

Discrete-Time Sliding Mode Control Strategies—State of the Art

Paweł Latosiński *  and Katarzyna Adamiak 

Institute of Automatic Control, Lodz University of Technology, 18 Bohdana Stefanowskiego St., 90-537 Lodz, Poland; katarzyna.adamiak@p.lodz.pl

* Correspondence: pawel.latosinski@p.lodz.pl

Abstract: Variable structure control systems are known to provide a high level of robustness to external disturbances and modeling uncertainties with comparably low computational complexity. Thanks to these features, they have found applications in various fields, such as power engineering, electronics, robotics, and aviation. In recent decades, the field of sliding mode control has developed significantly. Therefore, this study aims to discuss the basic concepts and design methodology of such strategies. Although in the 20th century, continuous-time sliding mode control has been the center of the control engineering society's attention, it has certain major shortcomings. In particular, such control schemes result in undesirable high-frequency oscillations when applied digitally. Therefore, the more recent discrete-time approach to sliding mode control has gained recognition in the 21st century. Since the introduction of discrete-time sliding mode control strategies, the reaching law-based controller design method has been designed, within which two main paradigms may be named: the switching type and the nonswitching type quasi-sliding mode. This paper presents a broad review of the discrete-time sliding mode control strategies, starting from the definition of sliding mode through the controller design procedures and up to potential applications. The aim of this study is to provide an up-to-date state of the art and introduce readers to the newest trends and achievements in the field of sliding mode control.

Keywords: control design; discrete-time systems; reaching-law approach; sliding mode control; variable structure control



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1. Introduction

The goal of this paper is to present the concept of sliding mode control, its history, and various paths of development. In the presently computerized world, most control systems apply discrete-time measurement and electronic control, and all signals are processed digitally, therefore requiring discrete-time analysis. Consequently, the main focus of this paper is the discrete-time sliding mode control. This paper will clearly present the history of its development, most popular definitions and design methods, including a profound review of the reaching-law approach and various reaching functions. Finally, the paper points out and briefly discusses the newest concepts in sliding mode control, such as higher relative degree outputs or event-triggered control. It is worth pointing out that the selection of the presented works is arbitrary, as the topic of sliding mode control is extremely popular, and it is not possible to mention all the noteworthy papers in the field.

The remainder of this paper is organized as follows. Section 2 presents the idea of sliding mode control in the continuous-time domain and briefly discusses its history and most commonly encountered problems. Section 3 introduces the discrete-time sliding mode control and its design methodology. Finally, Section 4, divided into two sections, discusses the most common design method through the reaching-law approach, considering two alternative quasi-sliding mode definitions. Section 5 aims to point out the most promising novel concepts in the discrete-time sliding mode control, and Section 6 provides final comments and remarks.

2. The Idea of Sliding Mode Control

Variable structure control (VSC) strategies have their origins in the mid-20th century in the works of Emelyanov [1,2]. Thanks to the multiple favorable properties they provide, they have been thoroughly studied ever since. The key idea behind variable structure controllers is to switch the structure of the system according to its actual state obtained in a feedback loop. Therefore, they use a characteristic type of control laws in the form of nonlinear, discontinuous state feedback.

Along with the development of the method, the branch of sliding mode control has spread the most. The term sliding mode first appeared in the works of Utkin [3–6] in the Soviet Union. The first works in this area were purely theoretical. They showed that switching between two or more unstable feedback control systems can result in a stable closed-loop system. However, it is a challenge to determine the right switching strategy. For this purpose, the sliding mode control method assumes limiting the dynamics of the system to a predefined hyperplane in the state space. The selection of the switching hyperplane is crucial in achieving a given control goal, such as stability, quick disappearance of transients, or elimination of the steady-state error. Moreover, when the dynamics of the system are confined to the sliding hyperplane, its order is reduced, and it becomes completely insensitive to the influence of external disturbances and model inaccuracies. This undeniable advantage of the sliding motion was analyzed in detail by Draženović in [7]. It was the insensitivity to external disturbances that pushed many authors in Utkin's footsteps and contributed to the rapid development of the strategy, as demonstrated by numerous literature items devoted to this topic [8–16], articles in journals [17–20] and their Special Issues [21,22].

As already mentioned, in sliding mode control, the correct selection of switching hypersurface plays a key role. The selected hypersurface, described by a certain linear or nonlinear combination of state variables, should contain a given operating point of the system. Its selection methods were widely studied in [23–26]. The position of the representative point of the system in relation to this hypersurface is described with the so-called sliding variable or switching variable, defined as $s(t) = f[x(t)]$, where $s(t)$ denotes the sliding variable, $x(t)$ is the state vector of the system, and t is the time. Therefore, the sliding hypersurface is a set of points where the sliding variable equals zero. For third-order systems, the sliding hyperplane reduces to a plane, and for second-order dynamical systems it becomes a line. Therefore, the task of the sliding mode controller is, first, to bring the representative point of the system to the sliding hyperplane in a finite time and then to ensure a stable sliding motion on this plane. The sliding hyperplane divides the state space into two regions, therefore determining the switching condition of the controller structure. In the first region, the sliding variable has positive values, $s(t) > 0$, and the control must ensure that $\dot{s}(t) < 0$. In the second one, the sliding variable is negative, $s(t) < 0$, and the controller must guarantee that $\dot{s}(t) > 0$. The above conditions clearly indicate the structure of the controller, which usually contains two opposite-in-sign feedback values, switched as follows:

$$u(t) = \begin{cases} U^+ & \text{for } s(t) > 0 \\ U^- & \text{for } s(t) < 0 \end{cases} \quad (1)$$

where $u(t)$ is the scalar control signal, $s(t)$ denotes the sliding variable, and U^+ and U^- are constants. The control process may be divided into two stages:

- The reaching phase, when the representative point of the system moves towards the sliding surface.
- The sliding phase, when the representative point of the system moves along the sliding plane $s(t) = 0$. The choice of the sliding plane must ensure the global asymptotic stability of the closed-loop system.

The phases of the sliding motion are presented in Figure 1a, which shows the trajectory of the second-order plant with a sliding mode controller. For this example, a double

integrator was considered, and the sliding plane took the form of a straight line with the equation $0.5x_1(t) + x_2(t) = 0$.

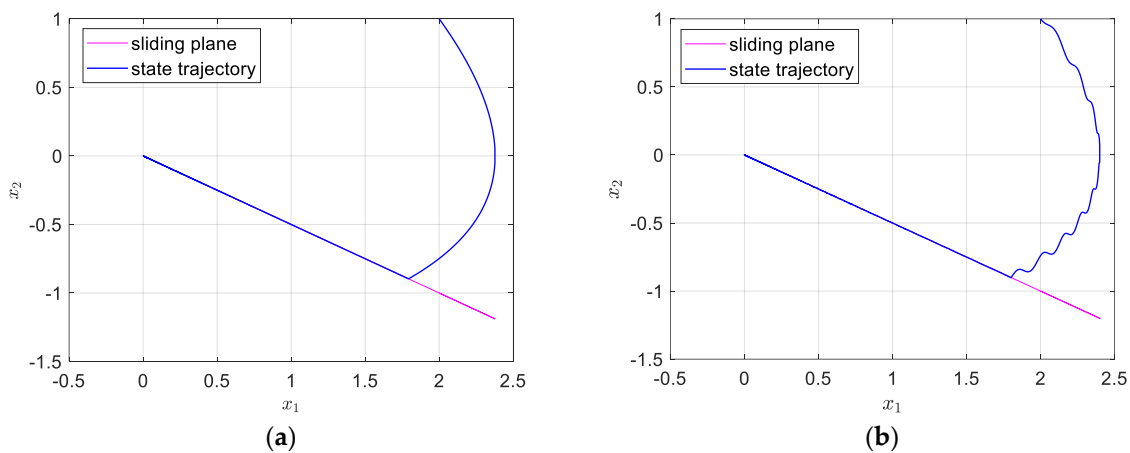


Figure 1. State trajectory. (a) without disturbance. (b) with a sinusoidal disturbance.

This method of control ensures a reduction of the order of dynamical problems. In the sliding phase, the trajectory of the closed-loop system is limited to a predefined sliding hyperplane of a lower order than the order of the original plant. Moreover, any disturbance causing a change in the current value of the sliding variable forces a change in the value of the control signal. Additionally, if the acting disturbances increase the distance of the representative point of the system from the sliding plane, they force a change in the control sign and, thus, a change in the structure of the system in such a way that the trajectory is immediately brought back to the switching plane. Thanks to this, sliding mode-controlled systems are insensitive to external disturbances and modeling uncertainties that meet the so-called matching conditions, i.e., affect the system through the same channel as the control. An example of the trajectory of a second-order system under the influence of sinusoidal disturbances is shown in Figure 1b. In the reaching phase, the representative point of the system is always located on one side of the sliding plane. Therefore, the control structure does not change, and the disturbances disrupt the system's trajectory. However, in the sliding phase, this effect is eliminated by continuous switching of the control signal. This undeniable advantage was thoroughly analyzed by Draženović [7].

Apart from several advantages of sliding mode control, a few drawbacks of this methodology have also been noticed. The introduction of discrete state measurements and digital control systems drew more attention to the so-called “chattering phenomenon”. The implementation of sample-and-hold devices enforces that the switching frequency may not be higher than the sampling frequency and the control signal remains constant during each sampling period. Therefore, in the sliding phase, we encounter a discontinuous control signal, which causes the system to oscillate. The performance of the second-order system, considered before, with a discrete-time sliding mode controller is presented in Figure 2. The control switching frequency was limited to 20 Hz. From the figure, it is clearly visible that the discretization of the control signal causes the system to oscillate around the sliding plane, and the convergence to the equilibrium point is only asymptotic.

The “chattering phenomenon” was analyzed by many authors [27–31]. One way to cope with limited switching frequency is to replace the popularly used signum function through saturation. This leads to introducing additional intermediate control signal values. Hence, it is allowed for the representative point of the system to slightly deviate from the sliding plane. Such movement is called a quasi-sliding mode around the switching hyperplane, where the system is not insensitive but only partially resistant to the influence of external disturbances. Observations regarding “chattering” and the occurrence of the quasi-sliding mode (QSM) contributed to the consideration of sliding mode control in the discrete-time domain, which began at the end of the 20th century.

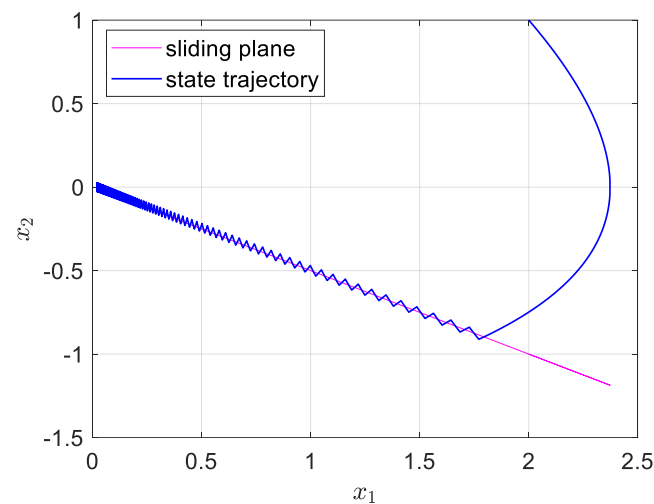


Figure 2. State trajectory with a discretized control signal.

The aim of this paper is to provide a wide review of well-known quasi-sliding mode control methods and provide a brief overview of the novel trends in this field. In particular, this paper covers such topics as:

- Definition and achievability of the quasi-sliding mode
- Switching-type reaching-law approach
- Nonswitching type reaching-law approach
- Higher relative degree sliding mode
- Multirate output feedback approach
- Event-triggered sliding mode control
- Model reference sliding mode control

3. Early Stages of Quasi-Sliding Mode Control

The idea of discrete-time sliding mode control was first presented by Milosavljević in 1985 [32]. The author considered a linear continuous-time system of n -th order, $\dot{x}(t) = f[x(t), u(t)]$, with a discrete error measurement, where T is the discretization period. In such a system, the controller must use a discrete state vector $x(kT)$, where k is a positive integer, instead of a continuous state vector $x(t)$. By analogy to continuous-time systems, Milosavljević defined the linear switching surface as

$$s(kT) = cx(kT) = 0, \quad (2)$$

where c is a $1 \times n$ vector, x is the $n \times 1$ state vector, T represents the discretization period, and k is a positive integer. By analogy to continuous-time systems, the control signal was defined as

$$u(t) = w[s(kT)]x_1(kT) \text{ for } t \in [kT, (k+1)T], \quad (3)$$

where w is a switched gain. The control signal remains constant between instants kT and $(k+1)T$. Control changes occur with a finite frequency, maximum equal to $1/T$. Therefore, as noted by Milosavljević, ideal sliding motion on the selected switching hyperplane is unattainable in the discrete-time case. The author defined the novel quasi-sliding motion as a state when the representative point of the system moves in the finite vicinity of a selected sliding hyperplane and defined the conditions necessary for its occurrence. According to his work [32], quasi-sliding motion on the hyperplane (2) occurs if

$$\lim_{s(kT) \rightarrow 0^+} \nabla s(kT) < 0, \quad \lim_{s(kT) \rightarrow 0^-} \nabla s(kT) > 0, \quad (4)$$

where $\nabla s(kT)$ is a forward difference defined as $\nabla s(kT) = s[(k+1)T] - s(kT)$. The above condition can be interpreted graphically. When the representative point of the system approaches the sliding hyperplane with a positive value, i.e., $\nabla s(kT)$ is negative, the trajectory is brought towards the switching plane. On the other hand, when the point describing the state of the system approaches the sliding hyperplane with a negative value, then $\nabla s(kT)$ is positive, and therefore the trajectory is again brought towards the switching plane. Based on condition (4), the author proposed a method of selecting the value of the switched gain to ensure the occurrence of quasi-sliding motion. Milosavljević's work established a necessary condition for the occurrence of quasi-sliding motion on a given hyperplane. However, it did not consider the conditions for bringing the system's trajectory from any starting point to the vicinity of this hyperplane.

Sufficient conditions for the occurrence of the quasi-sliding mode were presented in the later work of Sarpturk, Istefanopulos, and Kaynak [33], where the authors considered the issue of convergence of the system's trajectories to the sliding hyperplane. A discrete-time system with an external disturbance $d(kT)$ was taken into account. The system is described by the following state equation.

$$x[(k+1)T] = Ax(kT) + bu(kT) + d(kT), \quad (5)$$

where x is the system's state vector, A is the state matrix, b represents the control distribution vector, u is the scalar control signal, and d is the disturbance vector. For the above plant, a sliding hyperplane has been defined according to (2), and the necessary and sufficient condition for the occurrence of quasi-sliding motion is

$$|s[(k+1)T]| \leq |s(kT)|. \quad (6)$$

The above may be split into two inequalities. The first one,

$$\{s[(k+1)T] - s(kT)\} \text{sign}[s(kT)] < 0, \quad (7)$$

ensures the convergence of the representative point of the system to the vicinity of the sliding hyperplane. If the value of $s(kT)$ is close to zero, the condition should be extended to the form $\{s[(k+1)T] - s(kT)\} \text{sgn}[s(kT)] \leq 0$. The second condition, in form

$$\{s[(k+1)T] + s(kT)\} \text{sign}[s(kT)] \geq 0, \quad (8)$$

ensures that the representative point will move in the vicinity of the sliding hyperplane of a non-increasing radius. Only the simultaneous fulfillment of inequalities (7) and (8) guarantees the occurrence of quasi-sliding motion. According to [33], the control signal can be defined by analogy to continuous-time systems as a switched gain state feedback:

$$u(kT) = -k[x(kT), s(kT)]x(kT), \quad (9)$$

where the elements of vector k are changed according to the following rule

$$k_i = \begin{cases} k_i^+ & \text{for } s[x(kT)]x_i(kT) > 0 \\ k_i^- & \text{for } s[x(kT)]x_i(kT) < 0 \end{cases} \quad (10)$$

and k_i^+ and k_i^- are constants. The values of individual elements of vector k can be calculated by substituting the selected sliding hyperplane, defined in (2), and the state Equation (5) into the conditions (7) and (8). The authors also proved that unlike continuous-time systems, where the control signal is upper bound, in the case of discrete-time systems, the control is limited both from above and from below. These limitations depend on the discretization period T and the uncertainty of the system parameters and, as shown by Kotta in [34], on the initial distance of the representative point from the selected switching hyperplane. The achievability of quasi-sliding motion was also considered in the works [35,36]. It is worth

noticing that discrete-time sliding mode control may be applied both to continuous-time systems with input and output sampling or discrete-time systems. A schematic diagram of a system with sliding mode control is depicted in Figure 3. The symbols represent $\mathbf{x}$ —state vector, u —control signal, d —external disturbance, and y —output signal for continuous-time and discrete-time systems, respectively. The continuous-time state matrix and input distribution vector are denoted with Φ and Γ , whereas the discrete-time state matrix and input vector are A and b . The discretization period is denoted with T and k is a positive integer. The sampling frequency in the input and output channel is ideally $1/T$. However, in such methods as multirate output feedback, the output sampling frequency may be higher than the input. The demand state vector is $\mathbf{x}_d(kT)$, and $s(kT)$ represents the sliding variable.

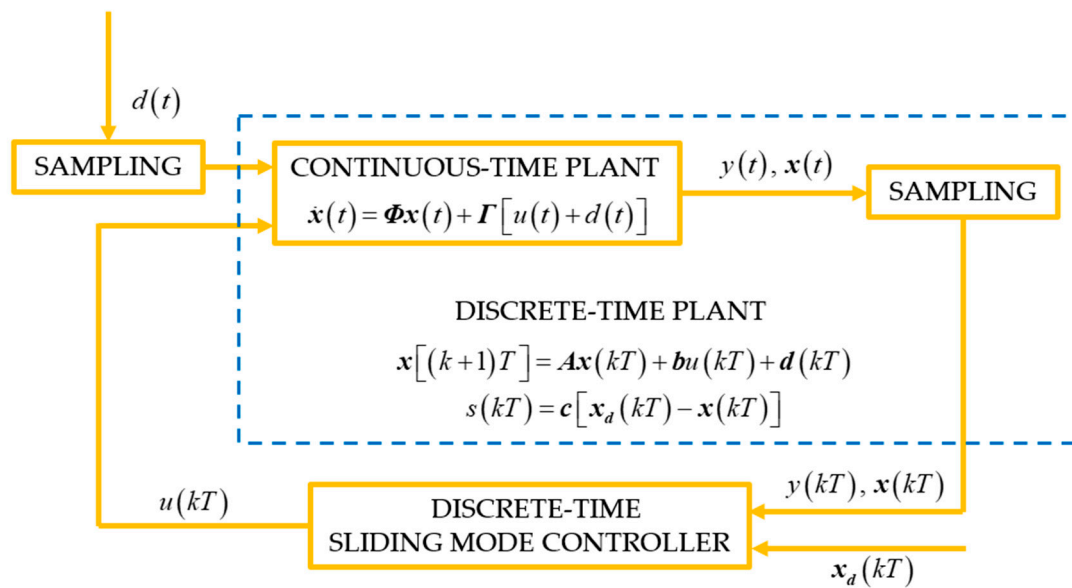


Figure 3. Schematic diagram of a system with sliding mode control.

A completely different approach to the definition of discrete-time sliding mode control was presented by Furuta in [37]. The author assumed driving the representative point of the system to a specific sector in the state space instead of a sliding hyperplane. Within such a sector, a linear control signal ensures the convergence of the control error to zero. Consequently, the need to switch the control signal within the selected sector is eliminated. Unlike previous works, Furuta used Lapunov's principles to demonstrate that the proposed control ensures the asymptotic stability of the closed-loop system. For continuous-time systems, the Lapunov function is customarily defined as $V(t) = s^2(t)/2$, i.e., it is always positive, and the asymptotic stability of the system is ensured if the derivative of the Lapunov function is negative, i.e., $\dot{V}(t) < 0$. In his work, Furuta used a discrete equivalent of this function. According to his work, quasi-sliding motion inside a specific sector is asymptotically stable if $V[(k+1)T] - V(kT) < 0$, where $V(kT) = s^2(kT)/2$. Furuta's strategy was further studied in [38–40] but has not been popularized on a wider scale.

Research on discrete-time sliding mode control continued in various parts of the world. Subsequent works focused on limiting the control magnitude. A universal algorithm for calculating the control values was presented by Utkin and Drakunov [41] in 1989. The authors considered an LTI system, described by (5), with a sliding surface determined by (2). The choice of elements of vector c had to ensure the desired dynamics of the closed-loop system. The authors specified that the control signal should be calculated in such a way as to bring the representative point of the system to the sliding plane in one step. Therefore, the following equation must be satisfied:

$$s[(k+1)T] = c\mathbf{x}[(k+1)T] = 0, \quad (11)$$

which defines the trajectory of the sliding variable. After substituting the system's state equation, the control signal can be calculated as follows:

$$u_{eq}(kT) = (cb)^{-1}[cAx(kT) + cd(kT)]. \quad (12)$$

However, it should be noted that the control (12) contains an unknown external disturbance vector. Hence, it cannot be applied directly, and it is called the equivalent control. The study also analyzed the case when the control signal is limited by an a priori known constant M . To satisfy this limit, the control law was modified.

$$u(kT) = \begin{cases} M & \text{for } u_{eq}(kT) \geq M \\ u_{eq}(kT) & \text{for } |u_{eq}(kT)| < M \\ -M & \text{for } u_{eq}(kT) \leq -M \end{cases} \quad (13)$$

where u_{eq} is the equivalent control, calculated according to (12), and M is a positive constant. Such a modification can be considered to be adding a saturation function to the control signal. The consequence of limiting the control value may be the extension of the regulation process. The representative point of the system may not reach the sliding hyperplane in a single step, as would result from (11). In 1995, Bartolini, Ferrara, and Utkin [42] attempted to solve a similar problem, in this case extending the approach to multiple input plants. According to their work, the equivalent control vector, $u_{eq}(kT)$, is calculated as in (12). Then, its value is compared with the limit vector u_0 . If the signal value remains within the admissible range, i.e., $\|u_{eq}(kT)\| \leq \|u_0\|$, then it can be directly applied. If the signal exceeds it, it must be replaced by $u(kT) = -u_0[u_{eq}(kT)/\|u_{eq}(kT)\|]$, which, when reduced to a single input, is equivalent to the control (13). The authors proved that after such a control modification, monotonic convergence of the system's representative point to the sliding hyperplane is still ensured. However, if the system is subject to unknown disturbances or model uncertainties, the condition $s(kT) = 0$ may not be met. Then, the motion of the system will remain within the δ -vicinity of the sliding hyperplane, where the value of δ results from the sampling period T . To minimize the radius of this band, the authors of [42] introduced an adaptive control strategy based on the reference model (MRAC). A similar approach was also proposed by Kaynak and Denker in [43]. In the work from 1993, a system subject to unknown disturbances and model uncertainties was considered. The authors presented a method of estimating the impact of disturbances on the trajectory based on state feedback and the distance of the system's representative point from the sliding plane. This led to the derivation of a new, adaptive, nonlinear control law that allows the elimination of chattering. Adaptive control was also used in further works [44,45].

In the studies [46,47], the authors analyzed the relationship between the discretization period of the system and the range of oscillation around the sliding hyperplane. As they showed, the radius of the quasi-sliding mode band is directly proportional to the discretization period, which gives the $O(T)$ relationship. In [48], this phenomenon was demonstrated in the example of servo control. The authors of [49] went a step further. Using the expansion of the discrete disturbance signal into a Taylor series, they presented additional conditions allowing the compensation of external disturbances, therefore obtaining a quadratic relationship between the width of the quasi-sliding mode band and the discretization period T . This procedure results in narrowing the quasi-sliding mode band for small sampling periods, which contributed to the popularization of the method. In later years, numerous authors used the Taylor series expansion and introduced more and more far-reaching assumptions regarding the rate of change in external disturbances, which led to the derivation of an exponential relationship between the discretization period and the quasi-sliding mode band radius, achieved in [49–61]. Particular attention should be paid here to numerous works by Ma and his colleagues, which showed that the use of the m -th-order disturbance difference from subsequent discrete-time instants ensures a

quasi-sliding mode bandwidth of $O(T^m)$ radius, where m is a positive integer. However, this result is only achievable under the assumption that the continuous-time disturbance signal has limited derivatives of all orders up to m .

The considerations so far have been devoted to the general principles of the existence of sliding modes for continuous and discrete-time control systems. The presented rules may be applied both to SISO and MIMO plants, where hierarchical control applies. Separate sliding planes are defined for each of the input channels, and the sliding motion occurs at the intersection of these planes. Such control strategies were considered for both linear and nonlinear systems in the works [62–64].

4. Reaching-Law Approach

4.1. Switching Type

A real breakthrough in the discrete-time sliding mode control was brought by Gao, Wang, and Homaifa in 1995 [65]. The authors referred to the work [20], which presented the reaching-law concept in the continuous-time domain and proposed an adaptation of these laws for discrete-time systems. First, however, they introduced a clear definition of the quasi-sliding mode. According to their work, the quasi-sliding mode emerges when:

- The representative point of the system moves monotonically from any initial point towards the sliding plane and crosses it in finite time.
- After the first intersection of the sliding plane, the representative point moves along it in a characteristic zigzagging motion, crossing it again at each step.
- After the sign of the sliding variable changes for the first time, it will change again in each subsequent step, and its absolute value will not exceed an a priori known value.

Furthermore, the authors proposed the design of quasi-sliding mode controllers based on predefining the trajectory of the sliding variable, similar to the works [19,20] for continuous-time systems. While for continuous-time systems, convergence to the switching hyperplane is ensured by an appropriate relationship between the sliding variable $s(t)$ and its derivative $\dot{s}(t)$, for discrete-time systems, the relationship between the sliding variable at two consecutive discrete-time instants must be defined, $s(kT)$ and $s[(k+1)T]$. Thus, the authors proposed a function describing the course of the sliding variable in the form: $s[(k+1)T] = f[s(kT)]$. On its basis, it is possible to determine the value of the control signal in subsequent steps by substituting the system dynamics and the switching hyperplane definition. The reaching function f must be defined in such a way as to ensure all three conditions for the existence of quasi-sliding motion. The authors considered a discrete-time system free from any external disturbances, described by the following state equation.

$$x[(k+1)T] = Ax(kT) + bu(kT). \quad (14)$$

First, they assumed that the controller's task is to bring the sliding variable to the sliding plane in a single step, according to (11). Consequently, the control signal was expressed as $u(kT) = (cb)^{-1}cAx(kT)$, and the new state matrix of the closed-loop system took the form $[1 - b(cb)^{-1}c]A$. The stability of this matrix can be ensured using the pole placement method, optimization, or by designing deadbeat behavior. In the second stage, the authors proposed the following definition of the course of the sliding variable.

$$s[(k+1)T] = (1 - qT)s(kT) - \varepsilon T \operatorname{sgn}[s(kT)], \quad (15)$$

where q and ε are positive constants and $(1 - qT) > 0$. This function contains an element proportional to the sliding variable with a proportionality coefficient q , which determines the rate of convergence to the sliding hyperplane, and a discontinuous element with gain ε , which is designed to ensure the intersection of the sliding hyperplane. The work proves that the presented strategy (15) provides all the properties of the quasi-sliding motion. Additionally, as calculated in later work [66], in the sliding phase, the value of the sliding variable satisfies the inequality $|s(kT)| < \varepsilon$. The study [65] also provided a solution for

a disturbed system. In the second part of the article, the authors took into account that the system (14) was subjected to an external disturbance $d(kT)$ acting through the control channel, and the elements of the state matrix A were subject to modeling uncertainty of the form ΔA . These disturbances were assumed to be limited, and their upper and lower limits were known in advance. On their basis, the impact of the disturbance and uncertainty of the model on the sliding variable was defined as $D(kT) = c b d(kT)$ and $S[x(kT)] = c \Delta A x(kT)$. Moreover, their mean values D_1, S_1 and maximum deviations from the mean values D_2, S_2 were calculated. These values were used to partially compensate for the disturbance in a modified reaching law.

$$s[(k+1)T] = (1-qT)s(kT) - \varepsilon T \operatorname{sgn}[s(kT)] + D(kT) + S(kT) - (D_1 + S_1) - (D_2 + S_2) \operatorname{sgn}[s(kT)]. \quad (16)$$

As the disturbance impact may cause the representative point of the system to deviate from the sliding plane, the control parameters must be carefully selected to ensure that the quasi-sliding motion occurs. The principles of selecting these parameters were partially presented in [65] and developed in [66]. Based on [66], the width of the quasi-sliding mode band for the disturbed system was determined as $\varepsilon + 2(D_2 + S_2)$. Both presented studies clearly show that the system is highly robust to external disturbances and model uncertainties. Moreover, the reaching law-based approach allowed for a separate analysis of the dynamics in the reaching phase, where the proportional term plays a key role, and in the quasi-sliding phase, where the value of the ε parameter directly determines the width of the quasi-sliding mode band. Thanks to these properties, the strategy of [65] became the foundation for the further development of discrete-time sliding mode control. To clearly present the properties of a switching-type reaching law, the evolution of the sliding variable $s(kT)$ in this type of motion is presented in Figure 4. In this case, a second-order integrator was taken into account and discretized with the discretization period of $T = 1$ s. It may be seen that the representative point of the system is driven towards the sliding hyperplane, defined as $s(kT) = 0$. Once the representative point crosses the sliding plane for the first time, it recrosses it in each following step. The range of the deviations from the sliding plane is strictly dependent on the range and sign of the external disturbances.

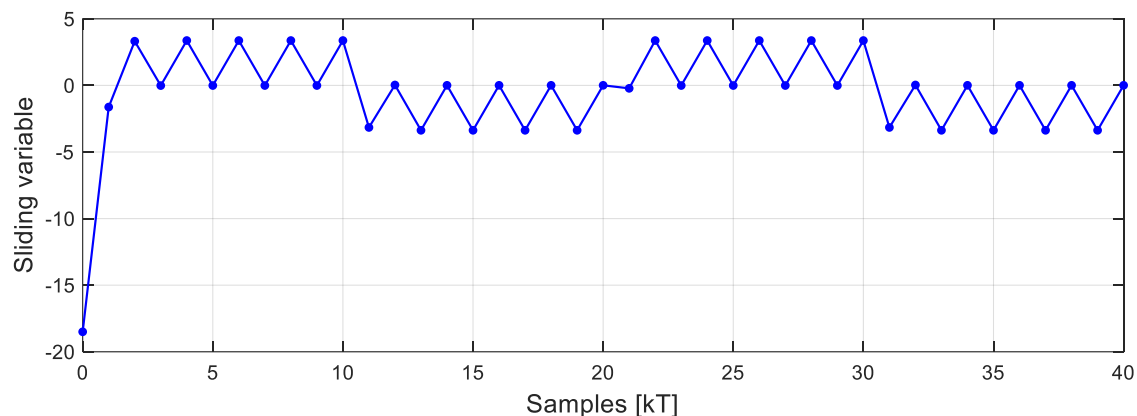


Figure 4. Evolution of the sliding variable in switching-type quasi-sliding mode.

Over the following decades, many authors have analyzed the strategy of Gao and his colleagues. Bartoszewicz in [66] derived additional conditions necessary to ensure sliding motion under the influence of external disturbances. However, Chakrabarty and Bandyopadhyay [67] considered the impact of quantization errors on the dynamics of the system. In addition, in later years, many new reaching laws were introduced. In [68], an adaptive version of Gao's strategy was proposed. It was noticed that large values of gain at the signum element can cause large oscillation amplitudes around the sliding hyperplane. Therefore, it was proposed to use an adaptive algorithm to make the gain value dependent on the current distance of the system's representative point from the sliding hyperplane. Consequently, the constant value of ε was replaced with a function $\varepsilon = |s(kT)| / \rho$, where ρ

is a positive constant. When the distance between the system's representative point and the sliding plane is large, the gain of the switching element also has large values, which increases the convergence rate. However, when the trajectory remains in the vicinity of the switching hyperplane, the gain of the switching element is reduced to minimize the oscillation amplitude. The effects of such modification were verified with an example of an optoelectronic system control with a servo drive. A similar procedure was used in [69]. Here, however, the authors focused on adapting the gain of the switching element at the expense of eliminating the proportional component. In this paper, the system's trajectory is divided into two zones: a fast convergence zone and a slow convergence zone. In the first one, the gain of the switching element is constant, equal to σ_1 . This value determines the boundary between the areas. When the sliding variable enters the second region, the gain is scaled by $1/\alpha$, which slows down the convergence process but, at the same time, reduces the chattering effect around the sliding hyperplane, which reduces the quasi-sliding mode bandwidth. In further studies [70,71], the same strategy was applied to control the position of an induction motor.

The reaching law presented by Gao and his colleagues has also been used in the previously mentioned disturbance compensation strategies. In the works [47,48], the authors aimed to increase the robustness of the system, assuming that the maximum rate of change in the disturbance in a single step is known. Thanks to this assumption, by applying a Taylor series expansion of the disturbance signal, it is possible to introduce a compensating element based on the difference between the sliding variable from two consecutive time instants. This allows the reduction of the quasi-sliding mode bandwidth compared to Gao's original method. In [52], it was proposed to replace the switching element in the reaching law with an approximating function $F(kT) = \varepsilon Ts(kT) / [|s(kT)| + \rho]$, which, after adding a proportional element and a disturbance compensator, allowed for quasi-sliding motion inside a layer narrower than the work [47]. In later works by Ma and his colleagues [53–58], the disturbance compensator was successfully used in combination with new reaching laws. Paper [53] introduced a reaching law based on the exponential function of the sliding variable in the simplified form

$$s[(k+1)T] = s(kT) - \frac{\lambda}{\beta + (1-\beta)e^{-\varphi|s(kT)|^\gamma}} \operatorname{sgn}[s(kT)] + \delta_m(kT) \quad (17)$$

where $\lambda, \gamma, \varphi, >0, 0 < \beta < 1$ and δ_m is the disturbance compensation term using m -th order difference. This modification of the switching term allows for the adaptation of the convergence rate depending on the value of the sliding variable. The larger the value of the sliding variable, the greater the gain, which results in faster convergence to the sliding manifold than Gao's original strategy. Moreover, in the vicinity of the sliding hyperplane, the introduced exponential function reduces to one, which results in the reaching law as in (15). Additionally, using a third-order difference to compensate for disturbances, a quasi-sliding mode band $O(T^3)$ width was achieved, which was confirmed by a numerical example. Article [54] uses the same exponential function, adding an element of the power function of the sliding variable at the switching element in the form: $|s(kT)|^\alpha$, where $\alpha \in (0, 1)$. This modification allows for a smoother adjustment of the convergence rate depending on the distance of the representative point of the system from the sliding hyperplane than in [53]. This led to the definition of the so-called power-rate reaching laws, defined as

$$s[(k+1)T] = (1-qT)s(kT) - k_0|s(kT)|^\alpha \operatorname{sgn}[s(kT)] + \delta_m(kT), \quad (18)$$

where $0 < (1-qT) < 1$, k_0 is positive and $\alpha \in (0, 1)$. A similar procedure was used in [72]. The works [55,73] abandoned the exponential function at the expense of the power function of the sliding variable, achieving similar results in the form of adjustable gain of the switching element, dependent on the distance from the sliding hyperplane. Accordingly, in [73], a single element $|s(kT)|^\alpha \operatorname{sgn}[s(kT)]$ was used, and in [55], two elements with such

a structure were used. In [74], a comparison of different power-based reaching laws was presented. The paper considered an exponential reaching law, a single power function, and a double power function, showing that the use of two power elements ensures the fastest convergence to the quasi-sliding manifold. Scaling the switching element through a power function of the form $\varepsilon|s(kT)|^\alpha \text{sgn}[s(kT)]$, where $0 < \alpha < 1$, appeared in [75], which allowed for a significant reduction in the width of the quasi-sliding mode band compared to the strategy of Gao. Furthermore, in [75], the authors proposed the use of a digital filter to reduce the impact of noise and model uncertainty on the system. They presented a method for designing a quasi-sliding mode controller using a Kalman filter. As a result, the signum function was replaced by saturation, which allowed for effective reduction of chattering. The effectiveness of this procedure in minimizing chattering effects for continuous-time systems contributed to the use of a similar solution for discrete-time systems, which was analyzed in [76]. Same as in the previous article, the authors decided to replace the signum function with saturation in the reaching law (15). It has been proven that this modification ensures monotonic convergence of the trajectories to the sliding hyperplane. Moreover, near the switching plane, the regulation process is slowed down, which eliminates chattering. On the other hand, in [56], the advantages of the exponential function were combined with a new dead zone function, which eliminates the switching element inside a very narrow layer around the switching hyperplane. This allows a further reduction of the width of the quasi-sliding mode band, which directly increases the robustness of the system. Moreover, it is proposed here to use an n -th order estimator to compensate for disturbances, which results in a quasi-sliding mode bandwidth of $O(T^{n+1})$ order. An identical compensation method was also used in [57], where the power-rate reaching law was applied, and the results were tested in simulation on a magnetic railway model.

The next stage in the development of the quasi-sliding mode control was the introduction of the general form of reaching law by Bartoszewicz and Leśniewski in [77].

$$s[(k+1)T] = s(kT) - f[s(kT)] - \varepsilon \text{sgn}[s(kT)], \quad (19)$$

where f is an a priori designed reaching function and ε is a positive constant. The authors proposed a universal method describing how to construct functions describing the sliding variable profile to guarantee all the attributes of the quasi-sliding motion defined by [65]. According to them, the reaching function should contain a proportional element, which is responsible for bringing the system's trajectory from any initial position to the sliding manifold, and a signum element with appropriate gain, which is to ensure a change in the sign of the sliding variable at each step in the sliding phase and determines the width of the quasi-sliding mode band. Moreover, the proportional factor q , proposed in [65], can be replaced by an appropriately selected function $f[s(kT)]$. This function must be non-decreasing and symmetrical with respect to the sliding variable. Moreover, its derivative with respect to $s(kT)$ cannot exceed unity, and the maximum absolute value of the function must be limited. The last of the mentioned features results in a limited rate of change in the sliding variable. The following works propose several example functions that meet the above requirements and can be used to generate trajectories of $s(kT)$. In [78], the use of an inverse tangent function was suggested, and the proposed strategy ensures a reduction of the quasi-sliding mode band and, consequently, an improvement of the system's robustness compared to [65]. Moreover, since the inverse tangent function is both upper and lower bounded, it allows limiting the maximum rate of change in the sliding variable in a single step. A similar structure of the reaching law was presented by Latosiński in [79]. The author proposed modifying the strategy (16) by replacing the proportional element q with the function $\min\{1, |s(kT)|/q\}$. Thanks to this, for absolute values of $s(kT)$ greater than q , the proportionality coefficient is constant and equal to one. Naturally, for $s(kT) = 0$, the proportionality coefficient is 0. Consequently, the proposed strategy ensures a limitation of the rate of change in the sliding variable, which results in a limitation of the control signal. Limited change in sliding variables in a single step and reducing the width of the quasi-sliding mode band were also achieved by Bartoszewicz and Latosiński [80].

Limiting the rate of change in the sliding variable in the initial phase of the movement was the motivation for the creation of a new reaching law presented in [81]. The authors proposed a modified version of Gao's strategy, in which they replaced the proportional element q with the function $q[s(kT)] = s_0/[s_0 + |s(kT)|]$. Consequently, the maximum rate of change in the sliding variable, occurring in the initial phase of regulation, has been limited, which directly reduces the value of the control signal. As the value of $s(kT)$ decreases, the coefficient q increases, thus ensuring that the system converges to the sliding plane faster than with a constant value of q in Gao's original strategy. Moreover, the introduced modification also reduces the width of the quasi-sliding mode band and limits the control signal, which was not achieved in [65].

In 2012, Chakrabarty and Bandyopadhyay proposed a new approach to analyzing the strategy proposed by Gao. In [82], they presented the design of a sliding mode controller in accordance with the so-called band approach. Gao's approach assumed defining a function describing the trajectory of the sliding variable, determining the control signal and the width of the quasi-sliding mode band on its basis. In [82], the authors started by determining the desired width of the quasi-sliding mode band and then used Gao's reaching law to determine the parameters and values of the control signal. For the needs of such a strategy, the authors divided the state space into several areas:

- The B_d decrement band is the area where the value of the sliding variable is monotonically reduced. For any $|s(kT)| > B_d$, its absolute value in the next instant satisfies: $|s[(k+1)T]| < |s(kT)|$.
- The limit to cross band B_c has been defined as the value that cannot be exceeded after the sliding variable changes sign for the first time starting from the decrement band, which can be described by the relationships: if $\text{sgn}[s(kT)] = -\text{sgn}[s[(k+1)T]]$ and $|s(kT)| > B_d$ then $|s[(k+1)T]| < B_c$.
- The maximum crossing band B_s is the value that cannot be exceeded by the value of the sliding variable after the representative point of the system intersects the sliding plane, regardless of the starting point.
- The ultimate band δ is a value such that if $|s(kT)| \leq \delta$ then $|s[(k+1)T]| \leq \delta$.

By solving the inequalities describing the subsequent layers, the authors determined the required values of the control parameters. Their strategy results in a sliding mode band with a width between $D_{\max}$ and $2D_{\max}$, which is smaller than in Gao's method. The approach based on the width of the quasi-sliding mode band was used to control a continuous-time system subject to external disturbances and discretization [83]. The authors first determined the required width of the quasi-sliding mode band, used the sliding variable trajectory proposed by Gao, and, based on it, calculated the necessary values of the control parameters. As a result, they obtained a strategy in which the gain of the switching element in Gao's strategy should be adapted according to the current value of the sliding variable. Their strategy was extended to a general form in [84,85]. First, in [84], a generalized reaching law was proposed. It should contain a function of the sliding variable f_1 , a function of another known system variable f_2 , and a function or constant f_3 as the gain of the switching term. For this general form of the strategy, a bandwidth-based approach was used, and the required properties of individual reaching-law elements were determined. As a result, the obtained trajectory of the sliding variable was limited by functions known to the designer. The same general form of the reaching function was considered in [85] to determine the effect of different rates of convergence to the sliding hyperplane on the system dynamics and the width of the quasi-sliding mode band. As follows from [85], the greater the convergence rate defined by the f_1 function, the greater the width of the quasi-sliding mode band. Therefore, the selection of reaching-law parameters is always a compromise between the convergence rate and robustness level.

4.2. Nonswitching Type

Another breakthrough in the history of discrete-time sliding mode control was brought by the nonswitching type definition of the quasi-sliding mode. A quasi-sliding motion,

where the representative point of the system does not need to cross the sliding surface, was first used by Utkin in [41]. However, a clear definition of such a case was not presented until [86], where Bartoszewicz defined the nonswitching quasi-sliding mode, therefore eliminating the chattering problem. According to his definition, the representative point of the system must monotonically converge to a specified vicinity of the sliding manifold. Once it enters a specified neighborhood of the switching plane, it will never leave it again. However, it is not required that the representative point of the system intersects the sliding hyperplane and that the sliding variable changes sign in each subsequent step. The new definition of quasi-sliding motion initiated the development of a new trend of reaching laws of a nonswitching nature. In [86], the author proposed a reaching function based on a predetermined reference sliding variable trajectory and defined the requirements that the given trajectory must meet to ensure monotonic convergence of the system to the quasi-sliding mode band. Moreover, the author noted that the width of this band can be reduced for slowly varying disturbances using a disturbance estimator based on the difference between the current value of the sliding variable and the value calculated from the reaching law in subsequent steps. The results of [81,86] were extended in [87], where the author compared his strategy with Gao's original method and showed that the use of nonswitching reaching laws allows for narrowing the sliding mode band, improving the system's robustness and eliminating chattering. To better explain the evolution of the sliding variable $s(kT)$ in a nonswitching type of quasi-sliding motion, Figure 5 presents an example. Here, a second-order integrator system with a discretization period of $T = 1$ s was considered. In this case, the representative point of the system is driven towards the sliding hyperplane. However, contrary to the switching type of motion, the representative point does not have to cross the sliding plane. Instead, it must stay in its vicinity, whose size is strictly dependent on the range of external disturbances. Crossing of the sliding plane may occur due to changes in the sign of the disturbance, but the sign of the sliding variable does not have to change in each consecutive time instant.

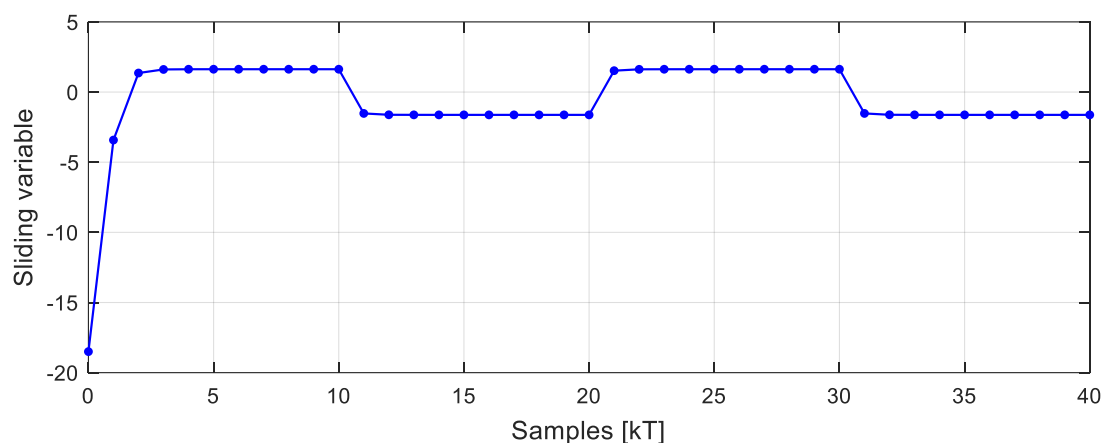


Figure 5. Evolution of the sliding variable in nonswitching type quasi-sliding mode.

The definition of a nonswitching type of quasi-sliding motion quickly gained popularity for practical reasons. Eliminating the requirement to change the sign of the sliding variable at each step of the sliding phase reduces the wear of electronic switching elements and reduces the likelihood of mechanical damage. Thanks to these advantages, such a definition was also adopted by Golo and Milosavljević in [88,89]. The authors took into account a continuous-time system without disturbances, discretized using the progressive difference method. For such a system, an adaptive reaching law was used, resulting in the following class of systems:

$$s[(k+1)T] = s(kT) - T\Phi[s(kT), x(kT)], \quad (20)$$

where Φ is a function chosen to divide the regulation process into two stages. In the first stage, the control signal operates in a nonlinear mode to bring the system's trajectory to the vicinity of the sliding hyperplane. When the representative point of the system enters the vicinity of the sliding hyperplane, the control signal switches to linear mode, ensuring that the trajectory of the system will move onto the sliding plane in one step. Then, the representative point will move along it in subsequent steps. The work also showed that in the case of systems subject to external disturbances, perfect sliding motion cannot be guaranteed. Therefore, the strategy ensures that the representative point of the system remains within a specific quasi-sliding mode band with a non-increasing radius. The authors of [89] showed that the system's robustness increases with the reduction of the sampling period, and the results of their work were confirmed by an example of a DC drive control. Another adaptation of the nonswitching definition of quasi-sliding motion was considered in 2010 [90]. The authors proposed a reaching law based only on the mean disturbance compensation term and a signum term scaled by the maximum disturbance deviation from the mean value. The authors proved that their strategy reduces the width of the quasi-sliding mode band compared to Gao's control strategy [65].

Discrete-time sliding mode control based on the reaching laws of both switching and nonswitching nature has become so popular that in 2012, an extensive review of the quasi-sliding mode control design methods was carried out [88,91]. Predefined sliding variable trajectories of switching and nonswitching types were also compared in [81], where a new sliding variable trajectory was presented. The authors not only used scaling in the proportional part of the reaching law but also proposed the elimination of the switching element, therefore adopting a new nonswitching definition of quasi-sliding motion. Their strategy ensures that the trajectory of the system converges to the quasi-sliding mode band from any initial state and remains there. The work showed that the use of nonswitching quasi-sliding mode control strategies allows the achievement of a narrower sliding mode band, limits the value of the control signal, and eliminates the undesirable chattering effect. The nonswitching version of the proposed strategy was used to control data flow in a telecommunications network in [92] and for supply control in the logistics system, as described in [93]. Moreover, in the above works, deadbeat selection of the sliding hyperplane was used, which ensures the convergence of the system state to the set position in a finite time, no longer than n steps, where n is the order of the system. Another nonswitching type function was presented in [78,94], where the authors proposed the use of the inverse tangent function, which results in limiting the rate of change in the sliding variable and, consequently, limiting the control signal. On the other hand, in work [95], the same results were obtained with a hyperbolic tangent function. Similar properties were also achieved in [96] with the application of an exponential function.

The reaching law presented in [81] was also used in [97], where a modified proportional part was combined with a minimum function for the gain of the switching element. The adaptive nature of the proportional part ensures that the control signal is limited in the initial control phase, and the minimum function used allows for the elimination of switching within the sliding mode band. A similar procedure was used in [98], where a hybrid structure of the reaching law was proposed, combining the advantages of Gao's strategy [65] with the nonswitching sliding motion proposed by Utkin in [41]. The minimum function in the gain of the switching element was also used in [99], where the considerations were supplemented with a state estimator.

5. Recent Trends

The goal of the discrete-time sliding mode control design is narrowing the quasi-sliding mode band, which directly reflects the level of robustness of the system to external disturbances and modeling uncertainties. As described in the previous chapter, this may be obtained with various reaching laws and disturbance compensation methods. However, recent years have brought some alternative control schemes that allow robustness improvement.

5.1. Higher Relative Degree Sliding Mode

All the works presented above used first-order sliding variables with respect to the control signal. This means that the dynamics of the system, including the control $u(kT)$, allow the calculation of the variable $s[(k+1)T]$, and the trajectories of the sliding variable were described by a function of the form $s[(k+1)T] = f[s(k)]$. However, in recent years, several authors have worked on sliding mode control strategies using higher-order sliding variables. The concept of higher relative degree output constitutes a new reaching-law definition, where the control signal at step k is designed to impact the sliding variable at step $k+r$, where r is the relative degree. As it turns out, such a procedure results in narrowing the quasi-sliding mode band, which is a direct manifestation of the system's robustness. This idea was first presented in [100]. The work took into account LTI systems with a relative order of output r , assuming that $r \leq n$, and n represents the order of the system's dynamics. The authors decided to expand the basic definition of the system's trajectory, in which the sliding variable is influenced by the disturbance from the last step, i.e., $s[(k+1)T] = d_1(kT)$, to the following form $s[(k+r)T] = d_r(kT)$. It is worth noting that as the order of the sliding variable changes, the impact of the disturbance on the system's operation, and therefore its maximum values, also changes. Therefore, it is necessary to use the correct letter designations. The term $d(kT)$ should not be confused with $d_r(kT)$. Then, the authors presented a detailed analysis of the control strategy for second-, third-, and n -th-order sliding variables. They compared the obtained widths of quasi-sliding mode bands with the results of the original strategy with a relative degree output one and showed that with the appropriate selection of the sliding hyperplane, systems with higher-order sliding variables show better robustness to disturbances. The work also proves that the use of higher-order sliding variables allows the state variables to converge to zero. In the general case, first-order sliding variable strategies guaranteed that the sliding variable converged to zero in a finite time while the system state converged only asymptotically. The authors of [100] showed that the use of higher-order sliding variables ensures that the system's state converges to zero in a finite time.

In [101], Chakrabarty and Bartoszewicz developed this idea for second-order sliding variables. For this purpose, they took into account the switching-type control strategy for systems with disturbances presented by Gao in [65] and the nonswitching strategy from [87] and modified them to the form $s[(k+2)T] = f[s(kT)]$, whereas in the relative degree one approach, the reaching law describes the relation $s[(k+1)T] = f[s(kT)]$. To design a second-order sliding variable, the sliding manifold must be selected so that $c_2b = 0$ and $c_2Ab \neq 0$, so that the control $u(kT)$ does not impact the sliding variable in the nearest step $(k+1)T$, but two steps ahead, so at $(k+2)T$. After a proper selection of the sliding manifold, the author has adapted the original reaching law of Gao [65] for the second-order sliding variable, obtaining:

$$s_2[(k+2)T] = q^2 s_2(kT) - q\epsilon_2 \operatorname{sgn}[s_2(kT)] - \epsilon_2 \operatorname{sgn}\{s_2[(k+1)T]\} + D_2(kT). \quad (21)$$

With this definition of the sliding variable profile, the control signal $u(kT)$ directly affects the state of the system two steps forward, i.e., $s[(k+2)T]$. Consequently, such a strategy cannot guarantee monotonic convergence of the sliding variable to the switching plane. As shown by the authors of [101], the control signal forces the absolute value of the sliding variable to decrease in every second step, i.e., in the reaching phase, the inequality $|s[(k+2)T]| < |s(kT)|$ is satisfied. Such evolution of the sliding variable is presented in Figure 6.

In the first part of the work, it was shown that in the absence of disturbances, the use of a second-order sliding variable results in achieving stability of the system state in a finite time. However, in the case of a system with disturbances, in the quasi-sliding phase, the state of the system is limited from above and below. Then, a comparison of the two analyzed strategies was made with their counterparts using first-order sliding variables. It

is shown that in both cases, the appropriate selection of vector c allows the reduction of the width of the quasi-sliding mode band.

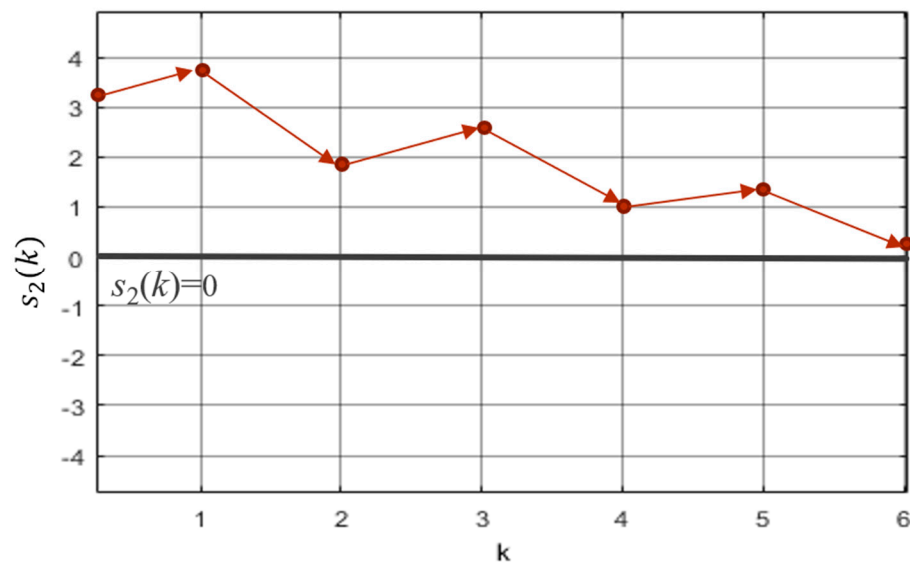


Figure 6. Evolution of the relative degree two sliding variable.

Gao's strategy was taken into account again in the next work [102], where it was modified for a sliding variable of any order. The authors derived a general form of the rule for selecting vector c_r for a sliding variable of order r . According to them, the selection of elements of the vector c_r should satisfy $c_r A_i b = 0$ for $i = 1, \dots, r - 2$ and $c_r A_{r-1} b \neq 0$. Then they proposed a new general form of the reaching law for sliding variable $s_r[(k + r)T]$ consisting of a proportional element and signum elements related to the values of the sliding variable in subsequent steps $k, k + 1, \dots, k + r - 1$. The work showed that in the worst case, the representative point of the system intersects the sliding plane for the first time between steps $k + r - 1$ and $k + r$ and then remains within the quasi-sliding mode band until the end of the control process. Moreover, to eliminate the reaching phase and ensure quasi-sliding motion from the very beginning of the control process, the authors of [102] proposed combining the idea of higher-order sliding variables with a non-stationary sliding hyperplane.

Subsequent studies focused on the development of new predefined trajectories for higher-order sliding variables. In 2016, Bartoszewicz and Latosiński proposed a new reaching law for a system with a second-order output [103]. The authors analyzed a modified version of the Gao strategy, which contains an adapted proportional element in the form $s(kT) |s(kT)| / s_0$ for $s(kT) > s_0$ and $s(kT)$ for $s(kT) \leq s_0$, a switching element with an appropriate amplification and an element compensating for average disturbance. The authors first applied the strategy to a system with a first-order sliding variable and proved that it ensures the convergence of the system's trajectory to the sliding hyperplane in a finite time. Moreover, it was shown that if the sliding variable is in a specific environment of the sliding hyperplane, it will never leave this environment, and the sign of the sliding variable will change in each subsequent step, which meets the definition of quasi-sliding motion presented by [65]. It was also shown that if the new state matrix of a closed-loop system is a nilpotent matrix, i.e., it was selected using the deadbeat method, the values of the system's state variables in the sliding phase are limited. In the next part of [103], the authors focused on systems with a second-order sliding variable and proposed a new predefined trajectory of this variable consisting of functions and switching elements related to $s_2(kT)$ and $s_2[(k+1)T]$ and a component compensating the average disturbance. It was shown that the representative point of the system is brought inside the quasi-sliding mode band in a finite time and remains there until the end of the control process. Moreover, in accordance with the definition of quasi-sliding motion, after the first change in the sign

of the sliding variable, subsequent sign changes will occur in each subsequent step. The authors demonstrated that the resulting width of the sliding mode band for their strategy is smaller than in the case of a first-order sliding variable, which translates into increased system robustness.

The results of [103] were further developed in [104], where it was shown that the use of a second-order sliding variable with the appropriate selection of control parameters ensures a limitation of the values of all state variables from above and below in the sliding phase, and the values of these limitations result directly from the width of the sliding mode band. Therefore, the state variables fall into a narrower layer than in the case of a first-order sliding variable strategy. The strategy was extended to multiple input systems in [105].

5.2. Multirate Output Feedback

A popular problem in sliding mode control applications is the availability of the full-state vector. The sliding mode control concept, both in continuous-time and in the discrete-time domain, is based on full-state feedback. In real systems, we often encounter an input-output description, and measurement of the full-state vector is not possible. A solution to such a problem was presented by Bandyopadhyay and his colleagues in strategies based on fast output sampling or multirate output sampling techniques. This method involves sampling the output signal at a higher frequency than the changes in the input channel. In [106], Saaj, Bandyopadhyay, and Unbehauen combined this idea with Gao's reaching law [65]. Their work assumes controlling the system with a frequency of $1/T$ and measuring the output with a frequency of N/T , thanks to which N values of the output signal are obtained during each discretization period T . Therefore, the authors were able to reconstruct the state vector, considering the system's description and the output measurements. A schematic diagram of a multirate output feedback system is presented in Figure 7.

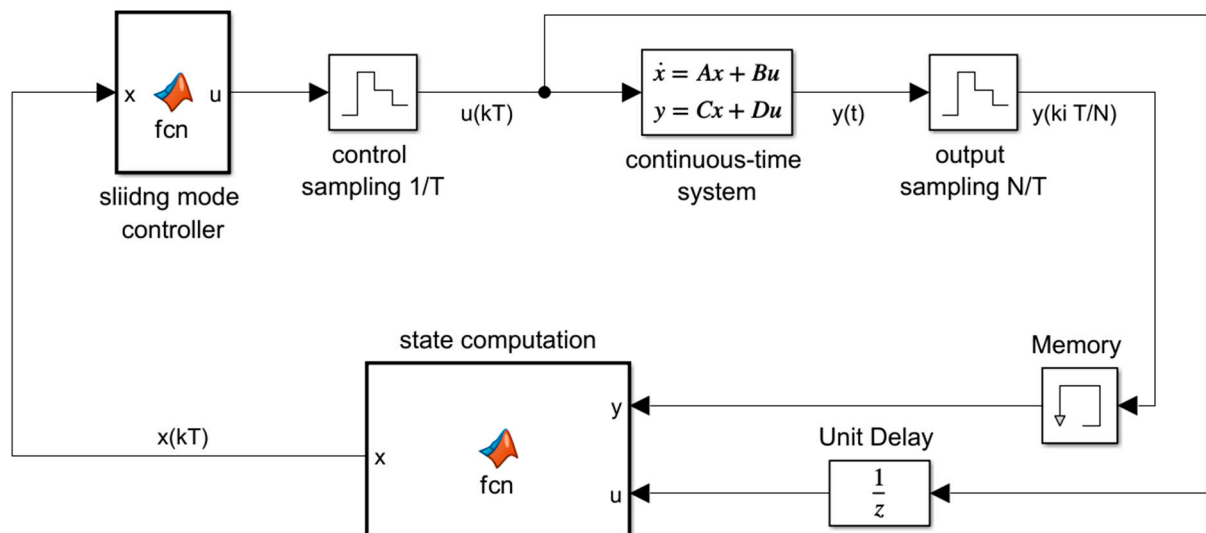


Figure 7. Schematic diagram of a multirate output feedback system.

It may be seen from the figure that during one control period, i.e., between kT and $(k + 1)T$, N output samples are obtained. Furthermore, the authors proposed to use a linear combination of those N samples in a state computation block, which reconstructs the full-state vector using output measurements and the last control value. Having the reconstructed state vector, a sliding mode control scheme may be applied, where the control is constant in the period T . The evolution of the control signal with respect to the following state measurements is depicted in Figure 8. In this case, $N = 2$, i.e., two output samples are used to compute the state vector in each control period T .

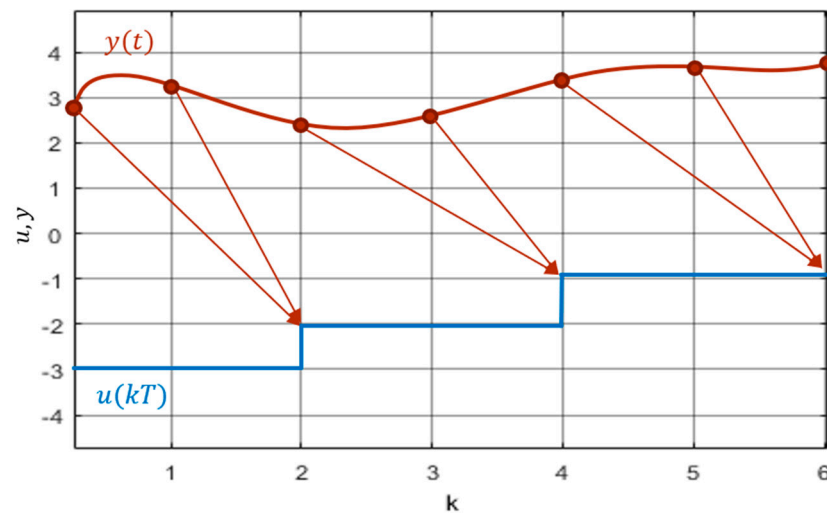


Figure 8. The evolution of the output signal and the control signal in a multirate output feedback control scheme.

The authors showed that the proposed strategy ensures stable quasi-sliding motion, consistent with Gao's definition, despite the lack of availability of the full system state vector. Therefore, the multirate output feedback technique allows the reconstruction of the full-state vector from the output samples only, making the sliding mode control applicable even when full-state measurement is not possible. The multirate output feedback method presently plays a key role in sliding mode control applications and contributes to its popularization. The multirate output feedback strategy was strongly developed in later years, which is confirmed by numerous articles [107–111] and a book [112] devoted to this issue.

5.3. Event-Triggered Sliding Mode Control

One of the newest directions of development in discrete-time sliding mode control is event-triggered control. The idea of event-triggering assumes that the state measurement and control calculation occur at each step $k, k + 1, k + 2, \dots$, etc. However, the actuator shall only update the control signal applied to the plant when it results in a significant change in the output. Therefore, an error vector is calculated at each step k :

$$e(kT) = x(k_iT) - x(kT), \quad (22)$$

where $e(kT)$ represents the state error between discrete-time instants k_iT and kT , and k_i denotes the last instant when the control signal was updated. The state error is used to evaluate a certain stabilization rule at each moment kT . The stabilization rule is constructed so that it interprets the sliding variable error in the following step, i.e., $|cAe(kT)| \leq \alpha$. The control signal is updated only at those triggering instants k_i when the stabilizing rule is violated, not at each sampling instant k . At the instants $k \in [k_i, k_{i+1})$ the control remains constant. In other words, the control triggering instants are detected according to the condition:

$$k_i + 1 = \inf[k > k_i : |cAe(kT)| \geq \alpha], \quad (23)$$

where α is a positive constant. The evolution of a control signal, with marked triggering instants, is presented in Figure 9. Not every state measurement leads to an update of the control value.

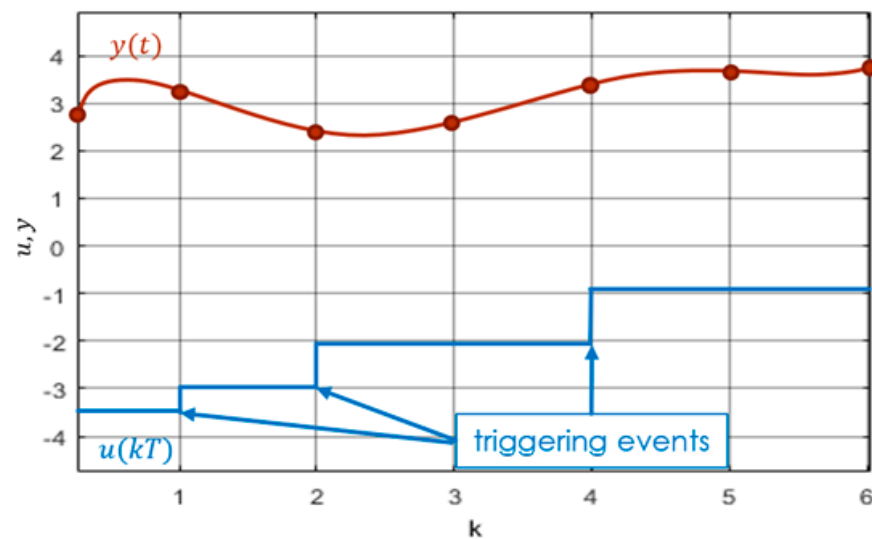


Figure 9. The evolution of the system output and the control signal in an event-triggered control scheme.

Event-triggered sliding mode control is a solution for large, spatially distributed systems where communication time is considerable, such as, for example, communication networks. On the contrary to the previous methods, the event-triggering scheme does not improve the robustness of the system. In fact, the width of the quasi-sliding mode band is enlarged by α . However, it reduces the data exchange and, therefore, communication time. This allows for more optimal use of the resources without destabilizing the system. This is a strategy introduced in 2016 [113] aimed at limiting communication between the controller and the facility. The authors proposed that the control signal should not be updated at each subsequent discrete-time moment kT but only at those selected moments when a specific condition of system stability is not met. This condition is derived based on the assumed width of the required quasi-sliding mode band, which is larger than in the traditional case. Detailed studies on this method were presented in the book [114], where the law of achieving the sliding hyperplane from [41,87] was considered, and in the articles [114–119], where special classes were taken into account regulation systems. Although triggering by events generally worsens the system's robustness, this approach can be used in the case of large, distributed control facilities, where communication delays become important.

5.4. Model Reference Sliding Mode Control

Another recently studied topic is quasi-sliding mode control with a reference sliding variable profile. The concept aims to minimize the impact of disturbances on the system by applying a new trajectory following the reaching law for the disturbed plant and using an external trajectory generator. The reference sliding variable profile, denoted with $s_d(kT)$:

$$s_d[(k+1)T] = f[s_d(kT)], \quad (24)$$

may be generated using a mathematical model of the considered system, as proposed in [120], or through any chosen reaching law, as in [121], and is not influenced by any external disturbances. The strategy aims to drive the system's sliding variable to its desired value at each step with a novel reference sliding variable following the reaching law in the form:

$$s[(k+1)T] = s_d[(k+1)T] - D(kT), \quad (25)$$

where $D(kT)$ represents the disturbance impact. On the other hand, classical reaching laws are defined as:

$$s[(k+1)T] = f[s(kT)] - D(kT). \quad (26)$$

At first glance, it may be seen that in the conventional control concept, the sliding variable bears and accumulates the impact of disturbances, as each next step calculation uses the previous sliding variable value, also influenced by the previous disturbance value. On the other hand, the model reference approach replaces the sliding variable with the ideal reference value. As the control is based on the reference position, all the disturbance values from the beginning of the control process are rejected except for the last step. Conventionally, on the contrary, the sliding variable is calculated according to a chosen reaching law and accumulates the disturbance impact throughout the control process. The model reference approach assumes the plant's sliding variable is controlled according to the desired trajectory, which does not depend on disturbances. Therefore, the sliding variable $s(k+1)$ only bears the impact of the disturbance $D(k)$ from one control step and is not affected by the previous disturbance values $D(0), D(1), \dots, D(k-1)$. As a result, the model-based approach guarantees an improvement in the robustness of the system by reducing the ultimate bandwidth and state error. A schematic diagram of a reference sliding variable-based sliding mode control system is shown in Figure 10.

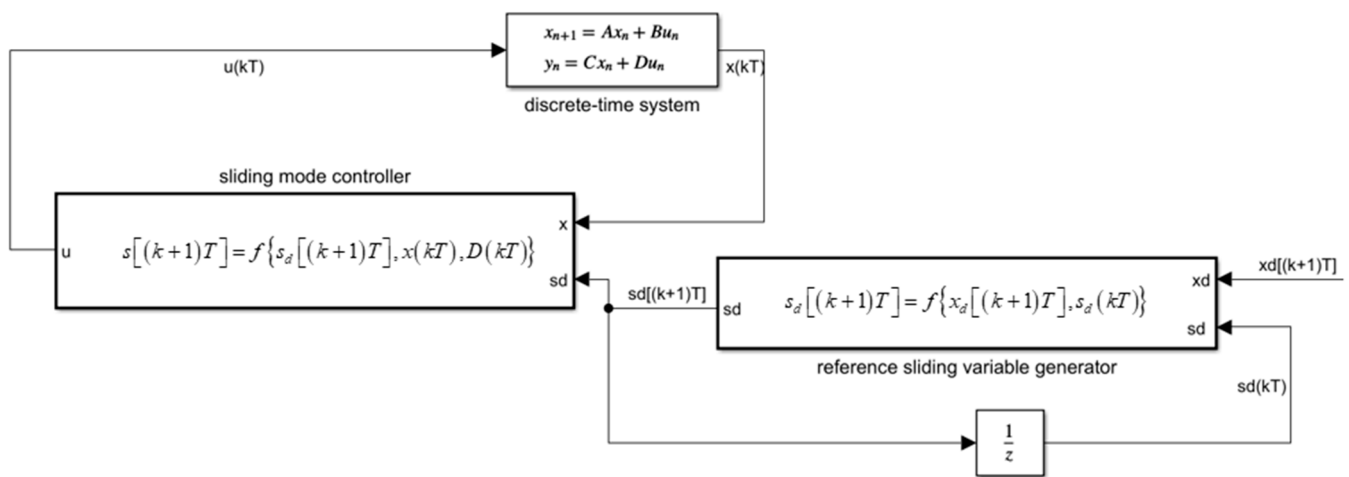


Figure 10. A schematic diagram of a model reference-based sliding mode control system.

The idea of a reference sliding variable profile was first proposed in [120], where a mathematical model of the considered system was applied together with the conventional reaching law of [65]. The study showed that the model reference strategy ensures a smaller quasi-sliding mode bandwidth while satisfying all the switching-type quasi-sliding mode principles. Later, in [121], the model was eliminated, and reference trajectory was obtained by simple calculation of a selected reaching function. Next, a new reference sliding variable following the reaching law was proposed so that the representative point of the system is driven to the demand position with an accuracy of one time instant disturbance value only. Therefore, the impact of external disturbance from all previous steps is rejected, which results in a smaller quasi-sliding mode bandwidth. The paper also proved that the reduction of the quasi-sliding mode bandwidth results in smaller absolute errors of all state variables. In later studies, various demand profiles of the sliding variable were proposed, resulting in switching [122–124] and nonswitching types of quasi-sliding motion [125–128]. The idea was also applied to systems with mismatched disturbances [129]. On the other hand, in [130], the authors proposed a relative degree two reaching law for the reference sliding variable generator. In [131,132], the idea was applied to inventory system control. On the other hand, in [133], the model reference control scheme was used for a DC-DC buck converter.

6. Conclusions

The above study considered the basics of sliding mode control in the discrete-time domain. As in the discrete-time case, the frequency of the control signal updates is lim-

ited, and the ideal sliding mode in the presence of external disturbances or modeling uncertainties is not achievable. Therefore, the manuscript described the conditions of the existence of sliding modes for discrete-time systems and presented different design methodologies. The focus of the paper was two alternative quasi-sliding mode definitions. The first one, presented by [65] is a direct interpretation of continuous-time sliding motion. It assumes that in the sliding phase, the representative point of the system must intersect the sliding hyperplane between each consecutive time instant. Therefore, in the obtained quasi-sliding motion, the sign of the sliding variable changes in each step. The other mainstream definition removes the switching condition, stating that in the quasi-sliding mode, the representative point of the system moves in the vicinity of the sliding hyperplane. However, changing the sign of the sliding variable is not enforced. As it turns out, the removal of the switching requirement, in general, leads to smaller widths of the quasi-sliding mode band, which improves the system's robustness. For the quasi-sliding mode definitions, the study reviewed numerous design methods, mainly based on the reaching-law approach. The paper showed the advantages of different control laws and disturbance compensation methods. Finally, the study presented some new trends in the quasi-sliding mode control, such as the multirate output feedback method, event-triggered control, or applications of higher relative degree sliding variables.

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