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Photovoltaic Capacity Management for Investment Effectiveness

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Abstract: The production of photovoltaic utility varies within the day/night cycle. At night, photovoltaic cells do not produce anything. However, their day-light production, unconsumed on a current basis and exported to the grid, is compensated for with supply from the grid at night. This scheme of exploitation is called net-metering and is considered herein. Solar energy produced by a prosumer and fed into the grid has to be equal to the electricity supplied from the grid at night; otherwise, a shortage or waste of photovoltaic production occurs. The above finding leads us to the proposition for the optimal solution of photovoltaic capacity. We derived a closed-form capacity solution to the maximized non-linear profit function. It solves harmonic and 2-point production functions that vary symmetrically around the mean production. To verify the solution methodology, harmonic and 2-point models from empirical production data are estimated. Then, the solution is presented together with its return rate and internal return rate. The main finding is that the unit cost of the grid electricity, photovoltaic capacity unit cost and exploitation time all affect the total profit and return rate values while not impacting the optimal capacity of the photovoltaics. The optimal capacity depends on the prosumer's energy consumption volume and on the natural conditions of production captured here by the technology efficiency coefficient estimated from the production time series.

Keywords: operations research; photovoltaics; capacity management; profit maximization; effectiveness

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1. Introduction

Many papers concern photovoltaic capacity and its expansion due to expected trends in electricity consumption or cost and price. They are focused on the minimization of the expenditure and operation costs of dual- or multi-source energy supplies. Usually, minimizing costs should be coherent with return rate maximization. However, both objectives are non-linear, and the return rate function of capacity is more complex. For a prosumer, being a household, the main cost is PV investment expenditure, to which an alternate is the deposit at interest rate. Similarly, a small manufacturer needs to choose between PV and alternative business development projects. They also compare return rate to credit cost. Hence, the methodology of capacity management should take into account opportunity cost and capture for the return rate, money value in time and internal return rate.

Measuring the effectiveness is challenging because the investment in photovoltaics (PV) should maximize total profits and the return ratio.. This ratio, by general definition, is a non-linear function of the capacity. The calculus should also account for money value over the long term until PV exploitation ends. Such measures like the return period, rate of return, break-even point, NPV and IRR for deterministic as well as for uncertain cashflow investments are recommended [1–3]. Under analysis here is the net-metering scheme in which PV production surplus is fed into the grid not sold. Equivalent energy is available to be taken back by a prosumer to supply their needs. So, the revenue is defined as the PV opportunity cost that is equal to paid traditional electricity bills in case there is no PV investment. In Section 3.3, the decision model of the capacity volume is formulated in

order to maximize total profit. Additionally, the return rate from the investment in PV is assessed ex post. Return non-negativity is guaranteed with the solution of the model. Hence, in practice, it is more important to check whether the return from the PV investment maximizing profit exceeds the risk-free market interest rate. The last is an alternative investment of a prosumer's money. This research focuses on effective investing in PV by a prosumer, being a household or a small business. Each market parameter and consumption parameter of the model is settable by a prosumer at a feasible level at their installation place. Production efficiency is not a direct function of weather and is treated as known. We estimated the efficiency from the production data of an existing installation [4,5].

The integration of operational and investment effectiveness approaches is desired for effective PV planning, while in the literature, total cost minimization is usually discussed [6–8]. Yang et al. [9] propose a two-step procedure for modeling and solving the maximum rate of return with adaptive weighted particle swarm optimization. Their model concerns a net-billing scheme. The authors of [9] assume a general PV efficiency value equal to 0.95. We use the day/night cycle for PV production modeling like in [8,9]. However, the model of [8] explains the production with climate explanatory variables, while our model is a production time series based directly on production data at a given geolocation.

The 2-point model of production is the simplest to calibrate. Its deterministic setting follows a day/night variation in production. Its properties and results are easily comparable with the uniform distribution of output commonly used in the literature to model energy consumption, energy production or its shortage [10,11]. We examine the harmonic model of PV production and the two-state PV production model determined by a day/night cycle. Although the proposed capacity model captures for investment effectiveness aspects, it simplifies the consumption impact on capacity by treating it as a constant. Therefore, the model holds for a household that stabilizes electricity consumption due to pricing policy [12]. The model also holds for an enterprise prosumer that applies production planning to stabilize electricity consumption [13]. Energy storage is not captured in the model either as an energy storage system (ESS) or a heat pump [14,15]. These are usually used in big buildings that are constructed in a contemporary manner from greenfield for institutions [16]. A wind/photovoltaic/battery hybrid system is usually infeasible in old buildings with traditional heating installations. Moreover, the authors of [17] show that photovoltaics with an energy storage system (ESS) are profitable when demand fluctuates a lot. That is not the case for the prosumer types analyzed here.

The proposed model of a single PV installation connected to the grid provides investment efficiency for a daily cycle and is extendable with other seasonal effects. Huge seasonal variabilities in PV production are noted in [18,19]. Additionally, the model framework is flexible enough to capture the optimal capacity and its sensitivity to systemic provision to the auto-consumption of grid-stored PV production and to technology production efficiency comparison.

Model terms are presented in Section 2. Next is considered PV output modeling with the 2-point function and the harmonics and their optimal capacity solving propositions. PV production is estimated from empirical data in Section 4. This paper concludes with Section 5.

2. Methodology of PV Capacity Planning

PV production modeling and capacity planning processes should consist of the stages presented in Figure 1.

The consumption and production notation applied in Figure 1 and further in the model sets the following variables:

- Hourly electricity constant consumption b.
- PV 24 h cyclical production function $X(t)$. Two functions further considered. The harmonic discrete 24 h cycle production is $X_1(t)$, and the 2-point day/night production is denoted with $X_2(t)$.
- Hourly average production $\overline{x}$.

- PV capacity x, note that it is identical with the maximal production hourly output.
- The optimal PV capacity x^*.

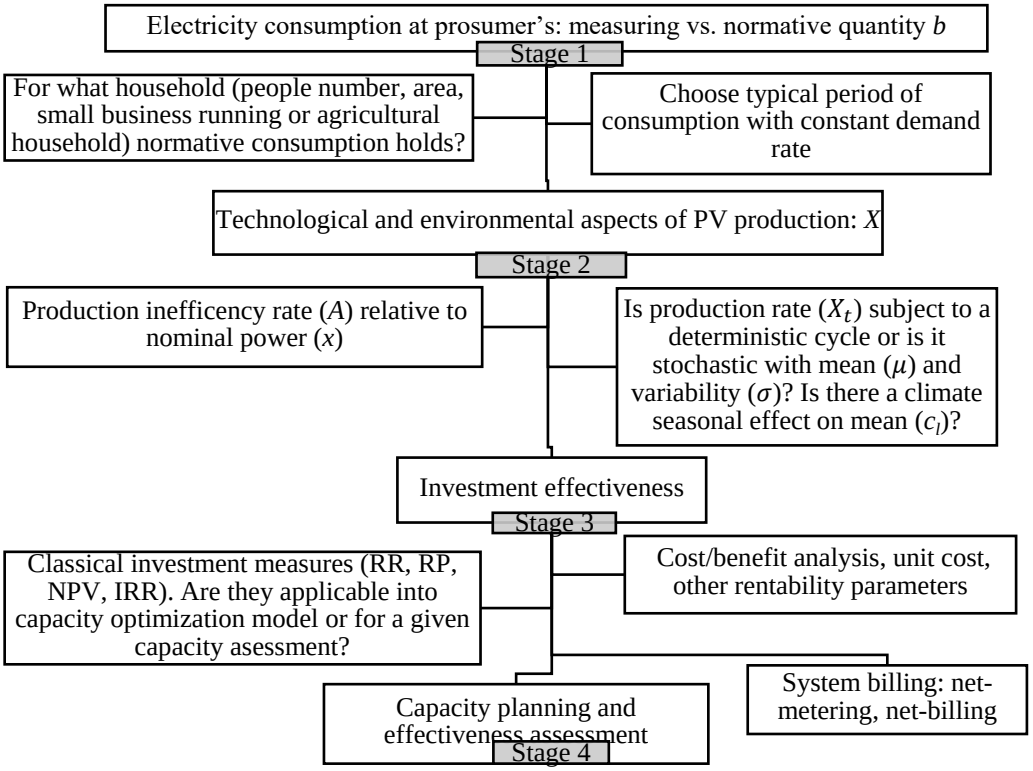


Figure 1. PV capacity planning process.

Consumption is assumed to be constant due to balancing electricity usage intra-24-h cycles by a prosumer. Oppositely, they could adjust consumption to the current fluctuations of the supply from a single PV installation. The latter would result in a 24-h cycle consumption pattern that is identical with the production cycle. No shortages and no surpluses of PV production would occur. However, this is an unrealistic assumption as people leave households for work or other activities during the day instead of waiting for sunny moments to immediately consume energy produced from PV. Moreover, some businesses and industry branches do not stop electricity consumption at night.

Figure 2 presents examples of production from a PV capacity of 8 (kWp) that is to supply constant hourly consumption $b = 5$ (kWh). There is harmonic X_1(t) with an average $\overline{x}_1 = 5$ and the highest output that is equal to the capacity so $x_1 = \sup\{X_1\} = 8$. The lowest output is 2. A production process $X_2(t)$ is also presented. It has a 2-point output with the highest value of 6.5 and the lowest value of 3.5. The higher is assumed to remain constant during 12 h day-light and the lower during 12 h nighttime. So, the output on average is $\overline{x}_2 = 5$ with an hourly standard deviation that equals $s_2 = 1.5$. In Section 4, estimation methods will be applied to obtain the best fitted model of observations including zero output at night.

The parameters of the optimal capacity model are the following:

- c_1 is the unit cost of electricity production from PV; during a PV's whole life-cycle, unit cost c_1 is independent from the variability in the current production rate X and from capacity volume x.
- Unit cost c_2 holds for electricity supplied from the grid, $c_2 > c_1$. Otherwise, it does not make sense to invest in a PV installation by a prosumer.
- The coefficient of the profit per unit of PV production equals opportunity cost $c_2 - c_1$.
- Provision rate d is applied on the surplus of PV production stored virtually in the electricity grid and reversed to a prosumer.

- The percent coefficient of technology inefficiency due to operating conditions for PV installed capacity is A. Let us note that $A = 100a$ where a is the real number counterpart of A:

$$a = \frac{x - \overline{x}}{x}, 0 \leq a \leq 0.5 \quad (1)$$

After considering the non-negativity of the production hourly rate, we have $(1 - 2a)x \leq X(t) \leq x$, and we obtain that $a \leq 0.5$ and equivalently $A \leq 50\%$ in (1). The effective rate of production fluctuates in the range $[1 - 2a, 1]$ or the percent range $[100 - 2A, 100]$. So, the average rate equals $1 - a$ or $100 - A$ in percent. This measure is applicable not only to symmetrical $X_1(t)$ and $X_2(t)$ production processes. Assumption (1) that upper bounds to the inefficiency coefficient is 0.5 is relaxable for models with negative theoretical values of production. In Figure 2, the average percent loss of the nominal capacity for X_1 equals

$$A_{X1} = \frac{x - \overline{x}}{x} * 100 = \frac{8 - 5}{8} * 100 = \frac{300}{8}\% = 37.5\%, \; so \; a_{X1} = \frac{3}{8}$$

and for X_2,

$$A_{X2} = \frac{x - \overline{x}}{x} * 100 = \frac{6.5 - 5}{6.5} * 100 = \frac{300}{13}\%, \; a_{X2} = \frac{3}{13}.$$

Let us note that under the presented framework, the unit production cost c_1 is determined with PV capacity investment unit cost c_0, with the number T of all PV operating 24-h cycles and technology inefficiency coefficient a

$$c_1 = \frac{c_0 x}{24xT(1 - a)} = \frac{c_0}{24(1 - a)T} \quad (2)$$

where the life-cycle PV production is $24\overline{x}T = 24xT(1 - a)$ because $\overline{x} = x(1 - a)$ from the definitions of A and a.

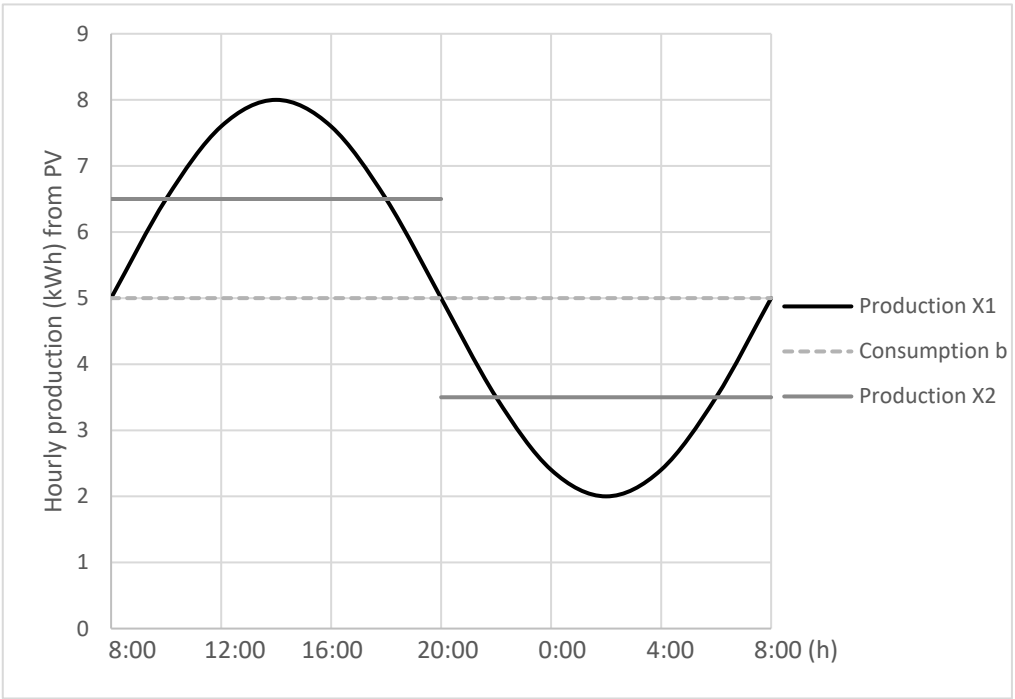


Figure 2. PV capacity 8 kWp 24-h cycle production functions: harmonic (X_1) vs. 2-point output (X_2).

This modeling framework also ensures pricing arbitrage for two PV technologies with daily output functions $X_1(t)$ and $X_2(t)$. The unit costs of capacity should exactly follow

the proportion of effectiveness coefficients $\frac{c_{0(X1)}}{c_{0(X2)}} = \frac{1-a_{X1}}{1-a_{X1}}$ for technologies number 1 and 2. This finding is derived from (2) being equal for both technologies:

$$\frac{c_{0(X1)}}{24(1-a_{X1})T} = \frac{c_{0(X2)}}{24(1-a_{X2})T}. \tag{3}$$

Otherwise, a prosumer that buys technology X_1 at the same cost as technology X_2 loses $24\left(c_{1(X1)} - c_{1(X2)}\right)bT$.

Current PV production surpluses are transferred to the grid. Two energy meters are installed at a prosumer's place. One measures flow-out from the grid, and another one measures flow-in. Flow-out occurs when there is shortage of PV output. Flow-in occurs when the current PV production exceeds demand. Two accounting systems are choice alternatives for the grid rule of cooperation with PV:

- **Net-metering** is the net-energy flow usually measured yearly. The surplus produced on a monthly basis is to be compensated within a year with monthly shortages. The positive balance of months is lost. We will model further on a daily basis. We aggregate days into a month using a linear time scale.
- **Net-billing** is when in each accounting period inflow should be paid, and outflow should be sold by a prosumer [20]. Net payment is to be made. This scheme is not considered in this paper.

3. Solving the Optimal Capacity

3.1. No Provision on PV Production Returned by System Operator

In this sub-section, the grid acts as an ideal cost-free energy storage, so there is no provision on PV production surplus.

Proposition 1. *The optimal capacity x^* is given with the following formula:*

$$x^* = \frac{b}{1-a}, \quad c_2 > c_1 \tag{4}$$

Proof. If the total of production hourly surpluses is lower (higher) than the total of hourly shortages during the 24-h cycle, a prosumer can benefit from higher (lower) capacity. So, profit is the highest when the highest surplus of production $x^* - b$ is equal to symmetrical shortage:

$$x^* - b = b - (1-2a)x^*.$$

After manipulations, Equation (4) is obtained. □

Example 1. *For any production process with inefficiency coefficients calculated above for X_1 and X_2, Formula (4) gives the optimum capacity as the following:*

$$x^* = \frac{b}{1-a} = \frac{5}{1-\frac{3}{8}} = 8 \text{ and } x^* = \frac{b}{1-a} = \frac{5}{1-\frac{3}{13}} = 6.5.$$

It turns out that for production X_1 and a consumption of 5, the optimal capacity is 8, as it is shown in Figure 2. It is best to invest in a capacity of 8 kWp within technology specified with a production inefficiency of 37.5%. For the production of a type like X_2, the optimal capacity is 6.5. The higher the inefficiency coefficient a, the higher the optimal capacity for the same electricity consumption.

3.2. A Prosumer That Pays Provision on PV Production Stored in the Grid

Proposition 2. *The optimal capacity x^* of the PV 2-point production process $(X_2(t))$ is as follows:*

$$x^* = b\frac{2-d}{2-d-2a},\ d \geq 0,\ c_2 > \frac{c_1}{1-d} \tag{5}$$

Proof. This time, the PV production surplus is devaluated with a commission coefficient d before it supplies symmetrical shortage: $(1-d)(x^* - b) = b - (1-2a)x^*$. After manipulations, (5) is obtained. If $d = 0$, Equation (5) can be simplified to (4). □

Example 2. *The optimal capacity for technology X_2 and commission $d = 0.2$ is given with Formula (5) and equals the following:*

$$x^* = 5 \cdot \frac{2-0.2}{2-0.2-2\cdot\frac{3}{13}} \approx 6.724.$$

To illustrate this example, Figure 3 is provided. The optimum production process is $X_2(t) = \{3.62; 6.72\}$ where $\overline{x}_2 = 6.724 \cdot (1 - \frac{3}{13}) \cong 5.172$. The lower output was calculated using the assumed symmetry of extreme outputs

$$5.172 - (6.724 - 5.172) = 3.62.$$

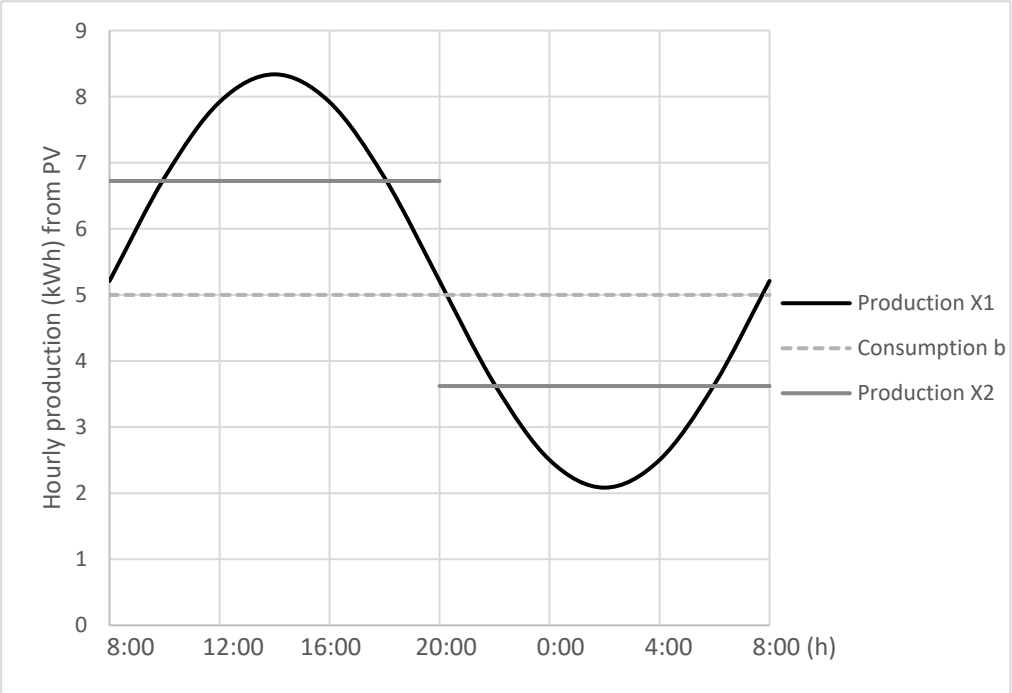


Figure 3. Optimum PV capacity under PV surplus reverse supply provision $d = 0.2$.

Proposition 3. *For harmonic production $X_1(t)$, the optimum capacity presented in Figure 3 should balance the output surplus devaluated by the rate of provision d coming from the first part of the cycle with supply shortages occurring at night. The optimum capacity x^* is calculated from the following formula*

$$x^* = \frac{\overline{x}}{1-a}\ d \geq 0,\ c_2 > \frac{c_1}{1-d} \tag{6}$$

in which the hourly mean output $\overline{x}$ exceeds hourly fixed consumption b by the provision to be paid on the total surplus in a cycle:

$$\overline{x} = b + \frac{7.582bda}{12(2-d)(1-a)}\ . \tag{7}$$

The optimal capacity raises in provision rate d.

Proof. In cycle X_1, surpluses of output occur in 12 h. Within each hour t, surplus is calculated relatively to peak surplus and summed as $\sum_{t=1,2,\ldots,12}\left(sin\frac{2\pi t}{24}\right)$ =7.582. The peak surplus is related to the mean according to equation $\acute{a} = \frac{a}{1-a}$. It is introduced into the first component in the following expression of the sum of surpluses after provision:

$$7.582b\grave{a}(1-d) + 12(\overline{x} - b).$$

The last component accounts for the average $\overline{x}$ which exceeds consumption b due to provision.

On the other hand, the total shortage has the following symmetrical formula:

$$7.582b\grave{a} - 12(\overline{x} - b)(1-d).$$

Balancing the sum of provision net surpluses with the sum of shortages defined above and conducting manipulations, we obtain (7). □

Example 3. *According to (7), the average output $\overline{x} \approx 5.211$ is enough to supply the constant consumption of 5. Following (6), we obtain that the optimal capacity equals*

$$x^* = \frac{5.211}{1-0.375} \approx 8.337.$$

Respecting symmetry around the calculated mean, the output values are within the following range: $2.084 \leq X_1 \leq 8.337$. It is presented in Figure 3. We conclude that the higher commission d, the higher the optimal capacity, although there is a threshold of commission that makes lagged auto-consumption not rentable. In such a case, the capacity is equal to the demand level. See (8).

Proposition 4. *For the relation of costs $c_2 \leq \frac{c_1}{(1-d)}$, lagged auto-consumption is not rentable. This results in the optimum capacity that is equal to the demand level:*

$$x^* = b,\ \ 0 > d \geq 1 - \frac{c_1}{c_2},\ \ c_2 > c_1. \tag{8}$$

3.3. Total Profit

Profit Z from any capacity x is the opportunity cost of using traditional electricity instead of PV during the whole life-cycle of PV installation. The formula is the following:

$$\max Z(x),\ \ x \geq 0$$

$$Z(x\} = 24Tbc_2 - c_0x - uTc_2\sum_{t=1}^{24}[(1-y)(b - X(t)) - y(1-d)(X(t) - b)], \tag{9}$$

where y is a dummy variable that indicates whether the surplus of PV production occurs at the current hour or not:

$$y = \begin{cases} 0,\ X(t) < b \\ 1,\ X(t) \geq b \end{cases}$$

and u is a variable that indicates net shortage after the summation of 24-hourly balances. Such a whole cycle net shortage costs c_2 per unit.

$$u = \begin{cases} 0, \sum_{t=1}^{24} [(1-y)(b-X(t)) - y(1-d)(X(t)-b)] \leq 0, \\ 1, \sum_{t=1}^{24} [(1-y)(b-X(t)) - y(1-d)(X(t)-b)] > 0 \end{cases}$$

If $\sum_{t=1}^{24} [(1-y)(b-X(t)) - y(1-d)(X(t)-b)] \leq 0$, the net PV production occurs as a result of overinvestment in the PV capacity. Such a positive balance is transferred into the grid without benefit for a prosumer under net-metering. Therefore, it does not affect the total profit in (9).

Note that when $x = 0$ in (9), we have $X(t) = 0$, $t = 1, 2, \ldots, 24$; $y = 0$, $u = 1$ and finally $Z(x) = 0$. So, (9) gives 0 profit when investment is 0. The total profit function (9) is maximized by the above propositions of the optimal capacity, balancing PV productions with consumptions within a daily cycle. For the capacity being smaller than the optimal one, shortages are at least partly supplied from the grid at a higher cost than the PV production cost is. It is enough to rearrange (2) into the following:

$$c_0 = c_1 24(1 - \mathrm{a})\mathrm{T} \leq c_2 24(1 - \mathrm{a})(1 - d),$$

Substituting c_0 and the optimal capacity x^* from the appropriate proposition into (9) results in the component of the surplus and shortage total balance being equal to 0, and the maximal total profit value is as follows:

$$Z(x^*) = \max_x Z(x) = 24Tbc_2 - c_0 x^* = 24T\{bc_2 - (1-a)c_1 x^*\}. \quad (10)$$

Formulas (9) and (10) are easy to disaggregate into a sequence of operating periods of equal, dynamic or heterogenic cashflows. This is especially useful in the case of the increasing trend of c_2. Moreover, various discount rate time structures can be applied as well as in investment expansion planning presented by [8]. Hence, a classical investment measure is easy to integrate into the presented model. The return rate yearly is given with the following formula

$$r(x) = \frac{360Z(x)}{Tc_0 x}, x > 0. \quad (11)$$

Formula (11) is appropriate for the assessment of capitalized return on some positive capacity x and positive expenditure xc_0 to investment in PV. No investment means that $x = 0$, so Formula (11) is mathematically infeasible.

3.4. Optimization Results for Theoretical PV Production Value

To illustrate the results of Formulas (9)–(11) for theoretical values of PV production, we provide Figure 4. In this calculus model, the parameters are set to the following values: $c_0 = 7000$, $c_2 = 1$, $b = 5$, $T = 3600$, $a_{X1} = \frac{3}{8}$, $a_{X2} = \frac{3}{13}$. Note that the discount is considered at two levels $d = 0.2$ or $d = 0$ for both production functions.

The results led to the following conclusions about the impact of the capacity volume on profit and return:

- The more technologically efficient PV (the lower the technology inefficiency ratio a is) is, the smaller the optimal capacity is and simultaneously the higher total profit as well as return rate. The rates of return of both or more available technologies should be calculated for rational decision making.
- Overinvestment by a unit of capacity results in total profit being a little smaller than the maximal profit.

- Underinvestment by a unit of capacity hardly affects the return rate, while it results in a total profit much lower than the maximum.
- The capacity that maximizes return is less or equal to the capacity with the maximal profit. Also, when one substitutes c_0 and rearranges (2) into (10), one achieves that profit increases nominally in capacity and faster than the linear increase in expenditure to capacity.
- The capitalization return rate was calculated from the formula $r_c(x) = \sqrt[\frac{360}{T}]{\frac{Z(x)}{c_0 x} + 1} - 1$. Of course, the resulting values of return are smaller than for return without capitalization (11). The not-capitalized return is appropriate as savings/profit from PV are received by a prosumer on a schedule of bills payment as periodical cashflows. They cannot be reinvested. Expenditure in PV investment is not allowed to be reinvested in capacity expansion as in our model, electricity consumption stays constant. A prosumer saves up to 84.9% of the amount from bills received from the operator if the grid supply is used. However, the capitalization return rate better shows time perspective and compares PV investment with traditional financial investment. We mean that expenditure is conducted in the beginning, so it can be compared to a deposit with the capitalization of interests.
- The higher the **provision** d, the lower the total profit and the lower the return rate.
- For **application** purposes, a prosumer should choose a PV output model that fits the best to production data from a place being similar to theirs. We discuss the issue further using some empirical data. Taking into account real production data, we obtained lower profits and returns.

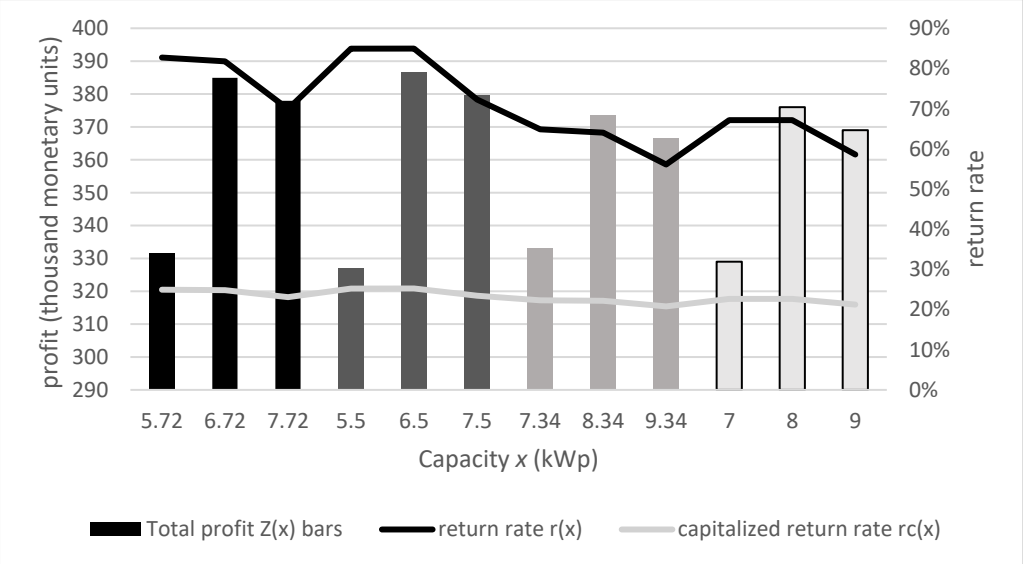


Figure 4. Profit vs. return for the optimal and sub-optimal PV capacities. For the 2-point production and discount $d = 0.2$, black bars are appropriate. For $d = 0$, see dark gray bars. For the harmonic, gray bars show results for $d = 0.2$ and light gray bars under the discount $d = 0$.

4. Empirical Data Fitted PV Production Models

4.1. Calibration of PV Production Process for Prosumers

The estimation of production models from data needs the introduction of a stochastic error component capturing for weather and unmodeled determinants of the production. These are considered in the following model of a day/night 24 h production cycle from 1 kWp of PV installation:

$$X_1(t) = \alpha + \beta sin\left(\frac{2\prod t}{24} + \gamma\right) + \varepsilon, \tag{12}$$

$t = 0, 1, 2, 3, \ldots, 23$—time (hour) within a daily cycle;
$\alpha > 0$—constant, mean hourly output within a cycle;
$\beta > 0$—production hike in relation to the mean within a cycle;
γ—radian shift in the sinusoid from starting hour 0:00 ($t = 0$), γ'—shift in hours;
ε—stochastic component.
We estimated Model (12) with the following methods:

- Ordinary Least Squares with $\varepsilon \sim N(0, \sigma)$ with negative model output and $0 \leq \alpha \leq 1$;
- Restricted OLS to $\alpha - \beta \geq 0$;
- Tobit estimation with negative model output and $0 \leq \alpha \leq 1$.

Figure 5 presents production data recalculated per unit of capacity collected by the authors from a big institutional installation with a power of 198 kWp at geolocation N52°23′. Of course, production data series should come from a geographical location that is close to the one considered by a decision maker. The estimated harmonics are presented. Alternatively, the 2-point production output model was also estimated:

$$X_2(t) = \begin{cases} 0, & t = \{0, 1, 2, 3, 4, 5, 6, 7, 20, 21, 22, 23\} \\ \theta = \frac{1}{N}\sum_{i=1}^{n} x_i, & t = \{8, 9, \ldots, 19\} \end{cases} \quad (13)$$

where x_i is observation i of non-zero production.

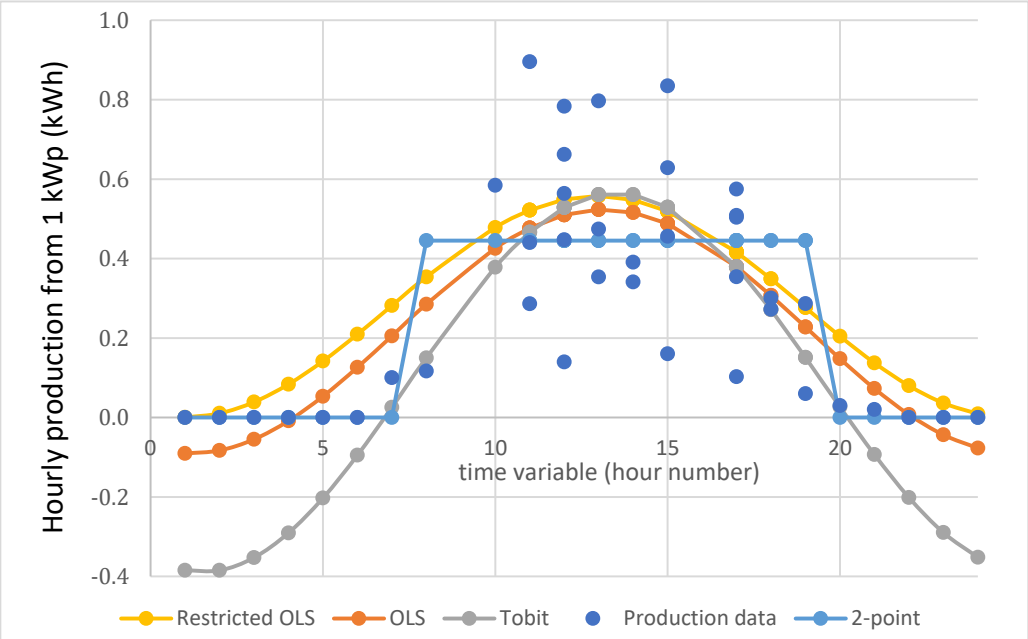


Figure 5. Estimated PV production models.

Descriptive statistics of hourly production data collected in April 2023 and their model estimates are presented in Table 1. Note the very high volatility of data. Tobit's estimation really fits the best positive values of production. Moreover, for the analyzed geolocation, the yearly production standard is 1000 kWh from 1 kWp. From that comes the hourly average rounded to 0.12 kWh to which Tobit model's alpha is the closest. This standard yearly production-to-capacity ratio is used commonly in practice to calculate capacity and here is further called the practical approach and stands for a benchmark.

Table 1. Hourly production data estimated 24 h models.

Data		Model				
Descriptive	**Statistics**	**Parameter/Model Value** [1]	**Harmonic (12) Estimation with:**			**2-Point (13)**
			OLS	**Restr. OLS**	**Tobit**	
n	40	α (kWh) [3]	0.229	0.279	0.088	N/A [2]
m (kWh)	0.312	β (kWh)	0.363	0.279	0.477	
s (kWh)	0.280	γ (rad)	4.410	4.458	4.318	
SD	0.895	γ' (HH:MM)	16:50	17:02	16:27	
lowest	0	lowest $\hat{x}^l$	−0.134	0	−0.385	0
highest	0.895	highest $\hat{x}^h$	0.592	0.558	0.561	0.445
Month	4/2023	θ (kWh)	N/A [2]			0.445
-	-	R-square	0.798	N/A [2]	N/A [2]	0.518
-	-	SE (kWh)	0.127	0.139	0.194	0.191

[1] insignificant at p-value = 0.05, estimates were not obtained for OLS and restricted OLS; [2] not applicable; [3] from (12), we remember that harmonic α gives average hourly production of PV during daily cycle.

4.2. Solving the Optimal PV Capacity Model

Having estimated a daily cycle, production models' efficiency and inefficiency coefficients are calculated in Table 2. The data-calibrated values of parameters are as follows:

- Coefficient a has a limit (1) that holds for models excluding negative model production. Note also that from (2), it follows in this case that $c_1 = \frac{7000}{24 \cdot (1-0.64) \cdot 3200} \leq 0.23$.
- Unit PV production cost does not directly impact profit (9). Assumption (5) is satisfied as $\frac{c_1}{1-d} \approx 0.29 < 1$. So, the supply of electricity from the grid is less profitable than from PV. In other words, all supply from the grid should be compensated with PV surplus production after commission.
- The empirical results are not exact due to the asymmetry of the production daily cycle. About 10 h that account for at least −54.91 kWh solved to be supplied at night from the grid and not compensated with PV surpluses.

Table 2. PV production planning and investment effectiveness for empirical production data and parameters: $b = 5$, $d = 0.2$, $c_2 = 1$, $T = 3600$, $c_0 = 7000$.

Result	Practical Approach	Harmonic (12)			2-Point (13)
		OLS	**Restr. OLS**	**Tobit**	
a [1]	0.885 [2]	0.61	0.50	0.84	0.50
x^*	43.20 [3]	14.35	10.70	43.77	11.25
Daily prod. $= 24(1-a)x^*$ (kWh)	119.23	78.99	71.66	92.65	60.08
$c_0 x^*$	302,400.00	100,419.00	74,914.00	306,397.00	78,750.00
Net-metering(kWh) [4]	−0.77	−32.69	−48.34	88.53	−59.93
$Z(x)$	126,835.05	199,494.08	180,157.08	125,603.66	137,466.85
$r(x)$	0.04	0.20	0.24	0.04	0.18
IRR	0.07	0.27	0.32	0.07	0.24

[1] Assumption (1) has to be relaxed to $0 \leq a \leq 1$ for models in which the smallest theoretical value of production is negative (see Table 1). We substitute regression equation into (1) and obtain $a = \frac{\hat{x}^h - \hat{\tilde{x}}}{\hat{x}^h}$. For any harmonic, $a = \frac{\beta}{\alpha+\beta}$ and $\hat{\tilde{x}} = X_1(0) = \alpha$. If one takes into account asymmetrical models, all hour outputs within a daily cycle should be taken into account for $\hat{\tilde{x}}$. For the Tobit model, $\hat{x}^h - \hat{\tilde{x}} = 0.561 - 0.2 \neq \beta$. In the 2-point model, we have $a = 0.5$ as 0 is the smallest output in (12). [2] Note that data from 1000 kWh yearly from 1 kWp follow that

$a = 1 - 1000/360/24$. [3] In practice, PV capacity is often calculated as $x^*_{practical} = \frac{yearly\ consumption}{1000} = \frac{24 \cdot 360 \cdot b}{1000}$. Indeed, daily production is over 127.5 kWh when summing the smallest observation of output at each hour within a cycle. In data, 9 AM and 4 PM hourly productions are not observed. Instead, the smallest output rates during 8–10 AM and 3–5 PM were used. The hourly average is 5.3 kWh from the capacity of 43.2 kWp calculated from the formula; [4] daily energy balance between PV production and consumption by prosumers. A negative value means an energy deficit that is to be supplied from the grid. Note that for models with non-negative theoretical production and for the practical approach, the sum of net-metering and daily production equals a total consumption of 120 kWh. We assumed that the hourly deficit cannot be smaller number than -5 at night and includes PV negative theoretical production.

4.3. Return Rate Sensitivity to Grid Market Parameters

The proposed approach advantage is that the optimal capacity value is insensitive to parameters such as PV installation durability measured with the number of cycles T, capacity unit cost c_0 and grid electricity price c_2. However, all these are subject to changes in the market and affect the return rate. Each parameter's new value impact on the results is analyzed separately from other changes. See values in Table 3.

Table 3. The sensitivity of the optimal solution return rate to each parameter separately: T, c_0, c_2.

Parameter New Value	Practical Approach	Harmonic (12)			2-Point (13)
		OLS	Restr. OLS	Tobit	
$T = 7200$	0.09	0.25	0.29	0.09	0.23
$c_0 = 5000$	0.10	0.32	0.38	0.10	0.28
$c_2 = 0.5$	-0.03	0.05	0.07	-0.03	0.0

5. Conclusions and Prospects for Further Research

The optimization of the PV capacity discriminates between efficient financial investing or wasting resources, especially under subsidized renewable energy investments. The higher the capacity of PV cells sold to a prosumer, the higher the profit of the seller and carbon dioxide emission due to cell production and transportation. The more electricity prosumers produce, the more intertemporal re-balancing of the electricity system is needed, while its load is hard to forecast. These are important reasons for the optimization of capacity as well as its return rate. If the market participants neglect optimization, then by the law regulation, PV capacity overinvestment should be prevented. The proposed approach under the net-metering system scheme results in the optimal capacity being independent from variable parameters such as the PV exploitation period, PV capacity unit cost and the grid supplied energy unit cost.

The proposed optimal capacity approach also impacts grid operator and energy system. The system benefits double from the overcapacity at a prosumer if they bear the risk of the redistribution of the current PV production surplus from prosumers to other electricity consumers. However, the overcapacity of PV at prosumers makes the operator lose from each unit of PV production surplus that is not stored or redistributed on a current basis to customers. Therefore, the current adjustments of systemic plants' electricity production to prevent photovoltaic surplus loss is the main condition for the grid operator profitability under net-metering. Therefore, the net-metering market rule under yearly balanced production and consumption at each prosumer does not solve the grid operator problem, while it stops overinvestment in capacity and limits the scale of PV surplus that introduces uncertainty to income and expenditure cashflow analysis by a prosumer but also for the grid operator [21].

The same PV production and energy consumption balancing rule is incorporated into the proposed approach as it aims at the maximal total profit of a prosumer under net-metering. Summarizing, the optimal PV capacity model is provided by this research to

balance PV installation production with a prosumer's electricity consumption in a typical day/night prosumer production and consumption cycle.

Finally, photovoltaic cells' capacity optimization prevents a prosumer from overinvestment in terms of a smaller total profit as well as return. This finding is backed with total profit versus return analysis for theoretical and empirical data. The "practical approach" used by PV sellers is shown to result in overcapacitation and even negative return from a prosumer's investment in PV.

However, our model also has limitations due to the assumption of the constant hourly consumption of electricity and the symmetrical variability of PV production in a day/night cycle. The latter one has already been addressed by applying different estimation methods and checking if they fit the data. The model can estimate the cost of imbalance between PV production and electricity consumption due to the fluctuations of production and consumption. Therefore, our model is appropriate for households and devices that stabilize energy consumption. Electricity consumed by devices used during the day such as heat pumps, computers, electrical heating, cookers and water-boilers can be easily balanced with illumination, night-running dishwashers, programmed washing, and electric car chargers. A tariff including a cheaper unit cost for night shortage would also confirm our findings while limiting the opportunity cost and the profit from the optimal PV capacity. Therefore, we would recommend our model especially to households that manage their electricity consumption and also to smart houses and enterprises working two or three shifts or implementing aggregating production planning.

The arising limitations and aspects for further research are the following:

- Econometric modeling of production. Using harmonics and 2-point econometrics, models (13) and (14) were backed with their properties and analytics matched to the concept of the optimal capacity model. However, the estimated models moderately fit to the analyzed empirical hourly production data and their asymmetry.
- The enhanced analysis of the seasonal PV production effects on the volume of electricity supplied from the electrical grid. Of course, seasonal variability compensates to nothing in a yearly scale by definition. Our model lacks seasonal effects and uses April production parameters as a proxy for the average daily cycle for the whole year within net-metering compensation to be conducted. Therefore, there is a need for some correction for provision on seasonal variability.
- For the practical verification of the results, a simulation of net-metering total profit and return should be run, for the optimal capacity solution using full yearly data. We verified the sample of hourly production in April presented in Figure 5 as well as theoretical production values fitted to data.
- Calculating the measure of effectiveness like IRR and NPV examines the impact of fast deteriorating production rate PV cells. It can also be used to measure the effectiveness of the prosumer using a hybrid financing structure. These measures also provide a general benchmark of the effectiveness of the investment.

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