

Article

Constrained Model Predictive Control for Generation Power Distribution on Aircraft Engines

Lingfei Xiao ^{1,*} , Yushuo Tan ¹, Robert R. Sattarov ²  and Ye Wei ¹

¹ College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China; ystan18@nuaa.edu.cn (Y.T.); yeewei@nuaa.edu.cn (Y.W.)

² Institute of Digital Systems, Automation and Energy, Ufa State Petroleum Technological University, 450064 Ufa, Russia; sattar.rb@gmail.com

* Correspondence: lfxiao@nuaa.edu.cn

Abstract: Aiming at the increasing demand for electric energy in aircraft in the future, a multi-objective optimization aircraft engine constrained model predictive control method based on generation power distribution is proposed. Firstly, based on the aircraft engine component level model and the equilibrium manifold theory, the aircraft engine equilibrium manifold expansion model is established. Secondly, the influence of the power generation is modeled, and the influence of the low- and high-pressure shaft generators on the normal operation of the aircraft engine is studied and compared. The control variables such as fuel flow and total generation power are taken as the constraint conditions to design the constraint model predictive controller. Furthermore, the multi-objective grey wolf optimization algorithm is introduced to intelligently optimize the parameters of the designed controller. At last, the simulation based on the component level model shows that the high-pressure shaft generator has less influence on the state quantity, including engine thrust, than the low-pressure shaft generator. The proposed control method using the multi-objective grey wolf optimization (MOGWO) algorithm has rapid response and no steady-state error.

Keywords: aircraft engine; high-pressure synchronous generator; low-pressure generator; power distribution; model predictive control; multi-objective grey wolf optimizer



Citation: Xiao, L.; Tan, Y.; Sattarov, R.R.; Wei, Y. Constrained Model Predictive Control for Generation Power Distribution on Aircraft Engines. *Energies* **2024**, *17*, 4533. <https://doi.org/10.3390/en17184533>

Academic Editor: Frede Blaabjerg

Received: 13 July 2024

Revised: 17 August 2024

Accepted: 30 August 2024

Published: 10 September 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

With the increasing requirements for aviation engines from military and civil aircraft, the performance of aviation engines has gradually improved, and their structures have become increasingly complex. This has led to a continuous increase in the demands on the control systems of aviation engines. Additionally, aviation engines have a wide operating range and are subject to numerous constraints and limitations in practical operations [1]. In the early 21st century, there emerged more electric aircraft (MEA) such as the B787 and F35, which gradually unified the secondary power sources of traditional aircraft into electrical energy [2]. In these MEA systems, sufficient electrical power is obtained through more electric engines (MEEs) [3]. MEEs utilize high-power generators to provide enough electricity for onboard systems and adopt a multi-axis structure to form a multi-axis power generation system. Each axis can bear the corresponding electrical power, giving rise to power distribution issues among the axes [4]. Moreover, the extraction of power from the engines also has a certain impact on their normal operation, and this impact increases with the increase in the power extraction [5].

Model Predictive Control (MPC) was introduced in the 1970s, and its principles can be summarized as model prediction, rolling optimization, and feedback correction [6,7]. One major advantage of MPC is its ability to explicitly handle constrained problems, which has led to its research and application in various fields such as power electronics and vehicles [8,9]. MPC has also been studied and applied in the field of aviation engines. Jonathan et al. [10] developed a linear parameter varying (LPV) model for the

blade tip clearance of a fan engine and proposed a rate-based predictive control method. Hanz et al. [11] proposed a multiplexing predictive control method to reduce the computational burden of predictive control algorithms and applied it to aviation engine control. Gu et al. [12] designed a predictive controller based on the LPV model for a turbofan engine and demonstrated the superiority of the proposed method compared to traditional PI controllers. Du et al. [13] designed a corresponding controller for a certain type of civil turbofan engine and achieved self-adjustment of predictive controller parameters using a proposed parameter scheduling method. Fan Fei et al. [14] presented a cooling EVTMS model, considering battery pack temperature, and utilizes the prediction of a neural network as a feedforward in a fuzzy PI controller to compensate for the model temperature variations. Shan et al. [15] applied the alternating direction method of multipliers to the rolling optimization part of predictive control to improve its real-time performance. Yang et al. [16] addressed the issue of potential mismatching between multiple models in engine control and proposed a predictive control method based on an augmented model to eliminate steady-state errors. Jinwoo [17] proposed centralized and distributed predictive control strategies for systems including aviation engines, generators, and energy storage devices. Huang et al. [18] explored the electric power system of more electric aircraft, proposing an energy storage-based design and control strategy that improves energy efficiency and reduces bus current fluctuations, thereby enhancing aircraft economy and supporting long-range flight capabilities. However, both of these studies were based on linearized models at specific operating points and did not investigate the effects of power extraction from the high-pressure shaft (HPS) or low-pressure shaft (LPS) on the self-operation of aviation engines.

As the number of factors considered in the control process increases, the adjustable parameters also gradually increase, and many scholars have combined intelligent optimization algorithms with predictive control. The Grey Wolf Optimization algorithm (GWO) [19,20] exhibits excellent global search performance and can achieve optimization of the objective function. For instance, Peng et al. [21] integrated predictive control and the grey wolf optimization algorithm to track multiple unmanned aerial vehicles in urban environments.

Although recent domestic and international research has achieved certain success in applying predictive control methods to aircraft engine control, the increasing impact of power extraction on aircraft engine operation has been less considered. Furthermore, research on generator installation positions from an energy consumption perspective is also relatively rare. Inspired by the aforementioned literature, this paper proposes a novel constrained predictive control method for the power distribution problem in aircraft engines. First, an equilibrium manifold expansion model is established based on a component-level model of the aircraft engine and the theory of equilibrium manifolds. Next, considering the impact of the generator on the aircraft engine, the control variable is treated as a constraint, leading to the design of a constrained predictive controller. A power distributor is designed based on the influence of electrical power generation. Additionally, a Multi-Objective Grey Wolf Optimizer (MOGWO) is introduced for the intelligent optimization of the controller parameters. Finally, simulation verification is conducted using the component-level model. The simulation results indicate that the proposed controller exhibits faster response times, more stable control performance, and quicker convergence compared to conventional controllers.

2. Aerospace Engine Equilibrium Manifold Expansion Model

Real-time models of aircraft engines form the basis of aircraft engine control and are essential for applying model-based modern control theory to aircraft engine control. A typical general structure of the twin-shaft turbofan aircraft engine is shown in Figure 1. Toolbox for the Modeling and Analysis of Thermodynamic Systems (T-MATS) is an open-source component-level model developed by the National Aeronautics and Space Administration (NASA), known for its modular and widely used characteristics and advantages. Therefore,

this study utilizes T-MATS as the component-level model in the design of engine control to verify the feasibility and superiority of the control algorithm.

In order to apply the predictive control algorithm proposed in this paper to control the aircraft engine based on the T-MATS component-level model, it is necessary to obtain a linearized model for the control algorithm design from the component-level model of the aircraft engine. Yu et al. [22] investigated the design of a shock position control system utilizing the equilibrium manifold method. Sui and Yu [23] introduced the EME model, conducted a thorough analysis of the model's errors, and addressed the ill-posed problem of the identification matrix by applying the constraint conditions inherent to the equilibrium manifold. Lv et al. [24] developed a nonlinear equilibrium manifold expansion (EME) model for a multiple input multiple output turbofan engine, capable of simulating variations in key engine parameters. This paper establishes this model based on equilibrium manifold theory.

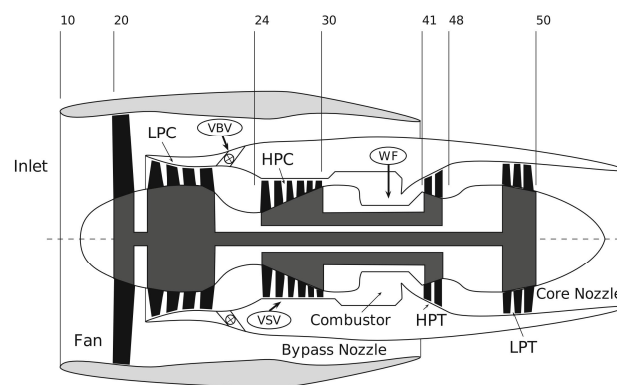


Figure 1. General structure of the aircraft engine [25].

2.1. Equilibrium Manifold Theory

By parameterizing the dynamic matrix coefficients to obtain the LPV model, and then parameterizing the equilibrium point, the Equilibrium Manifold Expansion (EME) model [26] is obtained. Since the EME model is an improvement of the LPV model, it has a similar structure to the LPV model, making it easier to analyze and design, and it has higher accuracy than the LPV model. In general, a nonlinear system can be described as follows:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases} \quad (1)$$

where $x \in R^m$ is the system state quantity, $u \in R^n$ is the system input, $y \in R^s$ is the system output, and f and g are the system nonlinear functions.

The set of all equilibrium points within Equation (1) is the equilibrium manifold of the system [27], which can be described as follows:

$$\{(x_e, u_e, y_e) | f(x_e, u_e) = 0, y_e = g(x_e, u_e)\} \quad (2)$$

Introducing a scheduling variable θ with the same dimension as the system input, the system described by Equation (2) can be represented as follows:

$$\begin{cases} x_e = x_e(\theta) \\ u_e = u_e(\theta) \\ y_e = y_e(\theta) \end{cases} \quad (3)$$

For a system described by Equations (1) or (2), the linearized model at the equilibrium point can be obtained using the Taylor expansion method near a certain steady state (x_0, u_0, y_0) , neglecting higher-order terms, as follows:

$$\begin{cases} \dot{x} - \dot{x}_0 = A(x - x_0) + B(u - u_0) \\ y - y_0 = C(x - x_0) + D(u - u_0) \end{cases} \quad (4)$$

where $A = \frac{\partial \dot{x}}{\partial x} \Big|_{(x_0, u_0)}$, $B = \frac{\partial \dot{x}}{\partial u} \Big|_{(x_0, u_0)}$, $C = \frac{\partial y}{\partial x} \Big|_{(x_0, u_0)}$, and $D = \frac{\partial y}{\partial u} \Big|_{(x_0, u_0)}$.

This paper focuses on the common dual-shaft turbofan engine in multi-electric aircraft, considering the main fuel flow rate W_f as the engine input and the low-pressure shaft speed n_L and high-pressure shaft speed n_H as the state and output parameters.

After selecting the scheduling variable, the equilibrium manifold expansion model of the aircraft engine can be obtained:

$$\begin{cases} \begin{bmatrix} \dot{n}_L \\ \dot{n}_H \end{bmatrix} = A(\theta) \begin{bmatrix} n_L - n_{Le}(\theta) \\ n_H - n_{He}(\theta) \end{bmatrix} + B(\theta)(W_f - W_{fe}(\theta)) \\ \begin{bmatrix} n_L \\ n_H \end{bmatrix} = C(\theta) \begin{bmatrix} n_L - n_{Le}(\theta) \\ n_H - n_{He}(\theta) \end{bmatrix} + D(\theta)(W_f - W_{fe}(\theta)) + \begin{bmatrix} n_{Le}(\theta) \\ n_{He}(\theta) \end{bmatrix} \end{cases} \quad (5)$$

In the engine equilibrium manifold model, the low-pressure shaft speed or high-pressure shaft speed is often chosen as the scheduling variable. In this study, the low-pressure shaft speed n_L is chosen as the scheduling variable, i.e.,

$$\theta = n_L \quad (6)$$

where $A(\theta) = \begin{bmatrix} a_{11}(\theta) & a_{12}(\theta) \\ a_{21}(\theta) & a_{22}(\theta) \end{bmatrix}$, $B(\theta) = \begin{bmatrix} b_1(\theta) \\ b_2(\theta) \end{bmatrix}$, $C(\theta) = \begin{bmatrix} c_{11}(\theta) & c_{12}(\theta) \\ c_{21}(\theta) & c_{22}(\theta) \end{bmatrix}$, and $D(\theta) = \begin{bmatrix} d_1(\theta) \\ d_2(\theta) \end{bmatrix}$.

2.2. Dynamic–Static Separation Method Modeling and Accuracy Checking

The dynamic–static separation two-step method involves dividing the modeling process into two steps: steady-state modeling and dynamic modeling. Steady-state modeling mainly involves identifying the equilibrium manifold and obtaining the parameters of the model at steady state. Dynamic modeling is mainly aimed at obtaining dynamic matrix parameters that vary with the equilibrium point, i.e., state-space matrix parameters. Finally, the results of both parts are combined to form a complete equilibrium manifold model.

Considering the strong nonlinearity of multi-electric aircraft engines, multiple stepwise step input signals, each with a step size of 1%, are used to obtain sufficient data. The fuel flow rate is initially set at 100% and decreases by 1% every 10 s until reaching 60%, and the resulting changes in physical quantities such as W_f , n_L , and n_H are recorded in the engine component-level model. The normalized variation of the main fuel flow rate W_f is shown in Figure 2.

From the data obtained above, steady-state data are extracted and fitted using scheduling parameters. In this paper, the polyfit function is used to perform polynomial fitting on the data with a selected order of 3. Taking the high-pressure shaft speed as an example, it is fitted using the scheduling variable, yielding the following fitting equation:

$$n_H(\theta) = p_1\theta^3 + p_2\theta^2 + p_3\theta + p_4 \quad (7)$$

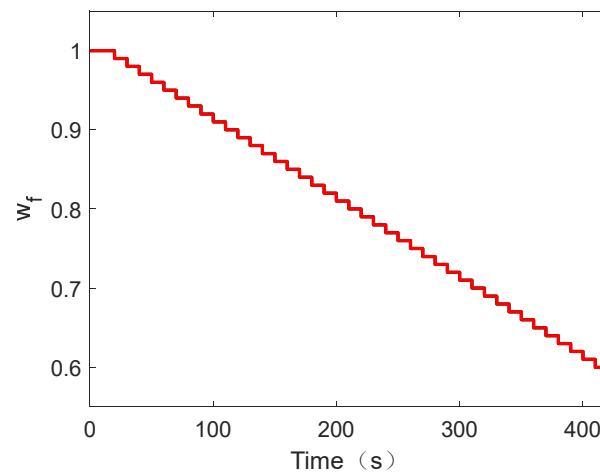


Figure 2. The change of the main fuel flow rate.

Dynamic modeling is primarily aimed at obtaining dynamic parameters that vary with the equilibrium point, i.e., state-space matrix parameters. At each equilibrium point, the equilibrium manifold expansion model can be represented as follows:

$$\begin{cases} \begin{bmatrix} \Delta \dot{n}_L \\ \Delta \dot{n}_H \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta n_L \\ \Delta n_H \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Delta W_f \\ \begin{bmatrix} \Delta n_L \\ \Delta n_H \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta n_L \\ \Delta n_H \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Delta W_f \end{cases} \quad (8)$$

where $\Delta \dot{n}_L$, $\Delta \dot{n}_H$, Δn_L , Δn_H , and ΔW_f denote the amount of deviation from the equilibrium point.

At each equilibrium point of the engine, a small perturbation is applied to only one relevant parameter while keeping the rest unchanged to obtain the dynamic matrix at that equilibrium point. For ease of observation and perturbation design, Equation (8) is expanded as follows:

$$\begin{cases} \Delta \dot{n}_L = a_{11} \Delta n_L + a_{12} \Delta n_H + b_1 \Delta W_f \\ \Delta \dot{n}_H = a_{21} \Delta n_L + a_{22} \Delta n_H + b_2 \Delta W_f \end{cases} \quad (9)$$

For example, at a certain state described by Equation (9), in order to obtain the parameter a_{11} at that state, a small perturbation Δn_L is introduced while keeping Δn_H and ΔW_f constant. Thus, the equation for solving a_{11} from Equation (9) under the perturbation simplifies to the following:

$$a_{11} = \frac{\Delta \dot{n}_L}{\Delta n_L} \quad (10)$$

Similarly, other dynamic parameters can be obtained in the same manner. By employing this method, the linear model's dynamic parameters at a certain equilibrium point are obtained. After obtaining the parameters at all points, the dynamic parameters at all equilibrium points are integrated using n_L as the scheduling variable.

After obtaining the steady-state and dynamic data of the equilibrium manifold model, the overall accuracy is validated, and the errors are shown in Figure 3.

It can be observed from Figure 3 that the EME model established using the dynamic-static separation two-step method has higher accuracy compared to the original nonlinear component level (NCL) model. The error in the low-pressure shaft speed is maintained within 2%, and the relative error in the high-pressure shaft speed is within 0.5%.

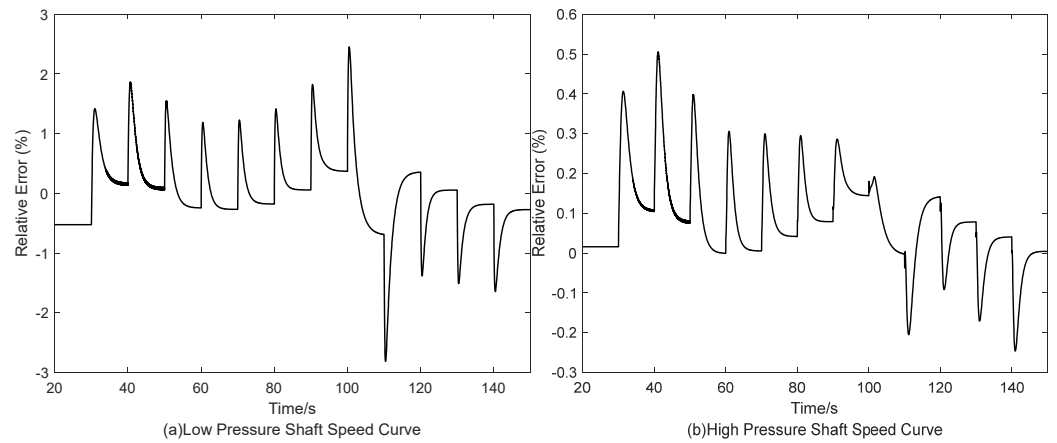


Figure 3. Output relative error between the NCL and EME models.

3. Predictive Controller Based on Distributed Power Generation

As the demand for electrical power in multi-electric aircraft continues to increase, and the total electrical power generation also increases, the impact of electrical loads on the mechanical system of the engine, especially the engine shaft operation, becomes significant [27–29]. Therefore, it is necessary to study the effect of electrical power extraction on engine operation and consider it during control. This paper focuses on the study of the Mechanical-Electrical Energy (MEE) extraction using installed generators. There are various possible installation methods for generators on actual aircraft engines. Therefore, considering the installation of generators on both the LPS and HPS, they are respectively referred to as the Low-Pressure Shaft Generator (LPSG) and the High-Pressure Shaft Generator (HPSG). The LPSG and HPSG maintain consistent speeds with the corresponding engine shafts and feedback torque to the respective shafts. The LPSG and HPSG can work independently to extract electrical power, or work together to extract electrical power, as shown in Figure 4.

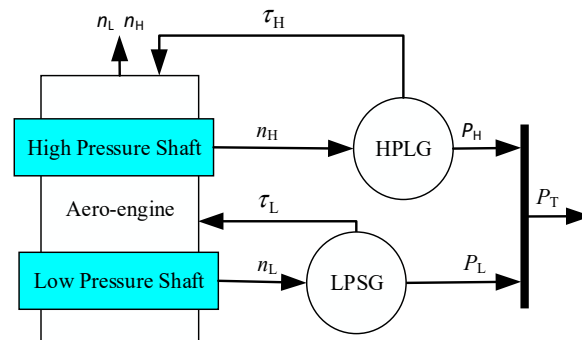


Figure 4. Aircraft engine system with generators [30].

The relationship between the engine shaft speed and the generator power is described as follows:

$$\begin{aligned} P_L &= n_L \times \tau_L \times \eta_L \\ P_H &= n_H \times \tau_H \times \eta_H \end{aligned} \quad (11)$$

where P_L and P_H denote the output power of the LPSG and HPSG, τ_L and τ_H denote the resistance torque applied by the generator to the LPSG and HPSG shafts, and η_L and η_H denote the efficiency of the LPSG and HPSG.

When only the LPSG or HPSG is operational, the total electrical power is the power generated by the LPSG/HPSG. When the LPSG and HPSG work simultaneously, the total output power (P_T) is as follows:

$$P_T = P_L + P_H \quad (12)$$

Therefore, the torque of the generator on the corresponding engine shaft is as follows:

$$\begin{aligned}\tau_L &= \frac{P_L}{n_L \times \eta_L} \\ \tau_H &= \frac{P_H}{n_H \times \eta_H}\end{aligned}\quad (13)$$

3.1. Design of the Power Divider Based on the Distribution Principle

The proposed structure in this article is shown in Figure 5.

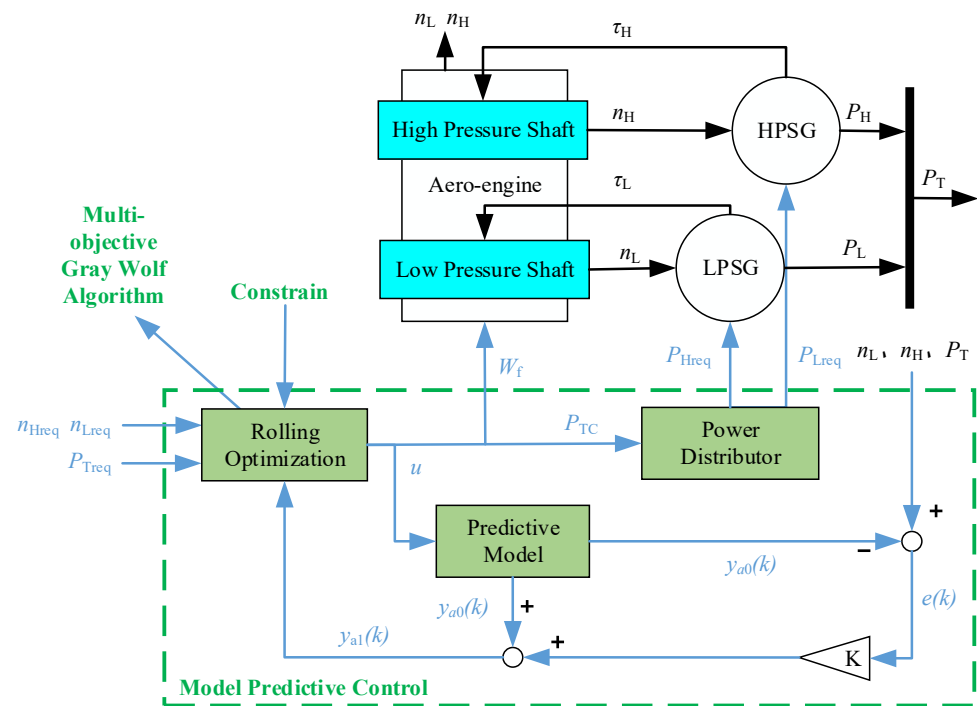


Figure 5. Aircraft engine constrained model predictive control based on generation power distribution.

The proposed control algorithm considers the low-pressure shaft speed (n_L) and high-pressure shaft speed (n_H) of the aircraft engine as state variables, and the fuel flow rate (W_f) and electrical power (P_{Lreq}, P_{Hreq}) as control variables. After obtaining the required low- and high-pressure shaft speeds (n_{Lreq}, n_{Hreq}), as well as the total required power generation rate (P_{Treq}), the fuel flow rate (W_f) and the total power generation rate (P_{TC}) are obtained through rolling optimization considering the constraints on control variables. In the rolling optimization process, the multi-objective grey wolf optimization algorithm is employed to optimize the parameters in the predictive controller. The total electrical power (P_{TC}) is distributed into P_{Lreq} and P_{Hreq} through the power distribution unit, and then W_f , P_{Lreq} , and P_{Hreq} are fed into the engine, low-pressure shaft generator, and high-pressure shaft generator to complete the entire control process.

Based on the relationship between the engine shaft and the generator mentioned above, the impact of the LPSG and HPSG is studied in the component-level model state simulation. A comparative analysis is conducted at the rated power generation, i.e., 250 kW, as shown in Figure 6.

From Figure 6, it is evident that when the electrical power extraction is 250 kW, the impact of the HPSG on the engine is smaller compared to the LPSG, resulting in less loss of thrust (or energy). This phenomenon holds true when extracting power other than 250 kW as well. It is speculated that when using the HPSG, the impact on the low-pressure shaft speed is smaller. Therefore, under the same power extraction, the low-pressure shaft speed is higher when using the HPSG compared to using the LPSG, resulting in a larger outflow of air from the engine and ultimately a greater thrust. This phenomenon is similar

to the findings in the literature [5], where typical MEE extraction is also mostly done on the high-pressure shaft for power generation [31].

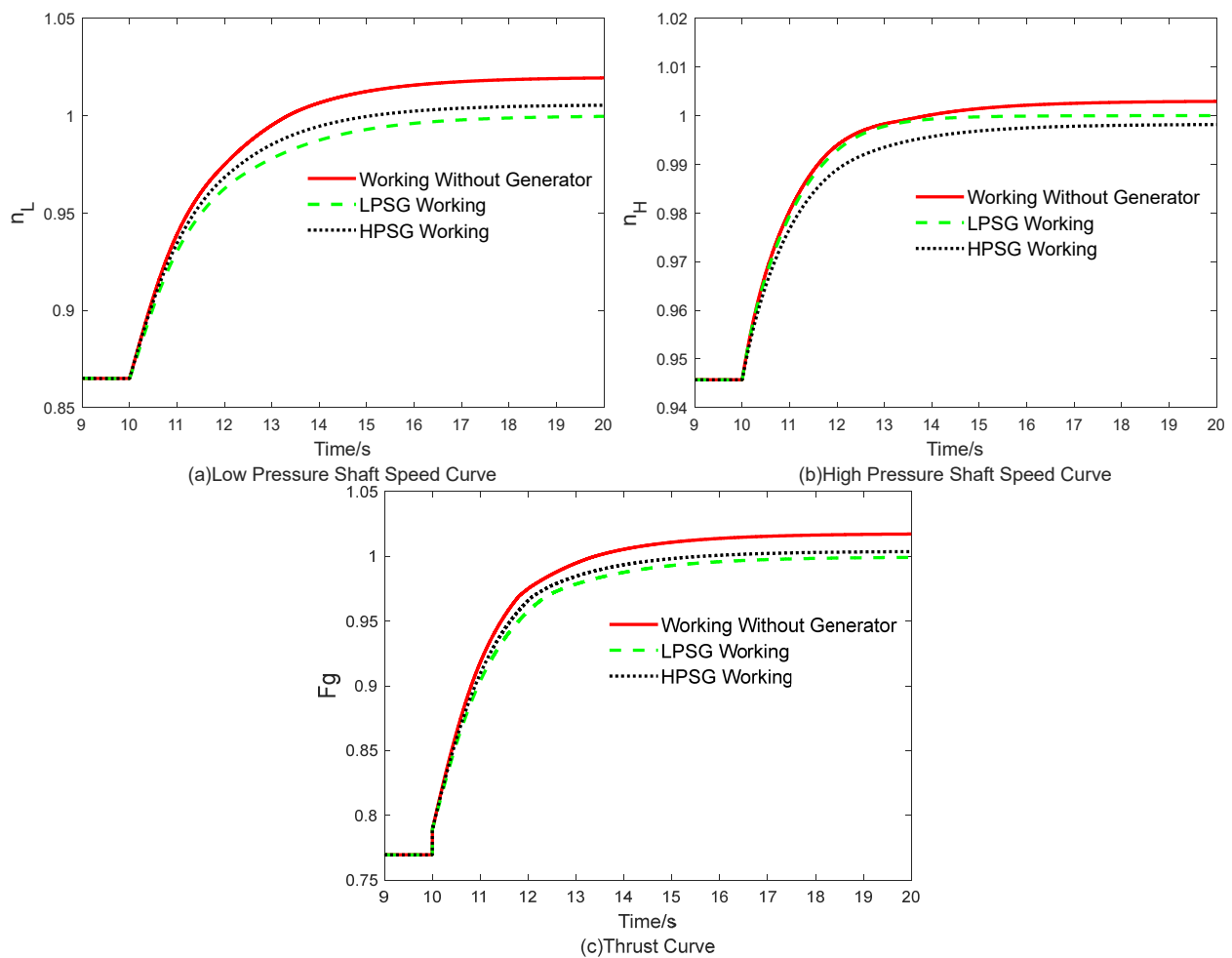


Figure 6. Comparisons between the LPSG and HPSG at $P_T = 250$ kW.

As the situation involves multiple generators working together to collectively provide the required power, it is necessary to design a power distribution unit to allocate the generated power. The principles for distribution are as follows: (1) meeting the total power demand, (2) ensuring that the power of each generator does not exceed its maximum capacity, and (3) selecting the extraction method that minimally impacts the engine operation. Therefore, it is essential to consider the practical constraints and requirements reflected in principles 1 and 2, while also taking into account the minimal impact reflected in principle 3. Thus, power distribution rules are proposed to determine the distribution factors between the two generators based on the above principles, as shown in Figure 7.

In Figure 7, P_{smax} represents the maximum power that a single generator can provide, and D_p is the power distribution factor. Through this distribution rule, the design of the power distribution unit aims to minimize the impact on the operation of the multi-electric aircraft engine due to power extraction while enhancing the power supply capacity to meet the increasing demand for electrical power in aircraft.

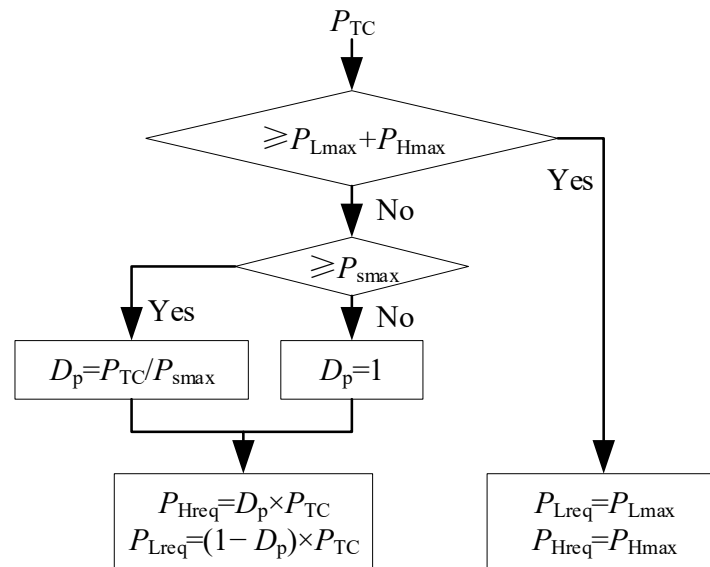


Figure 7. Distribution rules for power generation.

3.2. Aircraft Engine Constrained Prediction Controller Design

The equilibrium manifold model can be represented as follows:

$$\begin{aligned}\Delta \dot{x} &= A(\theta)\Delta x + B(\theta)\Delta u \\ \Delta y &= C(\theta)\Delta x + D(\theta)\Delta u\end{aligned}\quad (14)$$

where $x = [n_L \ n_H]^T$, $u = [W_f \ P_T]^T$, and $y = [n_L \ n_H]^T$ are the state, control, and output, respectively. The θ denotes the scheduling variable, $\Delta x = x - x_j$, $\Delta y = y - y_j$, and $\Delta u = u - u_j$ denote the increments, and j denotes the equilibrium point.

Equation (14) represents the engine model as a set of continuous equations, which when discretized yields Equation (15) as follows:

$$\begin{aligned}\Delta x(k+1) &= A\Delta x(k) + B\Delta u(k) \\ y(k) &= y(k-1) + C\Delta x(k) + D\Delta u(k)\end{aligned}\quad (15)$$

To meet the demand for power tracking, the total power generation is considered as an output, resulting in the following equation:

$$\begin{aligned}\Delta x(k+1) &= A\Delta x(k) + B\Delta u(k) \\ \begin{bmatrix} y(k) \\ P_{TC}(k) \end{bmatrix} &= \begin{bmatrix} y(k-1) \\ P_{TC}(k-1) \end{bmatrix} + C_a\Delta x(k) + D_a\Delta u(k)\end{aligned}\quad (16)$$

Equation (16) is abbreviated as follows:

$$\begin{aligned}\Delta x(k+1) &= A\Delta x(k) + B\Delta u(k) \\ y_{a0}(k) &= y_{a0}(k-1) + C_a\Delta x(k) + D_a\Delta u(k)\end{aligned}\quad (17)$$

By recursion, the predictive model is obtained as follows:

$$Y_{a0}(k) = G\Delta x(k) + S\Delta U(k) + \xi y_{a0}(k)\quad (18)$$

$$\text{where } Y_{a0}(k) = \begin{bmatrix} y_{a0}(k+1) \\ y_{a0}(k+2) \\ \vdots \\ y_{a0}(k+P) \end{bmatrix}, G = \begin{bmatrix} C_a A \\ \sum_{i=1}^2 C_a A^i \\ \vdots \\ \sum_{i=1}^P C_a A^i \end{bmatrix}_{P \times 1}, \Delta U(k) = \begin{bmatrix} \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix}, \text{ and}$$

$$\xi = \begin{bmatrix} I_{ny \times ny} \\ I_{ny \times ny} \\ \vdots \\ I_{ny \times ny} \end{bmatrix}_{P \times 1}.$$

Introducing a feedback correction step, the predictive tracking error is given as follows:

$$e_k(k) = y(k) - y_{a0}(k) \quad (19)$$

where $y(k)$ is the actual output of the engine at time k , and $y_{a0}(k)$ is the predicted output at time k before correction.

The corrected predicted output is as follows:

$$Y_{a1}(k) = Y_{a0}(k) + KE_k(k) \quad (20)$$

where $E_k(k) = [e_k(k), e_k(k+1), \dots, e_k(k+P)]^T$ and $K = [k_1, k_2, \dots, k_{ny \times P}]^T$.

The reference command at time k is given as follows:

$$r(k) = [n_{\text{Lref}}(k) n_{\text{Href}}(k) P_{\text{Tref}}(k)]^T \quad (21)$$

assuming the specified output sequence is as follows:

$$r(k) = [n_{\text{Lref}}(k) n_{\text{Href}}(k) P_{\text{Tref}}(k)]^T \quad (22)$$

The objective function is then defined as:

$$J = (R_s - Y_{a1})^T Q (R_s - Y_{a1}) + \Delta U^T R_d \Delta U \quad (23)$$

where Q and R_d are the output and control volume incremental weighting matrices, respectively.

Considering that the actual engine fuel supply rate is limited and the maximum value of fuel flow can be limited to ensure that the aircraft engine does not overrun, W_f is used as a constraint. Additionally, in multi-electric aircraft, electrical power equipment is predominantly supplied by the generators, and the stability of electrical power significantly impacts the power electronic equipment. Therefore, the extracted total power P_{TC} is also imposed as a constraint. Hence, the consideration is focused on constrained control quantities, as follows:

$$u_{\min} \leq u \leq u_{\max} \quad (24)$$

To address this constraint, it is divided into two parts, as shown in Equation (25):

$$\begin{aligned} u_{\min} &\leq u \\ u &\leq u_{\max} \end{aligned} \quad (25)$$

By derivation, the following equations can be obtained:

$$\begin{aligned} -C_2 \Delta U &\leq -u_{\min} + C_1 u(k-1) \\ C_2 \Delta U &\leq u_{\max} - C_1 u(k-1) \end{aligned} \quad (26)$$

$$\text{where } C_1 = \begin{bmatrix} I_{nu \times nu} \\ I_{nu \times nu} \\ \vdots \\ I_{nu \times nu} \end{bmatrix}_{M \times 1}, U_{min} = \begin{bmatrix} u_{min} \\ u_{min} \\ \vdots \\ u_{min} \end{bmatrix}_{M \times 1}, \text{ and } C_2 = \begin{bmatrix} I_{nu \times nu} & \mathbf{0} & \cdots & \mathbf{0} \\ I_{nu \times nu} & I_{nu \times nu} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ I_{nu \times nu} & I_{nu \times nu} & \cdots & I_{nu \times nu} \end{bmatrix}_{M \times M}.$$

In accordance with Equation (26), the considered constraints can be expressed as follows:

$$M_1 \Delta U \leq N_1 \quad (27)$$

$$\text{where } M_1 = \begin{bmatrix} -C_2 \\ C_2 \end{bmatrix} \text{ and } N_1 = \begin{bmatrix} -U_{min} + C_1 u(k-1) \\ U_{max} - C_1 u(k-1) \end{bmatrix}.$$

To transform the problem into a standard QP problem for solving, let

$$\begin{cases} H = 2(G^T Q G + R_d) \\ f^T = -E^T Q S \end{cases} \quad (28)$$

This transformation turns the problem into a QP problem, which seeks to minimize the objective function subject to constraints of the form $Ax \leq b$, i.e.,

$$\begin{aligned} \min_x & \frac{1}{2} x^T H x + f^T x \\ \text{s.t.} & M_1 \Delta U \leq N_1 \end{aligned} \quad (29)$$

By Equation (29), obtaining ΔU , and to acquire the control quantity at time k , there is the following equation:

$$u(k) = u(k-1) + \Delta u(k) \quad (30)$$

3.3. Intelligent Optimization of Controller Parameters Based on MOGWO

To address the difficulty of tuning constrained predictive controller parameters, we introduce the globally efficient MOGWO algorithm, which is a heuristic algorithm inspired by the hunting process of grey wolves [20].

This study considers two objectives. The first objective function represents the tracking effect of speed over the entire simulation period, specifically as follows:

$$J_1 = \int_0^T (|n_L - n_{Lref}| + |n_H - n_{Href}|) dt \quad (31)$$

where t is the simulation time.

The second objective function represents the fuel supply and its variations over the entire simulation period, specifically as follows:

$$J_2 = \int_0^T (W_f + |\Delta W_f|) dt \quad (32)$$

where ΔW_f is the amount of fuel change.

The optimization logic using the MOGWO algorithm is illustrated in Figure 8.

In Figure 8, W_L , W_H , W_P and W_{fuel} represent the weighting coefficients for low-pressure shaft speed, high-pressure shaft speed, total power generation, and fuel flow rate, respectively. There are six controller parameters to be optimized. In predictive control, the predicted time domain length and the control time domain are necessarily integers; thus, the Round function is used to round the length of these two time domains. Finally, the optimized results are passed to the corresponding parts of the MPC, where the prediction horizon length is used for model derivation and the weighting coefficients are used for rolling optimization solutions.

The selection of objective functions and the setting of parameter ranges reflect the designer's emphasis on different parameters. Unlike the traditional empirical setting of adjustable parameters for predictive controllers, using the MOGWO for parameter tuning only requires specifying approximate ranges for each parameter. The MOGWO algorithm

will automatically search for the optimal solution within these ranges, significantly reducing the difficulty of parameter tuning.

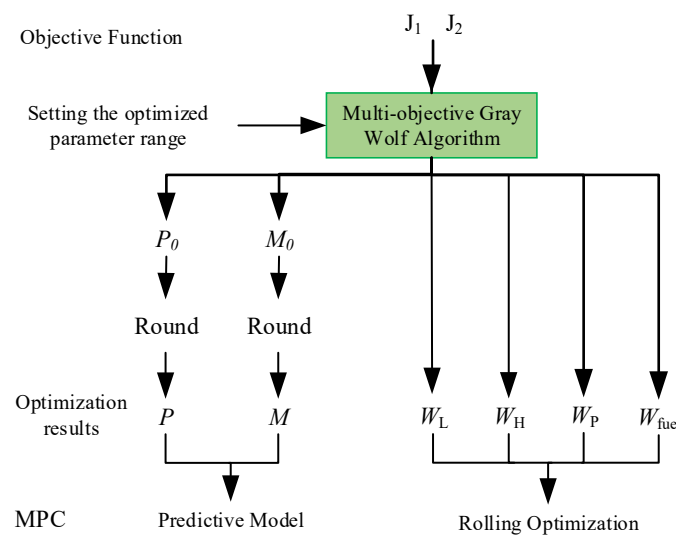


Figure 8. Multi-objective optimization using MOGWO.

4. Simulation and Analysis

To validate the effectiveness and feasibility of the control algorithm proposed in this chapter, simulation studies are conducted based on the MATLAB R2023a/Simulink platform using the T-MATS toolbox on a component-level model of a multi-electric aircraft engine. The data in this section are normalized, and the simulation sampling time is 1×10^{-4} s.

During the experimental validation of the control algorithm, the simulation model is first run without a controller, and then the MPC control strategy is introduced at 10 s. To demonstrate the control effect, simulation results for the time interval 9–15 s are presented. Additionally, power generation extraction begins at 10 s.

Since the low-pressure shaft speed is a scheduling variable in the equilibrium manifold model, involving the accuracy of the predictive model in the predictive controller, it is assigned the highest weighting coefficient, with a reference speed of 0.96. In the state of $H = 10\text{km}$ and $Ma = 0.8$, MOGWO optimization parameters are selected, as shown in Table 1, for the simulation experiment.

Table 1. Parameters of MOGWO.

Parameter	Minimum	Maximum
Predictive Time Domain	5	50
Control Time Domain	2	10
Weight Factor for n_l	0	10^6
Weight Factor for n_h	0	10^3
Weight Factor for P_{TC}	0	10^3
Weight Factor for W_f	0	10^3

4.1. HPSG Working Alone

In Section 3.1, it has been established that the HPSG has a lesser impact on engine operation. Therefore, the scenario of HPSG operation alone is initially investigated. The predictive control algorithm without constraints, the traditional constrained predictive control algorithm used in [32], and the algorithm proposed in this paper are denoted as MPC-1, MPC-2, and MPC-MOGWO respectively.

To validate the effectiveness of the proposed algorithm, the control inputs obtained from the three algorithms are applied to an aircraft engine, as shown in Figure 9.

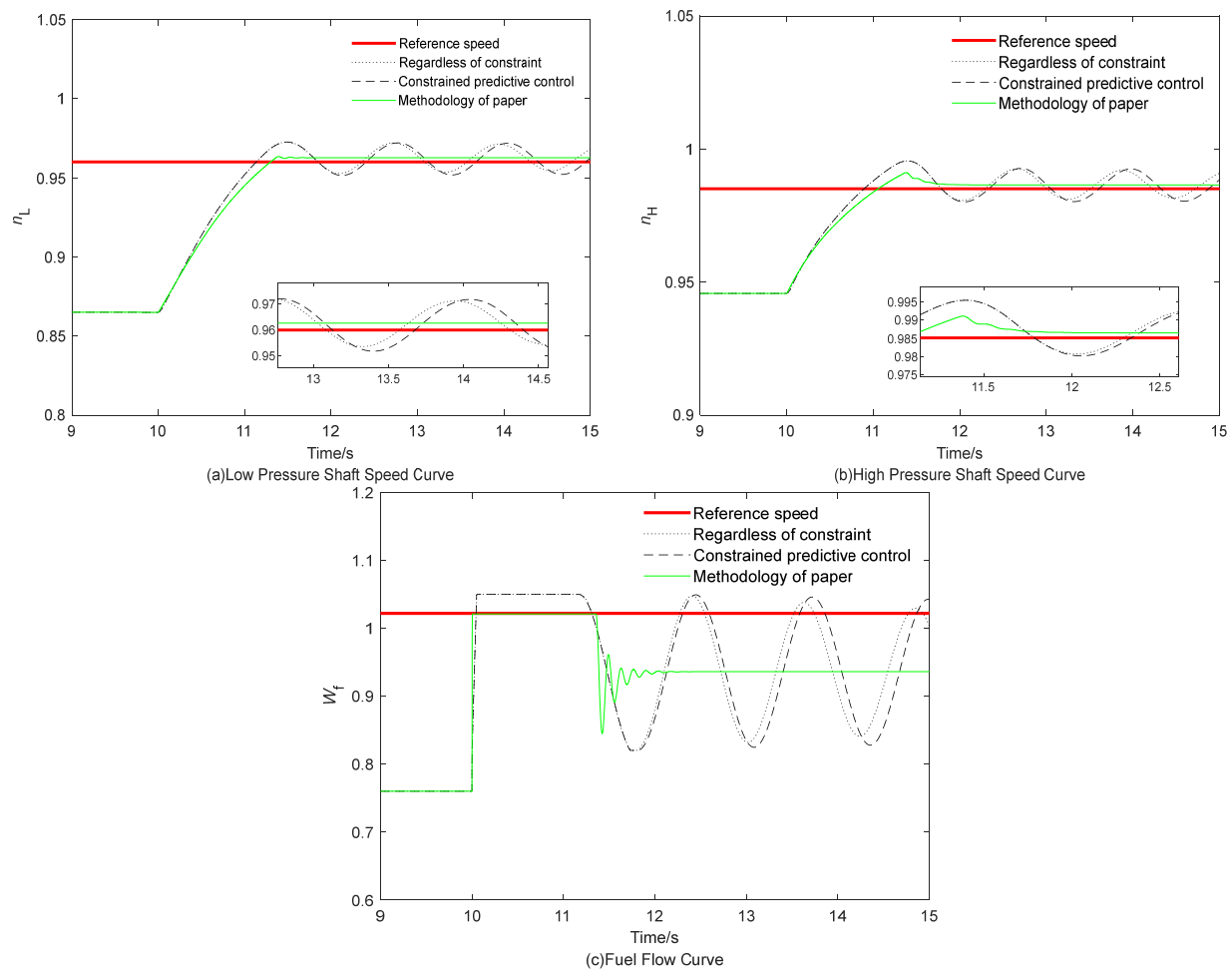


Figure 9. Comparison of three model predictive control methods.

From Figure 9a, it is evident that all three methods can track the low-pressure shaft speed near 96%. However, the speed fluctuates and exhibits instability around 96% when constraints are not considered in MPC-1, with a slow tracking response. In contrast, MPC-2 and MPC-MOGWO, which consider constraints, exhibit stable and rapid responses. MPC-2 achieves steady-state control within 1.68 s at 95.67% speed, with a steady-state error of 0.34%, meeting the tracking requirements to a large extent but still exhibiting some steady-state error. On the other hand, MPC-MOGWO follows a trajectory identical to MPC-2 until 11.38 s, after which it continues to rise and reaches steady-state at 11.45 s with almost no steady-state error, demonstrating the best performance among the three methods.

Similar to Figure 9a, Figure 9b indicates that the MPC-1 method, without considering constraints, results in speed fluctuations while tracking the reference speed, which is avoided by the constraint-aware solution methods.

Figure 9c shows the variation in fuel flow rate. The proposed MPC-MOGWO algorithm reaches the constraint and ceases to increase fuel supply from 10 s to 11.44 s, maintaining it at 1.021. At this point, in conjunction with Figure 9a, the target low-pressure shaft speed is achieved. Subsequently, the fuel flow rate decreases to a minimum of 0.9287, gradually rises again, and eventually stabilizes at 0.9488. This demonstrates the necessity of considering constraints in controller design and the significance of explicitly considering constraints when applying MPC to engine control.

Using these three algorithms, the objective function values calculated based on Equations (31) and (32) are presented in Table 2.

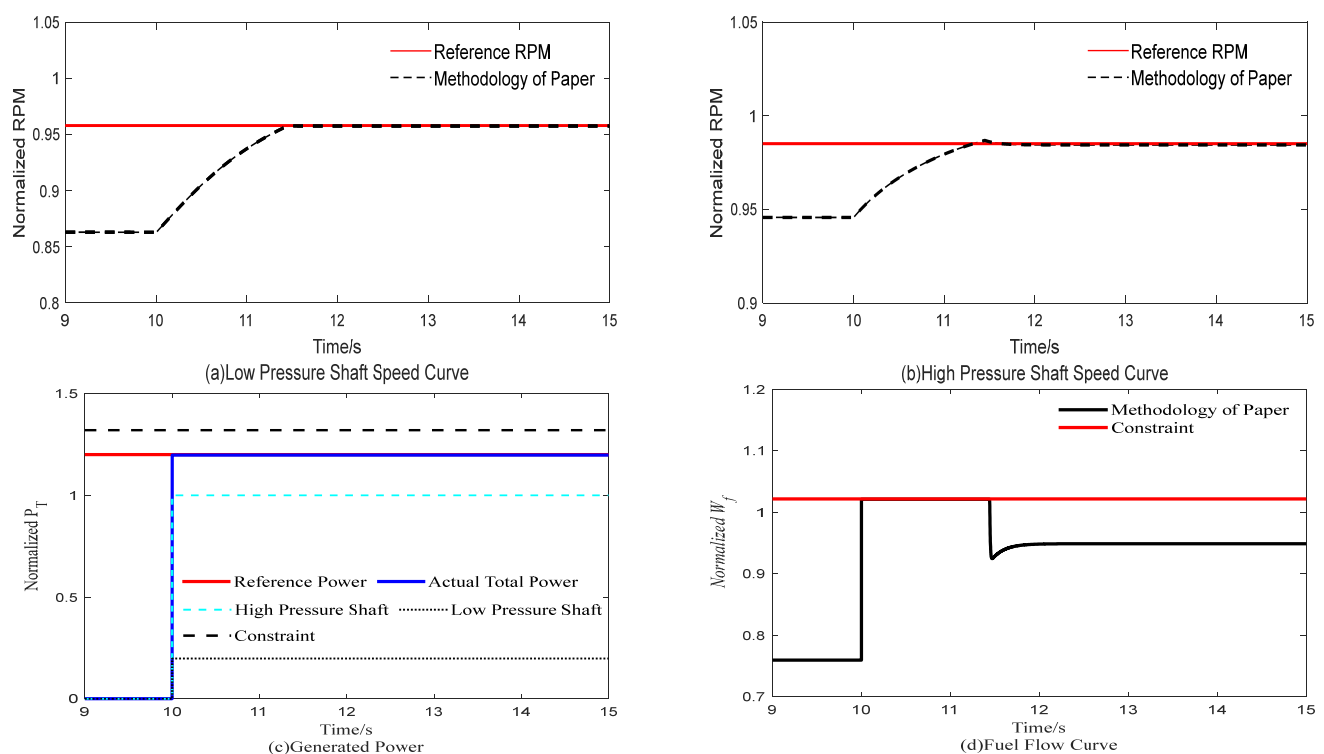
Table 2. Objective Function Values of Three Algorithms.

Parameter	MPC-1	MPC-2	MPC-MOGWO
J_1	0.020	0.018	0.018
J_2	528.1	408.3	384.1

From Table 2, it is evident that the proposed algorithm outperforms previous algorithms in speed tracking and fuel supply, demonstrating the superiority of the proposed algorithm.

4.2. LPSG and HPSG Working Together

Considering the scenario of the LPSG and HPSG working together, and given the demonstrated superiority of the proposed method over others, the MPC-MOGWO algorithm is directly employed for control, with the results shown in Figure 10.

**Figure 10.** Effect curves using the proposed algorithm.

As depicted in Figure 10a, the reference speed is achieved at 11.44 s, with a settling time of less than 2 s and no steady-state error, aligning closely with the results shown in Figure 9a. Figure 10b indicates that the maximum value is reached at 11.45 s, and from 11.56 s onwards, it remains consistent with the reference value. In this simulation, due to the high power requirement, a single generator is insufficient, necessitating the combined operation of two generators. Figure 10c illustrates that when the LPSG and HPSG work together to extract shaft power for electricity generation, the predictive control algorithm proposed in this paper continues to function effectively, ensuring the total power generation matches the required electrical load. Furthermore, as shown in Figure 10d, the fuel flow rate remains at 1.021 from 10 s to 11.45 s, subsequently dropping to a minimum of 0.9268, and gradually rising back to 0.949. Additionally, both the actual power generation and fuel flow rate adhere to the specified constraints.

Figures 9 and 10 demonstrate the high control quality of the proposed aviation engine constrained predictive control algorithm based on power generation allocation. From the comprehensive analysis above, it is evident that the proposed method achieves error-free

tracking of the engine's high- and low-pressure shaft speeds while satisfying constraints related to fuel flow and power generation, meeting the power generation requirements.

5. Conclusions

This paper introduces a novel aviation engine constrained predictive control method based on power generation allocation, yielding the following conclusions:

1. A comparison of the impact of high-pressure shaft generators and low-pressure shaft generators on aviation engine operation indicated that high-pressure shaft generators have a smaller impact, resulting in lower energy loss.
2. The proposed control algorithm exhibits rapid response and excellent control performance, with a settling time of less than 2 s when applied to aviation engine control, meeting power generation requirements.
3. Using a multi-objective grey wolf optimization algorithm for parameter tuning in constrained predictive control results in more stable control performance and faster convergence rates.

The predictive control algorithm adopted in this study possesses various advantages, yet it encounters inherent challenges regarding slow computational speed during application. Therefore, the focus of our future research lies in developing faster predictive control algorithms or solution techniques.

Author Contributions: L.X.: conceptualization and original draft preparation; Y.T.: simulation, review, and editing; R.R.S. and Y.W.: investigation and software. All authors have read and agreed to the published version of the manuscript.

Funding: This manuscript is supported by the National Natural Science Foundation of China (No.62373185, No.51876089).

Data Availability Statement: The data presented in this study are available in the article.

Acknowledgments: The authors would like to thank all of our lab-mates. This work is partially supported by the High Performance Computing Platform of Nanjing University of Aeronautics and Astronautics.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Csank, J.; May, R.; Litt, J.; Guo, T.H. Control design for a generic commercial aircraft engine. In Proceedings of the 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, Nashville, TN, USA, 25–28 July 2010.
2. Zhang, Z.; Li, J.; Han, J.; Lu, J.; Shi, H. Research and implementation of high-power high-voltage DC brushless stator generator system for more electric aircraft. *Acta Aeronaut. Astronaut. Sin.* **2020**, *41*, 324–335.
3. Hirst, M.; McLoughlin, A.; Norman, P.J.; Galloway, S.J. Demonstrating the more electric engine: A step towards the power optimised aircraft. *IET Electr. Power Appl.* **2011**, *5*, 3–13. [[CrossRef](#)]
4. Yu, S. Research on Internal Fault-Tolerant Permanent Magnet Starter Generator System. Master's Thesis, Nanjing University of Aeronautics and Astronautics the Graduate School College of Automation, Nanjing, China, 2016.
5. Huang, X. The Study of Energy Efficiency of Adaptive Power and Thermal Management System. Master's Thesis, Nanjing University of Aeronautics and Astronautics College of Energy and Power Engineering, Nanjing, China, 2018.
6. Xi, Y.; Li, D.; Lin, S. Model predictive control—Status and challenges. *Acta Autom. Sin.* **2013**, *39*, 222–236. [[CrossRef](#)]
7. Dong, X.; Gan, J.; Wu, H.; Deng, C.; Liu, S.; Song, C. Self-Triggered Model Predictive Control of AC Microgrids with Physical and Communication State Constraints. *Energies* **2022**, *15*, 1170. [[CrossRef](#)]
8. Wang, L.; Wang, Y.; Liu, C.; Yang, D.; Chen, Z. A power distribution strategy for hybrid energy storage system using adaptive model predictive control. *IEEE Trans. Power Electron.* **2020**, *35*, 5897–5906. [[CrossRef](#)]
9. Xu, F.; Zhang, J.; Hu, Y.; Qu, T.; Qu, Y.; Liu, Q. Lateral and longitudinal coupling real-time predictive controller for intelligent vehicle path tracking. *J. Jilin Univ. (Eng. Technol. Ed.)* **2021**, *51*, 2287–2294.
10. Decastro, J.A. Rate-Based Model Predictive Control of Turbofan Engine Clearance. In Proceedings of the 42nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, Sacramento, CA, USA, 11 July 2006.
11. Richter, H.; Singaraju, A.V.; Litt, J.S. Multiplexed Predictive Control of a Large Commercial Turbofan Engine. *J. Guid. Control Dyn.* **2008**, *31*, 273–281. [[CrossRef](#)]
12. Gu, N.; Wang, X. Model Predictive Controller Design based on the Linear Parameter Varying Model Method for a class of Turboshaft Engines. In Proceedings of the 2018 Joint Propulsion Conference, Cincinnati, OH, USA, 9–11 July 2018.

13. Du, X.; Guo, Y. Design of model predictive controller for commercial turbofan engine. *Aeroengine* **2013**, *39*, 27–30.
14. Fei, F.; Wang, D. Application of Neural Network Feedforward in Fuzzy PI Controller for Electric Vehicle Thermal Management System: Modeling and Simulation Studies. *Energies* **2024**, *17*, 9. [[CrossRef](#)]
15. Shan, R.; Li, Q.; He, F.; Feng, H.; Guan, T. Model predictive control based on ADMM for Aero-engine. *J. Beijing Univ. Aeronaut. Astronaut.* **2019**, *45*, 1240–1247.
16. Yang, S.; Lu, F.; Huang, J. Multivariable constrained model predictive control of Aero-Engine based on augmented predictive model. *J. Propuls. Technol.* **2019**, *40*, 2579–2586.
17. Seok, J.; Kolmanovsky, I.; Girard, A. Coordinated model predictive control of aircraft gas turbine engine and power system. *J. Guid. Control. Dyn.* **2017**, *40*, 2538–2555. [[CrossRef](#)]
18. Huang, T.; Liu, Y.; Xue, L.; Tao, Y.; Yuan, B. Research on Multi-objective Optimization Methods for Aircraft Energy Conversion System Control Unit. In Proceedings of the 6th China Aeronautical Science and Technology Conference, Wuzhen, China, 26–27 September 2023; CASTC 2023; Lecture Notes in Mechanical Engineering. Springer: Singapore, 2024. [[CrossRef](#)]
19. Mirjalili, S.; Mirjalili, S.M.; Lewis, A. Grey wolf optimizer. *Adv. Eng. Softw.* **2014**, *69*, 46–61. [[CrossRef](#)]
20. Mirjalili, S.; Saremi, S.; Mirjalili, S.M.; Coelho, L.D.S. Multi-Objective grey wolf optimizer: A novel algorithm for multi-criterion optimization. *Expert Syst. Appl.* **2016**, *47*, 106–119. [[CrossRef](#)]
21. Yao, P.; Wang, H.; Ji, H. Multi-UAVs tracking target in urban environment by model predictive control and Improved Grey Wolf Optimizer. *Aerosp. Sci. Technol.* **2016**, *55*, 131–143. [[CrossRef](#)]
22. Yu, D.R.; Cui, T.; Bao, W. Equilibrium manifold linearization model for normal shock position control systems. *J. Aircr.* **2005**, *42*, 1344–1347.
23. Sui, Y.F.; Yu, D. Identification of expansion model based on equilibrium manifold of turbojet engine. *Hangkong Xuebao/Acta Aeronaut. Astronaut. Sin.* **2007**, *28*, 531–534.
24. Lv, C.; Wang, Z.; Dai, L.; Liu, H.; Chang, J.; Yu, D. Control-Oriented Modeling for Nonlinear MIMO Turbofan Engine Based on Equilibrium Manifold Expansion Model. *Energies* **2021**, *14*, 6277. [[CrossRef](#)]
25. Richter, H. *Advanced Control of Turbofan Engines*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2011.
26. Zhao, H. Research on Nonlinear Control for Aeroengines Based on Equilibrium Manifold Expansion Model. Ph.D. Thesis, Harbin Institute of Technology School of Energy Science and Engineering, Harbin, China, 2011.
27. Pluijms, A.; Schmidt, K.J.; Stastny, K.; Chibisov, B. Performance comparison of more electric engine. In Proceedings of the ASME Turbo Expo 2008: Power for Land, Sea, and Air, Berlin, Germany, 9–13 June 2008.
28. Ahumada, C.; Garvey, S.; Yang, T.; Wheeler, P.; Morvan, H. Impact of electric loads on engine shaft dynamics within More Electric Aircraft. In Proceedings of the SAE 2015 AeroTech Congress & Exhibition, Seattle, WA, USA, 22–24 September 2015.
29. Roberts, R.A.; Decker, D.D. Control architecture study focused on energy savings of an aircraft thermal management system. *J. Dyn. Syst. Meas. Control* **2013**, *136*, 41003. [[CrossRef](#)]
30. Xiao, L.; Sattarov, R.R.; Liu, P.; Lin, C. Intelligent Fault-Tolerant Control for AC/DC Hybrid Power System of More Electric Aircraft. *Aerospace* **2022**, *9*, 4. [[CrossRef](#)]
31. Madonna, V.; Giangrande, P.; Galea, M. Electrical power generation in aircraft: Review, challenges, and opportunities. *IEEE Trans. Transp. Electr.* **2018**, *4*, 646–659. [[CrossRef](#)]
32. Rasmussen, H. *Advanced Control of Turbofan Engines*; Springer: New York, NY, USA, 2012.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.