1} = Q_r^*$, the periodic components of the load angle $\Delta\delta_p$ and angular velocity $\Delta\omega_p$ can be expressed as

$$\left. \begin{aligned} \Delta\delta_p(t) &= 4\Delta m_m \operatorname{Re} \left[\frac{Q_r}{b_r} e^{\alpha_h t} (\cos\omega_h t + j\sin\omega_h t) \right] \\ \Delta\omega_p(t) &= 4\Delta m_m \operatorname{Re} [Q_r b_r e^{\alpha_h t} (\cos\omega_h t + j\sin\omega_h t)] \end{aligned} \right\} \quad (50)$$

From Equations (43) and (45), it follows that during a transient following a step change in the driving torque, the steady-state component of the power angle $\Delta\delta_s$ and the aperiodic components of the power angle $\Delta\delta_a$ and angular velocity $\Delta\omega_a$ are determined by all the roots of the denominator polynomial in Expression (37). In contrast, the periodic components of the power angle $\Delta\delta_p$ and angular velocity $\Delta\omega_p$ are defined by two complex conjugate roots with positive real parts. From the relationship in Equation (49), the electromechanical swing frequency is equal to twice the product of the real and imaginary parts of the complex root with a positive real part, and the damping coefficient equals the difference in the squares of its real and imaginary parts. The periodic components of the responses of the power angle and the angular velocity will decay if the damping coefficient in Equation (49) is negative, which occurs when the real part of the root with a positive real part is smaller than its imaginary part.

The calculations of the power angle waveform were also performed for the lumped-parameter circuit model using the data presented in Table 2. Taking into account the relation in Equation (22), the operational expression of the power angle in Equation (36) for the lumped-parameter model, under a step change in the turbine torque, takes the following form:

$$\Delta\delta(p) = \frac{\Delta m_m d_1 p^{r-2} + c_2 p^{r-3} + \cdots + c_{r-2} p + c_{r-1}}{p(c_1 p^r + c_2 p^{r-1} + \cdots + c_r p + c_{r+1})} = \Delta m_m \sum_{i=1}^r \frac{Q_i}{p(p - q_i)} \quad (51)$$

where r is the order of the denominator polynomial, and $r = n_d + n_q + 2$.

The roots of the polynomial of Equation (51) are presented in Table 3. The time-domain form of Equation (51) is defined by the relation

$$\Delta\delta(t) = -\Delta m \sum_{i=1}^r \frac{Q_i}{q_i} + \Delta m \sum_{i=1}^{r-2} \frac{Q_i}{q_i} e^{q_i t} + \Delta m \sum_{i=r-1}^r \frac{Q_i}{q_i} e^{q_i t} \quad (52)$$

The first term of Equation (52) represents the steady-state component. The second term represents the sum of the aperiodic components associated with the real roots. The third term of Equation (52) defines the periodic component of the power angle and is associated with a pair of complex conjugate roots $q_r = q_{r-1}^*$. The real part of these roots determines the damping factor, while the imaginary part determines the frequency of the electromechanical oscillations. The values of the damping factor and the oscillation frequency are presented in Table 4.

Table 4. Values of the damping factor and the pulsation of electromechanical swings.

Parameter	Fractional-Order Model		Lumped-Parameter Model (Figure 1)
	One Circuit in the q-Axis (Figure 2b)	Two Circuits in the q-Axis (Figure 2c)	
α_h [p.u.]	−0.00565	−0.0035094	−0.0046353
ω_h [p.u.]	0.04168	0.038585	0.040803
T_h [s]	0.563	0.907	0.687
f_h [Hz]	2.08	1.93	2.04

The waveforms of the load angle and angular velocity, as well as the damping coefficient and electromechanical swing frequency, were calculated for the TWW_200 turbogenerator with the following rated parameters: $P_N = 200$ MW, $U_N = 15.75$ kV, $I_N = 8625$ A, and $\cos\varphi_N = 0.85$.

It was assumed that the time constant of the turbine generator unit $T_m = 6.8$ s, the induced voltage in the armature winding $e_{fm} = 2.46$ p.u., the load angle $\delta_0 = 34^\circ$, and the step change in the driving torque $\Delta m_m = 0.2$.

For the fractional-order model, Figures 9 and 10 show the load angle and angular velocity waveforms, while Figure 11 presents the angular velocity–load angle trajectory.

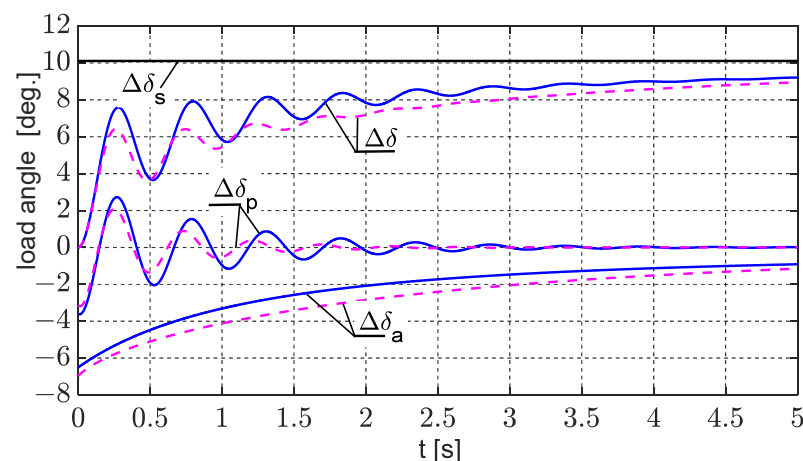


Figure 9. Waveform of the load angle $\Delta\delta$: $\Delta\delta_s$ —steady-state component, $\Delta\delta_a$ —aperiodic component, $\Delta\delta_p$ —periodic component, full line—two equivalent circuits, and dashed line—one equivalent circuit in the rotor q-axis.

From the waveforms presented in Figure 9, it follows that the number of equivalent circuits (Figure 2) used in the rotor affects the load angle waveform, as well as the values of the damping factor and the frequency of electromechanical swings (Table 4).

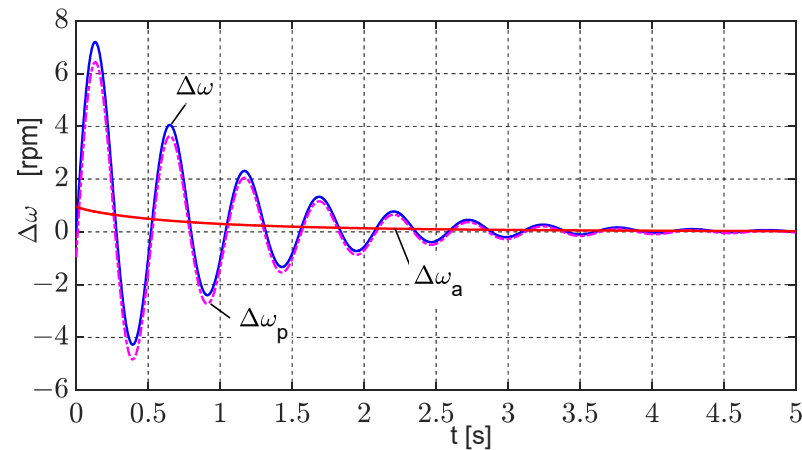


Figure 10. Waveform of the rotor angular velocity $\Delta\omega$. $\Delta\omega_a$ —aperiodic component, and $\Delta\omega_p$ —periodic component.

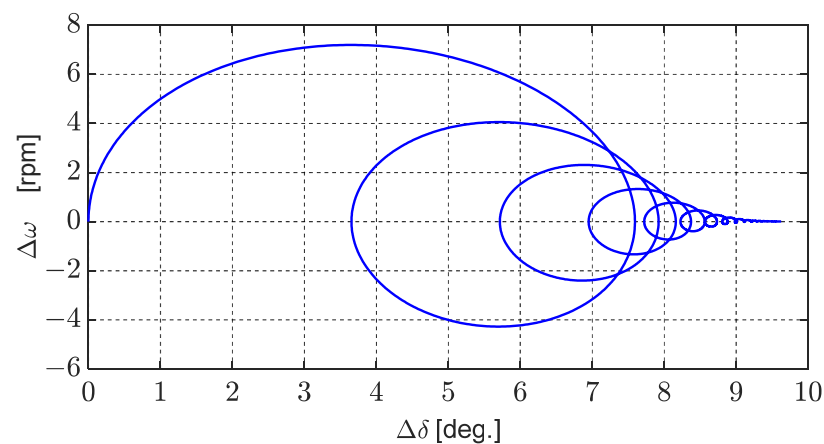


Figure 11. Trajectory of angular velocity $\Delta\omega$ versus load angle $\Delta\delta$.

The values of the damping coefficient and the pulsation of electromechanical swings, calculated from Equation (49), are $\alpha_h = 0.00351$ pu and $\omega_h = 0.03858$ p.u., which correspond to the damping time constant and the electromechanical swing frequency of the turbogenerator: $T_h = 0.907$ s and $f_h = 1.929$ Hz.

Figure 12 compares the power angle waveforms calculated based on the fractional-order model and those determined from the lumped-parameter circuit model.

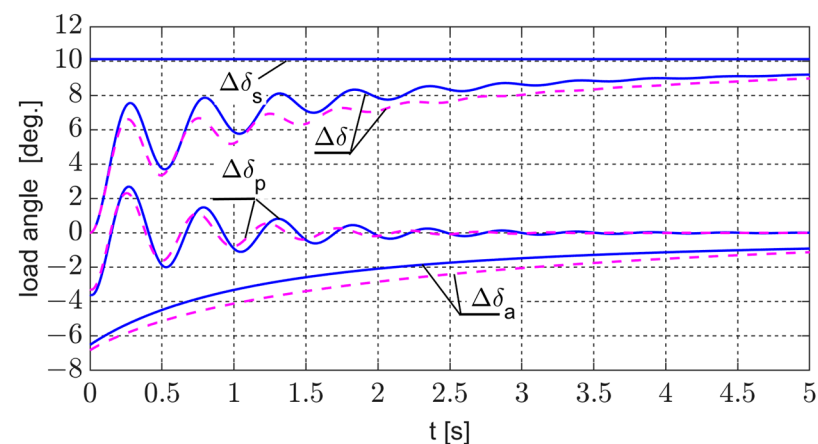


Figure 12. Waveform of the load angle $\Delta\delta$: $\Delta\delta_s$ —steady-state component, $\Delta\delta_a$ —aperiodic component, $\Delta\delta_p$ —periodic component, full line—fractional-order model (two equivalent circuits in the rotor q-axis), and dashed line—lumped-parameter model.

Table 4 presents the values of the damping factor α_h and the pulsation ω_h , as well as the time constant of damping T_h and the frequency f_h of the electromechanical swings (quantities marked as p.u. are expressed in per unit, i.e., relative to the selected base values).

The values of the swing frequency f_h determined from the fractional-order model and the lumped-parameter model are similar.

5. Conclusions

From the conducted considerations and performed calculations, it follows that it is possible to determine the variation in the load angle of a synchronous generator connected to an infinite bus under small disturbances from the equilibrium state by applying fractional derivatives of order 0.5. In this case, analytical relations are obtained, on the basis of which the damping coefficient and the frequency of electromechanical swings can be determined.

The presented fractional-order model of the synchronous generator can be used to analyze small disturbances, such as those that normally occur in a power system. These disturbances are sufficiently small to allow the linearization of the differential equations around the equilibrium point. Therefore, this model cannot be applied to simulations of large disturbances, such as faults in the transmission system. Due to its computational complexity, it is also not suitable for real-time simulation.

Commonly used models of synchronous generators for power system analysis are based on equivalent circuit models with lumped parameters. They typically include one damper circuit in the d-axis and one in the q-axis, or two damper circuits in the d-axis and the q-axis. A drawback of lumped-parameter models is the difficulty of measurement-based identification of more than two equivalent circuits in the rotor in the d-axis and q-axis.

A key advantage of the fractional-order model is its accuracy in the frequency domain. The results indicate that the fractional-order model achieves a more precise fit of the frequency characteristics compared with the classical model with the same number of equivalent circuits in the rotor. Specifically, using one equivalent circuit in the rotor d-axis and two circuits in the q-axis for the fractional-order model corresponds to the need for two equivalent circuits in the d-axis and four circuits in the q-axis in the lumped-parameter model to attain comparable approximation accuracy. The correctness of the obtained results can be verified by calculating the steady-state value of the load angle using Equation (29) based on the final value theorem in the Laplace domain, and by calculating it from Equation (43) in the time domain. In both cases, the same values were obtained.

Future work will involve developing a cylindrical rotor synchronous generator model and performing both simulation and experimental studies under large disturbances, employing nonlinear integer-order and fractional-order models. The test setup will consist of a synchronous generator with a power of approximately 10 kW driven by a separately excited DC motor and a high-power single-phase sinusoidal voltage generator for measuring frequency characteristics of spectral transmittances in the range of 0.001–200 Hz.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in this article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The author declares no conflicts of interest.

Abbreviations

The following abbreviations were used in this manuscript:

FEM Finite Element Method
SSFR Standstill Frequency Response

Appendix A

The differential voltage Equations (1) and (2), as well as the flux linkage equations for the armature and excitation windings, expressed in operational form for a stationary rotor ($\omega = 0$), are as follows:

$$\left. \begin{aligned} U_{sd}(p) &= -R_s I_{sd}(p) + p\Psi_{sd}(p) \\ U_{sq}(p) &= -R_s I_{sq}(p) + p\Psi_{sq}(p) \end{aligned} \right\} \quad (A1)$$

$$U_f(p) = R_f I_f(p) + p\Psi_f(p) \quad (A2)$$

$$\left. \begin{aligned} \Psi_{sd}(p) &= -L_{sd0}(p)I_{sd}(p) + L_{df}(p)I_f(p) \\ \Psi_{sq}(p) &= -L_{sq}(p)I_{sq}(p) \end{aligned} \right\} \quad (A3)$$

$$\Psi_f(p) = L_{f0}(p)I_f(p) - L_{fd}(p)I_{sd}(p) \quad (A4)$$

where L_{sd0} and L_{sq} are the operational inductances of the armature winding in the d- and q-axes, L_{f0} is the operational inductance of the field winding, and L_{fd} and L_{df} are the mutual operational inductances between the armature winding and field windings in the d-axis.

$$\left. \begin{aligned} L_{sd0}(p) &= -\frac{\Psi_{sd}(p)}{I_{sd}(p)} \Big|_{I_f=0} & L_{sq}(p) &= -\frac{\Psi_{sq}(p)}{I_{sq}(p)} \\ L_{df}(p) &= \frac{\Psi_{sd}(p)}{I_f(p)} \Big|_{I_{sd}=0} & L_{fd}(p) &= -\frac{\Psi_f(p)}{I_{sd}(p)} \Big|_{I_f=0} \\ L_{f0}(p) &= \frac{\Psi_f(p)}{I_f(p)} \Big|_{I_{sd}=0} \end{aligned} \right\} \quad (A5)$$

The inductances L_{sd0} and L_{f0} should be increased by the end-winding leakage inductance of the armature and field windings, respectively. These inductances are determined from empirical formulas [29].

The inductances $L_{d0}(p)$ and $L_{fd}(p)$ correspond to the condition with the field winding open, whereas $L_{f0}(p)$ and $L_{df}(p)$ correspond to the condition with the armature winding open.

By calculating the field current from Equation (A2), incorporating the expression for the flux linkage of the field winding in Equation (A4), and substituting it into Equation (A3), the expression for the flux linkage of the armature winding in the d-axis is obtained as follows:

$$\begin{aligned} \Psi_{sd}(p) &= -\left\{ L_{sd0}(p) - \frac{pL_{fd}L_{df}(p)}{R_f + pL_{f0}(p)} \right\} I_{sd}(p) + \frac{L_{df}(p)}{R_f + pL_{f0}(p)} U_f(p) = \\ &= -L_{sd}(p)I_{sd}(p) + G_{fd}(p)U_f(p) \end{aligned} \quad (A6)$$

where L_{sd} is the operator inductance of the armature with the field winding short-circuited, and $G_{fd}(p)$ is the transfer function of the field circuit, with

$$\left. \begin{aligned} L_{sd}(p) &= -\frac{\Psi_{sd}(p)}{I_{sd}(p)} \Big|_{U_f=0} = L_{sd0}(p) - \frac{pL_{fd}L_{df}(p)}{R_f + pL_{f0}(p)} \\ G_{fd}(p) &= \frac{\Psi_{sd}(p)}{U_f(p)} \Big|_{I_{sd}=0} = \frac{L_{df}(p)}{R_f + pL_{f0}(p)} \end{aligned} \right\} \quad (A7)$$

From the relationship in Equation (A7), it follows that the transfer functions $L_{sd}(p)$ and $G_{fd}(p)$ can be determined from the inductances $L_{d0}(p)$, $L_{df}(p)$, $L_{f0}(p)$, and $L_{fd}(p)$, which are obtained under open-circuit conditions of either the armature or field winding. These transfer functions, along with the operator inductance $L_q(p)$, form the core parameters used in the analysis of transient states in synchronous machines. By substituting the Laplace operator $p = j\omega$, where $\omega = 2\pi f$, into relationships Equations (A5) and (A7), the spectral inductances are obtained. Calculations performed for frequencies in the range of 0–1000 Hz yield the frequency characteristics of these spectral inductances. The spectral inductances were determined based on FEM simulations of electromagnetic field

distribution with the machine at standstill. These calculations were carried out using the FEMM 4.2 software, applying sinusoidal currents to the armature or field windings over the 0–1000 Hz frequency range.

The spectral armature inductance L_{sd0} in the d-axis and the mutual inductance between the armature winding and the field winding are determined by supplying two phases of the armature winding, with the rotor direct axis aligned with the resultant stator flux Ψ ($\vartheta = 90^\circ$) (Figure A1a), and the excitation winding open.

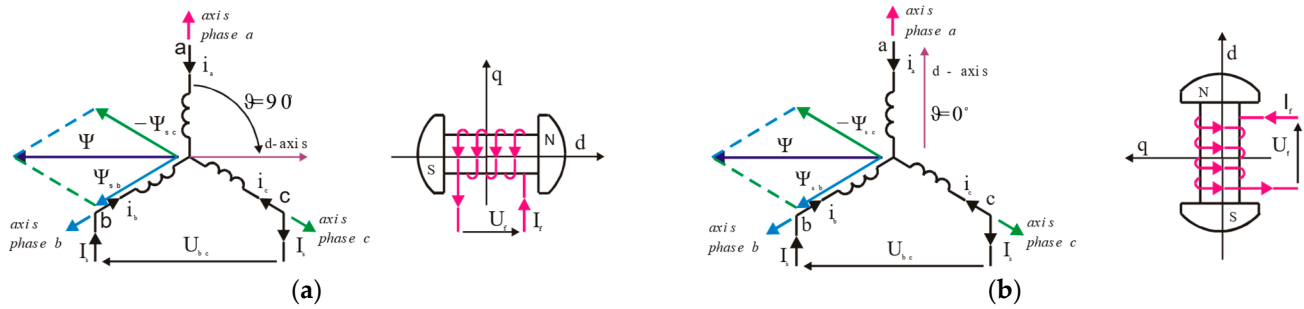


Figure A1. Diagrams for identifying the spectral inductances of the synchronous machine at standstill: (a) rotor in direct alignment; (b) rotor in quadrature alignment.

The following conditions apply: $\vartheta = 90^\circ$, $i_{sa} = 0$, $i_{sc} = -i_{sb}$, $u_{sa} = 0$, and $u_{sb} - u_{sc} = -u_{bc}$. Applying the Park transformation to Equation (8) under these conditions gives

$$\left. \begin{aligned} i_{sd} &= \frac{2}{\sqrt{3}} i_{sb} = \frac{2}{\sqrt{3}} i_s \\ i_{sq} &= 0 \end{aligned} \right\} \quad (\text{A8})$$

$$\left. \begin{aligned} u_{sd} &= -\frac{u_{bc}}{\sqrt{3}} \\ u_{sq} &= 0 \end{aligned} \right\} \quad (\text{A9})$$

Using relation Equation (A5), and taking into account Equations (A1), (A8), and (A9), an expression is obtained for the spectral inductance L_{sd0} with the field winding open-circuited:

$$L_{sd0}(j\omega) = -\frac{1}{j\omega} \frac{\Psi_{sd}(j\omega)}{I_{sd}(j\omega)} = \frac{1}{j\omega} \left(-\frac{U_{sd}(j\omega)}{I_{sd}(j\omega)} - R_s \right) = \frac{1}{j\omega} \left(\frac{1}{2} \frac{U_{bc}(j\omega)}{I_b(j\omega)} - R_s \right) \quad (\text{A10})$$

By introducing the notation

$$Z_s(j\omega) = \frac{U_{bc}(j\omega)}{I_b(j\omega)} \quad (\text{A11})$$

of the spectral inductance in Equation (A10), it can be written in the following form:

$$L_{sd0}(j\omega) = \frac{1}{2} \left[\frac{Z_s(j\omega) - 2R_s}{j\omega} \right] = \frac{1}{2} L_s(j\omega) \quad (\text{A12})$$

Taking into account Equation (A8), the mutual spectral inductance L_{df} is determined from the following relationship:

$$L_{fd}(j\omega) = -\frac{\Psi_f(j\omega)}{I_{sd}(j\omega)} \Big|_{I_f=0} = -\frac{\Psi_f(j\omega)}{I_{sd}(p)} = -\frac{\sqrt{3}}{2} \frac{\Psi_f(j\omega)}{I_s(j\omega)} \quad (\text{A13})$$

In the linear case, the spectral inductance L_{df} is equal to the inductance L_{fd} . The FEMM 4.2 software enables direct computation of the spectral inductance L_s , impedance Z_s , and resistance R_s in Equation (A12), as well as the associated flux Ψ_f in Equation (A13) for a given stator current frequency.

The spectral inductance L_{sq} in the q-axis is determined by supplying two phases of the armature winding, with the rotor q-axis aligned with the resultant stator flux ($\vartheta = 0^\circ$) (Figure A1b). The following conditions apply: $\vartheta = 0$, $i_{sa} = 0$, $i_{sc} = -i_{sb}$, and $u_{sa} = 0$, $u_{sb} - u_{sc} = -u_{bc}$. Applying the Park transformation in Equation (8) under these conditions gives

$$\left. \begin{aligned} i_{sd} &= 0 \\ i_{sq} &= \frac{2}{\sqrt{3}} i_{sb} = \frac{2}{\sqrt{3}} i_s \end{aligned} \right\} \quad (\text{A14})$$

$$\left. \begin{aligned} u_{sd} &= 0 \\ u_{sq} &= -\frac{u_{bc}}{\sqrt{3}} \end{aligned} \right\} \quad (\text{A15})$$

Following the same procedure as for the d-axis, the spectral inductance in the q-axis is determined by

$$L_q(j\omega) = \frac{1}{2} \left[\frac{Z_s(j\omega) - 2R_s}{j\omega} \right] = \frac{1}{2} L_s(j\omega) \quad (\text{A16})$$

where the spectral impedance Z_s , incorporating Equations (A14) and (A15), is defined by

$$Z_s(j\omega) = \frac{U_{bc}(j\omega)}{I_b(j\omega)} \quad (\text{A17})$$

The self-spectral inductance L_{f0} of the excitation winding under open-circuit armature conditions is determined from the relationship in Equation (A5):

$$L_{f0}(j\omega) = \frac{\Psi_f(j\omega)}{I_f(j\omega)} \quad (\text{A18})$$

The excitation flux Ψ_f and excitation current i_f appearing in the above relationships are quantities referring to the armature side. In contrast, FEMM 4.2 uses the actual quantities Ψ_f^* and i_f^* in its magnetic field analysis. To refer the field circuit quantities to the armature side, the voltage transformation ratio k_u and current transformation ratio k_i are applied [28].

$$\left. \begin{aligned} k_u &= \frac{N_s k_{ws}}{N_f k_{wf}} \\ k_i &= \frac{m}{2} \frac{N_s k_{ws}}{N_f k_{wf}} = \frac{m}{2} k_u \end{aligned} \right\} \quad (\text{A19})$$

where N_s and k_{ws} represent the number of series turns of one stator phase and the winding factor of the stator; N_f and k_{wf} represent the number of series turns and winding factor of the field winding; and m is the number of stator winding phases.

The flux linkage Ψ_f^* and the field current i_f^* , referring to the armature winding side, are, respectively, equal to

$$\left. \begin{aligned} \Psi_f &= k_u \Psi_f^* \\ i_f &= \frac{i_f^*}{k_i} \end{aligned} \right\} \quad (\text{A20})$$

where Ψ_f^* and i_f^* are real quantities, and Ψ_f and i_f are referred quantities.

Based on the computed spectral inductances L_{d0} , L_{fd} , L_{f0} , and L_{df} , determined under open-circuit conditions of the field winding (L_{d0} and L_{fd}) or the armature (L_{f0} and L_{df}), the armature inductance in the d-axis with short-circuited field winding L_{sd} and the transfer function G_{fd} are determined.

$$\left. \begin{aligned} L_{sd}(j\omega) &= L_{sd0}(j\omega) - \frac{j\omega L_{fd} L_{df}(j\omega)}{R_f + j\omega L_{f0}(j\omega)} \\ G(j\omega) &= \frac{L_{df}(j\omega)}{R_f + j\omega L_{f0}(j\omega)} \end{aligned} \right\} \quad (\text{A21})$$

All calculations were performed using FEM-based electromagnetic field simulations over the frequency range of $0 \div 1000$ Hz.

Appendix B

Appendix B.1

For steady-state conditions, Equations (1) and (4) take the following forms:

$$\left. \begin{aligned} U_{sd0} &= -\omega_s \Psi_{sq0} \\ U_{sq0} &= +\omega_s \Psi_{sd0} \end{aligned} \right\} \quad (\text{A22})$$

$$\left. \begin{aligned} \omega_s \Psi_{sd0} &= -\omega_s L_{sd} I_{sd0} + E_{fm} = U_{sq0} \\ \omega_s \Psi_{sq0} &= -\omega_s L_{sq} I_{sq0} = -U_{sd0} \end{aligned} \right\} \quad (\text{A23})$$

From the relationship in Equation (10), it follows that

$$\left. \begin{aligned} U_{sd0} + \Delta u_{sd} &= U_{sm} \sin(\delta_0 + \Delta\delta) = U_{sm} \sin\delta_0 \cos\Delta\delta + U_{sm} \cos\delta_0 \sin\Delta\delta \\ U_{sq0} + \Delta u_{sq} &= U_{sm} \cos(\delta_0 + \Delta\delta) = U_{sm} \cos\delta_0 \cos\Delta\delta - U_{sm} \sin\delta_0 \sin\Delta\delta \end{aligned} \right\} \quad (\text{A24})$$

Assuming small deviations in the power angle increment $\Delta\delta$, $\cos\Delta\delta \approx 1$ and $\sin\Delta\delta \approx \Delta\delta$, the relationships from Equation (A24) take the following form:

$$\left. \begin{aligned} U_{sd0} + \Delta u_{sd} &= U_{sm} \sin\delta_0 + U_{sm} \cos\delta_0 \Delta\delta \\ U_{sq0} + \Delta u_{sq} &= U_{sm} \cos\delta_0 - U_{sm} \sin\delta_0 \Delta\delta \end{aligned} \right\} \quad (\text{A25})$$

Thus,

$$\left. \begin{aligned} U_{sd0} &= U_{sm} \sin\delta_0 \\ U_{sq0} &= U_{sm} \cos\delta_0 \end{aligned} \right\} \quad (\text{A26})$$

$$\left. \begin{aligned} \Delta u_{sd} &= U_{sm} \cos\delta_0 \Delta\delta = U_{sq0} \Delta\delta \\ \Delta u_{sq} &= -U_{sm} \sin\delta_0 \Delta\delta = -U_{sd0} \Delta\delta \end{aligned} \right\} \quad (\text{A27})$$

The transformation voltages appearing in Equation (25) can be neglected, since the transients associated with electromechanical swings can reasonably be approximated as slowly varying processes [23]. Equation (25), written in operator form and incorporating relationships Equations (26) and (A26), becomes

$$\left. \begin{aligned} U_{sq0} \Delta\delta(p) &= -\omega_s \Delta\Psi_{sq}(p) - p\Delta\delta(p) \Psi_{sq0} \\ -U_{sd0} \Delta\delta(p) &= \omega_s \Delta\Psi_{sd}(p) + p\Delta\delta(p) \Psi_{sd0} \end{aligned} \right\} \quad (\text{A28})$$

The deviations in the flux linkages in the d- and q-axes are given by Equations (A3) and (A6):

$$\left. \begin{aligned} \Delta\Psi_{sd}(p) &= -L_{sd}(p) \Delta i_{sd}(p) + G_{fd}(p) \Delta u_f(p) \\ \Delta\Psi_{sq}(p) &= -L_{sq}(p) \Delta i_{sq}(p) \end{aligned} \right\} \quad (\text{A29})$$

Assuming that the field winding voltage is unregulated, $u_f = U_{f0}$, and the voltage deviation Δu_f in Equation (A29) is zero. Using Equation (A29), and applying the relationship in Equation (A28), the armature currents in the direct axis Δi_{sd} and quadrature axis Δi_{sq} can be determined.

$$\left. \begin{aligned} \Delta i_{sd}(p) &= -\frac{\Delta\Psi_{sd}(p)}{L_{sd}(p)} = \frac{U_{sd0} + p\Psi_{sd0}}{\omega_s L_{sd}(p)} \Delta\delta(p) \\ \Delta i_{sq}(p) &= -\frac{\Delta\Psi_{sq}(p)}{L_{sq}(p)} = \frac{U_{sq0} + p\Psi_{sq0}}{\omega_s L_{sq}(p)} \Delta\delta(p) \end{aligned} \right\} \quad (\text{A30})$$

Taking into account the relationships in Equations (A23), (A27), (A29), and (A30), the expression for the electromagnetic torque (Equation (27)) becomes

$$\Delta M_e(p) = \frac{p_b}{\omega_s} \left\{ \begin{aligned} &Q_{e0} + \frac{3}{2} U_{sm}^2 \left(\frac{\cos^2 \delta_0}{X_{sq}(p)} + \frac{\sin^2 \delta_0}{X_{sd}(p)} \right) + \\ &-\frac{p}{\omega_s} \left[P_{e0} + \frac{3}{4} U_{sm}^2 \left(\frac{1}{X_{sq}(p)} - \frac{1}{X_{sd}(p)} \right) \sin 2\delta_0 \right] \end{aligned} \right\} \Delta\delta(p) \quad (\text{A31})$$

where P_{e0} and Q_{e0} are quantities representing active and reactive power, respectively.

$$\left. \begin{aligned} P_{e0} &= \frac{3}{2} \frac{E_{fm} U_{sm}}{X_{sd}} \sin \delta_0 + \frac{3}{4} U_{sm}^2 \left(\frac{1}{X_{sq}} - \frac{1}{X_{sd}} \right) \sin 2\delta_0 \\ Q_e &= \frac{3}{2} \frac{E_{fm} U_{sm}}{X_{sd}} \cos \delta_0 - \frac{3}{2} U_{sm}^2 \left(\frac{\sin^2 \delta_0}{X_{sq}} + \frac{\cos^2 \delta_0}{X_{sd}} \right) \end{aligned} \right\} \quad (\text{A32})$$

Appendix B.2

The inverse Laplace transforms of the operator expressions appearing in Equations (39) and (40) take the following form [30]:

$$\left\{ \begin{aligned} \frac{1}{p(p^\lambda + b)} &= \frac{1}{b} [1 - E_\lambda(-bt^\lambda)] \\ \frac{1}{p^\lambda + b} &= t^{\lambda-1} E_{\lambda, \lambda}(-bt^\lambda) \end{aligned} \right\} \quad (\text{A33})$$

where $E_{\alpha, \beta}(z)$ is the two-parameter Mittag-Leffler function [30,31], where $E_\alpha = E_{\alpha, 1}(z)$.

$$E_{\alpha, \beta} = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \quad (\text{A34})$$

For the specific case $\lambda = 0.5$, the inverse transforms in Equation (A33) take the following form [11,30,32]:

$$\left\{ \begin{aligned} \frac{1}{p(p^{0.5} + b)} &= \frac{1}{b} [1 - e^{b^2 t} \operatorname{erfc}(b\sqrt{t})] \\ \frac{1}{p^{0.5} + b} &= \frac{1}{\sqrt{\pi t}} - b e^{b^2 t} \operatorname{erfc}(b\sqrt{t}) \end{aligned} \right\} \quad (\text{A35})$$

If a complex root of q_i of the denominator of the polynomial $M(q)$ in Formula (40) has a positive real part, the inverse Laplace transform is obtained from the following relationship:

$$\left\{ \begin{aligned} L^{-1} \left\{ \frac{1}{p(p^{0.5} - b)} \right\} &= \frac{1}{b} [1 - e^{b^2 t} \operatorname{erfc}(b\sqrt{t}) + 2(e^{b^2 t} - 1)] \\ L^{-1} \left\{ \frac{1}{p^{0.5} - b} \right\} &= \frac{1}{\sqrt{\pi t}} - b e^{b^2 t} \operatorname{erfc}(b\sqrt{t}) + 2b e^{b^2 t} \end{aligned} \right\} \quad (\text{A36})$$

where $\operatorname{erfc}(z)$ is the complement of the Gaussian error function [32].

The product of the complex function $\exp(z^2) \cdot \operatorname{erfc}(z)$ for a complex argument z is calculated from the following series [11,30,32]:

$$e^{z^2} \operatorname{erfc}(z) = e^{z^2} - \frac{2}{\sqrt{\pi}} \left\{ z + \sum_{k=1}^{\infty} \frac{2^k z^{2k+1}}{1 \cdot 3 \dots (2k+1)} \right\} = \gamma_1 + j\gamma_2 \quad (\text{A37})$$

For large arguments $|z|$, for which $|\arg(z)| < \pi/2$, the relationship [11,30,32] may be applied:

$$e^{z^2} \operatorname{erfc}(z) = \frac{1}{\sqrt{\pi}} \left\{ \frac{1}{z} + \sum_{k=1}^{\infty} (-1)^k \frac{1 \cdot 3 \dots (2k-1)}{2^k z^{2k+1}} \right\} \quad (\text{A38})$$

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