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Optimal PMU Placement to Enhance Observability in Transmission Networks Using ILP and Degree of Centrality

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Abstract: The optimal PMU placement problem is placing the minimum number of PMUs in the network to ensure complete network observability. It is an NP-complete optimization problem. PMU placement based on cost and critical nodes is solved separately in the literature. This paper proposes a novel approach, a degree of centrality in the objective function, to combine the effect of both strategies to place PMUs in the power network optimally. The contingency analysis and the effect of zero-injection buses are solved to ensure the reliability of network monitoring and attain a minimum number of PMUs. Integer linear programming is used on the IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, and IEEE 118-bus systems to solve this problem. The results are evaluated based on two performance measures: the bus observability index (BOI) and the sum of redundancy index (SORI). On comparison, it is found that the proposed methodology has significantly improved results, i.e., a reduced number of PMUs and increased network overall observability (SORI). This methodology is more practical for implementation as it focuses on critical nodes. Along with improvement in the results, the limitations of existing indices are also discussed for future work.

Keywords: phasor measurement unit (PMU); optimal PMU placement (OPP); integer linear programming (ILP); bus observability index (BOI); sum of redundancy index (SORI)

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1. Introduction

In a power system, energy is transformed from different energy sources to electrical energy. A power system has four critical electrical parameters: voltage, current, phasors, and frequency. The frequency parameter stays constant in the system, while the remaining parameters are not constant. These parameters should function between defined threshold values [1] for regular power system operation. A precise and accurate monitoring system is necessary for the power system's efficient operation, risk assessment, and restoration after failure. Conventional monitoring is carried out through a supervisory control and data acquisition system (SCADA), which is inadequate to meet the requirements of modern electrical systems [2]. Researchers have started to investigate a better monitoring system to overcome the limitations of SCADA. The Virginia Tech professor Phadke introduced the concept of phasor monitoring in 1980 [3]. Later, in 1991, commercial production of phasor measurement unit (PMU) was started on phasor monitoring concept by Macrodyne and Virginia Tech.

Installation of PMUs in the power system has enabled smart grid implementation, real-time monitoring, dynamic control, enhanced state estimation, adaptive protection, system restoration, postdisturbance analysis, and state estimation. Their installation on each bus for complete network observability will incur substantial costs. To minimize this cost, a critical role is played by Ohm's law. So, by strategically placing PMUs in the network, numerical and topological observability can be achieved [4]. This strategic placement of PMUs in the network to minimize the cost is called the optimal PMU placement (OPP) problem. Based on the research findings presented in [5], it is found that OPP is an NP-complete problem.

Despite this challenge, Abur et al. have made notable progress in solving OPP through integer linear programming (ILP) [6]. Since their groundbreaking work, numerous researchers have used ILP to solve the OPP problem. Abbasy et al. have solved the OPP problem using ILP and considered the effect of conventional measurements and PMU loss, ensuring complete network observability [7]. Enshaee et al. investigated OPP using ILP and solved PMU loss or line loss, the effect of zero-injection buses, and channel limitation. Amare et al. considered various observability scenarios, including full observability, redundancy, and multistage installation of PMUs [8]. Sarailoo et al. proposed PMU allocation, ensuring data availability despite communication interruptions and synchrophasor availability using ILP [9]. Ahmed et al. have considered measurement redundancy to solve the OPP problem using ILP [10]. Ruben et al. proposed a multiobjective model that accounts for PMU installation costs, system observability, and gross error detection using a mixed ILP approach [11]. Elimam et al. proposed a model that accounts for electrical parameters in OPP formulation considering constraints of measurement redundancy, PMU channel limitation, preexisting conventional measurements, and small signal stability of power network and solved this proposed model using the ILP technique [12]. Several scholars have expanded the scope of their research to address OPP using different optimization techniques, such as greedy algorithm [13], nonlinear programming [14], graph theory [15], fuzzy decision [16], tabu search [17], cellular learning automata [18], cuckoo search [19], gravitational search [20], artificial bee colony [21], ant colony [22], genetic algorithm [23], and particle swarm optimization [24]. In a review article [25], Ahmed et al. discussed the stated objective functions and constraints in detail to achieve complete network observability.

Islam et al. proposed an observability-aware PMU networking framework, optimizing data transfer from PMUs to phasor data concentrator [26]. PMU roles and resilient routing schemes help monitor the grid in real time. Results obtained help to minimize end-to-end delay and maintain grid observability. Mandal et al. proposed a framework for smart grid monitoring, integrating system-aware data pruning and optimal PMU placement [27]. This method helps to analyze spatial error propagation and node-level data pruning and saves the cost of bandwidth and resource utilization. Perl et al. used the neural additive model to place PMUs with improved performance and better computational efficiency [28]. A global explainable artificial intelligence model is used to identify fault location using phasor measurement units. Asadzadeh et al. addressed inaccuracies in power network state estimation using the probabilistic model [29]. PMU placement to improve state estimation is solved using K-medoids and binary particle swarm optimization. Zhou et al. gave novel PMU placement using a reinforcement learning graph convolutional network–deep deterministic policy gradient algorithm [30]. Under complex operating conditions, system graphs and PMU states are considered for various networks to improve state estimation. Zhang et al. proposed an attacked resilient approach for PMU placement using the reinforcement learning guided tree search method and prioritized the vulnerable buses by employing sequential decision making to improve state estimation [31]. Cojoaca et al. investigated multiagent approach to solve PMU placement problem [32]. PMUs are modeled as agents for real-time state estimation to achieve local observability and enhanced monitoring efficiency. Using deep learning methods, as described in [33], may provide an effective approach for improved PMU placement. Furthermore, machine learning and deep learning techniques increase the efficiency of power system observability.

Observability is the determination of the internal states of the power system. It can be achieved through two methods: numerical and topological methods. Numerical methods are complex, time-consuming, and prone to errors for large networks. Topological methods are faster and more accurate for large networks. The scope of this paper is limited to topological observability only.

In topological observability, the decoupled measurement model and graph theory are used. The decoupled measurement model refers to the condition that each measurement obtained from the PMU is treated independently. It minimizes the coupling effect between different variables and allows a more straightforward interpretation of each measurement [34]. In graph theory, graphs are formed of vertices v and edges e. Each edge is connected to an unordered pair of vertices [35]. In power networks, buses are represented as vertices $V = \{v_1, v_2, \ldots, v_n\}$ and lines are represented as edges $E = \{e_1, e_2, \ldots, e_m\}$. Here, n and m represent the cardinality of the V and E sets. In the context $e_1 = (v_1, v_2)$, e_1 is identified as an incident to both v_1 and v_2. Notably, v_1 and v_2 are adjacent, and their interconnectivity is presented through the connectivity matrix. The decisions are based on logical operations based on the information of the connectivity matrix, type of measurement devices, and location of devices. The system will be completely observable only if the current measurement set can make the full rank-spanning tree. This paper uses the following rules to place PMU for complete network observability:

1. If a PMU is installed on a bus, all its connected branch currents and bus voltage are known. It is a direct measurement.
2. If the current phasor and bus voltage information are available at one end of a branch, then the bus voltage phasor can be calculated at the other end of the branch. It is a pseudomeasurement, as shown in Figure 1a.
3. If information on bus voltages at both ends of a branch is known, then the current phasor of that branch can be calculated. It is a pseudomeasurement, as shown in Figure 1b.

This paper not only focuses on the topological observability of the network but also considers contingency analysis and the effect of zero-injection buses in PMU placement. A zero-injection bus (ZIB) is one where no active and reactive power is injected or withdrawn [36]. In OPP, a ZIB is observed using Kirchhoff's laws. For complete network observability, the ZIB rules can be written as follows:

1. If the current phasors of all branches except one, the one connected to the ZIB, are known, then Kirchhoff's current law (KCL) can be used to figure out the unknown branch current phasor as shown in Figure 1c.
2. If the voltage phasors of all incident buses except one, the one connected to the ZIB are known, then Kirchhoff's voltage law (KVL) can be used to figure out unknown bus voltage phasor as shown in Figure 1d.
3. If a group of adjacent ZIBs exists, provided that the voltage phasor of adjacent buses and current phasors of connecting branches to the group of ZIB are known, the bus voltage and the branch current phasors of the group of ZIBs in the network can be calculated. If the current phasors of all branches except one and the voltage phasors of all incident buses except one connected to a group of ZIBs are known, then using rule 4 and rule 5, the unknown branch current and unknown bus voltage can be calculated, as shown in Figure 1e. The measurements obtained using the above three ZIB rules are called extended measurements.

Understanding the rules of observability and ZIB is vital to solving the OPP problem. The literature reveals that there are two strategies to solve OPP. One approach focuses on the economic aspects of installing, maintaining, and allocating resources in OPP [37]. The second strategy focuses on the network attributes to identify critical nodes such that nodes with better connectivity are considered more valuable [38]. Despite the benefits of the above-stated strategy, there is an opportunity to integrate both concepts for improved results. The main contributions of this paper are stated below:

1. A proposed modified objective function incorporates the degree of centrality to solve the optimal PMU placement (OPP) problem.
2. The $N-1$ contingency of PMU and the effect of zero-injection buses are incorporated to solve the OPP problem.
3. The OPP results are evaluated based on measures of observability and redundancy.
4. The results are improved as the network's overall observability is enhanced and the number of PMUs is reduced.
5. For future work, the limitations of conventional PMU placement and existing performance measures are addressed.

This paper is divided into several sections. Section 2 highlights the importance of proposed formulation and the normalized degree of centrality. Section 3 discusses the integer linear programming technique to solve the OPP problem. Section 4 evaluates different OPP solutions based on observability and redundancy measures with limitations. Section 5 presents results and discusses, four major cases to solve the OPP problem on IEEE test-bed systems. Finally, in Section 6, a conclusion is made with a summary of the significant findings and their implications.

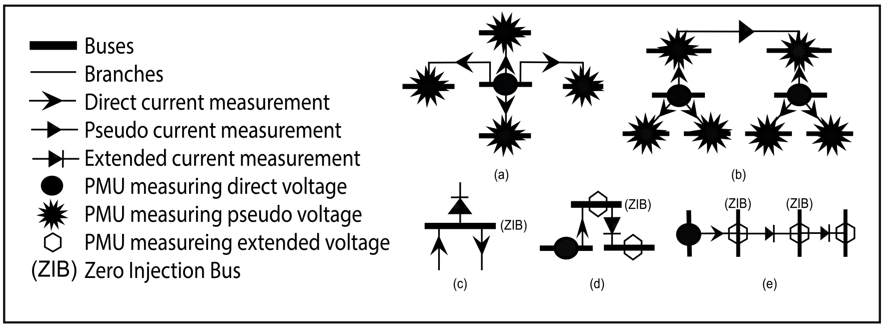

Figure 1. Visual representation of PMU placement rules: (**a**) present rule 2 for complete network observability, (**b**) present rule 3 for complete network observability, (**c**) present ZIB rule 1 for complete network observability, (**d**) present ZIB rule 2 for complete network observability, and (**e**) present ZIB rule 3 for complete network observability.

2. Proposed OPP Formulation with Normalized Degree of Centrality

2.1. Normalized Degree of Centrality

Degree centrality is the measure of node centrality that gives a quantitative number of connections a particular node has in the network. The formula to calculate the degree of centrality is as follows:

$$D_i = \sum_{j=1}^{n} (a_{ij} - \delta_{ij}) \tag{1}$$

Buses with high centrality play a crucial role in network connectivity and communication. These buses are essential for fault detection, flow stability, and network reliability. A new factor, normalized degree of centrality or zeta ζ, is introduced to incorporate the degree centrality of buses into the objective function. The degree centrality value is first normalized such that the weighted sum of zeta of all buses is equal to 1. Although the degree and normalized degree provide the same information about the bus criticality, handling with the normalized degree of centrality is more manageable and offers practical advantages in optimization. The normalized value is also robust to the changes in the network if the network expands or is reduced in a real-life scenario. For any bus i, the value of ζ is calculated using the following formula:

$$\zeta_i = \frac{D_i}{\sum_{i=1}^{n} D_i} \tag{2}$$

The midrange normalized degree of centrality $\overline{\zeta}$ across all buses indicates the equilibrium point between higher and lower centrality levels, serving as a reference for assessing the distribution of centrality values. It can be calculated using the following formula:

$$\overline{\zeta} = \frac{\zeta_{\max} + \zeta_{\min}}{2} \quad (3)$$

For illustration, an example of a 7-bus system is shown in Figure 2a. Let $A = [a_{ij}]_{n\times n}$ is a $n \times n$ matrix, where a_{ij} is defined as follows:

$$a_{ij} = \begin{cases} 1 & \text{If vertex } v_i \text{ is connected to itself or adjacent vertex } v_j \\ 0 & \text{Otherwise} \end{cases}$$

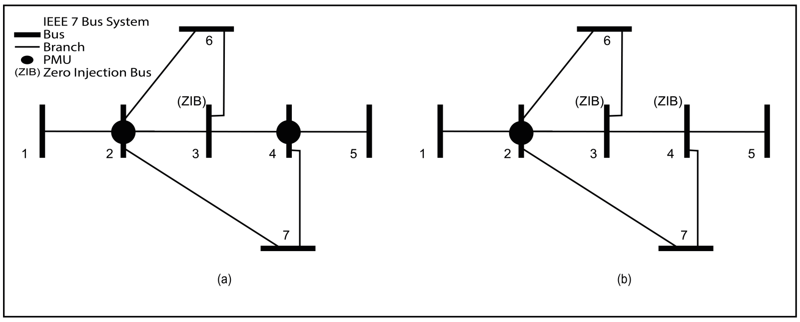

Figure 2. IEEE7-bus systems: (**a**) Original IEEE 7-bus system. (**b**) Modified IEEE 7-bus system (dual ZIB).

From Figure 2a, a bus-to-bus connectivity matrix can be formed as follows:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{n\times n}$$

For Figure 2a, each bus's degree and normalized degree of centrality are calculated using Equations (1) and (2) from the connectivity matrix of the IEEE 7-bus system and presented in Table 1.

Table 1. Degree and normalized degree of centrality of IEEE 7-bus system.

$\mathbf{Bus_i}$	$\mathbf{D_i}$	ζ_i	$1-\zeta_i$
1	1	0.0625	0.9375
2	4	0.2500	0.7500
3	3	0.1875	0.8125
4	3	0.1875	0.8125
5	1	0.0625	0.9375
6	2	0.1250	0.8750
7	2	0.1250	0.8750

Variations in normalized degree centrality values highlight the differences in bus importance across the IEEE 7-bus network, as shown in Table 1. As seen, bus two has the highest value of normalized degree centrality ζ, so it will be prioritized in placing the first PMU. Then, as buses three and four have the same value of ζ, and it is seen that bus three is having pseudomonitoring as of the PMU placed on bus two, as shown in Figure 2a, PMU will next be placed on bus four to monitor the complete network. Calculating the midrange normalized degree of centrality from Equation (3) enables the identification of each bus criticality as shown in Figure 3. Buses with normalized degrees of centrality above midrange are more critical than remaining buses. They should be prioritized for PMU placement.

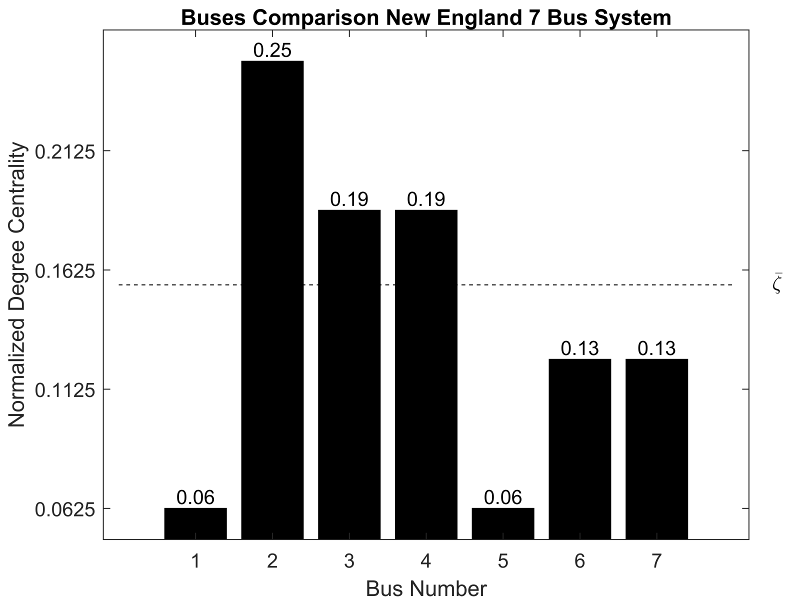

Figure 3. Normalized degree of centrality graph for IEEE 7-bus system.

2.2. Mathematical Formulation

This section proposes a new mathematical formulation to solve the OPP problem using normalized degree centrality. Normalizing the value of degree centrality helps quantify node importance between 0 and 1. It is evident that the cost problem is a minimization problem, and critical bus observability is the maximization problem. So, optimization duality is used to convert acute bus observability maximization to a minimization problem. For this purpose, each bus ζ_i is subtracted from 1: presented in Table 1. This factor $1 - \zeta_i$ is multiplied by the decision variable X_i for each bus, so the algorithm prefers the critical buses for placing a minimum number of PMUs for complete network observability. It helps to improve grid observability and economic sustainability of the placement strategy. The mathematical formulation to incorporate normalized degree centrality into the objective function is discussed below:

$$\min \sum_{i=1}^{n} C\dot{(}1 - \zeta_i) \cdot X_i \tag{4}$$

Subject to:

$$AX \geq Y \tag{5}$$

In Equation (4): C is the cost of the PMU device and is assumed to be constant. The value $1 - \zeta_i$ helps to make decisions by showing the importance of each bus.

$$(1 - \zeta_i) \rightarrow \begin{cases} 1 & \text{As } \zeta_i \rightarrow 0 \text{ (Least potential bus for PMU)} \\ 0 & \text{As } \zeta_i \rightarrow 1 \text{ (Highly potential bus for PMU)} \end{cases}$$

In Equation (5): connectivity matrix A reveals any bus's dependency on the remaining buses in the network. When Y is a set of ones, the constraint $AX \geq Y$ means that at least one PMU should monitor each bus. It protects the system against blind spots and ensures complete network observability. This formulation helps to identify critical nodes to be selected as potential candidates for placing PMUs. Each bus is assigned a logical weight based on its criticality, which helps in decision making according to the needs of the network observability.

To solve the OPP problem for the IEEE 7-bus system through the proposed formulation, first of all, calculate D_i and ζ_i, as shown in Table 1 from Equations (1) and (2). The objective function and constraints for IEEE 7-bus systems can be expressed as given below:

$$\min_{x_1,x_2,\ldots,x_7} \{(0.9375)\cdot x_1 + (0.7500)\cdot x_2 + (0.8125)\cdot x_3 + (0.8125)\cdot x_4 + (0.9375)\cdot x_5 + (0.8750)\cdot x_6 + (0.8750)\cdot x_7\} \quad (6)$$

Subject to:

$$x_1 + x_2 \geq 1 \quad (7a)$$
$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 1 \quad (7b)$$
$$x_2 + x_3 + x_4 + x_6 \geq 1 \quad (7c)$$
$$x_3 + x_4 + x_5 + x_7 \geq 1 \quad (7d)$$
$$x_4 + x_5 \geq 1 \quad (7e)$$
$$x_2 + x_3 + x_6 \geq 1 \quad (7f)$$
$$x_2 + x_4 + x_7 \geq 1 \quad (7g)$$

These complete network observability constraints make the optimization problem harder to solve, but the optimization process becomes complete and well-rounded. It helps to find a solution that balances economic efficiency with comprehensive monitoring capabilities. Furthermore, two scenarios, i.e., $N-K$ contingency and the effect of zero-injection buses, are discussed below to address the reliability and financial perspectives.

2.2.1. $N-K$ Contingency Limitation

The observability constraint in the OPP problem mandates that at least one PMU must monitor every bus. Due to any unexpected failure of any device, the system becomes unobservable. The constraint is modified so that at least two or more PMUs must observe each bus. This reliability constraint is termed a $N-K$ contingency limitation. Although it is beyond the essential observability requirement, it is necessary for the network's reliability in terms of observability. It complicates problem-solving but helps improve operation resiliency by adding redundant information from PMU devices. To incorporate $N-K$ contingency, the modified version of Equation (5) is given below:

$$AX \geq K+1 \quad (8)$$

N is the total number of devices, and K is the number of failed devices, ranging from 1 to N. For the IEEE 7-bus system, the $N-K$ contingency is discussed below:

$$x_1 + x_2 \geq K+1 \quad (9a)$$
$$x_1 + x_2 + x_3 + x_6 + x_7 \geq K+1 \quad (9b)$$
$$x_2 + x_3 + x_4 + x_6 \geq K+1 \quad (9c)$$
$$x_3 + x_4 + x_5 + x_7 \geq K+1 \quad (9d)$$
$$x_4 + x_5 \geq K+1 \quad (9e)$$
$$x_2 + x_3 + x_6 \geq K+1 \quad (9f)$$
$$x_2 + x_4 + x_7 \geq K+1 \quad (9g)$$

In this paper, the observability constraint for $N-1$ contingency is considered. It helps to improve system strength and resilience against single-point failure. It improves real-time monitoring capabilities as at least two PMU devices monitor each bus simultaneously. It ensures that PMU deployment goes beyond nominal conditions to become resilient and reliable. As the number of PMUs is increased by incorporating $N-1$ contingency, the effect of zero-injection buses is addressed to reduce the number of PMUs in the network.

2.2.2. Effect of Zero-Injection Bus Limitation

In OPP, a critical constraint is the effect of zero-injection bus (ZIB) that further reduces the number of PMUs in comparison when the effect of ZIB is overlooked. Modeling ZIB in the ILP framework to allow some buses to remain unobserved selectively by using a set of rules [37].

1. All unobserved buses must belong to a cluster of ZIB or a cluster adjacent to ZIB.
2. For ZIB i, P_i is a set of buses adjacent to bus i. Let $Q_i = P_i \cup \{i\}$. The number of unobservable buses in cluster Q_i is at most one.

A generic formulation for ZIB is as follows:

$$A_m X_m \geq U \tag{10}$$

$$u_j = 1 \qquad \forall j \notin \bigcup_{z=1}^{\mathcal{Z}} Q_z \tag{11}$$

$$\sum_{k \in Q_i} u_k \geq |P_i| \qquad \forall i \in \mathcal{Z} \tag{12}$$

where

$$u_i = \begin{cases} 1 & \text{If bus } i \text{ is observed} \\ 0 & \text{Otherwise} \end{cases}$$

The constraint in Equation (11) ensures that buses not directly connected to the ZIB must be observed. It guarantees complete monitoring of buses near ZIB. For understanding, consider the modified IEEE 7-bus system shown in Figure 2b, which shows buses 3 and 4 as ZIBs. Thus, set $P_i = \{2,4,6\} \cup \{3,5,7\}$ and set $Q_i = \{2,3,4,6\} \cup \{3,4,5,7\}$. Thus, the additional constraint for ILP is

$$u_2 + u_3 + u_4 + u_5 + u_6 + u_7 \geq 5 \tag{13}$$

Equation (13) states that out of buses 2, 3, 4, 5, 6, and 7, at least five buses must be directly observed. The observability constraints are modified as follows:

$$x_1 + x_2 \geq 1 \tag{14a}$$
$$x_1 + x_2 + x_3 + x_6 + x_7 \geq u_2 \tag{14b}$$
$$x_2 + x_3 + x_4 + x_6 \geq u_3 \tag{14c}$$
$$x_3 + x_4 + x_5 + x_7 \geq u_4 \tag{14d}$$
$$x_4 + x_5 \geq u_5 \tag{14e}$$
$$x_2 + x_3 + x_6 \geq u_6 \tag{14f}$$
$$x_2 + x_4 + x_7 \geq u_7 \tag{14g}$$
$$u_2 + u_3 + u_4 + u_5 + u_6 + u_7 \geq 5 \tag{14h}$$

In the modified IEEE 7-bus system, two PMUs are needed if the effect of ZIBs is not considered. By addressing the effect of ZIB, the number of PMUs is reduced from two to one for complete network observability, as shown in Figure 2b. Notably, these results are improved on the modified IEEE 7-bus system. In the original IEEE 7-bus system, only bus 3 serves as ZIB, as shown in Figure 2a. In that case, the number of PMUs remains unchanged, regardless of whether ZIBs are considered or disregarded. The results Section 5 presents only the results of the original IEEE 7-bus system. The OPP problem, with its constraints, is an NP-complete problem. The branch and bound technique of integer linear programming is used to solve it.

3. Integer Linear Programming

Integer linear programming (ILP) is a mathematical optimization technique for solving problems where decision variables are restricted to integer values. The steps to solve ILP using the branch and bound method [39] are listed below:
Step 1: In the first iteration, generate the binary integer linear programming problem that gives all possible PMU placements. Solve the objective function for the initial problem and check if the results are integers:

- If yes, update the current best solution.
- If no, proceed to branching.

Step 2: Now, the iteration is incremented, and the decision variable having a value noninteger is used to make two subproblems, where the variable has a value of either 0 or 1. This process is repeated for each subproblem.
Step 3: Now, each subproblem is solved using LP relaxation. If this solution is worse than the current best integer solution, the branch is pruned; otherwise, the relaxed solution is better.
Step 4: Steps 2 and 3 are repeated for each subproblem. It is stopped when all subproblems are solved.
Step 5: During the given iteration, the best solution is updated if a better solution is found. The branch and bound process is updated based on the last best solution.
Step 6: Step 2 to 5 stop if the stopping criteria are met, i.e., the number of iterations exceeds the maximum iterations.
Step 7: During this process, the best integer solution gives the OPP adhering power system complete observability.

The flow chart of the proposed OPP is shown in Figure 4. It is a self-explanatory chart where data are loaded from Mat-Power [40] in MATLAB 2018a.All the necessary information about the network, contingency, ZIB buses, parameters, and constants was initialized. After initialization, the matrix A, X, U, and Y are extracted. The test bed system data helps to calculate each bus's degree and a normalized degree from Equations (1) and (2). The new objective function is introduced in Matlab using values of normalized degree of centrality and the observability constraint shown in Equations (4) and (5). Then, a check criterion is formed to evaluate for $N - K$ contingency. If contingency analysis is required, only the measurement vector is modified using Equation (8). After the $N - K$ contingency, a check is made on the availability of ZIBs. Suppose the effect of ZIBs needs to be addressed. In that case, the observability vector, connectivity matrix, and decision variable vector are modified using Equations (10)–(12). Then, the Matlab solver is used to solve the problem using the branch and bound technique. The results are displayed after solving OPP through the branch and bound technique of linear programming. These results are compared based on existing performance indices in the literature.

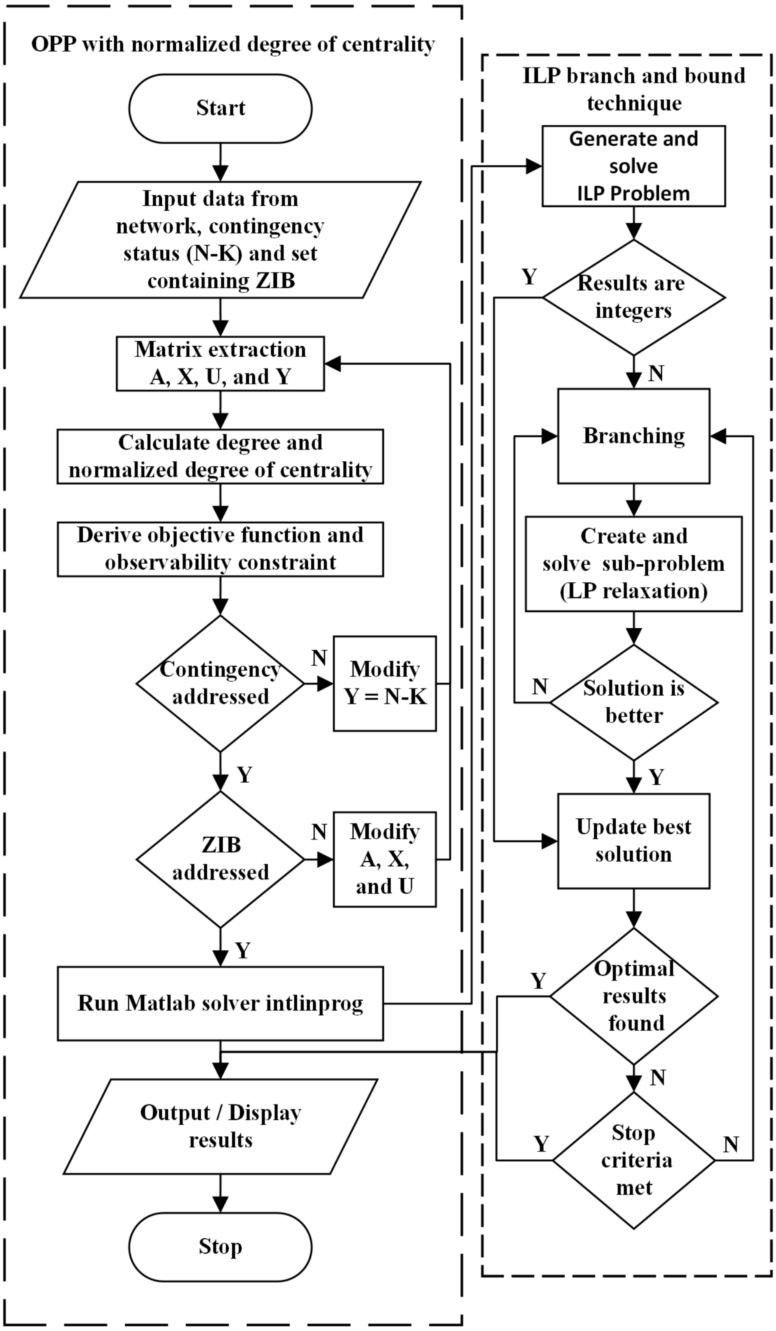

Figure 4. Flowchart of proposed cost-effective node-centric PMU placement.

4. OPP Evaluation

Solution sets achieved after implementing methods to solve the OPP problem are compared based on performance measures. These measures help researchers understand their strengths, weaknesses, trade-offs, and suitability for different application scenarios in power system operation and control. A few measures commonly used to evaluate these solution sets are discussed below:

4.1. Measure of Observability

A critical parameter to evaluate how well PMUs are monitoring the power network is the measure of observability. It gives information about bus visibility and its impact on network observability. It helps to quantify the observability of any bus after placing PMUs. The bus observability index (BOI) is the most commonly used observability measure.

A BOI is defined as the number of PMUs that observe data from a given bus [37]. The bus observability index for a bus i is calculated as follows:

$$BOI_i = \sum_{j=1}^{N} B_{ij} \tag{15}$$

This equation sums up the binary values in the i-th row of matrix B, representing the number of PMUs observing data from bus i. The maximum value of the bus observability index of any bus is $\mathrm{BOI}_{i(Max)}$, which is given by Equation (16), which means that maximum BOI is found if PMUs are placed at its connected buses. The addition of one implies that PMU is placed on that bus itself.

$$BOI_{i(Max)} = D_i + 1 \tag{16}$$

This index gives information about the monitoring capabilities of any PMU placed on a bus. A higher BOI means the PMU is well-positioned to monitor the network, providing redundant information for reliable operation. Considering the original IEEE 7-bus system and placing two PMUs at buses 2 and 4, as shown in Figure 2a, the matrix B is given below:

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}_{n \times N}$$

b_{ij} is a binary value indicating whether PMU j can observe data from bus i.

$$b_{ij} = \begin{cases} 1 & \text{If PMU } j \text{ can observe data from bus } i. \\ 0 & \text{Otherwise.} \end{cases}$$

In assessing the BOI for each bus, it is found that buses 1, 2, 4, 5, and 6 are observed by one PMU only, and buses 3 and 7 are observed by two PMUs, as can be seen in Figure 2a. So the BOI from bus 1 to bus 7 is $[1, 1, 2, 1, 1, 1, 2]$.

4.2. Measure of Redundancy

The sum of redundancy index (SORI), defined as cumulative redundancy observability achieved by placing PMUs in the network, is discussed in the literature [37]. It is the most popular index used to evaluate multiple solution sets of PMU placement based on redundant information provided by these solution sets. It is a metric that assesses the overall observability. Cumulatively, it quantifies the monitoring capability of PMUs placed among all buses. It is defined as the sum of BOI for each bus in the network.

$$SORI = \sum_{i=1}^{n} BOI_i \tag{17}$$

Solving Equation (17), the higher SORI value depicts a high redundancy level for a given PMU placement. A solution with high SORI is preferred when comparing different solution sets of PMU placement. As an example of the original IEEE 7-bus system, the SORI is calculated by adding the BOI of each bus. The BOI from bus 1 to bus 7 is $[1, 1, 2, 1, 1, 1, 2]$. SORI is found by adding the BOI of each bus and comes out to be 9.

4.3. Limitation of Performance Measures

The BOI and SORI are vital indices to quantify the redundancy of PMU placement. The BOI addresses the number of times a bus is observed, and the SORI addresses the total sum of observations for all buses. The limitation of these indices is that they only give system redundancy information and need to address whether the network is completely observed. It is worth noting that SORI is only adequate and comparable when the number of PMUs is the same when comparing solution sets of a given network. If the number of PMUs differs, then the SORI becomes an ineffective index.

5. Results and Discussion

PMU placement analysis provides a multifaceted approach to evaluating different results of strategies for placing PMUs. Various strategies include base case, contingency analysis, the effect of ZIBs, and the combined effect of contingency and ZIBs, as shown in Figure 5. Incorporating contingency enhances reliability, and the impact of ZIB makes the solution economical. These different cases are solved on the IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, and IEEE 118-bus systems.

A comparative analysis is made with previous works by Dua et al. [37], Hyacinth et al. [35], and Ahmadi et al. [41], focusing on PMU deployment and SORI metrics across various power system networks, offering valuable insights into PMU placement strategies' effectiveness and identifying areas for future exploration and refinement.

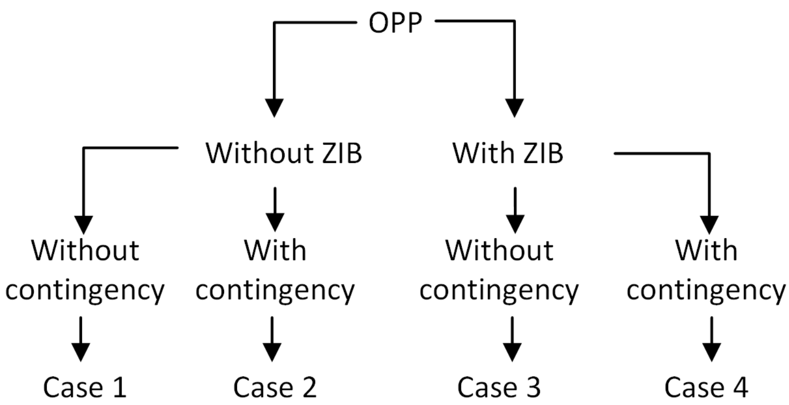

Figure 5. Four different cases in PMU placement.

5.1. Case I: Complete Network Observability in OPP

The base case is the placement of PMUs without considering any contingency and effect of ZIBs. It provides foundational insight into the network and is the basis for further analysis. The results of PMU location and BOI for different networks are given in Table 2.

The base case gives initial PMU placement and reveals insights into the foundational strategies to improve observability and control. The results of the base case are presented in Table 3. The proposed methodology for the base case results is better when compared with [37] integer linear programming (ILP), ref. [35] closed neighborhood search (CNS) and [41] binary particle swarm optimization (BPSO) techniques. The results are the same compared with integer linear programming proposed by Dua et al. in [37], except for the last network, the IEEE 118-bus system, where their SORI value is better. It is important to mention that verification of SORI value is not possible as BOI is not provided in [37].

Table 2. Exploration of PMU placement with complete observability constraint.

Bus System	PMU Location	BOI
IEEE 7	2, 4	1, 1, 2, 1, 1, 1, 2
IEEE 14	2, 6, 7, 9	1, 1, 1, 3, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1
IEEE 30	2, 4, 6, 9, 10, 12, 15, 20, 25, 27	1, 3, 1, 4, 1, 5, 1, 1, 3, 4, 1, 3, 1, 2, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1
New England 39	2, 6, 9, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29	1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 3, 2, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
IEEE 57	1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 41, 47, 50, 53, 57	2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1
IEEE 118	3, 5, 9, 12, 15, 17, 21, 25, 29, 34, 37, 40, 45, 49, 53, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	1, 1, 3, 1, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 3, 1, 2, 2, 1, 2, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 2, 1, 1, 3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 3, 2, 1, 2, 3, 1, 1, 3, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1, 3, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

Table 3. Comparison between ILP, CNS, BPSO, and proposed ILP results for the base case.

Complete Network Observability in OPP					
Bus System	**Parameters**	**[37] ILP**	**[35] CNS**	**[41] BPSO**	**Proposed ILP**
IEEE 7	PMUs SORI	- -	2 9	2 9	2 9
IEEE 14	PMUs SORI	4 19	4 19	4 19	4 19
IEEE 30	PMUs SORI	- -	10 50	10 52	10 52
New England 39	PMUs SORI	- -	- -	- -	13 52
IEEE 57	PMUs SORI	17 72	17 71	17 71	17 72
IEEE 118	PMUs SORI	32 164	32 156	32 148	32 162

5.2. Case II: Improving Monitoring Reliability in OPP

The second case is PMU placement, which considers reliability constraints by incorporating $N-1$ contingency of different networks. This case helps to adjust the PMU placement to ensure the robustness and reliability of monitoring. Table 4 gives PMU placement and BOI values of different networks under reliability constraints.

The contingency analysis gives insight into the system's reliable operation in case one or more PMUs are lost. This paper is confined to one device failure. The results reveal that the number of PMUs required increases compared with the base case. Despite the rise in PMUs when considering contingency into placement as a constraint, the benefit is that monitoring becomes more reliable. The results of the contingency analysis alone are presented in Table 5. Hyacinth et al. [35] and Ahmadi et al. [41] did not solve the contingency analysis, so the comparison between the proposed strategy and Dua et al. [37] reveals that the results obtained are the same.

Table 4. Exploration of PMU placement with complete network observability and reliability constraints.

Bus System	PMU Location	BOI
IEEE 7	1, 2, 3, 4, 5	2, 3, 3, 3, 2, 2, 2
IEEE 14	2, 4, 5, 6, 7, 8, 9, 11, 13	2, 3, 2, 5, 4, 4, 4, 2, 3, 2, 2, 2, 2, 2
IEEE 30	1, 2, 4, 5, 6, 9, 10, 11, 12, 13, 15, 17, 19, 20, 22, 24, 25, 26, 27, 28, 30	2, 5, 2, 4, 2, 6, 2, 2, 4, 6, 2, 4, 2, 2, 2, 2, 2, 2, 2, 3, 2, 3, 2, 3, 4, 2, 4, 3, 2, 2
New England 39	2, 3, 6, 8, 9, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39	2, 4, 2, 2, 2, 3, 2, 2, 3, 4, 3, 2, 3, 2, 2, 3, 2, 2, 4, 3, 2, 3, 3, 2, 4, 3, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2
IEEE 57	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 27, 28, 29, 30, 32, 33, 35, 36, 37, 38, 39, 41, 44, 46, 47, 50, 51, 53, 54, 56	2, 2, 3, 3, 2, 2, 2, 2, 3, 3, 3, 2, 4, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 3, 4, 4, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2
IEEE 118	1, 3, 5, 6, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 36, 37, 40, 42, 44, 45, 46, 49, 50, 51, 52, 54, 56, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 76, 77, 79, 80, 83, 85, 86, 87, 89, 91, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 115, 116, 117	2, 2, 4, 2, 4, 2, 2, 3, 2, 2, 3, 4, 2, 2, 3, 2, 4, 2, 3, 2, 2, 2, 4, 2, 2, 2, 4, 2, 2, 2, 4, 3, 2, 4, 2, 2, 3, 2, 2, 3, 2, 3, 2, 2, 4, 2, 2, 2, 7, 2, 3, 2, 2, 4, 3, 3, 2, 2, 4, 3, 4, 3, 2, 2, 3, 3, 2, 2, 5, 4, 3, 2, 2, 2, 3, 2, 4, 2, 2, 4, 2, 3, 2, 2, 4, 3, 2, 2, 3, 2, 2, 5, 2, 4, 2, 3, 2, 2, 2, 5, 2, 2, 3, 2, 3, 3, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2

Table 5. Comparison of ILP, CNS, BPSO, and proposed ILP results for N-1 contingency.

OPP with Complete Observability and Reliability Constraint					
Bus System	**Parameters**	**[37] ILP**	**[35] CNS**	**[41] BPSO**	**Proposed ILP**
IEEE 7	PMUs SORI	- -	- -	- -	5 17
IEEE 14	PMUs SORI	9 39	- -	- -	9 39
IEEE 30	PMUs SORI	- -	- -	- -	21 85
New England 39	PMUs SORI	- -	- -	- -	28 52
IEEE 57	PMUs SORI	33 130	- -	- -	33 130
IEEE 118	PMUs SORI	68 309	- -	- -	68 309

5.3. Case III: Cost Reduction with ZIBs in OPP

The third case deals with the effect of ZIBs without considering reliability constraints. ZIBs further reduce the number of PMUs when compared with the base case. Subsequent Table 6 gives information about PMU location and BOI value in different networks when considering the operational challenges of ZIB without compromising network observability.

Analysis under the ZIB constraint reveals that the number of PMUs reduce as compared with the base case. It helps to reduce the overall cost of the devices and installation costs. Although the problem becomes more complex, addressing the ZIB effect is worthwhile, as it reduces overall cost. The results of the ZIB constraint are presented in Table 7. For the modified IEEE 7-bus system, only one PMU is needed, and in the original IEEE 7-bus system, there is no effect on the number of PMUs compared with the base case. The remaining networks' overall number of PMUs is reduced compared with the base case in Table 3. The proposed strategy improves the overall SORI while considering complete network observability for IEEE 14-bus, IEEE 30-bus, and IEEE 118-bus systems. For the

IEEE 57-bus system, the number of PMUs is reduced compared with Dua et al. [37] and Ahmadi et al. [41]. These improved results enhance the reliability of the network monitoring and reduce the overall cost.

Table 6. Exploration of PMU placement with complete network observability and ZIB constraints.

Bus System	PMU Location	BOI
IEEE 7	2, 4	1, 1, 2, 1, 1, 1, 2
IEEE 14	2, 6, 9	1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1
IEEE 30	2, 4, 10, 12, 15, 18, 27	1, 2, 1, 3, 1, 3, 1, 1, 1, 1, 1, 3, 1, 2, 3, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
New England 39	2, 8, 12, 16, 20, 23, 25, 29	1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
IEEE 57	1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1
IEEE 118	3, 9, 11, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 59, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 2, 3, 2, 2, 2, 2, 3, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 3, 3, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 1, 3, 2, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 3, 1, 1, 2, 1, 1, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1

Table 7. Comparison of ILP, CNS, BPSO, and proposed ILP results for ZIB only.

OPP with Complete Observability and ZIB Constraint					
Bus System	**Parameters**	**[37] ILP**	**[35] CNS**	**[41] BPSO**	**Proposed ILP**
IEEE 7	PMUs SORI	- -	- -	2 9	2 9
IEEE 14	PMUs SORI	3 15	- -	3 16	3 16
IEEE 30	PMUs SORI	- -	- -	7 34	7 41
New England 39	PMUs SORI	- -	- -	- -	8 44
IEEE 57	PMUs SORI	14 61	- -	13 64	11 61
IEEE 118	PMUs SORI	29 152	- -	29 155	29 161

5.4. Case IV: Integrated Analysis of Complete Observability, Reliability, and ZIB in OPP

The last case deals with reliability constraint $N-1$ contingency and the effect of ZIBs. A comprehensive analysis of reliable and economized solutions in terms of contingency and ZIB, underscoring the commitment to system reliability and resilience, is presented in Table 8.

Table 8. Exploration of PMU placement with complete network observability, reliability, and ZIB constraints.

Bus System	PMU Location	BOI
IEEE 7	1, 2, 4, 5	2, 2, 2, 2, 2, 1, 2
IEEE 14	2, 4, 5, 6, 9, 10, 13	2, 3, 2, 4, 4, 3, 2, 1, 3, 2, 2, 2, 2, 2
IEEE 30	1, 2, 4, 7, 10, 12, 13, 15, 17, 19, 20, 24, 27	2, 3, 2, 3, 2, 4, 1, 1, 1, 3, 1, 4, 2, 2, 2, 2, 2, 2, 2, 3, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1
New England 39	2, 6, 8, 13, 16, 20, 23, 25, 26, 29, 34, 36, 37, 38	1, 2, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 2, 2, 4, 3, 1, 2, 3, 1, 1, 1, 1, 2, 1, 2, 2, 2, 1
IEEE 57	1, 3, 4, 9, 12, 14, 15, 18, 20, 25, 27, 29, 30, 32, 33, 36, 38, 41, 50, 51, 53, 54, 56	2, 2, 3, 3, 1, 1, 1, 1, 2, 3, 2, 2, 4, 2, 4, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 2, 2, 2, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 1
IEEE 118	1, 3, 6, 8, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 31, 32, 34, 35, 40, 42, 44, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 66, 69, 70, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117	2, 2, 3, 1, 4, 1, 2, 1, 1, 1, 2, 4, 2, 2, 3, 2, 3, 2, 3, 2, 2, 2, 4, 2, 2, 1, 3, 2, 2, 2, 4, 4, 1, 2, 1, 2, 3, 1, 1, 2, 2, 3, 2, 2, 4, 2, 3, 2, 8, 2, 3, 2, 2, 4, 3, 3, 2, 2, 3, 2, 2, 2, 1, 1, 1, 3, 2, 1, 5, 4, 1, 1, 1, 2, 4, 2, 6, 2, 2, 3, 1, 3, 2, 2, 4, 3, 2, 2, 4, 2, 2, 4, 2, 4, 2, 3, 2, 2, 2, 5, 2, 2, 3, 2, 3, 3, 2, 2, 2, 3, 2, 2, 2, 2, 2, 1, 2, 2

Analysis of both contingency and ZIB makes the problem more complex but helps in reliable monitoring and reduced cost. When ZIB is addressed with contingency, the system is wholly observed with fewer PMUs than the contingency analysis alone, as shown in Table 5. The results of contingency and ZIB are presented in Table 9. It is seen that for the results of the proposed strategy, when compared with Dua et al. [37], the number of PMUs is the same for the small networks, and the number of PMUs is reduced for the large network IEEE 57-bus and IEEE 118-bus systems while ensuring complete network observability. The simulations are run on the computer with the following specifications:

- CPU Intel(R) Core(TM) i7-5500U CPU @ 2.40GHz;
- Level L1 cache: 128 KB, L2 cache: 512 KB, L3 cache: 4.0 MB;
- Memory: 12.0 GB DDR3.

The solver-based approach is used to solve the PMU placement problem, and for the most extensive network IEEE 118 test bed, the simulation time for case 1 is 2.50 s, and for case 2, it is 2.61 s. For case 3, it is 2.47 s, and for case 4, it is 2.88 s.

The main findings of this paper yield that the proposed methodology is superior and gives improved results in terms of the number of PMUs and SORI. As for OPP with observability and ZIB constraint, they have improved results, as can be seen in Table 7; for the IEEE 57-bus system, the number of PMUs is reduced from 13 to 11, and for the IEEE 30- and IEEE 118-bus systems, the value of SORI is improved from 34 to 41 and from 155 to 161, respectively. The results of the OPP problem with combined observability, reliability, and ZIB constraints also have improved results using the proposed methodology, as seen in Table 9. It is shown that for the IEEE 57-bus and IEEE 118-bus systems, the number of PMUs is reduced from 29 to 23 and 64 to 59, respectively. For the IEEE 14-bus system, the value of SORI is improved from 33 to 34.

This paper focuses on two important parameters, reliability and economic aspects in PMU placement, and the results obtained using the proposed methodology are improved in terms of minimizing the number of PMUs and maximizing network observability, as can be compared through measures of observability and redundancy. The weakness of this paper is that it has compared the observability results based on existing measures only. These measures fail regarding information redundancy when the number of PMUs is not equal when comparing two placement results on the same network. There is a need to propose a new index that works well with two placement results when the number of PMUs is not equal for the same network.

Table 9. Comparison of ILP, CNS, BPSO, and proposed ILP results for N-1 contingency and ZIB.

OPP with Complete Observability, Reliability and ZIB Constraint					
Bus System	**Parameters**	**[37] ILP**	**[35] CNS**	**[41] BPSO**	**Proposed ILP**
IEEE 7	PMUs	-	-	-	4
	SORI	-	-	-	13
IEEE 14	PMUs	7	-	-	7
	SORI	33	-	-	34
IEEE 30	PMUs	-	-	-	13
	SORI	-	-	-	57
New England 39	PMUs	-	-	-	14
	SORI	-	-	-	58
IEEE 57	PMUs	29	-	-	23
	SORI	113	-	-	97
IEEE 118	PMUs	64	-	-	59
	SORI	297	-	-	280

6. Conclusions

This study presents a novel approach to the PMU placement problem, encompassing cost minimization and critical bus identification as a single problem in the objective function. Unlike previous approaches to minimize cost only, this paper addresses a new formulation where normalized degree centrality is used as a part of the problem. Four cases, base case, contingency analysis, the effect of ZIB, and the combined effect of contingency and ZIB, are solved for six test bed systems. The networks used as a test bed are the IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, New England 39-bus, IEEE 57-bus, and IEEE 118-bus systems. The integer linear programming is used to solve this problem. The main findings of this paper show that the proposed methodology is superior and gives improved results in terms of the number of PMUs and SORI. This research contributes to existing knowledge by introducing a methodology that significantly reduces the number of PMUs required for effective network observability.

However, it is essential to acknowledge the limitations of this paper. In all cases, results are either better or the same, except for the base case IEEE 118-bus system, where Dua et al.'s [37] SORI is better. There is also a limitation of indices available in the literature on BOI and SORI; they fail to guarantee complete observability of the network, which is the main requirement to solve the OPP problem. Furthermore, SORI fails when comparing two or more solution sets with differing PMUs for the same network with the same conditions. It will be more for the solution among all situation sets with more PMUs. It is against OPP requirements. In the future, there is ample scope to introduce performance indices that overcome the limitations of existing indices and develop comprehensive methodologies to increase the effectiveness of PMU placement strategies.

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

δ_{ij}	Kronecker delta, which equals 1 if $i = j$ and 0 otherwise.
$\mathcal{Z}$	Set of indices for zero injection busses.
ζ_i	Normalized degree of centrality of bus i.
ζ_{max}	Highest value of normalized degree centrality observed among all the buses.
ζ_{min}	Lowest value of normalized degree centrality observed among all the buses.
A	Binary connectivity matrix.
A_m	Modified binary connectivity matrix.
a_{ij}	Binary connectivity parameter between buses i and j.
B	Network observability matrix.
b_{ij}	Binary variable indicator to verify if PMU j can observe bus i.
$BOI_{i(Max)}$	Maximum value of the bus observability index of bus i.
BOI_i	Bus observability index of bus i.
C	Constant, cost of PMU device.
D_i	Degree of bus i.
E	Set of edges.
e_i	Edge representing line i.
K	Total number of failed measurement devices.
m	Total number of lines.
N	Total number of phasor measurement units.
n	Total number of buses.
P_i	Set of buses adjacent to the selected bus i.
Q_i	Set formed by union of set P and selected bus i.
$SORI$	Sum of redundancy index.
U	Set of bus observability status variable.
u_i	Bus observability status variable.
V	Set of vertices.
v_i	Vertex representing bus i.
v_j	Vertex representing bus j.
X	Set of decision variables.
x_i	Decision variables indicating the PMU placement status (1 / 0) for bus i.
X_m	Modified set of decision variables.
Y	Measurement vector.

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