

Article

Enhancing Heat Storage Cooling Systems via the Implementation of Honeycomb-Inspired Design: Investigating Efficiency and Performance

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Abstract: This study presents a novel approach inspired by the hexagonal honeycomb structure found in nature, leveraging image processing algorithms to precisely define complex geometries in thermal systems. Hexagonal phase change material containers and thermally conductive fins were meticulously delineated, mirroring the intricate real-world designs of honeycombs. This innovative methodology not only streamlines setup processes but also enhances our understanding of melting dynamics within enclosures, highlighting the potential benefits of biomimetic design principles in engineering applications. Two distinct honeycomb structures were employed to investigate their impact on the melting process within cavities subject to heating from the left wall, with the remaining walls treated as adiabatic surfaces. The incorporation of a thermally conductive fin system within the enclosure significantly reduced the time required for a complete phase change, emphasizing the profound influence of fin systems on thermal design and performance. This enhancement in heat transfer dynamics makes fin systems advantageous for applications prioritizing precise temperature control and expedited phase change processes. Furthermore, the critical role of the fin system design was emphasized, influencing both the onset and location of the final point of melting. This underscores the importance of tailoring fin systems to specific applications to optimize their performance. Our study highlights the significant impact of the Rayleigh (Ra) number on the melting time in a cavity without fins, revealing a decrease from 6 to 0.4 as the Ra increased from 10^2 to 10^5 ; the introduction of a fin system uniformly reduced the melting time to $Ste.Fo = 0.5$, indicating fins' universal effectiveness in optimizing thermal dynamics and expediting the melting process. Moreover, the cavity angle was found to significantly affect the fluid fraction diagram in unfanned cavities but had minimal impact when fins were present, highlighting the stabilizing role of fins in mitigating gravitational effects during melting processes. These insights expand our understanding of cavity geometry and fin interactions in heat transfer, offering potential for enhanced thermal system designs in various engineering applications. Decreasing thermal conductivity (λ) by increasing the fin thickness can halve the melting time, but the accompanying disadvantages include a heavier system and reduced energy storage due to less phase change material, necessitating a careful balance in decision-making.

Keywords: lattice Boltzmann method; phase change material; honeycomb structure



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1. Introduction

Due to their distinct ability to store and release thermal energy during phase transitions, phase change materials (PCMs) play a critical role in modern heat storage systems [1]. PCMs offer an efficient means of managing and optimizing thermal energy storage as the demand for energy rises and sustainable solutions become imperative [2]. PCMs maintain a constant temperature within a predetermined range by accumulating heat during the phase change from solid to liquid and releasing it during the reversal [3]. This capability is especially useful in applications such as solar thermal systems [4], where PCMs can store

excess solar energy during the day and discharge it during the night when temperatures drop. PCMs also improve energy efficiency by reducing temperature fluctuations [5] and preventing overheating [6] or refrigeration [7], thereby extending equipment life [8] and reducing energy consumption [9]. PCMs are crucial to the development of sustainable heat storage solutions for a greener future due to their adaptability to various environments, simplicity of integration, and potential for use with renewable energy sources [10].

Simulation methods have emerged as indispensable instruments for optimizing the performance of PCM-based systems and gaining a deeper understanding of their operation [11]. These simulation techniques enable scientists and engineers to model and analyze the intricate heat transfer and phase change phenomena that occur in PCM systems. Utilizing computational methods such as the Finite Volume Method (FVM) [12], Molecular Dynamics (MD) [13], and lattice Boltzmann method (LBM) [14], researchers are able to investigate the thermal behavior, transient responses, and overall efficacy of PCM-based systems under different conditions.

Numerous scholars have investigated different geometries in recent years to model the effects of phase change materials (PCMs) in various engineering applications using computational fluid dynamics (CFD) [15–20]. CFD approaches provide a potent tool for deciphering intricate heat and mass transport processes in PCM systems, and they are often applied using commercial software platforms like ANSYS CFD and COMSOL Multiphysics [21]. These studies include a broad variety of topics, such as energy storage [22], improved heat transport, thermal management [23], and more. Researchers are working to improve the performance of PCMs by using various geometries, including finned structures [24], enclosed containers [25], and composite materials [26].

The lattice Boltzmann method (LBM) has emerged as a versatile and powerful numerical technique for simulating complex fluid flow and heat transfer phenomena, making it a valuable tool in studying phase change material (PCM) melting and their interactions with various factors [27]. The LBM has distinct advantages when it comes to capturing intricate fluid–solid phase transitions, and it is well suited to dealing with complex geometries [28] and boundary conditions [29]. This method allows for a thorough examination of the dynamic interactions between fluid flow, heat transfer, and phase change phenomena in PCM systems. The LBM's distinct capabilities make it an essential approach for understanding the underlying mechanisms governing PCM melting processes and optimizing their use in a wide range of engineering applications.

The work by Huber et al. [30] introduces a pioneering lattice Boltzmann method that offers a fresh perspective on the intricate interplay between thermal convection and the melting process of pure substances. Their innovative approach adeptly captures the nuanced shift from heat transfer being primarily driven by conduction to the emergence of fully developed convection.

Wang et al. [31] introduced an innovative lattice Boltzmann method (LBM) model that encompasses a novel equilibrium distribution function. This pioneering approach enabled them to effectively solve complex two-dimensional (2D) and three-dimensional (3D) geometries associated with phase change phenomena.

Wei and Karimi [32] conducted a study using the lattice Boltzmann method (LBM) to investigate the impact of nanoparticles and expanded fins on the simulation of phase change material (PCM) behavior in a thermal storage device. Their findings indicated that the addition of nanoparticles, ranging from 0% to 3%, led to a decrease in the melting time. Similarly, increasing the number of fins in the system produced a similar outcome, highlighting the potential for enhanced heat transfer and accelerated melting processes.

Li and Yu [33] employed a three-dimensional lattice Boltzmann method to analyze the impact of fins on the phase change material melting process. They conducted a comprehensive parametric investigation, exploring different fin configurations while maintaining a constant total fin volume. Their study delved into the intricate interplay of fin arrangements with natural convection patterns, melting time, and interface area. Notably, the research highlighted that altering the fin structures could yield enhanced heat transfer rates and

improved energy storage capabilities, outperforming the simple strategy of increasing the number of fins.

Addressing the intrinsic limitation of PCMs' low thermal conductivity presents a considerable challenge in facilitating efficient heat transfer during the melting or solidification phases, consequently affecting the overall efficacy of energy storage solutions. As a response, a diverse array of strategies has emerged to augment the heat transfer capabilities of PCMs. These strategies encompass the utilization of various techniques such as incorporating metal foams [34], integrating nanoparticles [35,36], employing carbon nanotubes [36], introducing additional internal fins [37], as well as exploring the potential benefits of ultrasonic and magnetic fields [38]. These distinct methodologies collectively strive to enhance the heat transfer performance, thereby optimizing the effectiveness of thermal energy storage systems relying on PCMs.

Darzi et al. [39] addressed the low thermal conductivity of PCMs by proposing thermal fins in a cylindrical annulus setup. They investigated the fin geometry, quantity, and nanoparticle integration to enhance heat transfer, offering valuable insights for improved PCM-based thermal management applications.

Ren et al. [40] employed an enthalpy-based LBM model to simulate PCM melting enhanced by nanoparticles and fins. The study found that triangular fins outperformed nanoparticles in accelerating melting. At higher wall temperatures, an increased nanoparticle volume fraction decreased the LHTES energy storage efficiency due to the reduced capacity and heightened PCM viscosity, weakening natural convection. In another study, Ren et al. [41] explored electronic thermal management utilizing PCM-EG composites and pin fins. They extensively optimized the system under various parametric conditions to enhance its performance.

Zhang et al. [42] investigated Latent Heat Thermal Energy Storage (LHTES) systems with varied fin length ratios and no fins. Using a composite PCM of paraffin and copper oxide, they used an enthalpy-based LBM model to simulate phase change. Compared to finless designs, unequal fin lengths reduced the melting times by 16.7% to 33.06%, and solidification times by 47.33% to 57.54%. In terms of the overall heat storage and release, equal-length fins were found more effective.

Alizadeh et al. [43] conducted a numerical investigation on the solidification of a PCM within a hexagonal storage unit. They studied the impact of innovative branching fins and nanoparticles as direct and indirect heat transfer augmentation techniques. The study revealed a 3% reduction in release exergy efficiency when increasing the HTF temperature from 240 K to 260 K. Furthermore, enhancing the amplitude and duration of the cold wall was found to accelerate the solidification process.

Mostafa et al. [44] aimed to enhance the PCM melting efficiency in storage via the use of novel fins, combining rectangular and triangular shapes. They investigated the influence of triangular fin height on the melting rate, and explored the effects of various storage hot wall temperatures and PCM materials on the process.

Rapid PCM melting is crucial for applications like thermal energy storage, electronics cooling, and ice/snow removal. Fins enhance the heat transfer efficiency, crucial for quick melting, by increasing the surface area and promoting uniform heat distribution within the PCM material, benefiting systems requiring a swift phase change.

Honeycomb structures are advantageous in heat transfer due to their high surface-area-to-volume ratio, promoting efficient heat exchange and enhanced convection. They offer reduced weight, structural integrity, and uniform heat distribution, making them suitable for applications requiring both optimal thermal performance and structural support. Honeycomb structures also act as effective heat shields, offer low pressure drop, and improve the heat exchanger efficiency by facilitating convective heat transfer and large surface areas for fluid interaction. Additionally, they can provide acoustic damping properties. Overall, honeycomb structures offer a versatile solution for various heat transfer applications, including aerospace, automotive, electronics cooling, and thermal management.

Introducing a novel approach, this study draws inspiration from the hexagonal honeycomb structure seen in nature to enhance our proposed thermal system. We employ image processing algorithms to precisely define complex geometries, such as hexagonal phase change material (PCM) containers and thermally conductive fins, simulating intricate real-world designs. This innovative methodology not only streamlines the setup process but also contributes to a deeper understanding of the melting process within the enclosure, showcasing the potential benefits of biomimetic design principles in engineering applications.

In this research paper, our primary objective is to comprehensively explore the influence of hexagonal honeycomb structure fins on the phase change material (PCM) melting process. To achieve this goal, we have organized the paper into distinct sections to facilitate a clear and coherent presentation of our study. Commencing with a comprehensive background section, we provide the necessary context and foundational information relevant to our investigation. Following this, we delve into a detailed exposition of our methodology, elucidating the approach and techniques employed in our experimental design. The subsequent sections of the paper are dedicated to presenting the results obtained and conducting a thorough analysis of the data. Finally, we encapsulate our findings with a concluding section that not only summarizes the key outcomes but also engages in a thoughtful discussion regarding the broader implications and significance of our research within the relevant academic and practical domains.

2. Honeycomb Cooling Structure

Geometry

Inspired by bee honeycomb, we adopted a hexagonal structure for our study. The choice is rooted in the inherent benefits demonstrated by hexagonal cells in facilitating smooth movement, efficient use of space, and robust structural support. By emulating nature's design principles, we aim to enhance the performance of our proposed system.

In this study, two different honeycomb structures are used. In both cases, a hexagonal honeycomb structure is employed to examine its impact on the melting process within a cavity that experiences heating solely from its left wall, while the remaining walls are treated as adiabatic surfaces. Figure 1 illustrate the geometry adopted for this research. The fins in this configuration are presumed to be composed of a thermally conductive metal capable of efficiently distributing heat from the heat source to all regions within the cavity. The hexagonal cells contain PCMs. L_H represents the width of the hexagonal cells and L_F represents the width of the fins.

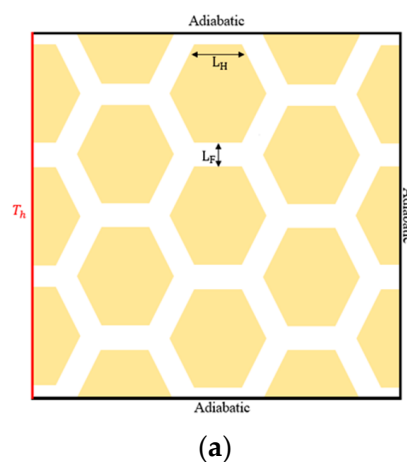


Figure 1. Cont.

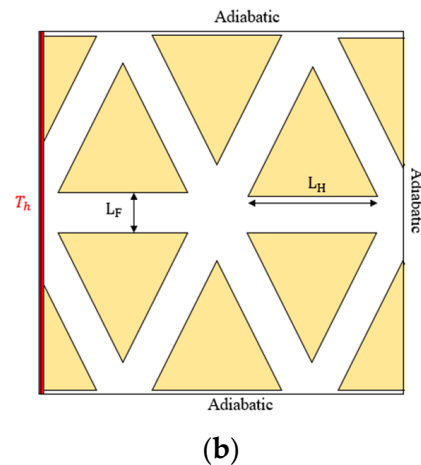


Figure 1. Visual representation of the utilized geometry in this study: (a) case 1 and (b) case 2.

To establish the parameters for the honeycomb phase change material (PCM) containers and fins within the newly devised code, an image processing algorithm is employed. This algorithm is designed to distinguish and classify regions within the image, assigning a value of zero to PCM zones and a value of one to solid regions. This technique aids in accurately demarcating the boundaries and characteristics of these components. By incorporating this algorithm, a precise and automated approach is achieved for delineating PCM containers and fins, enhancing the efficiency and accuracy of the overall process. The computer code utilized in this study includes a Python program for image processing algorithms, and FORTRAN code specifically developed for solving the lattice Boltzmann method (LBM) equations.

λ can be defined as the ratio between the PCM space and domain.

$$\lambda = \frac{S_{PCM}}{S_{total}}, \quad (1)$$

Table 1 displays the thermophysical properties of the phase change material (PCM).

Table 1. Thermophysical properties of phase change material (PCM).

Properties	Unit	Paraffin-Wax
c_p	(J/kg·K)	2000
ρ	(kg/m ³)	800
k	(W/mK)	0.2
L_f	(J/kg)	275
$\alpha = \frac{k}{\rho c_p}$	(m ² /s)	1.25×10^{-7}

3. Governing Equations

3.1. Lattice Boltzmann Method

The geometry described in Figures 1 and 2 contains two distinct zones which are the PCMs and metal fins, for which Navier–Stokes equations should be solved separately. For the PCM zone, momentum and energy equations should be solved, while for the metal fin zone, only an energy equation should be solved.

The equations for the PCM zone are provided below:

$$\nabla \cdot \vec{u}^* = 0, \quad (2)$$

$$\frac{\partial \vec{u}^*}{\partial t^*} + \vec{u}^* \cdot (\nabla \cdot \vec{u}^*) = -\nabla p^* + \text{Pr} \nabla^2 \vec{u}^* + \text{Pr} \text{Ra} \beta T^* \quad (3)$$

$$\frac{\partial T^*}{\partial t} + \vec{u}^* \cdot (\nabla T^*) = \alpha \nabla^2 T^* - \frac{1}{Ste} \frac{\partial h_l}{\partial t^*} \quad (4)$$

All variables are defined in the nomenclature section. These equations can be recovered using the lattice Boltzmann method. The lattice Boltzmann method (LBM) is a numerical technique widely utilized for simulating complex phenomena at the mesoscale, particularly in fluid dynamics. Employing a kinetic approach based on the Boltzmann equation, the LBM discretizes distribution functions on a regular lattice grid. Notable for its simplicity, scalability, and versatility, the LBM excels in handling large-scale problems via easy parallelization in high-performance computing environments. Its grid-based structure allows for the efficient representation of intricate geometries and irregular boundaries, making it suitable for multiphase flows. Operating at the mesoscale, the LBM proves advantageous for studying phenomena in diverse fields such as materials science, microfluidics, and biological systems. Its accessibility, owing to its straightforward implementation and clear physical interpretation, extends its utility to researchers across various disciplines.

Momentum:

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = f_i(\mathbf{x}, t) + \frac{\Delta t}{\tau_v} [f_i^{eq}(\mathbf{x}, t) - f_i(\mathbf{x}, t)] + \Delta t c_i F_i, \quad (5)$$

Energy:

$$g_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = g_i(\mathbf{x}, t) + \frac{\Delta t}{\tau_D} [g_i^{eq}(\mathbf{x}, t) - g_i(\mathbf{x}, t)] - w_i \frac{1}{Ste} (h_l^{n,k} - h_l^{n-1}), \quad (6)$$

where:

$$Ste = \frac{c_p \Delta T}{L_f} \quad (7)$$

To solve these equations, equilibrium distribution functions should be defined.

$$f_i^{eq} = w_i \rho \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{1}{2} \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{c_s^4} - \frac{1}{2} \frac{\mathbf{u}^2}{c_s^2} \right] \quad (8)$$

$$g_i^{eq}(\mathbf{x}, t) = w_i T \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} \right] \quad (9)$$

To recover the N-S equations, τ_v and τ_D should be defined as functions of viscosity and thermal diffusivity.

$$\tau_v = 3\nu + 0.5 \quad (10)$$

$$\tau_D = 3\alpha + 0.5 \quad (11)$$

It is important to emphasize that in the computational framework being utilized, the momentum equation is meticulously solved for employing a D2Q9 lattice structure. In parallel, the energy equation is addressed within the confines of a D2Q5 lattice arrangement. w_i and c_i are defined as:

$$\mathbf{c}_i = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \end{bmatrix} \quad (12)$$

$$w_i = \left[\frac{4}{9} \quad \frac{1}{9} \quad \frac{1}{9} \quad \frac{1}{9} \quad \frac{1}{9} \quad \frac{1}{36} \quad \frac{1}{36} \quad \frac{1}{36} \quad \frac{1}{36} \right] \quad (13)$$

F_i is representative of external forces which apply to the fluid domain such as gravity.

$$\vec{F} = \vec{F}_x + \vec{F}_y = -3w_i \rho \vec{g}_y \quad (14)$$

$\vec{g}_y$ can be calculated using the definition of the Ra number, which is defined as follows:

$$Ra = \frac{\vec{g} \beta (T_h - T_c) H^3}{\alpha \nu} \quad (15)$$

For an inclined cavity, $\vec{g} = g \cos(\theta)$ where θ is the angle of the cavity.

The term $w_i \frac{1}{Ste} (h_l^{n,k} - h_l^{n-1})$ in the energy equation is added to simulate the PCM melting process, for which a detailed description can be found in Ref. [45]. $h_l^{n,k}$ is the fluid fraction at time step n and iteration k in the i direction and can be updated as follows [46]:

$$f_l^{n,k} = \begin{cases} 0 & En^{n,k} < En_s = c_p T_m & \text{solid zone} \\ \frac{En^{n,k} - En_s}{En_l - En_s} & En_s \leq En^{n,k} \leq En_s + L_f & \text{mushy zone} \\ 1 & En^{n,k} > En_s + L_f & \text{liquid zone} \end{cases} \quad (16)$$

The fluid fraction, ranging from 0 to 1, is determinable based on the enthalpy value, enabling the automated calculation of the PCM front. The local enthalpy at time step n and iteration k can be computed as follows:

$$E_l^{n,k} = c_p T^{(n,k)} + L_f h_l^{(n,k-1)} \quad (17)$$

Solving Equations (6) and (7) will provide us with the opportunity to calculate the density, velocity, and temperature in the geometry using Equations (13)–(15).

$$\rho = \sum_i f_i \quad (18)$$

$$u_j = \sum_i f_i c_{ij} \quad (19)$$

$$T = \sum_i g_i \quad (20)$$

In the fin zone, the momentum equation will not be solved. The energy equation in the solid zone is presented in Equation (16):

$$\frac{\partial T^*}{\partial t} = \alpha_{fin} \nabla^2 T^* \quad (21)$$

To recover this equation, the following LBM equations are solved:

$$g'_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) = g'_i(\mathbf{x}, t) + \frac{\Delta t}{\tau'_D} [g_i^{eq}(\mathbf{x}, t) - g'_i(\mathbf{x}, t)] \quad (22)$$

where:

$$g_i^{eq}(\mathbf{x}, t) = w_i T \quad (23)$$

$$\tau'_D = 3\alpha_{fin} + 0.5 \quad (24)$$

Finally, the temperature in the fin area can be found using Equation (20).

$$T_{fin} = \sum_i g_i \quad (25)$$

3.2. Boundary Conditions

To model the lattice Boltzmann method (LBM) simulation for the phase change material (PCM) melting scenario, significant reliance is placed on the specification of the boundary conditions. The following outlines the boundary conditions employed for momentum and energy considerations.

Momentum:

In this simulation, all walls are considered to be stationary. To impose a no-slip boundary condition on the walls, a bounce-back boundary condition is utilized. Mathematically, this condition is expressed as:

$$f_i(\vec{x}, t) = f_{\tilde{i}}(\vec{x}, t) \quad (26)$$

in which $\tilde{i}$ is the opposite direction of i .

Energy:

For the energy equation, it is notable that interaction between the fins and PCMs should not be considered as a boundary because it is an interface wall, and instead two different energy equations should be solved for them. For the hot wall, we have:

$$\begin{aligned} g_1 &= T_h(w_1 + w_3) - g_3 \\ g_5 &= T_h(w_5 + w_7) - g_7 \\ g_8 &= T_h(w_8 + w_6) - g_6 \end{aligned} \quad (27)$$

And the rest of walls are adiabatic where:

$$g_{6,n} = g_{8,n}, \quad g_{5,n} = g_{7,n}, \quad g_{2,n} = g_{4,n} \quad (28)$$

3.3. Image Processing Algorithms

To employ image processing algorithms for importing the geometry into the lattice Boltzmann method (LBM) in Python, the OpenCV library proves to be a valuable tool. This methodology involves preparing an image that represents the fin system, where solid nodes are depicted in black and fluid nodes in white. The algorithm employed in this process typically converts black nodes into the numerical value '1' and white nodes into '0'. This binary representation simplifies the subsequent conversion of the image into a matrix. In the next steps of the workflow, this matrix, now representing the spatial distribution of solid and fluid nodes, is utilized in the LBM calculations. The lattice Boltzmann method, often used for fluid dynamics simulations, relies on a lattice-based grid where each node carries information about fluid density and velocity. By leveraging the capabilities of the OpenCV library, the image processing algorithm facilitates the seamless transition from a visual representation of the fin system to a numerical matrix that aligns with the requirements of the LBM. This integration of image processing and numerical simulation techniques plays a crucial role in accurately modeling and simulating fluid flows around complex geometries, such as fin systems, in a computationally efficient manner. The utilization of OpenCV in this context highlights the interdisciplinary nature of computational fluid dynamics, combining expertise in image processing with numerical methods to gain insights into the fluid behavior around intricate geometrical structures. This approach not only enhances the accuracy of simulations but also contributes to the broader field of fluid dynamics research. Figure 2 shows the image processing algorithm.

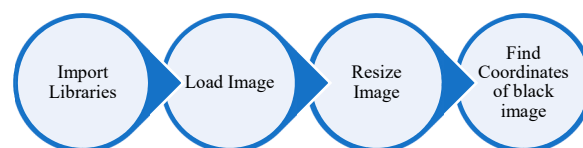


Figure 2. Image processing algorithm.

4. Grid Study

Based on an evaluation on the fluid fraction, different grid sizes are tested and confirmed to ensure that the solution's accuracy is not affected by the grid size chosen. As shown in Figure 3, there is a noticeable variation in melt fractions when using grid sizes of 50×50 , 100×100 , 150×150 , 200×200 , and 250×250 . However, the largest difference between a 200×200 grid and a 250×250 grid was reduced to just 2%. Consequently, we decided to use a 200×200 grid configuration for this study.

Ste.Fo is defined as a dimensionless time in our study:

$$t^* = Ste.Fo = \frac{c_p \Delta T}{L_f} \times \frac{t \alpha}{H^2} \quad (29)$$

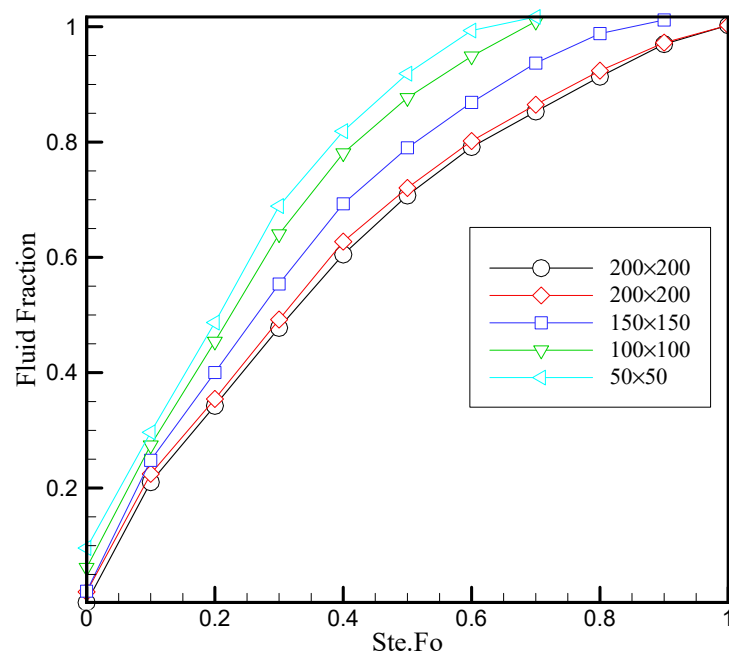


Figure 3. Grid study on different lattice sizes (50×50 , 100×100 , 150×150 , 200×200 , and 250×250).

Figure 4 provides a visual representation of the specifically selected grid configuration utilized in case 1, affirming our commitment to an optimal grid size that ensures both computational efficiency and solution accuracy in our simulation endeavors.

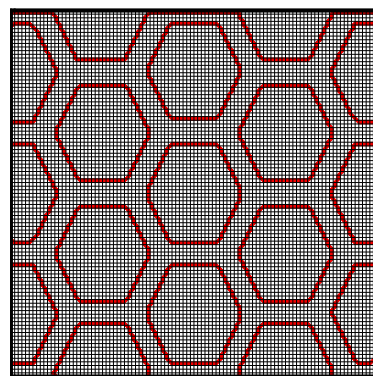


Figure 4. Used grid in simulation of case 1.

5. Validation

A validation study was conducted to assess the accuracy and reliability of newly designed numerical code in simulating the natural convection of phase change materials inside a finned cavity. The isothermal lines at a Rayleigh number (Ra) of 1.7×10^4 are reported in Figure 5 which is compared with the findings of Jourabian et al. [45]. The cavity in this article has been equipped with a metal fin that is attached to the heated wall of the cavity, namely located on the left side wall. The findings demonstrate a high level of concurrence and validate the implemented computer code.

The simulation results for a finned cavity, employing an image processing algorithm, reveal consistent isothermal lines across various lengths of fins at different dimensionless times ($Ste.Fo$). A comparative analysis is conducted for four distinct cases: the cavity without a fin and three cases with fins of lengths $l_f = 0.25, 0.5$, and 0.75 . Notably, a strong agreement is observed in both the melting pattern and the slope of the isothermal lines, underscoring the accuracy of the image processing method in effectively defining the geometrical features of the fins. This robust concurrence across different fin lengths substan-

tiates the reliability and precision of the employed image processing technique in capturing and representing the intricate details of the finned cavity, thereby validating its efficacy for simulations involving phase change materials. In this study, it is worth highlighting the utilization of an image processing algorithm as a novel approach to defining the intricate geometries of fins within the context of lattice Boltzmann method (LBM) simulations. The outcomes of our investigation unequivocally demonstrate that employing an image processing algorithm proves to be an effective and promising methodology for accurately characterizing complex geometrical features in LBM studies.

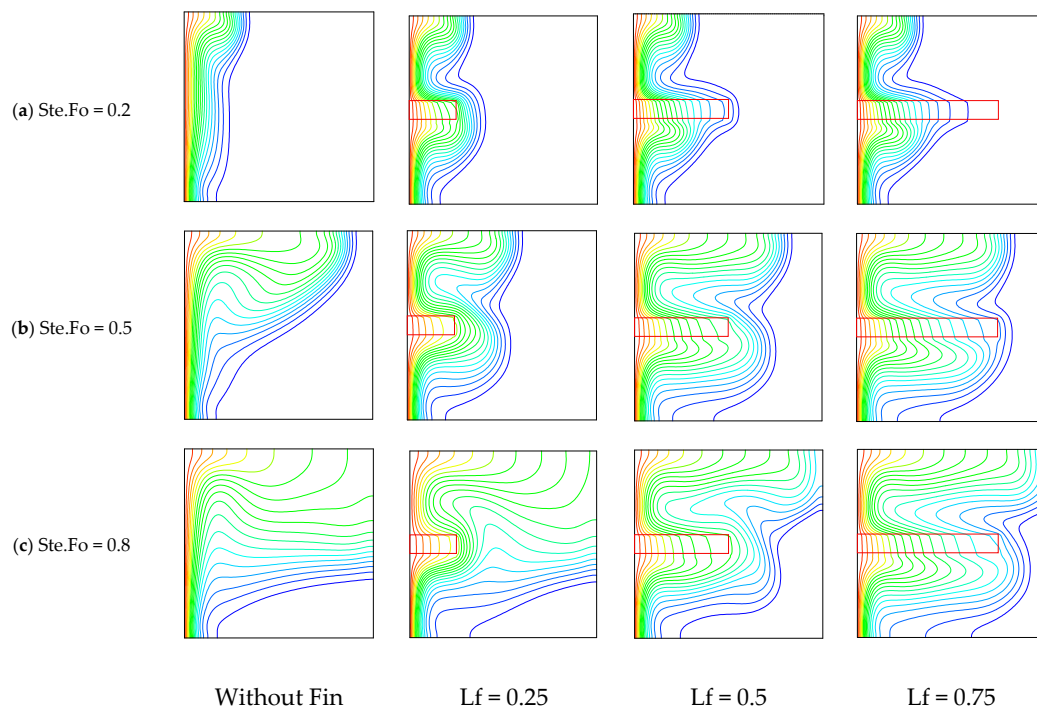


Figure 5. Results for a cavity equipped by metal fin as used by Jourabian et al. ((a) Ste.Fo = 0.2, (b) Ste.Fo = 0.5, (c) Ste.Fo = 0.8) at $Ra = 1.7 \times 10^4$ and Ste = 10.0 at three time steps.

6. Results

6.1. Effect of the Honeycomb Fin Structure

In this initial research endeavor, the influence of a honeycomb structure is investigated by contrasting its effects in two scenarios with varying Rayleigh (Ra) numbers within an equivalent domain size. The study employs visual aids including streamlines and fluid fraction diagrams. By comparing the results from cases with lower and higher Ra numbers, discernible patterns and differences in behavior emerge, shedding light on the structure's impact under differing thermal conditions. This investigation sets the groundwork for deeper insights into the intricate dynamics and serves as a springboard for subsequent studies.

Figures 6–8 illustrate isothermal lines within the cavity at a Ra value of 10^2 , showcasing two distinct scenarios: one with the presence of a honeycomb structure and the other without when $\lambda = 0.62$. Figure 6 illustrates that in the absence of fins, conductive heat transfer dominates over convective heat transfer because the slopes of the temperature profiles are very small, even though with the expansion of heat toward the center of the enclosure, the effect of gravity becomes such that the temperature profiles become slightly inclined. In honeycomb fin system 1, considering that the thermal conductivity coefficient of the fins is higher than that of the phase change materials (PCMs), heat reaches the center of the enclosure at a faster rate, causing each PCM compartment to melt at a faster pace because these compartments are exposed to heat from all sides and melt more quickly. In the case of system number 2, the design of the fin system plays a crucial role in the

heat transfer dynamics. In this system, it is observed that heat initially reaches the central fins at a higher rate due to their specific arrangement within the enclosure. However, the process of phase change, particularly the melting process, exhibits a similar behavior to that observed in system number 1.

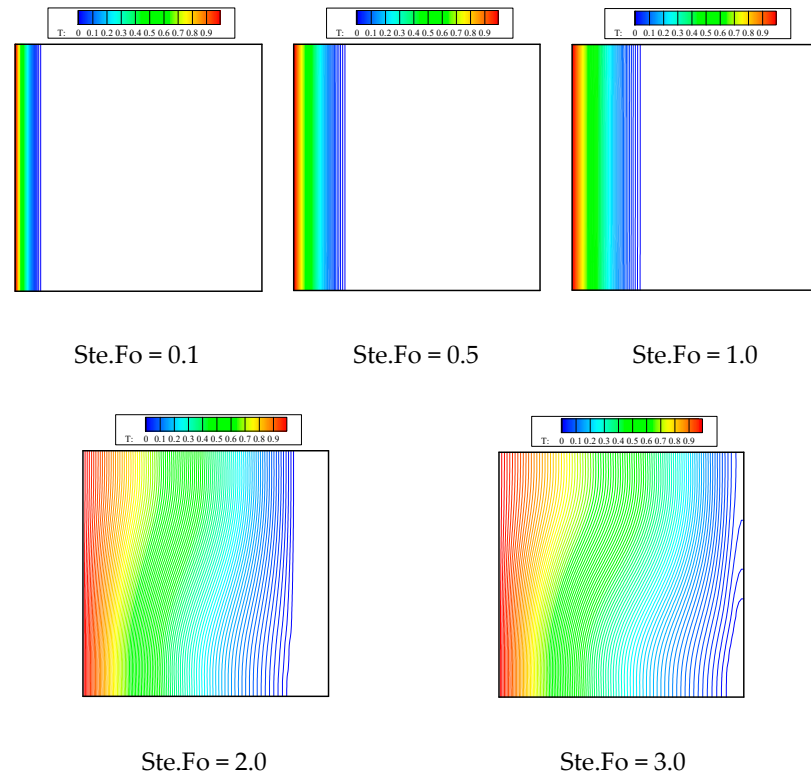


Figure 6. Isothermal line of cavity at $Ra = 10^2$ for a case without honeycomb structure.

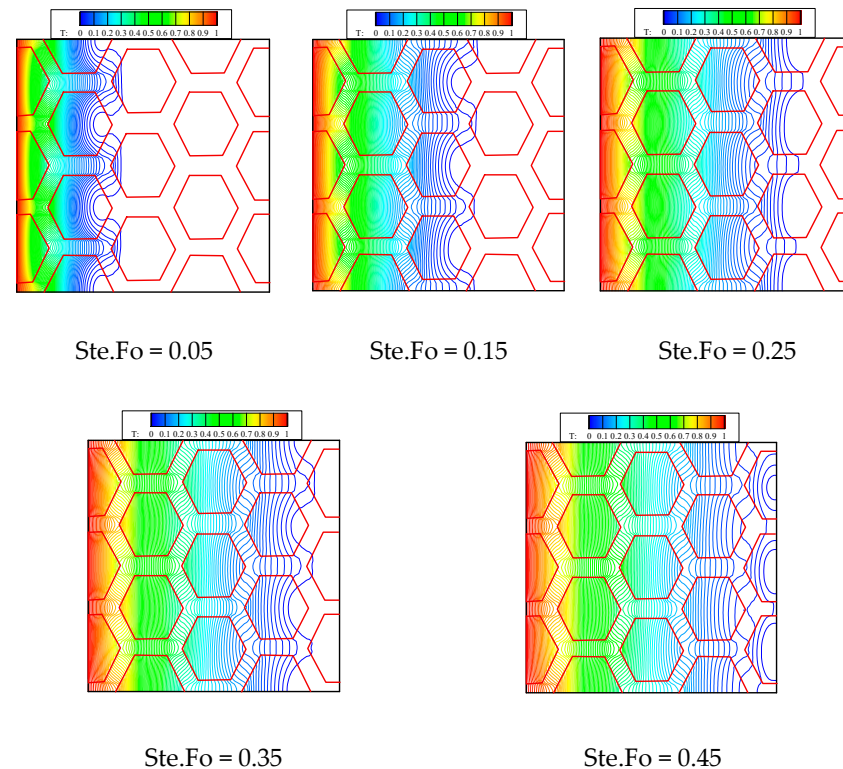


Figure 7. Isothermal line of cavity at $Ra = 10^2$ for honeycomb structure case 1 and $\lambda = 0.62$.

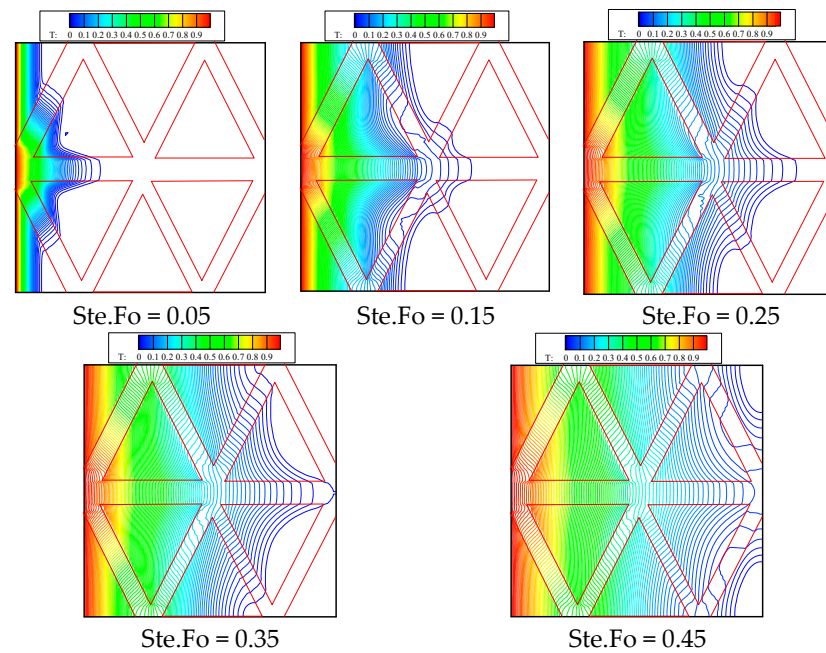


Figure 8. Isothermal line of cavity at $Ra = 10^2$ for honeycomb structure case 2 and $\lambda = 0.62$.

In a scenario where there is no fin system within the enclosure, the entire enclosure undergoes melting within a time interval known as $Ste.Fo$ (which appears to be equal to 3.0). This time interval signifies the duration required for the material to transition from one state (e.g., solid) to another (e.g., liquid) at lower temperatures.

However, when a fin system is present within the enclosure, this time interval significantly decreases to 0.45. This reduction indicates that the presence of the fin system has a substantial impact on its thermal design and performance. It is likely that the fin system enhances heat transfer and accelerates the melting process of the material within the enclosure. This is a crucial advantage, particularly in applications that require precise temperature control and expedited phase change processes.

In summary, the inclusion or exclusion of a fin system within the enclosure significantly influences the duration of the phase change process. The presence of the fin system can significantly improve heat transfer and expedite the melting process within the enclosure. This effect can be particularly significant in applications where precise temperature control and the timing of processes are crucial.

What stands out from comparing the last point of melting within the enclosure for all three scenarios is that the design of the fin system can significantly influence the location and timing of the final point of melting.

Figure 9 visually demonstrates that the presence of fins accelerates the onset of the liquid phase, leading to a significantly faster melting process compared to the scenario without fins. Nevertheless, in both scenarios with fins, the complete melting of the PCM material occurs at roughly the same time. This observation underscores the role of fins in expediting the phase change process and ensuring more efficient heat transfer in the enclosure.

Figures 10–12 display similar outcomes for $Ra = 10^5$. In both cases, with or without the fin system, a consistent result is observed. However, there is an interesting distinction when $Ra = 10^5$.

When the cavity lacks a fin system and $Ra = 10^5$, convection heat transfer takes precedence over heat conduction. This dominance of convection significantly accelerates the melt speed, resulting in a faster melting process compared to with lower Ra values. The increased Ra enhances the convective currents, leading to more efficient heat transfer and faster phase change within the enclosure.

In contrast, when the fin system is employed at $Ra = 10^5$, regardless of the specific configuration, the melting process mirrors that observed at $Ra = 10^2$. This suggests that

the presence of fins mitigates the impact of the increased Ra , and in both cases, the melting process remains consistent with the conditions observed at $Ra = 10^4$.

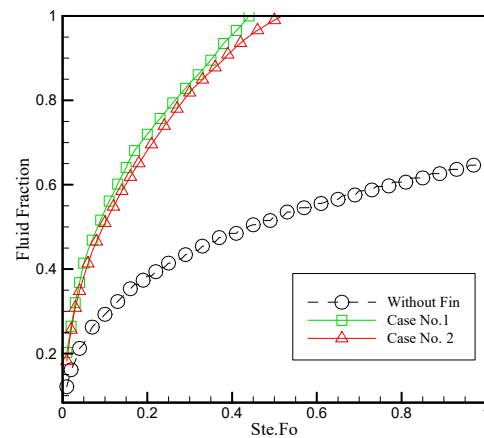


Figure 9. Fluid fraction diagram of cavity at $Ra = 10^2$ for honeycomb structures and cavity without fin structures.

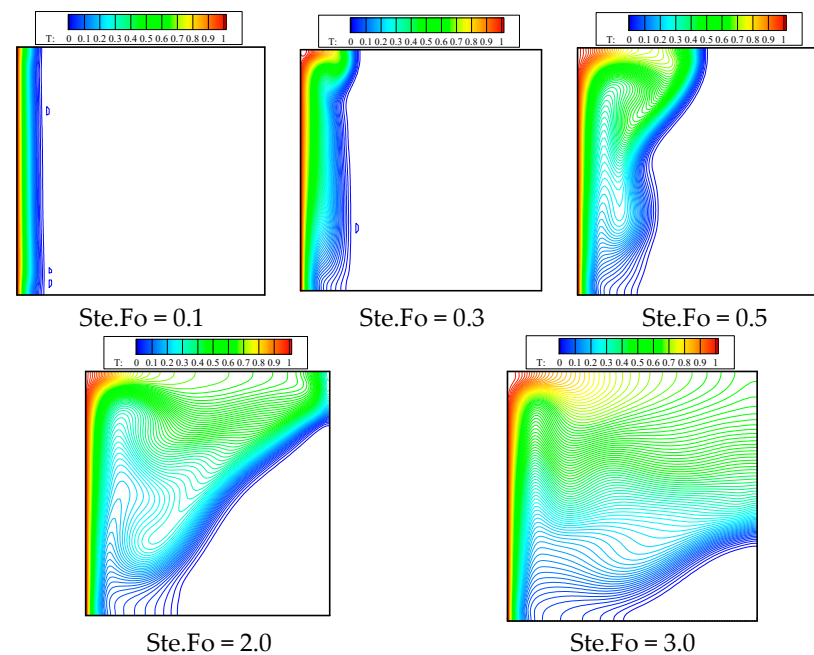


Figure 10. Fluid fraction diagram of cavity at $Ra = 10^2$ for cavity without fin structures.

In summary, Figures 10–12 demonstrate that at higher Rayleigh numbers ($Ra = 10^5$), the absence of fins results in convection heat transfer dominating heat conduction, leading to an accelerated melting process. However, the introduction of a fin system at $Ra = 10^5$ maintains the same melting behavior observed at lower Ra values. This underscores the stabilizing and standardizing effect of the fin system on the melting process under varying Rayleigh number conditions. The stabilization effect noted in enclosures with fins can be attributed to the dominance of conductive heat transfer over convective heat transfer. Fins are designed to enhance heat dissipation by increasing the surface area available for heat exchange. In this specific context, the fins facilitate a more efficient conduction of heat from the melting process to the surrounding medium. Conductive heat transfer is inherently more predictable and controllable than convective heat transfer. The presence of fins promotes a higher conductive heat transfer rate, leading to a more stable and consistent fluid fraction behavior during the melting process. This behavior is particularly pronounced across different Rayleigh numbers (Ra), indicating the robustness

of the observed stabilization effect. Figures 10–12 showcase a noteworthy observation—the temperatures within the fins exhibit a rapid ascent. This phenomenon has a profound impact on the surrounding phase change material (PCM) containers. The accelerated temperature rise in the fins plays a pivotal role in creating a conducive environment for enhanced heat transfer to the PCM containers. Consequently, the PCM experiences a faster and more substantial melting process. Notably, a larger quantity of heat is transferred and stored within the PCM in a significantly shorter period. This transient effect suggests that the inclusion of fins in the configuration not only expedites the melting process but also augments the overall heat storage capacity of the PCM. The dynamic nature of temperature distribution, as depicted in Figures 10–12, underscores the influence of fins on the temporal aspects of thermal behavior. These findings highlight the efficiency gains achieved by leveraging transient simulations and the strategic use of fins in optimizing heat transfer and storage in PCM systems.

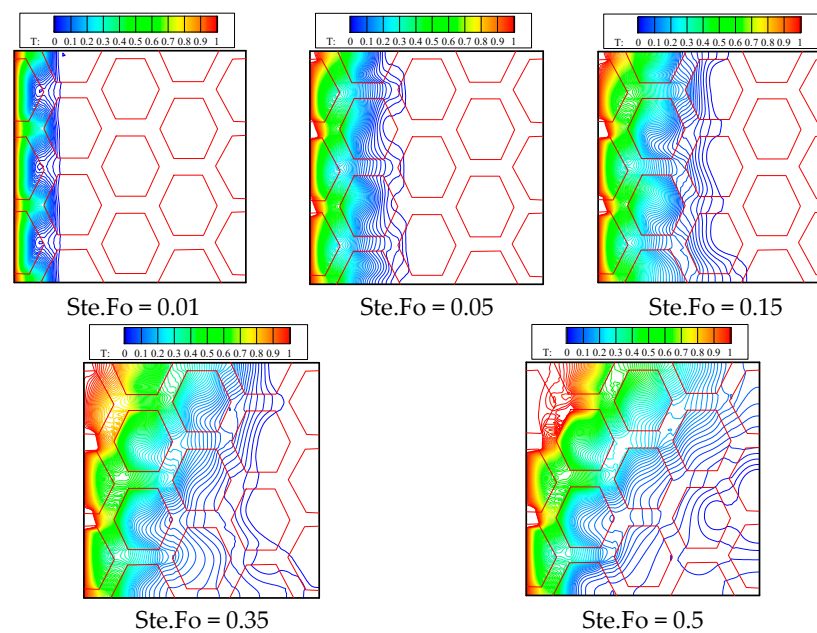


Figure 11. Isothermal line of cavity at $Ra = 10^5$ for a case without honeycomb structure.

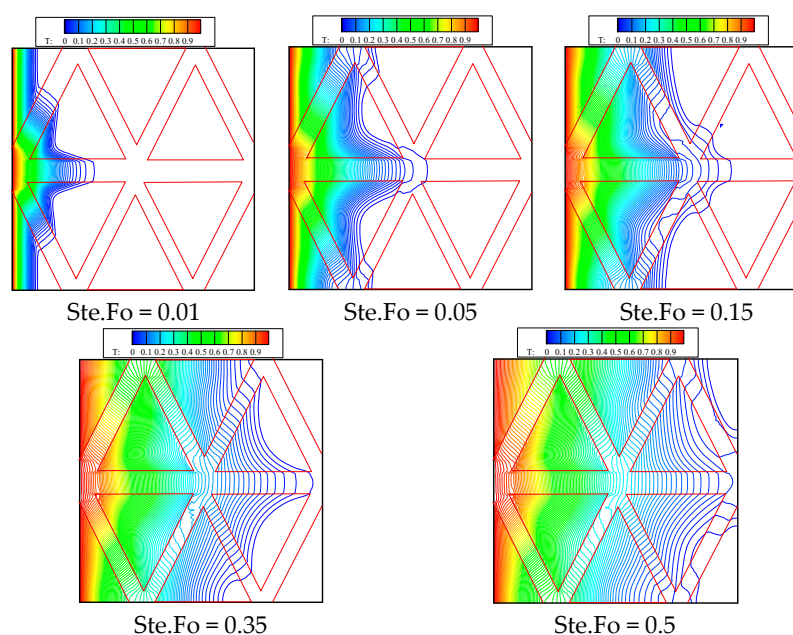


Figure 12. Isothermal line of cavity at $Ra = 10^5$ for a case with honeycomb structure case 1.

Figure 13 presents a fluid fraction diagram for the case where $Ra = 10^5$. Comparing this to the results at lower Ra values, it is evident that the fluid diagrams for systems with fins exhibit a consistent pattern. However, a notable difference is observed in the fluid fraction diagram when fins are absent. This indicates that the presence or absence of fins plays a crucial role in dictating the fluid behavior at high Ra numbers.

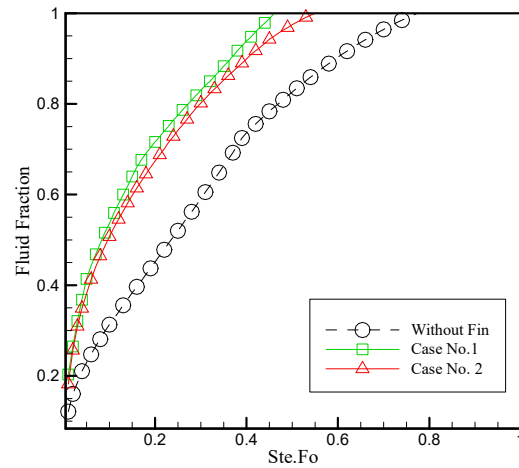


Figure 13. Fluid fraction diagram of cavity at $Ra = 10^5$ for honeycomb structures and cavity without fin structures.

Figures 14 and 15 further illustrate the impact of the Rayleigh number (Ra) on the fluid fraction diagram. Clearly, as the Ra number increases, the melting time decreases. This relationship is consistent with the expectation that higher Ra numbers result in more vigorous convective currents, leading to faster heat transfer and phase change within the enclosure.

However, in cavities equipped with fins, the change in Ra number does not yield a significant difference in the observed results. This suggests that the presence of fins stabilizes and standardizes the melting process, making it less sensitive to variations in the Ra number. The fins appear to enhance the heat transfer and maintain a consistent melting behavior, even as the Ra number increases.

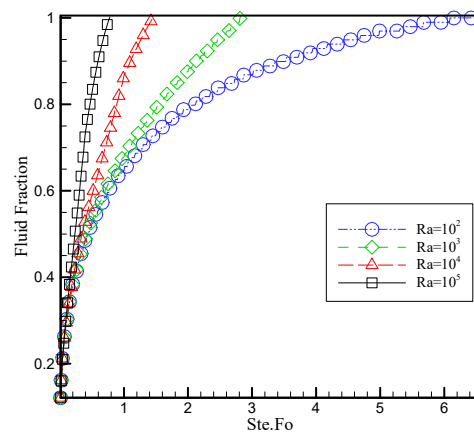


Figure 14. Fluid fraction diagram of cavity at different Ra numbers for the case without fin structure.

In summary, Figure 14 highlights that at $Ra = 10^5$, the fluid fraction diagram is different when fins are absent, while systems with fins exhibit consistent behavior. Figures 13–15 affirm the inverse relationship between the Ra number and melting time, but this effect is less pronounced in cavities equipped with fins, indicating the stabilizing influence of fins on the melting process, regardless of Ra variations.

In a cavity without a fin system, the study conducted by Yan et al. revealed a significant correlation between the Rayleigh (Ra) number and the melting time of the material.

The experiment demonstrated that as the Ra number increased from 10^2 to 10^5 , there was a notable decrease in the melting time, plummeting from 6 to 0.4. This observation underscores the influential role of the Ra number in dictating the melting behavior within the cavity. Intriguingly, the introduction of a fin system proved to be a transformative factor, uniformly reducing the melting time to $Ste.Fo = 0.5$ across all cases. This finding underscores the effectiveness of a fin system in optimizing the thermal dynamics within the cavity, resulting in a substantial and consistent reduction in the melting time regardless of the Ra number. The utilization of fins appears to offer a promising and universal solution for enhancing heat transfer and expediting the melting process in such systems.

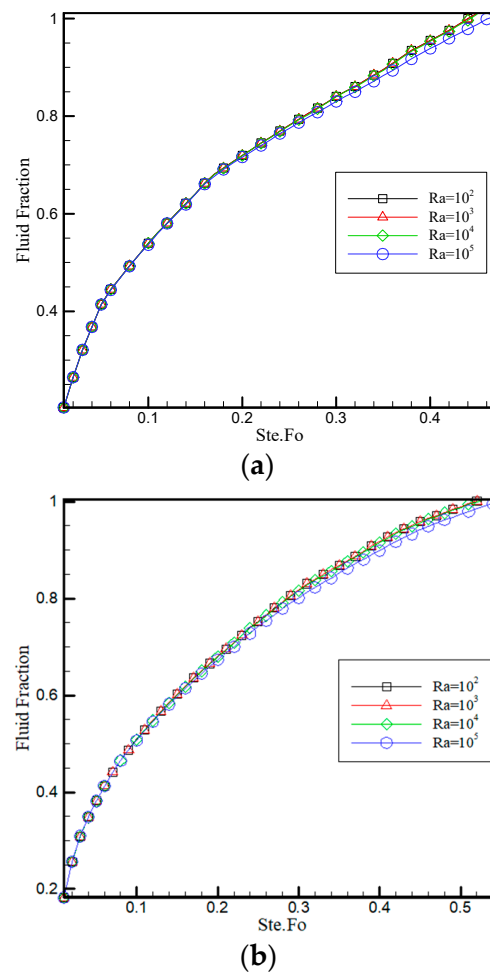


Figure 15. Fluid fraction diagram of cavity at different Ra numbers for honeycomb structures: (a) case 1 and (b) case 2.

The impact of fins on fluid behavior and their role in mitigating gravitational effects during melting processes is integral to our study. Fins enhance heat transfer by increasing the surface area, facilitating efficient phase change, and inducing favorable convective flow patterns. Moreover, strategically designed fins counteract the influence of gravity, minimizing undesired stratification and promoting uniform melting within the enclosure. The combined effects of enhanced heat transfer and gravity mitigation highlight the intricate interplay between fins and the enclosure, contributing to a more comprehensive understanding of thermal management systems and their applications in controlled phase change processes.

The parameter λ is defined as the ratio between the phase change material (PCM) space and the entire domain. In this section, we delve into the influence of λ on the melting process.

The images in the top row of Figures 16 and 17 illustrate the geometry definition within the enclosure. Using geometric calculations, various values of the dimensionless parameter λ can be defined. It is observed that in the study of case number 1, reducing the value of λ from 1 to 0.4 results in a more uniform temperature distribution within the enclosure. This change becomes more pronounced at higher Rayleigh numbers (Ra) because at higher Ra numbers, convective heat transfer dominates over conductive heat transfer, leading to more significant temperature variations within the enclosure.

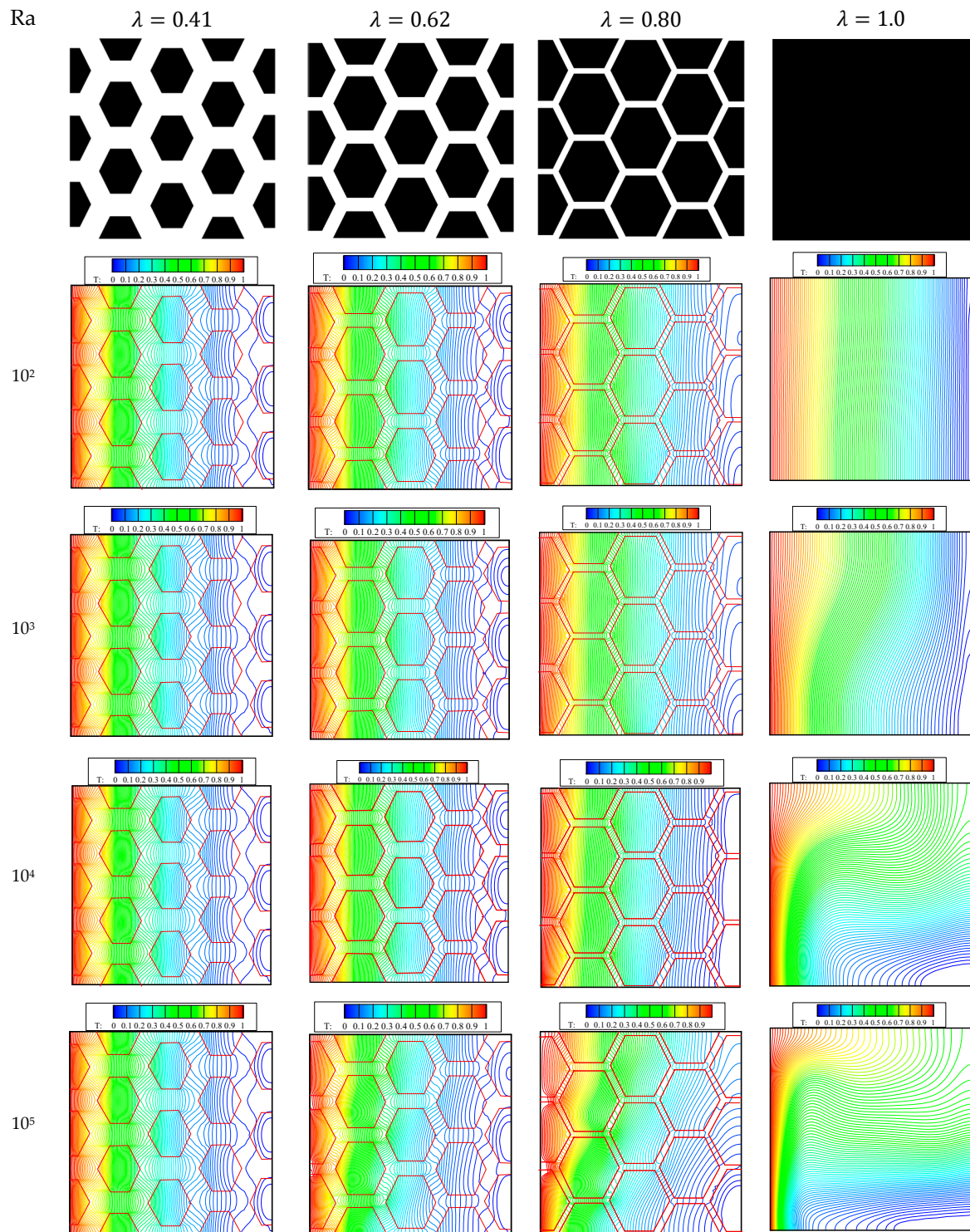


Figure 16. Impact of λ on PCM melting dynamics in honeycomb finned cavity (case 1) at different Ra numbers.

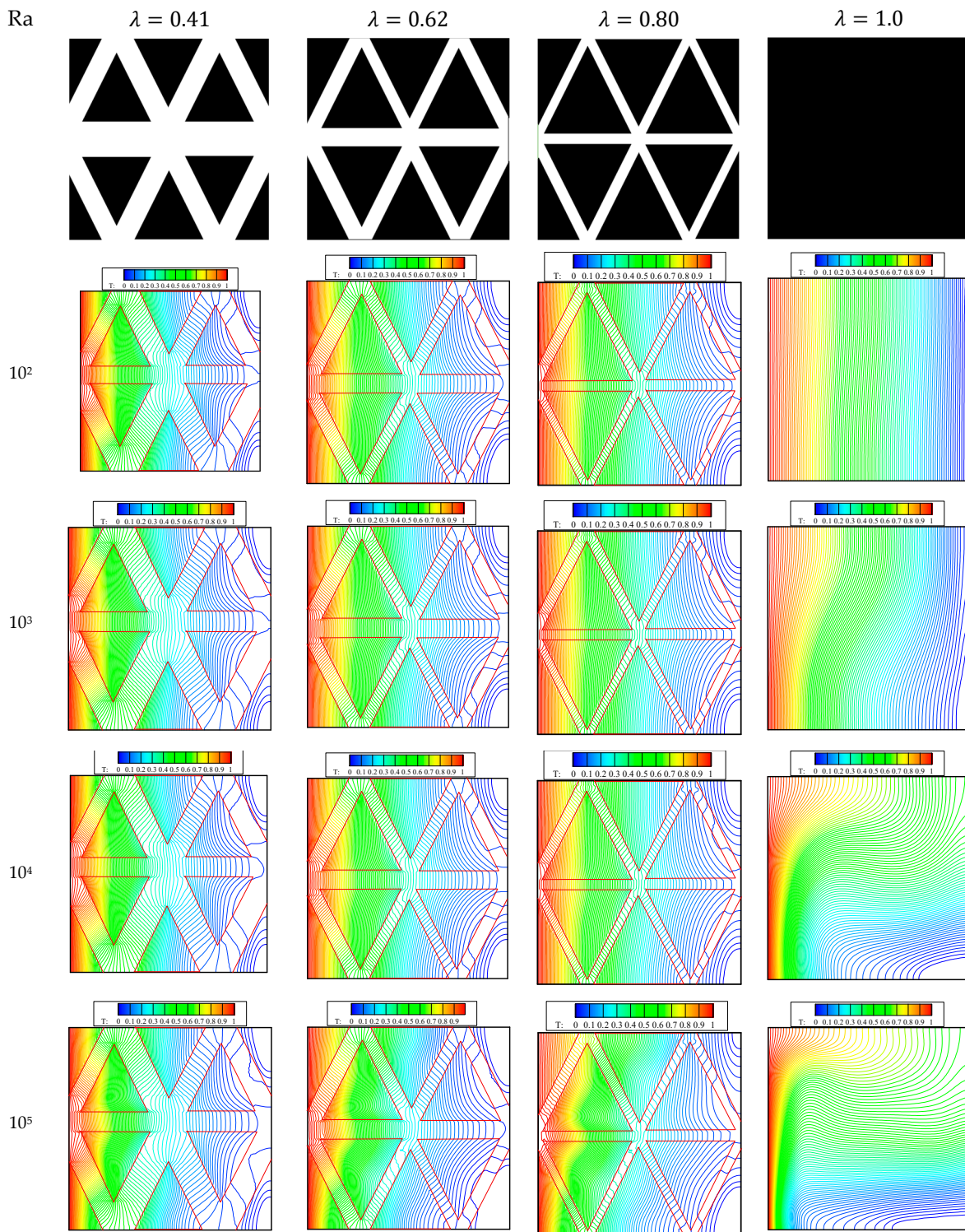


Figure 17. Impact of λ on PCM melting dynamics in honeycomb finned cavity (case 2) at different Ra numbers.

At lower λ values, due to the enhanced heat transfer effects of the fins, the volume of fluid within the enclosure decreases. This, in turn, accentuates the impacts of conductive heat transfer.

Figures 18 and 19 shows fluid fraction diagram versus $Ste.FO$ at different λ number at $Ra = 10^5$. Reducing λ by increasing the fin thickness can cut the melting time by 50%, but this comes with drawbacks. The thicker fins lead to a heavier system and reduced energy

storage due to a lower amount of phase change material (PCM). Decisions to decrease λ should balance the reduced melting time benefits against the increased weight and diminished energy storage concerns.

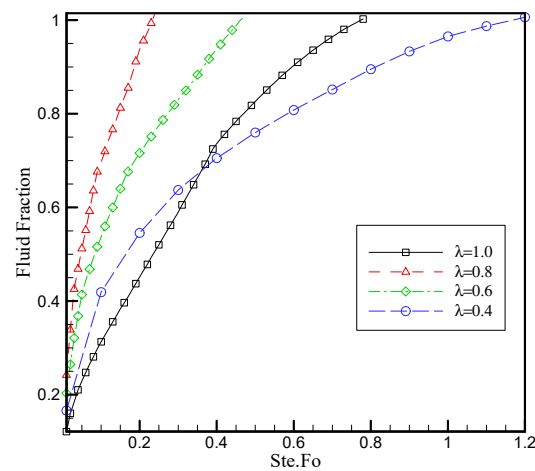


Figure 18. Fluid fraction diagram of cavity at different λ numbers for honeycomb structure case 1 at $Ra = 10^5$.

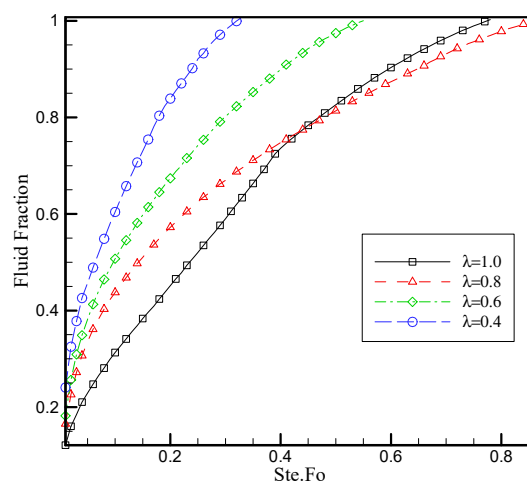


Figure 19. Fluid fraction diagram of cavity at different λ numbers for honeycomb structure case 2 at $Ra = 10^5$.

6.2. The Effect of Honeycomb Structure on Inclined Cavities

In the realm of heat transfer research, inclined cavities serve as essential tools for investigating heat exchange phenomena under non-standard gravitational conditions. These cavities, featuring various geometries, offer a unique platform for studying natural convection dynamics when buoyancy forces play a pivotal role, as seen in solar collectors and passive cooling systems. Additionally, they facilitate insightful analyses of the melt and solidification processes, influencing solidification fronts, heat transfer rates, and phase change behaviors. The insights gleaned from these studies significantly contribute to both theoretical advancements and practical applications in materials science and engineering.

Building upon the current research, this investigation aims to further examine the influence of the cavity angle on a honeycomb constructed finned cavity. The purpose of this investigation is to reach a deeper understanding of how the orientation of cavities affects the melting dynamics of phase change materials (PCMs). Thus, the fluid fraction diagram at $Ra = 10^5$ at different cavity angles $\theta = -90, 45, 0, 45, 90$ is presented in Figures 20–22. In all cases discussed, λ is considered to be 0.6. This parameter, representing the ratio

between the phase change material (PCM) space and the entire domain, is held constant at 0.6 for the purposes of the analysis and comparisons made in the study.

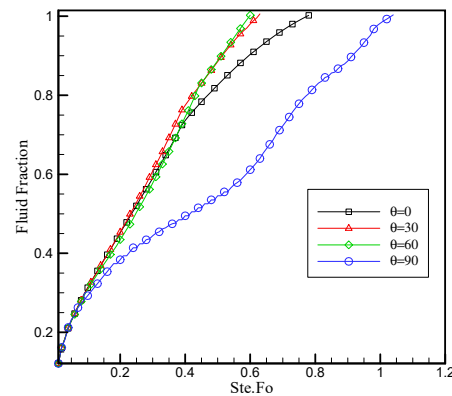


Figure 20. Impact of cavity angle on PCM melting dynamics at $Ra = 10^5$ in a cavity without fin.

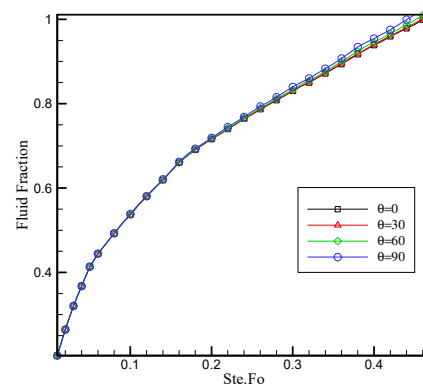


Figure 21. Impact of cavity angle on PCM melting dynamics in honeycomb finned cavity (case 1) at $Ra = 10^5$.

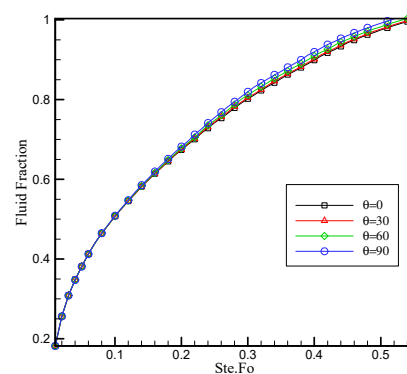


Figure 22. Impact of cavity angle on PCM melting dynamics in honeycomb finned cavity (case 2) at $Ra = 10^5$.

Figure 20 illustrates that in the case where the enclosure lacks fins, the angle of the enclosure has a significant impact on the fluid fraction diagram. However, with the presence of fins within the enclosure (Figures 21 and 22), it can be observed that the angle has a minimal effect in both cases. This phenomenon stems from the fact that the presence of the fin system within the enclosure minimizes the influence of gravitational forces within the enclosure. As a result, changes in the angle of the enclosure do not have a substantial impact on the melting process.

7. Conclusions

Our study drew inspiration from the efficiency of nature's hexagonal honeycomb structure to enhance the performance of our proposed system. Via the use of image processing algorithms, we precisely defined hexagonal phase change material (PCM) containers and thermally conductive fins, replicating the intricate geometry seen in honeycombs. This approach not only streamlined the setup process but also contributed to a deeper understanding of the melting process within the cavity, showcasing the benefits of biomimetic design principles in engineering applications.

The introduction of a fin system within the enclosure is associated with a substantial reduction in the time required for a complete phase change, with $Ste.Fo$ decreasing significantly to approximately 0.45, highlighting the profound impact of fin systems on the thermal design and performance of the enclosure.

Enhanced heat transfer dynamics and accelerated melting processes are enabled by fin systems, rendering them advantageous for applications prioritizing precise temperature control and expedited phase change. Moreover, the critical role of fin system design is emphasized, as both scenarios with fins exhibit a faster onset of the liquid phase, but the location and timing of the final point of melting can be significantly influenced by the fin system configuration. Thus, the choice of fin system design becomes pivotal, significantly affecting the location and timing of the final point of melting. These findings contribute to a broader understanding of thermal management systems and their potential applications in industries necessitating precise temperature control and rapid phase change processes.

Our study reveals that the presence of fins in enclosures at different Rayleigh numbers (Ra) leads to consistent fluid fraction behavior and stabilizes the melting process. Additionally, an inverse relationship between the Ra and melting time is observed, but this effect is attenuated in cavities equipped with fins. These findings underscore the critical role of fins in enhancing thermal stability during phase change process.

Our study reveals that the cavity angle significantly affects the fluid fraction diagram in cavities without fins but has minimal impact when fins are present. This underscores the stabilizing role of fins in mitigating gravitational effects during melting processes. These insights enhance our understanding of how cavity geometry and fins interact in heat transfer, offering potential for improved thermal system designs.

In conclusion, our study's outcomes not only advance the understanding of thermal dynamics but also hold significant implications for future engineering applications. The integration of hexagonal phase change material (PCM) containers and thermally conductive fins, inspired by nature's honeycomb structure, demonstrates a practical and efficient approach to biomimetic design. The streamlined setup process, facilitated by image processing algorithms, showcases the potential for precise temperature control and accelerated phase change in diverse industries. The observed reduction in the phase change time and the influential role of the fin design in shaping thermal outcomes underscore the transformative impact of our findings. Furthermore, the consistent fluid fraction behavior and stabilization of the melting process in enclosures with fins, irrespective of varying Rayleigh numbers, highlight the universal applicability of fin systems. This study not only contributes to our current understanding of thermal management systems but also lays the groundwork for future research, emphasizing the enduring importance of these results in shaping the future of thermal sciences and applications.

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Abbreviations

Nomenclature

c_i (m/s)	the discrete velocity of Boltzmann grid
c_p (m/s)	specific heat at constant pressure
c_s (m/s)	sound speed
F (J)	buoyancy force
f	the distribution function
$h^{n,k}$	fluid fraction at time step n and iteration k
f^{eq}	the local equilibrium distribution function
g_y (m/s ²)	the gravitational acceleration
g	temperature distribution function
g_i^{eq}	the local equilibrium distribution function of temperature
Gr	Grashof number
k (W/m ² ·K)	conduction heat transfer coefficient
L (m)	the width of the cavity
L_f (J/kg)	Latent heat
Pr	Prandtl number
Ra	Rayleigh number
T (K)	temperature
t (s)	time
u (m/s)	velocity component in the x-direction
v (m/s)	velocity component in the y-direction
u_0 (m/s)	the velocity of the top wall
w_i	the weight function of the i_{th} direction
Greek	
α (m ² /s)	thermal diffusivity
μ (m ² /s)	viscosity
ρ (kg/m ³)	density
τ_v	the relaxation time of the flow field
τ_D	the relaxation time of the temperature field
ν	kinematic viscosity

Footnotes

f	fluid
w	wall

Abbreviations

PCM	phase change material
LBM	lattice Boltzmann method

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