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Knowledge Discovery to Support WTI Crude Oil Price Risk Management

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Abstract: The high volatility of commodity prices and various problems that the energy sector has to deal with in the era of COVID-19 have significantly increased the risk of oil price changes. These changes are of the main concern of companies for which oil is the main input in the production process, and therefore oil price determines the production costs. The main goal of this paper is to discover decision rules for a buyer of American WTI (West Texas Intermediate) crude oil call options. The presented research uses factors characterizing the option price, such as implied volatility and option sensitivity factors (delta, gamma, vega, and theta, known as "Greeks"). The performed analysis covers the years 2008–2022 and options with an exercise period up to three months. The decision rules are discovered using association analysis and are evaluated in terms of the three investment efficiency indicators: total payoff, average payoff, and return on investment. The results show the existence of certain ranges of the analyzed parameters for which the mentioned efficiency indicators reached particularly high values. The relationships discovered and recorded in the form of decision rules can be effectively used or adapted by practitioners to support their decisions in oil price risk management.

Keywords: knowledge discovery; decision rules; crude oil price risk; commodity options; implied volatility; Greeks; decision making

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1. Introduction

Prices of energy resources are highly volatile, mainly due to events that cause disruptions in the supply chain and affect lead times, delivery times, and production costs. In addition, many unpredictable situations can disturb the demand for raw materials. This was clearly visible in 2020 when governments of many countries decided to freeze retail trade and other economic activities to limit the spread of the COVID-19 pandemic. In particular, various limitations and the resulting slowdown in production during the first COVID-19 wave (and the first lockdown) significantly reduced the demand for crude oil and that which follows reduced its price [1]. The COVID-19 pandemic also caused the crude oil market to become ineffective, and it was difficult to predict the crude oil prices [2]. Moreover, the commodity markets, including the crude oil market, are strongly interconnected with other financial markets [3–6]; therefore, they have a significant impact on the condition of the whole world economy [7,8]. There is also an observed transmission of volatility from the oil market to the stock market, which has become more prominent with the COVID-19 pandemic [9].

One of the instruments that was created to hedge against fluctuations in commodity prices is derivatives. There are financial instruments that derive their values based on the underlying assets. Furthermore, derivatives make it to possible to settle future deliveries at a price determined at the moment of concluding the contract. Thanks to this form of settlement, each counterparty entering into the contract knows exactly how much it will pay or what amount it will receive for future supplies of raw material, and is therefore able to plan future expenses. The very idea of derivatives was born in the 17th century, while the first stock exchange derivatives (futures contracts) were introduced to trading in the mid-19th century on the CBOT (Chicago Board of Trade) exchange. Currently, commodity derivatives play a very important role in global trading.

One of the main types of exchange-traded derivatives are options. An option is defined as a "contract between two parties, a buyer (holder, long position) and a seller (writer, short position), in which the buyer has a right, but not the obligation, to buy or sell a certain amount of the underlying asset (stock, bond, interest rate, commodity or other assets) at the denoted strike price within a specific time period" [10]. Sellers willing to accept the risk of unlimited loss as a result of fluctuations in the price of the underlying asset can write options, collecting the premium and taking advantage when options expire worthless. The premium collected by a seller is seen as a liability until the option either is offset (by buying it back) or expires [11].

Due to the specific structure (asymmetric risk), options offer opportunities for price risk management, including the risk of changes in energy commodity prices. The effective use of options in price risk management depends on parameters such as the strike price, the level of implied volatility, or the expiry date of the option. All of these parameters have a significant impact on the final result achieved by the buyer or option writer; however, only some of them (e.g., strike price) are set by those who open their positions in options, while the others (e.g., the level of implied volatility) are only observable. In this paper, we use association analysis to discover decision rules to adjust the price risk management parameters. We decided to use decision rules because they can be easily understood by business practitioners who do not always have much IT or statistical knowledge. The research focuses on WTI Crude Oil call options, i.e., call options written on the WTI crude oil and listed on the NYMEX exchange. The data used in the performed analyses cover a period of over 14 years (2008–2022) and concern the ATM (At-The-Money) option with the execution period from 1 to 90 days. The decision rules are discovered based on the levels of implied volatility, number of days to expiration, and the values of four option sensitivity coefficients (the Greeks): delta (Δ), gamma (Γ), vega ($\mathcal{V}$), and theta (Θ). The rules are optimized in terms of indicators related to the total return on long positions in call options and the costs incurred by the option buyer (total option premiums paid), i.e., we focus solely on the option buyer, for whom the maximum loss on the option market is limited and known on the day of opening the position. The decision to include only long positions is due to the fact that the rules are intended to hedge against the rise of oil prices and not to speculate on the commodity options market.

The approach proposed in this paper fills the research gap regarding the possibility of using commodity options in hedging against the risk of oil price fluctuations. There are only a few works that compare the effectiveness of these financial tools. Several articles [12–15] analyze the use of different types of options to construct buying strategies. However, the methods described therein do not consider market parameters (such as those used in our work) to find signals for taking a long position in the options market.

This work extends the research conducted in [16–18] which investigated the possibilities of using artificial neural networks (ANNs) for searching for buy signals for European WTI crude oil call options. In the approach presented here, we focus on discovering decision rules that are based on parameters, such as implied volatility, number of days to expiration, and the value of the Greeks, that are strictly linked to option premium. Moreover, we analyze American options that can be exercised at any time of option activity. Data used in the performed analysis cover the years used in [16–18] plus the years 2020–2022 (characterized by high volatility in the energy commodity markets; these last years had significant impact on the results obtained by participants of the options market).

The rest of the paper is organized as follows. Section 2 presents an overview of related works. The data and their properties are presented in Section 3. The proposed methods and efficiency indicators used in the performed analyses are provided in Section 4. The obtained results are presented in Section 5. The paper ends with discussion and concluding remarks.

2. Related Works

The most relevant parameter that affects an option's price is related to the volatility of the underlying asset. One particular type of volatility, extensively studied in financial literature, is the implied volatility, usually estimated from the Black–Scholes model [19,20]. The growing importance of implied volatility has led to the development of various implied volatility indices for different assets classes. There are a number of studies that investigate and document the information content of implied volatility indices for forecasting the realized volatility and the returns of the underlying asset [21,22]. Fassas and Siriopoulos [23] suggest that "implied volatility includes information about future volatility beyond that contained in past volatility". Furthermore, they find a significant relationship between changes in implied volatility and returns on the underlying assets, but at the same time show that implied volatility for commodities such as crude oil, among others, reacts differently to changes in the price of the underlying assets than to stocks. In turn, Slim et al. [24] showed that implied volatility index can be used as an input variable for VaR (value-at-risk) calculation. Furthermore, a growing number of studies (see, e.g., [25–27]) explore the transmission of implied volatility from international markets and various asset classes.

In the literature on implied volatility, studies can also be found on the phenomenon of the "implied volatility smile", which is an inconsistency with the assumptions of the Black–Scholes model (all options on the same underlying asset and with the same time to maturity should have the same implied volatility regardless of the strike price). The existence of this phenomenon in the real market has consequences when selecting options with different strike prices. Soini and Lorentzen [28] studied the determinants of WTI oil call option prices, focusing on the relationship between implied volatility and moneyness. They showed, in particular, that the curvature of implied volatility as a function of moneyness is positively and significantly correlated with the basis and hedging pressure of the underlying crude oil futures contract. In turn, Łamasz and Iwaszczuk [29] studied the impact of implied volatility on chosen spread option strategies on the WTI crude oil market. They showed that a change in the strike price significantly affects the level of implied volatility (according to the implied volatility phenomenon) and, thus, the effectiveness of hedging against volatility using option strategies.

Option market participants are also interested in the sensitivity of the option premium to the parameters that shape its value, i.e., the price of underlying asset, implied volatility, risk-free rate, and time to expiration. The Greeks, so named after the Greek letters, are basically derivatives of a certain order of the price of an option relative to its selected parameters [30]. They are financial measures that have a great impact on portfolio management and are considered by many investors to be essential for making informed decisions in options trading.

One of the main applications of the Greeks is to construct a hedging portfolio, consisting of an underlying asset and an option written on that underlying asset in such a way that a change in price does not change the value of the portfolio. This is achieved by holding selected Greeks at a fixed level and modifying the number of options in the portfolio hedging the underlying asset (delta-, delta–gamma-, delta–gamma–vega-hedging). However, the very idea of such an approach implies that the composition of the portfolio needs to be adjusted frequently, with the consequence that transaction costs may significantly reduce the potential gains from using such a form of hedging. Previous research has shown no significant differences in performance between delta-neutral portfolio construction based on parametric and nonparametric models, such as, for example, ANNs (cf. [31–36]). Moreover, nonparametric models, which performed better in option price prediction, did not necessarily produce better results in delta-hedging, mainly due to the high complexity and time-consuming nature of applying these models.

In this paper, we move away from the described classical application of the Greek coefficients and try to use their values to discover decision rules to take (or not) positions in the WTI crude oil options market.

3. Data

The data used in our research were obtained from QuikStrike—an options analysis platform available on the CME Group's website via Bantix Technologies LLC [37]. The data include ATM American options (with price rounded to USD 0.5/b) written on WTI crude oil between 26 August 2008 and 15 November 2022 (10,484 observations) with an expiry period of up to 90 days. The reason for choosing American options was their large volume and high liquidity. Options with different strike prices and different number of days to expiration have different levels of option premiums that must be paid by the buyer. The options available through QuikStrike can be divided (in terms of the execution period) into the following three categories:

- One-month option—with the execution period from 1 to 30 days;
- Two-months option—with the execution period from 31 to 60 days;
- Three-months option—with the execution period from 61 to 90 days.

The Bantix database also contains information on the level of implied volatility for options with different expiry dates and different strike prices, and provides Greeks values for each option. Below, we present background information on the Greeks because it is important for the discovery of the decision rules in the empirical part of our work [30].

- Delta (Δ) measures the change rate of the theoretical option value V with respect to changes in the underlying asset's price S:

$$\Delta = \frac{\partial V}{\partial S}. \tag{1}$$

- Gamma (Γ) measures the change rate of Δ with respect to changes in S:

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}. \tag{2}$$

- Vega ($\mathcal{V}$) measures sensitivity of V with respect to the volatility σ:

$$\mathcal{V} = \frac{\partial V}{\partial \sigma}. \tag{3}$$

  Vega can be an important Greek to monitor for an option trader, especially in volatile markets. For example, the values of the at-the-money options, which are the subject of this research, are extremely dependent on changes in volatility.
- Theta (Θ) measures the sensitivity of V to the "time decay" τ:

$$\Theta = -\frac{\partial V}{\partial \tau}. \tag{4}$$

  It is expressed in value per year. To obtain the amount by which the price of the option will fall in relation to the price of the underlying asset, the result is usually divided by the number of days in a year.

4. Methodology

In this section, we describe the proposed tool to support decision making in the crude oil options market. In order to hedge against the risk of oil price fluctuations, we took long positions in call options. The proposed tool uses association analysis which was carried out in relation to the three categories of options defined in Section 3. The rules were evaluated using the effectiveness indicators defined in Section 4.2. The pseudocode of the algorithm used to discover decision rules is presented in Section 4.3.

4.1. Association Analysis

The process of discovering valuable knowledge from a set of data is known as knowledge discovery in databases (KDD). This widely used data mining technique involves various steps such as data preparation, selection, cleansing, and interpretation of results. In addition to this, prior knowledge on datasets is also taken into account. KDD has a broad range of applications including marketing, fraud detection, telecommunication, and manufacturing. Extracting decision rules is one of the primary research focuses in knowledge discovery and data mining.

Association analysis, which is one of the KDD tools and has its origins in market basket analysis, identifies interesting relationships between items within (often extremely) large datasets in various types of databases and provides valuable information in assessing significant associative and correlative behaviors for new data. Association analysis algorithms have an advantage over conventional decision tree algorithms because they can identify connections between all attributes.

We use association analysis to discover the knowledge from the collected data on American WTI crude oil call options. This knowledge will be written in the form of decision rules, i.e., as implications of the form $X \Rightarrow Y$. The left-hand side (LHS) of the implication is called antecedent and the right-hand side (RHS) is called consequent. The statement $X \Rightarrow Y$ is often written as **IF** X **THEN** Y. Decision rules have been successfully applied, e.g., in economy [38], medicine [39], and logistics [40]. We believe that decision rules can be helpful in hedging against price fluctuations in the crude oil market. Since the stated problem (when to take the long position) determines the consequent ("buy option") of the decision rules, we focus on discovering the antecedents of the rules. In addition to the Greeks, we use the days to expiration (DTE) parameter which corresponds to the position of the hedge. The time horizon was "naturally" divided into monthly intervals since we analyze one-, two-, and three-month options (we also analyze the whole dataset, DTE $\leqslant 90$, for comparison purposes). Similarly, the range of each sensitivity parameter was divided into smaller intervals. The set of rules was determined for each interval of the parameters.

4.2. Indicators

The proposed construction of decision rules and the problem under consideration (when to take the long position in WTI crude oil call option) makes it impossible to evaluate the detected rules using standard indicators (e.g., the confidence level). This is due to the fact that the observations differ in terms of, e.g., payoff value. In addition to information on whether it is beneficial to buy collateral on a given day, it is also worth taking into account the payoff value, which may be used as the weight of a given observation. Therefore, the discovered rules were evaluated in terms of the following three indicators:

- Summary value of payoffs:
$$P = \sum_{c \in C} P_c, \tag{5}$$
where C is the set of all positions, $P_c = \max\{F - K, 0\} - op_c$ is the payoff of the position c, F is the future price of a good (here, crude oil) on the day of the option's expiration, K is the strike price of the option, and op_c is the option premium (for the strike price K) of the position c.
- Average payoff per one position:
$$AvgP = \frac{1}{|C|} \sum_{c \in C} P_c. \tag{6}$$
- Return of investment:
$$\mathrm{ROI} = \frac{\sum_{c \in C} P_c}{\sum_{c \in C} op_c} \times 100\%, \tag{7}$$
which is interpreted as costs resulting from an investment of some resources at a point in time.

4.3. Rules Discovery

To determine the decision rules, we use an algorithm consisting of searching all intervals of a given parameter that meet the given constraints. This strategy was adopted in order not to lose detailed observations during discretization (of continuous variables) used in such cases. The constraints that were imposed on the ranges of values for a given parameter are as follows:

- The number of observations (N) in a given interval must be not fewer than 10 ($N \geqslant N_{\min} = 10$)—this corresponds to the minimum level of support for the rule and was adopted at the level for which one can talk about the repeatability of the situation requiring the hedge.
- The maximum number of observations ($N_{\max}$) in the given interval was limited to 500 to obtain the result in a reasonable amount of computational time.

The pseudocode of the algorithm used to generate the rules is presented in Algorithm 1. For the purposes of the algorithm, we adopt the following notation:

$I = \{P, AvgP, ROI\}$;
$P = \{\text{implied volatility}, \Delta, \Gamma, \mathcal{V}, \theta\}$;
$DTE = \{\text{1month}, \text{2months}, \text{3months}, \text{from 1 to 3months}\}$;
R—set of decision rules;
N—number of observations, $N \in [N_{\min}, N_{\max}]$.

Algorithm 1 Pseudocode of rules generation algorithm.

Input: $dte \in DTE$, $p \in P$, $i \in I$
Output: Set of decision rules R
  $S \leftarrow \emptyset$
  $V_p^{dte} \leftarrow \{\text{values of parameter } p \text{ for execution period } dte\}$
  Sort V_p^{dte} in non-decreasing order
  Create all $s \leftarrow \{v_\alpha, v_{\alpha+1} \ldots, v_{\beta-1}, v_\beta \in V_p^{dte} : N_{\min} - 1 \leqslant \beta - \alpha \leqslant N_{\max}\}$, $S \leftarrow S \cup s$
  Evaluate all s with respect to the indicator i: $e_s \leftarrow i(s)$
  $S \leftarrow S \backslash \{s : e_s < 0\}$
  Sort S in non-increasing order of e_s
  for $k \leftarrow 2$ **to** $|S|$ **do**
    if $s_k \subset s_1$ or $s_k \subset s_2 \ldots$ or $s_k \subset s_{k-1}$ **then**
      $S \leftarrow S \backslash s_k$
    end if
  end for
  For each $s \in S$ create rule r and $R \leftarrow R \cup r$

The presented algorithm has three parameters: the analyzed market parameter, the option execution period, and the efficiency indicator. First, the values of the considered market parameter are sorted in nondecreasing order. Then, the parameter intervals are created so as to meet the condition of minimal number of cases. Each interval is then evaluated in terms of the efficiency indicator i. Next, the intervals with the indicator value lower than zero and the redundant (i.e., with lower value of the indicator and included in another interval) intervals are removed from S. The last step of the algorithm is to create the decision rules of the form $dte \wedge s \Rightarrow$ buy option for each $s \in S$.

The next section presents discovered decision rules grouped against the indicator they were supposed to optimize, and each group contains three best-rated rules. Detailed information on the value of each of the listed indicators for each of the presented rules is provided in Appendix A.

5. Results

In this section, we present the best-rated decision rules for each considered indicator (P, $AvgP$, ROI); they are sorted in nonincreasing order of the indicator value, i.e., the first rule gives the highest indicator value and the last rule gives the smallest indicator value. The first set of rules (R1–R5) maximizes the P indicator:

R1: **IF** DTE $\in [61, 90]$ **AND** $\mathcal{V} \in [3.7138, 8.3791]$ **THEN** Buy option.
R2: **IF** DTE $\in [1, 90]$ **AND** Implied Volatility $\in [0.5199, 0.6301]$ **THEN** Buy option.
R3: **IF** DTE $\in [61, 90]$ **AND** $\Theta \in [-0.0372, -0.0319]$ **THEN** Buy option.
R4: **IF** DTE $\in [61, 90]$ **AND** $\Delta \in [0.5446, 0.5621]$ **THEN** Buy option.
R5: **IF** DTE $\in [61, 90]$ **AND** $\Gamma \in [0.0367, 0.0436]$ **THEN** Buy option.

As can be seen, for four out of five analyzed parameters, the highest value of the P indicator was obtained for the longest analyzed position (options exercise period) (DTE ranging from 61 to 90 days). Only for implied volatility (rule R2) was the highest P value obtained for the DTE covering the widest time window, i.e., from 1 to 90 days. Figure 1 presents the values of all analyzed indicators achieved by using the rules R1–R5.

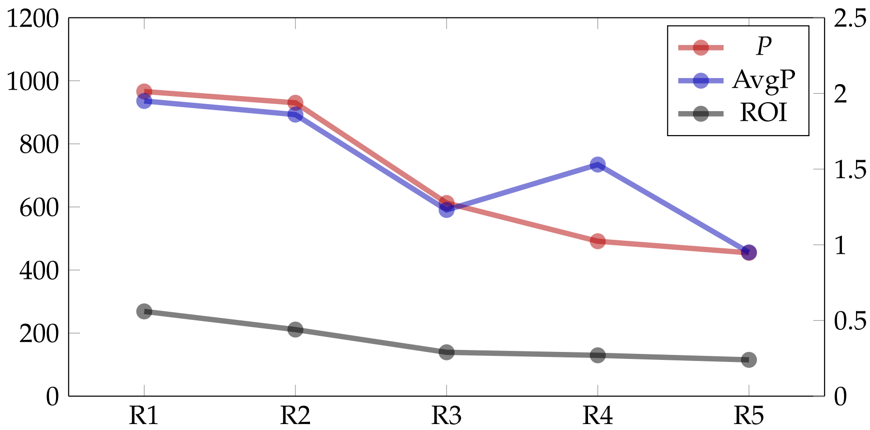

Figure 1. Values of P (left axis) and *AvgP* and ROI (right axis) indicators obtained using 5 best-rated rules determined to maximize P indicator.

It is clearly visible that the rules R1 and R2 stand out significantly from the remaining rules (R3–R5) in terms of the P indicator value. The rule R1 gives a value two times higher than the rule R5. Moreover, the rule R4 gives the highest AvgP indicator value, and there can be observed a strong positive correlation between P (whose value was maximized) and AvgP and ROI.

The next set of rules (R6–R10) was determined to maximize the AvgP indicator:

R6: **IF** DTE $\in [61, 90]$ **AND** $\mathcal{V} \in [3.7138, 4.4474]$ **THEN** Buy option.
R7: **IF** DTE $\in [1, 90]$ **AND** $\Delta \in [0.6121, 0.6592]$ **THEN** Buy option.
R8: **IF** DTE $\in [61, 90]$ **AND** Implied Volatility $\in [0.9679, 1.2842]$ **THEN** Buy option.
R9: **IF** DTE $\in [61, 90]$ **AND** $\Gamma \in [0.0245, 0.0247]$ **THEN** Buy option.
R10: **IF** DTE $\in [61, 90]$ **AND** $\Theta \in [-0.029, -0.0289]$ **THEN** Buy option.

Similarly to the case of the rules for the P indicator, for four out of five rules (R6, R8–R10), the highest value of the *AvgP* indicator was obtained for the longest analyzed position (options exercise period) (DTE ranging from 61 to 90 days). Only the R7 rule (cf. rule R2 for P indicator) covers options with DTE ranging from 1 to 90 days. Moreover, it is worth noting that for the AvgP indicator, the rule covering the widest time window (R7) depends on Δ (it does not involve the implied volatility). Figure 2 depicts the values of the analyzed parameters achieved using the rules R6–R10.

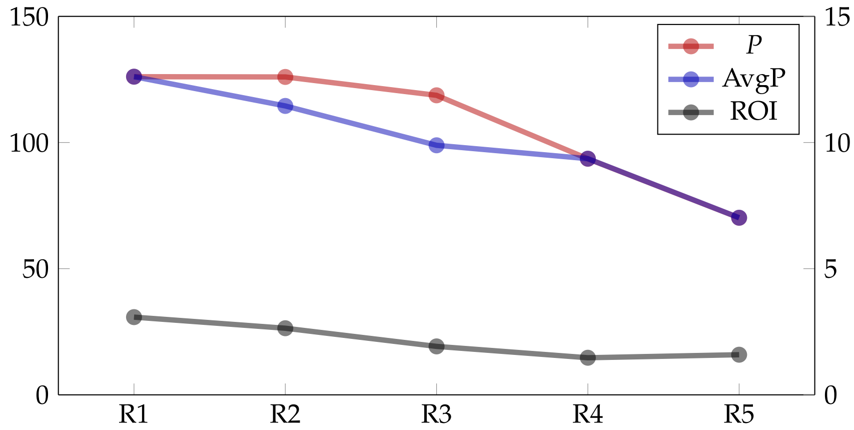

Figure 2. Values of P (left axis) and $AvgP$ and ROI (right axis) indicators obtained using 5 best-rated rules determined to maximize $AvgP$ indicator.

The last set of rules (R11–R15) is used to optimize the ROI indicator:

R11: **IF** DTE$\in [31,60]$ **AND** $\mathcal{V} \in [2.4251, 3.131]$ **THEN** Buy option.
R12: **IF** DTE$\in [1,30]$ **AND** $\Gamma \in [0.306, 0.3211]$ **THEN** Buy option.
R13: **IF** DTE$\in [1,90]$ **AND** $\Delta \in [0.6121, 0.6392]$ **THEN** Buy option.
R14: **IF** DTE$\in [1,30]$ **AND** $\Theta \in [-0.4815, -0.4531]$ **THEN** Buy option.
R15: **IF** DTE$\in [1,90]$ **AND** Implied Volatility$\in [1.792, 3.7777]$ **THEN** Buy option.

Also in this case, the highest value of the considered indicator (ROI) was obtained using the rule involving the $\mathcal{V}$ parameter (R11). However, contrary to P and $AvgP$, the R11 rule referred to two-month options. Only in this case did the set of rules cover the rules with the shortest DTE (one-month options). In addition, ROI is the only indicator for which none of the rules covered the maximum considered position (three months). Figure 3 illustrates the values of the analyzed parameters achieved using the rules R11–R15.

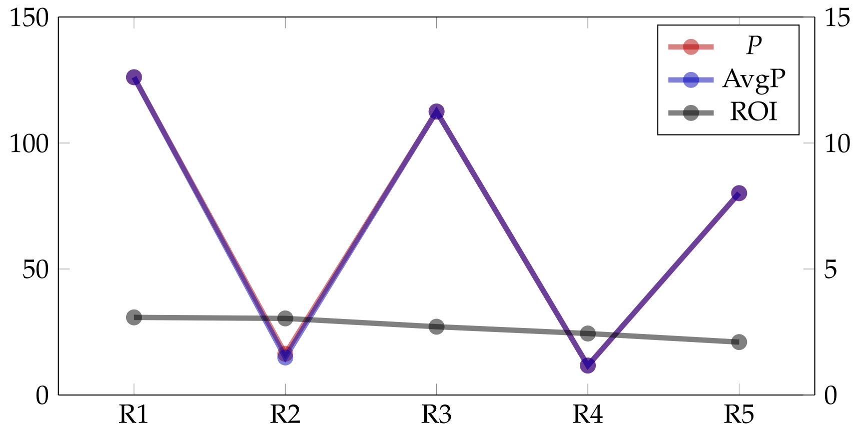

Figure 3. Values of P (left axis) and $AvgP$ and ROI (right axis) indicators obtained using 5 best-rated rules determined to maximize ROI indicator.

Figure 3 shows that the correlation of ROI with the remaining indicator is disturbed. However, it is worth noting that the values of P and $AvgP$ are strongly positively correlated. The rules for ROI can be divided into two groups:

I—with the rules R11, R13 and R15;
II — with the rules R12 and R14.

The rules from the first group give relatively high values of P and $AvgP$, for which values decrease along with the decrease of the ROI. In the case of the rules from the second group, a very clear decrease in P and $AvgP$ indicators can be observed, with a relatively high value of ROI. Based on this, it can be concluded that rules from the first and second group represent different buying strategies for achieving high level of return from investment. The first strategy assumes the purchase of even expensive options (in terms of option premium value), which, however, will generate potentially high profits. The second strategy is to buy relatively cheap options that generate small returns, but which are relatively high in relation to the investments made. Only for the rules of the second strategy were one-month options selected.

6. Discussion and Conclusions

The problem of high volatility of oil price is extremely important in times of uncertainty resulting, among others, from the COVID-19 pandemic or the war in Ukraine. In this paper, we analyzed the possibility of using decision rules to support price risk management using the example of WTI crude oil. We proposed an original basket-analysis-based method for discovering the decision rules. The proposed approach is distinguished by the fact that it does not discretize the domains of the parameter values and it does not omit any important information during the rules discovering process (which can happen when using discretization). The goal of the rules discovering process was to determine the predecessors of the rules where the decision (antecedent) was to "buy option". The predecessors of the rules were determined using the following parameters: the Greeks (delta, gamma, vega, and theta), implied volatility, and duration of the option (number of days to expiration). The conducted analysis shows that the abovementioned parameters enable us to determine the decision rules that can successfully support WTI crude oil price risk management. We analyzed 15 best-rated decision rules, five rules for each of the three investment efficiency indicators: payoff, average payoff, and return of investment. It turned out that the vega ($\mathcal{V}$) parameter is the best to use to construct the predecessors of the decision rules as it enabled us to obtain the highest values of all the indicators. Moreover, for most of the rules, there was a correlation between the values of the analyzed indicators. In addition, the results for the ROI indicator enabled us to observe two decision strategies to obtain high value of this indicator. The first decision strategy is to buy expensive options that generate high profits. The second decision strategy focuses on less profitable options which, however, are not that expensive. The main limitation of the proposed method is its high time-complexity. The computational time of the method grows exponentially with the number of observations and the number of explanatory factors.

To summarize, we showed that it is advisable to use data mining methods for the problem of decision support in price risk management. In our future works we plan to conduct further research on the effectiveness of applying business rules to support buying decisions, both in the case of oil and other energy resources.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

In this section, we provide the whole set of rules determined to maximize the P, $AvgP$, and ROI indicators. The rules (see Tables A1–A3) were selected from among the large number of rules according to the following rule: if the ranges of the indicator (included in the premises of the rule) overlapped, the rule with the higher value of the indicator was selected.

Table A1. Values of P (left axis) and $AvgP$ and ROI (right axis) indicators obtained using 3 best-rated rules determined to maximize P indicator.

DTE	Parameter	Parameter Value	P	AvgP	ROI
[1; 30]	Implied Volatility	[0.4737; 0.6831]	259.63	0.58	0.23
[1; 30]	Implied Volatility	[0.3778; 0.441]	134.75	0.29	0.15
[1; 30]	Implied Volatility	[1.0655; 6.6148]	101.10	1.68	0.63
[31; 60]	Implied Volatility	[0.4688; 0.694]	526.28	1.21	0.25
[31; 60]	Implied Volatility	[0.3342; 0.376]	314.19	0.74	0.23
[31; 60]	Implied Volatility	[0.9674; 2.1715]	113.98	2.85	0.66
[61; 90]	Implied Volatility	[0.5197; 0.6967]	530.74	2.67	0.42
[61; 90]	Implied Volatility	[0.3361; 0.381]	493.86	1.02	0.24
[61; 90]	Implied Volatility	[0.2159; 0.2391]	246.28	1.60	0.48
[1; 90]	Implied Volatility	[0.5199; 0.6301]	930.30	1.86	0.44
[1; 90]	Implied Volatility	[0.3452; 0.3609]	483.51	0.97	0.30
[1; 90]	Implied Volatility	[1.0008; 6.6148]	299.63	2.46	0.71
[1; 30]	Δ	[0.5346; 0.7569]	183.03	0.45	0.23
[1; 30]	Δ	[0.5281; 0.5317]	182.24	0.73	0.32
[1; 30]	Δ	[0.5124; 0.5146]	91.24	0.58	0.28
[31; 60]	Δ	[0.5366; 0.5494]	366.47	1.07	0.26
[31; 60]	Δ	[0.5281; 0.5302]	160.51	0.74	0.20
[31; 60]	Δ	[0.5747; 0.6592]	102.62	3.95	0.87
[61; 90]	Δ	[0.5446; 0.5621]	491.29	1.53	0.27
[61; 90]	Δ	[0.5357; 0.5416]	206.84	0.43	0.09
[61; 90]	Δ	[0.5779; 0.6199]	165.74	7.21	1.38
[1; 90]	Δ	[0.5446; 0.5566]	379.14	0.77	0.16
[1; 90]	Δ	[0.5746; 0.7569]	323.14	3.99	0.95
[1; 90]	Δ	[0.5379; 0.541]	320.02	0.70	0.16
[1; 30]	Γ	[0.0648; 0.0791]	245.65	0.53	0.24
[1; 30]	Γ	[0.1399; 0.2093]	122.32	0.25	0.26
[1; 30]	Γ	[0.0459; 0.0597]	105.66	0.28	0.09
[31; 60]	Γ	[0.0401; 0.0457]	331.53	0.85	0.23
[31; 60]	Γ	[0.0472; 0.0554]	215.83	0.49	0.16
[31; 60]	Γ	[0.0638; 0.0663]	81.07	0.51	0.21
[61; 90]	Γ	[0.0367; 0.0436]	454.57	0.95	0.24
[61; 90]	Γ	[0.0296; 0.0338]	375.81	0.87	0.18
[61; 90]	Γ	[0.0461; 0.0489]	251.05	0.82	0.25
[1; 90]	Γ	[0.0306; 0.0328]	286.87	0.78	0.16
[1; 90]	Γ	[0.0464; 0.0489]	284.90	0.59	0.18
[1; 90]	Γ	[0.0405; 0.0436]	282.72	0.57	0.15
[1; 30]	$\mathcal{V}$	[0.4139; 2.6585]	265.62	0.55	0.54
[1; 30]	$\mathcal{V}$	[3.1792; 4.1731]	191.66	0.39	0.25
[1; 30]	$\mathcal{V}$	[4.2237; 5.1401]	160.95	0.32	0.19
[31; 60]	$\mathcal{V}$	[2.4251; 6.3798]	524.69	1.08	0.40
[31; 60]	$\mathcal{V}$	[7.0044; 7.9097]	354.92	0.91	0.33
[31; 60]	$\mathcal{V}$	[9.7401; 11.1899]	193.74	0.49	0.14
[61; 90]	$\mathcal{V}$	[3.7138; 8.3791]	966.07	1.95	0.56
[61; 90]	$\mathcal{V}$	[9.5558; 10.2287]	256.58	1.06	0.31
[61; 90]	$\mathcal{V}$	[13.4029; 15.3858]	200.18	0.49	0.09
[1; 90]	$\mathcal{V}$	[4.6466; 5.3279]	462.91	0.93	0.44
[1; 90]	$\mathcal{V}$	[6.4968; 6.886]	374.27	0.92	0.36
[1; 90]	$\mathcal{V}$	[2.4251; 3.502]	361.77	0.76	0.53
[1; 30]	Θ	[−0.051; −0.0418]	306.04	0.62	0.33
[1; 30]	Θ	[−0.0657; −0.055]	114.37	0.24	0.12
[1; 30]	Θ	[−0.0952; −0.0877]	64.67	0.50	0.22

Table A1. *Cont.*

DTE	Parameter	Parameter Value	*P*	AvgP	ROI
[31; 60]	Θ	[−0.0372; −0.0319]	465.24	0.97	0.31
[31; 60]	Θ	[−0.0445; −0.038]	259.58	0.55	0.15
[31; 60]	Θ	[−0.0199; −0.0172]	98.37	0.87	0.45
[61; 90]	Θ	[−0.0306; −0.0255]	612.49	1.23	0.29
[61; 90]	Θ	[−0.0442; −0.0405]	284.86	1.94	0.31
[61; 90]	Θ	[−0.016; −0.0117]	182.78	1.65	0.68
[1; 90]	Θ	[−0.0442; −0.0411]	395.96	0.83	0.22
[1; 90]	Θ	[−0.0272; −0.0252]	394.09	0.84	0.28
[1; 90]	Θ	[−0.0306; −0.0289]	222.74	0.48	0.15

Table A2. Wartości wskaźników P, AvgP i ROI dla najlepszych reguł uzyskanych ze względu na maksymalizację wskaźnika AvgP.

DTE	Parameter	Parameter Value	*P*	AvgP	ROI
[1; 30]	Implied Volatility	[1.792; 6.6148]	79.88	7.26	1.76
[1; 30]	Implied Volatility	[0.4393; 0.441]	34.62	3.46	1.68
[1; 30]	Implied Volatility	[0.3869; 0.3878]	34.84	2.90	1.31
[31; 60]	Implied Volatility	[1.2468; 2.1715]	74.58	6.78	1.37
[31; 60]	Implied Volatility	[0.5735; 0.5792]	64.32	6.43	1.48
[31; 60]	Implied Volatility	[0.5905; 0.5986]	64.10	6.41	1.69
[61; 90]	Implied Volatility	[0.9679; 1.2842]	118.71	9.89	1.92
[61; 90]	Implied Volatility	[0.6207; 0.6355]	73.58	7.36	0.95
[61; 90]	Implied Volatility	[0.602; 0.6129]	86.68	7.22	1.04
[1; 90]	Implied Volatility	[1.792; 3.7777]	80.08	8.01	2.10
[1; 90]	Implied Volatility	[1.2097; 1.2491]	71.70	7.17	1.89
[1; 90]	Implied Volatility	[0.5641; 0.5657]	61.98	6.20	1.96
[1; 30]	Δ	[0.5944; 0.7569]	61.10	5.55	1.79
[1; 30]	Δ	[0.5292; 0.5293]	41.93	4.19	2.07
[1; 30]	Δ	[0.5438; 0.5447]	27.07	2.71	1.43
[31; 60]	Δ	[0.5889; 0.6592]	91.43	8.31	1.71
[31; 60]	Δ	[0.5423; 0.5427]	58.99	5.90	1.43
[31; 60]	Δ	[0.53; 0.5302]	55.69	5.57	1.31
[61; 90]	Δ	[0.5876; 0.6133]	101.91	10.19	2.06
[61; 90]	Δ	[0.5519; 0.5526]	92.04	8.37	1.62
[61; 90]	Δ	[0.5409; 0.541]	91.47	8.32	1.43
[1; 90]	Δ	[0.6121; 0.6592]	125.99	11.45	2.64
[1; 90]	Δ	[0.5873; 0.5937]	81.75	8.18	2.08
[1; 90]	Δ	[0.541; 0.541]	73.33	7.33	1.54
[1; 30]	Γ	[0.0716; 0.0718]	37.73	3.77	1.71
[1; 30]	Γ	[0.0693; 0.0696]	40.09	3.34	1.45
[1; 30]	Γ	[0.0481; 0.0483]	33.29	3.33	1.01
[31; 60]	Γ	[0.0324; 0.0326]	48.90	4.89	1.02
[31; 60]	Γ	[0.0441; 0.0441]	47.33	4.73	1.31
[31; 60]	Γ	[0.0305; 0.0308]	44.33	4.43	0.86
[61; 90]	Γ	[0.0245; 0.0247]	93.56	9.36	1.47
[61; 90]	Γ	[0.0321; 0.0322]	88.68	8.87	1.81
[61; 90]	Γ	[0.0312; 0.0313]	81.89	8.19	1.64
[1; 90]	Γ	[0.0321; 0.0321]	78.67	7.87	1.60
[1; 90]	Γ	[0.0313; 0.0313]	68.54	6.85	1.36
[1; 90]	Γ	[0.0381; 0.0382]	59.86	5.99	1.46
[1; 30]	$\mathcal{V}$	[13.4901; 13.5011]	83.82	8.38	1.86
[1; 30]	$\mathcal{V}$	[13.4079; 13.422]	57.54	5.75	1.31
[1; 30]	$\mathcal{V}$	[13.742; 13.7611]	56.37	5.64	1.21
[31; 60]	$\mathcal{V}$	[9.7062; 9.7704]	45.13	3.22	1.14
[31; 60]	$\mathcal{V}$	[0.4139; 0.7881]	28.12	2.81	2.11
[31; 60]	$\mathcal{V}$	[7.2653; 7.3058]	30.38	2.76	1.08

Table A2. *Cont.*

DTE	Parameter	Parameter Value	P	AvgP	ROI
[61; 90]	$\mathcal{V}$	[2.4251; 3.131]	126.09	12.61	3.08
[61; 90]	$\mathcal{V}$	[7.462; 7.4785]	52.79	5.28	1.83
[61; 90]	$\mathcal{V}$	[7.8068; 7.8368]	51.14	5.11	1.80
[1; 90]	$\mathcal{V}$	[3.7138; 4.4474]	129.46	12.95	3.02
[1; 90]	$\mathcal{V}$	[13.4806; 13.5011]	96.75	9.68	2.00
[1; 90]	$\mathcal{V}$	[4.4858; 4.8867]	92.79	7.73	1.94
[1; 30]	Θ	[−0.0481; −0.048]	30.42	3.04	1.67
[1; 30]	Θ	[−0.0458; −0.0455]	29.15	2.92	1.62
[1; 30]	Θ	[−0.0464; −0.0462]	28.51	2.85	1.39
[31; 60]	Θ	[−0.0367; −0.0366]	52.77	5.28	1.56
[31; 60]	Θ	[−0.0556; −0.0553]	47.19	4.72	1.00
[31; 60]	Θ	[−0.0462; −0.046]	45.44	4.54	1.21
[61; 90]	Θ	[−0.029; −0.0289]	70.17	7.02	1.59
[61; 90]	Θ	[−0.0367; −0.0367]	69.16	6.92	1.23
[61; 90]	Θ	[−0.0423; −0.0419]	70.67	6.42	1.00
[1; 90]	Θ	[−0.0367; −0.0367]	85.98	6.61	1.44
[1; 90]	Θ	[−0.0338; −0.0337]	68.70	6.25	1.25
[1; 90]	Θ	[−0.039; −0.039]	60.89	6.09	1.64

Table A3. Wartości wskaźników P, AvgP i ROI dla najlepszych reguł uzyskanych ze względu na maksymalizację wskaźnika ROI.

DTE	Parameter	Parameter Value	P	AvgP	ROI
[1; 30]	Implied Volatility	[1.783; 3.7777]	71.9	7.19	2.08
[1; 30]	Implied Volatility	[0.4393; 0.441]	34.62	3.46	1.68
[1; 30]	Implied Volatility	[0.3593; 0.3602]	22.16	2.22	1.34
[31; 60]	Implied Volatility	[0.5861; 0.5986]	76.79	6.4	1.73
[31; 60]	Implied Volatility	[0.2678; 0.2688]	55.94	4.3	1.54
[31; 60]	Implied Volatility	[0.5735; 0.5792]	69.56	6.32	1.5
[61; 90]	Implied Volatility	[0.9679; 1.2281]	98.6	9.86	1.96
[61; 90]	Implied Volatility	[0.2222; 0.2258]	64.96	5.41	1.82
[61; 90]	Implied Volatility	[0.3532; 0.3544]	63.93	6.39	1.59
[1; 90]	Implied Volatility	[1.792; 3.7777]	80.08	8.01	2.1
[1; 90]	Implied Volatility	[0.5641; 0.5657]	61.98	6.2	1.96
[1; 90]	Implied Volatility	[0.4872; 0.488]	59.77	5.98	1.95
[1; 30]	Δ	[0.5292; 0.5293]	41.93	4.19	2.07
[1; 30]	Δ	[0.5927; 0.6392]	60.34	5.03	1.94
[1; 30]	Δ	[0.4588; 0.4615]	10.05	1.01	1.76
[31; 60]	Δ	[0.5138; 0.514]	50.46	5.05	1.77
[31; 60]	Δ	[0.5889; 0.6592]	91.43	8.31	1.71
[31; 60]	Δ	[0.5423; 0.5427]	58.99	5.9	1.43
[61; 90]	Δ	[0.5853; 0.6059]	99.37	9.94	2.07
[61; 90]	Δ	[0.5519; 0.5526]	92.04	8.37	1.62
[61; 90]	Δ	[0.5379; 0.538]	61.32	6.13	1.61
[1; 90]	Δ	[0.6121; 0.6392]	112.49	11.25	2.71
[1; 90]	Δ	[0.5889; 0.5972]	80.91	8.09	2.31
[1; 90]	Δ	[0.4588; 0.4615]	10.05	1.01	1.76
[1; 30]	Γ	[0.306; 0.3211]	16.36	1.49	3.04
[1; 30]	Γ	[0.0716; 0.0718]	37.73	3.77	1.71
[1; 30]	Γ	[0.2246; 0.2285]	11.5	1.15	1.65
[31; 60]	Γ	[0.0576; 0.0577]	40.73	4.07	1.47
[31; 60]	Γ	[0.0805; 0.0809]	25.81	2.58	1.34
[31; 60]	Γ	[0.0441; 0.0441]	47.33	4.73	1.31
[61; 90]	Γ	[0.0321; 0.0322]	88.68	8.87	1.81
[61; 90]	Γ	[0.0673; 0.0677]	39.11	3.91	1.66
[61; 90]	Γ	[0.0312; 0.0313]	81.89	8.19	1.64

Table A3. *Cont.*

DTE	Parameter	Parameter Value	*P*	AvgP	ROI
[1; 90]	Γ	[0.306; 0.3211]	16.36	1.49	3.04
[1; 90]	Γ	[0.0717; 0.0718]	37.19	3.72	1.67
[1; 90]	Γ	[0.0658; 0.0659]	40.11	4.01	1.67
[1; 30]	$\mathcal{V}$	[0.4139; 0.7881]	28.12	2.81	2.11
[1; 30]	$\mathcal{V}$	[1.315; 1.3583]	16.45	1.65	2.11
[1; 30]	$\mathcal{V}$	[0.9284; 1.0249]	29.46	1.84	1.94
[31; 60]	$\mathcal{V}$	[2.4251; 3.131]	126.09	12.61	3.08
[31; 60]	$\mathcal{V}$	[7.462; 7.4785]	52.79	5.28	1.83
[31; 60]	$\mathcal{V}$	[7.8068; 7.8368]	51.14	5.11	1.8
[61; 90]	$\mathcal{V}$	[3.9195; 4.5307]	121.77	12.18	3.04
[61; 90]	$\mathcal{V}$	[13.4806; 13.5011]	96.75	9.68	2
[61; 90]	$\mathcal{V}$	[6.7133; 6.7597]	64.98	6.5	1.99
[1; 90]	$\mathcal{V}$	[0.4139; 0.7881]	28.12	2.81	2.11
[1; 90]	$\mathcal{V}$	[1.315; 1.3583]	16.45	1.65	2.11
[1; 90]	$\mathcal{V}$	[3.1866; 3.2061]	32.51	3.25	2.06
[1; 30]	Θ	[−0.4815; −0.4531]	11.69	1.17	2.44
[1; 30]	Θ	[−0.0241; −0.0182]	29.25	2.66	2.3
[1; 30]	Θ	[−0.0481; −0.048]	30.42	3.04	1.67
[31; 60]	Θ	[−0.0323; −0.0322]	41.94	4.19	1.58
[31; 60]	Θ	[−0.0367; −0.0366]	52.77	5.28	1.56
[31; 60]	Θ	[−0.0185; −0.0182]	24.62	2.46	1.38
[61; 90]	Θ	[−0.0151; −0.0149]	53.61	4.87	2.01
[61; 90]	Θ	[−0.029; −0.0289]	70.17	7.02	1.59
[61; 90]	Θ	[−0.0148; −0.0146]	38.44	3.84	1.54
[1; 90]	Θ	[−0.4815; −0.4531]	11.69	1.17	2.44
[1; 90]	Θ	[−0.0151; −0.0149]	53.61	4.87	2.01
[1; 90]	Θ	[−0.039; −0.039]	60.89	6.09	1.64

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