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A New Design for the Peer-to-Peer Electricity and Gas Markets Based on Robust Probabilistic Programming

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Abstract: This paper presents a fully-decentralized peer-to-peer (P2P) electricity and gas market for retailers and prosumers with coupled energy units, considering the uncertainties of wholesale electricity market price and prosumers' demand. The goal is to improve the overall economy of the proposed market while increasing its flexibility. In this market, the retailers are equipped with self-generation and energy storage units and can bilaterally negotiate for electricity and gas transactions with prosumers to maximize their profit. Furthermore, they can sell power to the upstream market in addition to prosumers. The prosumers have access to several retailers to supply their required electricity and gas and can freely provide their energy needs from every retailer, contributing to dynamicity in the proposed market. Given that they have an energy hub consisting of boiler units, combined heat and electricity (CHP) units, and electric pumps, they can switch their energy supply source from electricity to gas and vice versa. A robust possibilistic programming approach is applied to address the uncertainties. A fully-decentralized approach called the alternating direction method of multipliers (ADMM) is utilized to solve the presented decentralized robust problem. The proposed decentralized algorithm finds an optimum solution by establishing a smart balance between the average expected value, optimality robustness, and feasibility robustness. The feasibility and competitiveness of the proposed approach are evaluated through numerical studies on a distribution system with two retailers and three prosumers. The data analysis of the simulation results verifies the effectiveness of the proposed decentralized robust framework as well as the proposed decentralized solution. According to the maximum deviation, the expected optimal value in the robust case, the retailer's profit has decreased by 12.1 percent, and the prosumers' cost has increased by 27.4 percent due to the feasibility penalty term.

Keywords: peer-to-peer electricity and gas transactions; energy hub; decentralized robust optimization; fuzzy robust optimization; alternating direction method of multipliers

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1. Introduction

The energy distribution systems have increasingly changed in recent years with the rise of distributed energy resources and are evolving from centralized toward decentralized management to enable the participation of prosumers in the electricity markets. The peer-to-peer (P2P) energy transaction allows the prosumers to exchange energy needless of an intermediary entity. They can trade their surplus energy generation with other prosumers or even retailers. Such a system will create substantial economic, social, and environmental opportunities. From an economic perspective, it reduces operation and maintenance costs, peak demand, and energy losses. From an environmental view, it offers a greater opportunity to consume clean energy through renewable generation. Finally, from a social perspective, it can affect the lifestyle and cultural practices regarding electricity supply and demand and encourage the cooperation of community members to gain more profit [1,2]. The high penetration of renewable energy sources in these systems may introduce substantial uncertainties in their generation. In this regard, using multi-energy systems can help mitigate this problem. Equipped with various energy generation equipment such as CHP units, the prosumers can switch their energy source from electricity to gas and vice versa, which enhances the reliability and welfare of the whole system. Via an integrated demand response program, during peak electricity price and load periods, they purchase gas from the network instead of electricity to meet their needs. Thus, another challenge of these systems is demand uncertainty which causes an imbalance in supply and demand and signifies the appropriate tackling of these uncertainties [3].

1.1. Related Works

Numerous previous studies have concentrated on P2P energy trading and short-term scheduling of retail energy management. This study reviews the literature from three aspects: presence or absence of uncertainty, electricity trading, and electricity and gas trading.

Ref. [4] has presented a P2P market mechanism for energy and reserve markets to compensate for the uncertainties resulting from renewable generation. All agents can determine the price and amount of traded energy and reserve through negotiations. Versatile distribution is proposed to model the renewable generation uncertainty, and chance-constrained optimization is used to specify the reserve quantity. The proposed P2P market is then cleared by utilizing the ADMM theory. Ref. [5] has developed a P2P energy transaction mechanism to maximize the players' social welfare considering the uncertainty and fairness in players' profits. Based on the uncertainty characteristics, the P2P energy transaction performance is analyzed to find the optimality condition for social welfare maximization. The results suggest that social welfare is maximized when the producer and consumer pairs with similar demand characteristics are matched. Moreover, the results of centralized and decentralized methods are compared. Authors in [6] described a decentralized distributed (DD) adaptive robust optimization (ARO) approach for distributed scheduling of multi-microgrid systems. The modeling includes various uncertainties and accidental communication failures. DD-ARO optimization based on the parallelizing-distributed (PD) framework is applied to avoid the unavailable coordination center of the whole network. Tie-line power coordination in the virtual center is considered for neighboring stakeholders, and a column-and-constraint generation (CCG) algorithm is employed to solve the problem. Ref. [7] designed a novel decentralized periodic energy trading framework for pelagic islanded microgrids. The model has considered the limitations of energy resources, two types of consumers (normal and strategic), and the uncertainty of renewable energy sources. A robust optimization approach is used to address the corresponding uncertainties. The decentralized optimization problem derived from the game theory approach is solved using a decentralized bi-level iterative algorithm. The smart electricity exchange platform (STEP) is introduced in [8], which serves as an interface between the wholesale electricity markets and prosumers. The mentioned framework enables P2P energy trading among prosumers in a local market. The objective is to minimize the costs of all market players considering the uncertainties of prices and renewable generation. Furthermore, the contribution of battery storage systems to demand-side flexibility is investigated. Chen et al. studied P2P energy trading and energy conversion among multiple interconnected industrial, residential, and commercial microgrids [9]. Due to uncertainty and high-dimensional data in this problem, a multi-agent deep reinforcement learning approach is applied in conjunction with a multi-agent actor-critic algorithm. Additionally, the effect of carbon tax pricing is also taken into account. A P2P energy trading framework coordinates the demand response between the residential houses. Authors in [10] have modeled intraday and day-ahead energy management, considering the characteristics of energy storage systems and household appliances with demand response capability. A double auction mechanism is used for collaborative demand response schemes to counteract the disturbances. Ref. [11] developed a P2P energy trading system to increase resilience and self-sufficiency in a target community consisting of houses with micro fuel cells and CHP units. Each player can act as a buyer or seller at each time step. The problem objective is to minimize the costs from gas consumption and P2P electricity trading of houses and is solved using the ADMM algorithm. The risk-aversive P2P energy trading of small-scale producers and consumers is modeled in [12] based on the virtual power plants. Uncertainties in renewable energy sources are compensated by the diverse energy resources in virtual power plants. The physical electricity trading approach reduces the line congestion risk and promotes local electricity consumption. A two-stage stochastic game model is developed for the day-ahead energy bidding strategies where the Cournot Nash pricing mechanism is applied to balance the supply and demand. Ref. [13] has designed a sustainable microgrid using blockchain technology for P2P energy trading in the microgrid. By not requiring a central regulatory system, blockchain technology guarantees the sustainability and security of the microgrid participants. Moreover, the environmental and economic objectives along with the constraints related to maximizing the demand satisfaction of consumers, are considered. A fuzzy multi-objective programming model is utilized to capture the variations in the capacity of renewable distributed generation and consumers' demand. Uncertainties in the distributed energy sources impose a challenge for P2P energy trading incorporating these sources. A novel pricing strategy (uncertainty marginal price, UMP) is introduced in [14] to overcome the existing uncertainties. The final problem, formulated as a robust model, is decomposed into two sub-problems and solved using a column-and-constraint generation algorithm. Ref. [15] studied the energy hub concept in networked microgrids. Each microgrid has its specific objective function, including electricity, cooling, and heating demands. Moreover, each microgrid must coordinate its operation with other microgrids and the distribution network. The distributed model is solved using a robust ADMM scheme to ensure the privacy and independence of the entities. A scheme for price generation in a decentralized P2P energy transaction system integrated into an ADMM algorithm is developed in [16], aiming to improve the economic performance of the interconnected energy hub system. The configuration, inner operation strategy, and trading policy of each hub directly affect the transactive prices of each energy hub and result in optimal energy flow among hubs to achieve the global optimum. Ref. [17] presented a novel energy trading framework for a system with high PV penetration in distribution networks. In [18], a stochastic agent-based model is introduced for the coordinated scheduling of multi-vector microgrids considering interactions between electricity, hydrogen, and gas agents. A stochastic p-robust optimization algorithm is extended to tackle the uncertainties and obtain an optimal solution for the proposed management of multi-energy microgrids (MENG) [19]. In [20], three different modeling methods to construct the two-stage mean-risk stochastic minimum cost consensus models (MCCMs) with asymmetric adjustment cost are investigated. Ref. [21] proposed a supply chain consisting of a manufacturer with possible misreport behavior and one retailer with possible fairness concerns. A bilateral energy trading mechanism with an optimal power flow (OPF) technique is integrated to increase the economic benefits of individual participants. The problem is modeled by Nash bargaining theory and solved using the ADMM algorithm to guarantee privacy protection. In [22] have proposed an approach due to the co-simulation of distribution networks and P2P electricity trading. In addition, the technical impacts of P2P energy trading on LV network voltage, phase imbalance, and losses have been considered. In [23], a cooperative game theory framework has been proposed to encourage individual prosumers, which has presented different priorities at each time slot, such as geographic location, maximum energy demand, and pricing mechanism. Ref. [24] designed an electricity, gas, and heat market between retailers and prosumers using game theory. The retailer's objective is to maximize the profit from energy selling while the prosumers aim to minimize energy costs. The equilibrium point is achieved by simultaneously solving all MPECs. Ref. [25] developed a decentralized short-term multi-period and multi-stage parallel auction market, which is a pool-structured

P2P market to maximize the social welfare of prosumers. The market is cleared using a decentralized ant-colony optimization (DACO) method. The smart contract functionality is utilized to manage the balance of digital tokens called EuroTokens.

Authors in [26] presented the P2P energy transaction for multiple microgrids (MMGs) and studied the grid-oriented energy bidding problem considering uncertainties of renewable generation, demands, and market agents. The problem is modeled using a stochastic Cartel game-based strategy to minimize the total costs of MMGs under uncertainty. The problem is linearized via diagonal quadratic approximation and decomposed into subproblems for individual microgrids. The problem is solved in an iterative and distributed manner. An energy market is presented in [27] based on the concept of P2P negotiations to facilitate energy transactions between agents at the distribution level. Interdependencies of different energy carriers are studied. The linear optimization model is employed to find the optimal strategies of the agents. Furthermore, the effects of carbon emissions from CHP and boiler units on the optimal strategies of the market participants are discussed.

According to the reviewed studies, the following study gap is evident:

1. **A fully-decentralized P2P market model and a fully-decentralized P2P market-clearing approach:** In a group of previous studies, the players do not negotiate bilaterally in a P2P manner. Furthermore, some studies have used centralized methods or methods that do not have decentralized characteristics to clear the market.
2. **A dynamic and flexible model:** Many past studies lack a dynamic, flexible model and use conventional demand response programs, including load curtailment and shift. Nevertheless, the presence of energy coupling equipment and storage units and the use of various energy sources have contributed to the high flexibility of this model. Likewise, the participation of several prosumers and retailers ensures the dynamicity of the model.
3. **Considering the uncertainties of the wholesale electricity market price and prosumers' demand:** One key weakness of the previous studies is their failure to address the existing uncertainties. Considering the uncertainties has a huge implication for realistic modeling and can cause the players to have more robustness in the face of the uncertainties.

1.2. The Contribution of This Paper

This paper designs and models a novel market for fully decentralized P2P electricity and gas trading, considering the uncertainties of wholesale electricity market price and consumers' demand. Retailers aim to maximize their profit from selling electricity and gas to prosumers. In addition to purchasing electricity from the network, they are equipped with self-generation and storage units to increase their profit. Using these units allows them to exploit the price difference of the wholesale market to sell electricity to the network and gain more profit. The retailers also consider the uncertainty of wholesale market prices in their scheduling. More importantly, the presence of multiple retailers has led to the dynamicity of the model and higher competition for cost reduction. Prosumers negotiate individually over the price and quantity of their tradable electricity and gas to minimize their costs. They are equipped with CHP, boiler, and heat pump units and can benefit from the price difference between electricity and gas to switch the energy supply source to cut down their costs. Given the fully decentralized nature of the model and the use of the ADMM scheme to clear the market, the privacy and independence of players are guaranteed as the prices and amounts of energy purchases are the only information exchanged. Therefore, it prevents the disclosure of operational and commercial information. The robust possibilistic programming method is applied to tackle the uncertainties of wholesale electricity market price and prosumers' electrical demand. In summary, the contributions of this paper are outlined as follows:

1. A dynamic and flexible market for electricity and gas transactions among energy hub prosumers and retailers is modeled. The retailers are equipped with electrical storage

and self-generation units and can sell power to the upstream network in addition to prosumers.
2. The uncertainties of wholesale electricity market price and prosumers' electrical demand are considered using the robust possibilistic programming (RPP) approach.
3. A fully decentralized ADMM is utilized to clear the proposed electricity and gas market, guaranteeing the global solution for all players without needing a supervisory node. The obtained results are compared to those of a centralized approach.

1.3. Paper Organization

The present paper is organized as follows: Section 2: The concept and mathematical formulation of the P2P energy trading platform is described in this section. Section 3: This section provides the numerical studies of the proposed market and analyzes the simulation results in terms of two case studies. Section 4: This section draws conclusions.

2. Problem Definition

A decentralized P2P electricity and gas market between retailers with electrical storage and self-generation units and energy hub prosumers is modeled to maximize the retailers' profits while minimizing the prosumers' costs. The overall structure of the energy hub is shown in Figure 1. The retailers supply their required electricity and gas from the wholesale electricity and gas market. In addition to profit from energy selling to prosumers, the retailers gain extra profit by applying electrical storage and self-generation units, which allow them to leverage the price difference in the wholesale electricity market at different hours. When the electricity price is low, they purchase electricity to supply prosumers and charge their batteries. Conversely, during peak price periods, they sell the power they generate by self-generation units or discharge from the batteries to prosumers and the upstream network. The prosumers are also equipped with CHP, boiler, and heat pump units and leverage the price difference between electricity and gas to minimize their costs by switching their energy source supply. At off-peak hours, they supply their load by purchasing energy from retailers. Under these considerations, the presented model has dynamicity and flexibility to maximize the retailers' profits while minimizing the prosumers' costs. From a time aspect, a day-ahead market structure is considered. The platform structure of the proposed problem is as follows:

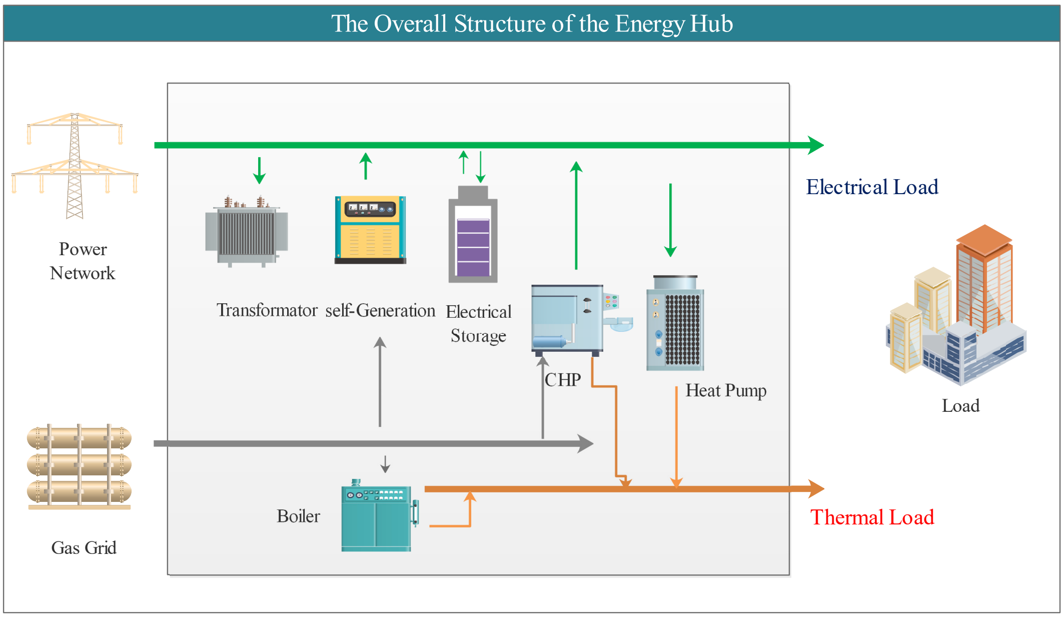

Figure 1. The overall structure of energy hubs containing both retailer and prosumer.

The retailers receive the demand signals of the prosumers. The retailers then determine and announce their selling price and quantity bids to prosumers. By receiving these values, the prosumers specify the generation level of their units and announce the amount of electrical energy and gas to be purchased from each retailer. The retailers then re-schedule their self-generation units and battery charge/discharge and update the amount of energy exchange with the upstream network. This cycle continues until the stopping condition for the proposed decentralized algorithm (reaching the convergence values) is met.

As stated, the prices and amounts of energy purchases are the only information exchanged. Thus, privacy is fully preserved, and operational and commercial information is not disclosed. Consequently, the proposed market is a fully decentralized electricity and gas market for P2P interactions between retailers and prosumers.

3. Mathematical Modeling

3.1. Assumptions

1. The proposed market is assumed to comprise of $\mathcal{N}_r=\{1,\ldots,N_r\}$ retailers and $\mathcal{N}_P=\{1,\ldots,N_P\}$ prosumers.
2. The prosumers cannot sell electricity to retailers and only have the "buyer" role. In other words, they are only price-takers and supply their required electricity and gas from the retailers.
3. The retailers can concurrently play the "buyer" and "seller" roles and purchase electricity and gas from the wholesale market to sell it to the prosumers. They can act as producers and trade energy with the upstream network and prosumers.
4. There is a bilateral P2P relationship between retailers and prosumers. All market players behave in a free and independent manner and manage their energy sources and loads to satisfy their objectives.

The main objective function of the proposed market is to maximize the retailers' profits and minimize the prosumers' costs:

$$\max\left(\sum_{i=1}^{\mathcal{N}_r} R_i - \sum_{j=1}^{\mathcal{N}_p} C_j\right) \tag{1}$$

where C_j and R_i are the gas and electricity cost of prosumer j and revenue of retailer i, respectively.

3.2. The Deterministic Model of the Retailer

Relations (2)–(10) formulate the profit maximization problem of each retailer. The first two terms of the objective function in relation (2) indicate the profits from selling electrical energy and gas to prosumers, respectively. The self-generation cost is given in the third term. The last two terms describe the purchase costs of electrical energy and gas from the wholesale market.

$$\max\ R_i = \sum_t \left\{ \sum_j \left(\gamma^e_{ijt} p^e_{ijt} + \gamma^g_{ijt} P^g_{ijt}\right) - C_i(p^{sg}_{it}) - \widetilde{\gamma}^{eDA}_t p^{eDA}_{it} - \gamma^{gDA} p^{gDA}_i \right\} \tag{2}$$

s.t.

$$\sum_j p^e_{ijt} + p^{BSS,\ dch}_{it} - p^{eDA}_{it} - p^{sg}_{it} - p^{BSS,ch}_{it} = 0 \tag{3}$$

$$p^{gDA}_{it} - \sum_j p^g_{ijt} = 0 \tag{4}$$

$$C\left(p^{sg}_{it}\right) = \Phi_i (p^{sg}_{it})^2 + \pi_i p^{sg}_{it} + \psi_i \tag{5}$$

$$E_{it}^{ESS} = \left(1 - Loss^{ESS}\right)E_{it-1}^{ESS} + \left(\eta^{ESS} \times p_{it}^{ESS,ch}\right) - \left(\frac{p_{it}^{ESS,dch}}{\eta^{ESS}}\right) \tag{6}$$

$$Cap^{ESS,min} \leq E_{it}^{ESS} \leq Cap^{ESS,max} \tag{7}$$

$$0 \leq P_{it}^{ESS,ch} \leq p_{ch}^{ESS,max} \times b_{it}^{ch} \tag{8}$$

$$0 \leq p_{it}^{ESS,dch} \leq p_{dch}^{ESS,max} \times b_{it}^{dch} \tag{9}$$

$$b_{it}^{ch} + b_{it}^{dch} \leq 1 \tag{10}$$

$$E_{i0}^{ESS} = E_{0}^{ESS} \tag{11}$$

$$E_{i24}^{ESS} \leq E_{0}^{ESS} \tag{12}$$

The retailers are equipped with electrical storage systems for optimal energy management. Given that more than one retailer exists in the market, the market is competitive, and the prosumers can purchase their required energy from a retailer with a lower price bid. γ_t^{eDA} and γ_i^{gDA} represent the wholesale market electricity and gas prices, respectively. The electrical power balance is established in relation (3). In addition to the upstream network, the retailers can also use their storage (q_{jt}^{dch}, q_{jt}^{ch}) and self-generation (p_{it}^{sg}) units to fulfill the prosumers' energy demands. According to relation (4), the amount of gas sold to the prosumers equals the amount purchased from the wholesale market. Relation (5) models the self-generation cost function for each retailer $C_i(p_{it}^{sg})$ [28]. The self-generation units are recognized as a generation of electrical energy whose output is electrical power. These units use fossil fuels as their input to generate power. Φ_i, π_i, and ψ_i are positive pre-determined parameters in the retailer's cost function and contain the private information of the retailer i. Relations (6)–(10) demonstrate the constraints on the energy storage systems [29]. The energy level of the energy storage system at each hour is obtained from relation (6), where the stored energy level depends on the charge and discharge levels and the remaining charge from the previous hour. Constraint (7) describes the minimum and maximum energy levels of the storage, and constraints (8) and (9) display the minimum and maximum amount of charge and discharge per hour. Constraint (10) states that every storage at each hour can operate in either charge ($b_{jt}^{ch} = 1$) or discharge ($b_{jt}^{dch} = 1$) modes.

3.3. The Deterministic Model of the Prosumer

Relations (13)–(19) present the minimization problem related to energy consumption costs of prosumers. The objective function in relation (13) minimizes the cost of purchasing electricity and gas from each retailer. Prosumers can purchase energy from the retailer with the lowest price bid. The electrical power balance is satisfied in relation (14). For optimal management of their energy consumption, the prosumers can purchase gas and produce heat and power using their CHP units, in addition to buying energy from retailers. The heat power supply is given in relation (15). Prosumers are equipped with CHP units [30], boilers [31], and heat pumps [32] to satisfy their heat demand. They consume the purchased gas in boiler and CHP units as shown in relation (16). The maximum gas input to the CHP and boiler units is modeled in relations (17) and (18). The relation (19) exhibits the highest electrical power of the heat pump.

$$\max\ C_j = -\left(\sum_i \left(\gamma_{ijt}^{e} q_{jit}^{e} + \gamma_{ijt}^{g} q_{jit}^{g}\right)\right) \tag{13}$$

s.t.

$$\sum_i q^e_{jit} + \eta_e^{chp} q^{gchp}_{jt} - a^h q^{e,hp}_{jt} = \tilde{q}^{De}_{jt} \tag{14}$$

$$\eta_h^{chp} q^{gchp}_{jt} + \eta^b q^{gb}_{jt} + q^{h,hp}_{jt} = \tilde{q}^{Dh}_{jt} \tag{15}$$

$$q^{gchp}_{jt} + q^{gb}_{jt} = \sum_i q^g_{jit} \tag{16}$$

$$q^{gchp}_{jt} \leq q^{gchp\_max}_j \tag{17}$$

$$q^{gb}_{jt} \leq q^{gb\_max}_j \tag{18}$$

$$q^{e,hp}_{jt} \leq q^{e,hp\_Max}_j \tag{19}$$

3.4. Uncertainty Modeling

Given that the presented schedule in this paper is for 24 h ahead, a time difference exists between the schedule and its realization time. This time difference causes uncertainties in some data. One input data of this model is power transactions with the network at the wholesale market price. Since the market the day before is not cleared, the wholesale market price is uncertain. Likewise, there is uncertainty in the power consumption of consumers, which is based on the day ahead. Various methods have been proposed in the literature to overcome these uncertainties. The current paper applies a robust possibilistic programming (RPP) model to tackle the introduced uncertainties [20]. The developed model attempts to achieve a trade-off between average performance, optimality robustness, and feasibility robustness. Thus, the objective function is modeled using the expected value operator and the uncertainty constraint using the necessity measure. As shown in Figure 2, the uncertain parameters are also modeled using trapezoidal possibility distribution, represented by four prominent points$(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$,$(d_1, d_2, d_3, d_4)$. The robust possibilistic programming (RPP) model of retailers and prosumers is described below.

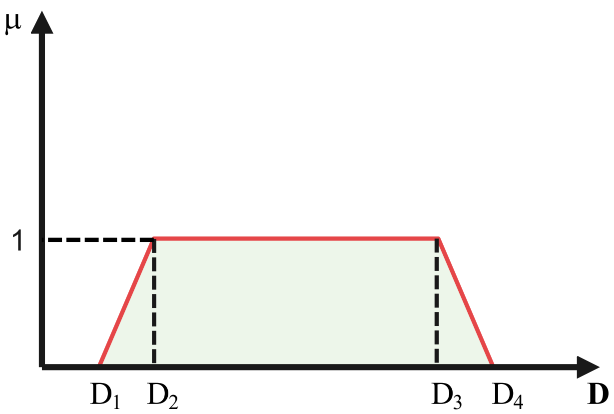

Figure 2. The trapezoidal possibility distribution of fuzzy parameter *D*.

3.5. Fuzzy Robust Model of the Retailer

The relation (20) presents the objective function of each retailer considering the uncertainty of wholesale market price $\widetilde{\gamma}_t^{eDA}$. The first term of this relation is the expected value of the objective function of each retailer and demonstrates the profit maximization of each retailer given the expected overall (average) price. The second term indicates the difference between the average and minimum expected values. Given that a maximum deviation below the expected optimal value is considered, this term maximizes the profit and ensures optimality robustness. ψ shows the significance or the weight of this term. Expected value and minimum expected profit is calculated from relations (21) and (22), respectively.

$$Max\ E[R_i] - \psi\left(E[R_i] - R_i^{min}\right) \tag{20}$$

$$E[R_i] = \sum_t \{ \sum_j \left(\gamma_{ijt}^{e} p_{ijt}^{e} + \gamma_{ijt}^{g} p_{ijt}^{g}\right) - C_i(p_{it}^{sg}) - \gamma^{gDA} p_i^{gDA} - p_{it}^{eDA}\left(\frac{\widetilde{\gamma}_t^{e_1DA} + \widetilde{\gamma}_t^{e_2DA} + \widetilde{\gamma}_t^{e_3DA} + \widetilde{\gamma}_t^{e_4DA}}{4}\right) \} \tag{21}$$

$$R_i^{min} = \sum_t \{ \sum_j \left(\gamma_{ijt}^{e} p_{ijt}^{e} + \gamma_{ijt}^{g} p_{ijt}^{g}\right) - C_i(p_{it}^{sg}) - \gamma^{gDA} p_i^{gDA} - p_{it}^{eDA} \widetilde{\gamma}_t^{e_4DA} \} \tag{22}$$

The electrical demand uncertainty in retailer's problem constraints can be modeled using the robust possibilistic programming:

$$Nec\{ \sum_i q_{jit}^{e} + \eta_e^{chp} q_{jt}^{gchp} - a^h q_{jt}^{e,hp} = \tilde{q}_{jt}^{De} \} \geq \Phi \tag{23}$$

$$\sum_i q_{jit}^{e} + \eta_e^{chp} q_{jt}^{gchp} - a^h q_{jt}^{e,hp} \leq \tilde{q}_{jt}^{D_3e} \Phi/2 + (1 - \Phi/2) \tilde{q}_{jt}^{D_4e} \tag{24}$$

$$\sum_i q_{jit}^{e} + \eta_e^{chp} q_{jt}^{gchp} - a^h q_{jt}^{e,hp} \geq \tilde{q}_{jt}^{D_2e} \Phi/2 + (1 - \Phi/2) \tilde{q}_{jt}^{D_1e} \tag{25}$$

The relation (23) shows the necessary measure of the power balance for which the deterministic model is formulated in relations (24) and (25). Φ relates to the confidence level. Thus, the objective function of each prosumer can be rewritten as below:

$$C_j = -\left(\sum_i \left(\gamma_{ijt}^{e} q_{jit}^{e} + \gamma_{ijt}^{g} q_{jit}^{g}\right)\right) - \delta_1 \left(\sum_{t=1}^{T} \tilde{q}_{jt}^{D_3e} \Phi/2 + (1 - \Phi/2) \tilde{q}_{jt}^{D_4e} - \tilde{q}_{jt}^{D_3e}\right) - \delta_2 \left(\sum_{t=1}^{T} -\tilde{q}_{jt}^{D_2e} \Phi/2 - (1 - \Phi/2) \tilde{q}_{jt}^{D_1e} + \tilde{q}_{jt}^{D_2e}\right) \tag{26}$$

The first term in relation (26) is related to energy purchase costs. The second term determines the confidence level of constraints under uncertainty. δ_1 *a*nd δ_2 are the penalty coefficients related to the possible deviation of each uncertain constraint and can be specified based on the demand deficiency or unsatisfied demand. $\sum_{t=1}^{T} \tilde{q}_{jt}^{D_3e} \Phi/2 + (1 - \Phi/2) \tilde{q}_{jt}^{D_4e} - \tilde{q}_{jt}^{D_3e}$ indicates the difference between the worst case value of the uncertain parameter and the value used in uncertain constraints. Therefore, this term reflects the feasibility robustness. Φ shows the minimum confidence level of uncertain constraints, which is a variable determined by the model.

Thus, the final deterministic centralized model considering the uncertainties in wholesale market price and prosumers' electrical demand is obtained from relations (27)–(29).

$$Max\ E[R_i] - \psi\left(E[R_i] - R_i^{min}\right) - C_j \tag{27}$$

s.t.

$$\sum_i q_{jit}^{e} + \eta_e^{chp} q_{jt}^{gchp} - a^h q_{jt}^{e,hp} \leq \tilde{q}_{jt}^{D_3e} \Phi/2 + (1 - \Phi/2) \tilde{q}_{jt}^{D_4e} \tag{28}$$

$$\sum_i q^e_{jit} + \eta_e^{chp} q^{gchp}_{jt} - a^h q^{e,hp}_{jt} \geq \tilde{q}^{D_2 e}_{jt} \Phi/2 + (1-\Phi/2)\tilde{q}^{D_1 e}_{jt} \tag{29}$$

Constraints (3)–(12)

3.6. *The Design of a Market Clearing Algorithm for the Proposed Decentralized Energy Trading*

Coupling Constraints

Relations (30) and (31) link the optimization problems of retailers and prosumers together and are known as coupled constraints in the optimization literature. The coupled constraints can be viewed as market clearing conditions and ensure that the amount of energy and gas that prosumer $\boldsymbol{j}$ buys at each time step equals the amount that retailer $\boldsymbol{i}$ sells at this period. Moreover, the dual of these constraints (γ^e_{ij} and γ^g_{ij}) presents the price of energy and gas transactions between prosumers and retailers.

$$p^e_{ijt} = q^e_{jit} : \gamma^e_{ij} \tag{30}$$

$$p^g_{ijt} = q^g_{jit} : \gamma^g_{ij} \tag{31}$$

The centralized implementation requires a central controller aware of the information of all market players, which makes the players prone to information disclosure and eliminates the possibility of fair competition. The present paper has tackled this issue by applying an ADMM algorithm for market clearing.

In the proposed decentralized ADMM, the players solve their optimization problems with minimal information exchange. Based on the dual decomposition principle [33], the main optimization problem can be divided into several sub-problems while respecting the coupling Equations (30), (31) and (24). The sub-problems are then solved in a decentralized manner.

The augmented Lagrangian for the retailer's optimization problem is given in Relation (32):

$$\mathcal{L} = \sum_{i=1}^{\mathcal{N}_S} E[R_i] - \psi\left(E[R_i] - R_i^{min}\right) - \sum_{j=1}^{\mathcal{N}_B} C_j + \kappa g(p,q) + \gamma^e_{ij}\left(p^e_{ijt} - q^e_{jit}\right) + \gamma^g_{ij}\left(p^g_{ijt} - q^g_{jit}\right) - \rho\left\|p^e_{ijt} - q^e_{jit}\right\|_2^2 - \rho\left\|p^g_{ijt} - q^g_{jit}\right\|_2^2 \tag{32}$$

For simplification, a Lagrangian coefficient κ is used for the model's $g(p,q)$ in addition to the coupled constraints. The retailer's and prosumer's problems are coupled via γ^e_{ij} and γ^g_{ij}, which are the coupling duals of the electrical and gas parts, respectively. The term $-\rho\left\|p^e_{ijt} - q^e_{jit}\right\|_2^2 - \rho\left\|p^g_{ijt} - q^g_{jit}\right\|_2^2$ (augmented Lagrangian) in the Relation (32) where ρ is the penalty parameter and a positive number guarantees the robustness and convergence [34]. In Relation (33), the optimization problem of each retailer is rewritten using the augmented Lagrangian:

$$\max_{p^e_i, p^g_i} \; R_i = E[R_i] - \psi\left(E[R_i] - R_i^{min}\right) - \rho\left\|p^e_{ijt} - q^e_{jit}\right\|_2^2 - \rho\left\|p^g_{ijt} - q^g_{jit}\right\|_2^2 \tag{33}$$

s.t.

Constraints (3)–(10)

Likewise, the optimization problem of prosumers is modeled in Relation (34) using the augmented Lagrangian:

$$\max \; C_j = \sum_t \left(\sum_i \left(\gamma^e_{ijt} q^e_{jit} + \gamma^g_{ijt} q^g_{jit}\right)\right) - \delta_1\left(\sum_{t=1}^{T} \tilde{q}^{D_3 e}_{jt}\Phi/2 + (1-\Phi/2)\tilde{q}^{D_4 e}_{jt} - \tilde{q}^{D_3 e}_{jt}\right)$$
$$- \delta_2\left(\sum_{t=1}^{T} -\tilde{q}^{D_2 e}_{jt}\Phi/2 - (1-\Phi/2)\tilde{q}^{D_1 e}_{jt} + \tilde{q}^{D_2 e}_{jt}\right) - \rho\left\|p^e_{ijt} - q^e_{jit}\right\|_2^2 - \rho\left\|p^g_{ijt} - q^g_{jit}\right\|_2^2 \tag{34}$$

s.t.

$$\text{Constraints (14)–(22)}$$

In the presented optimization problem, p^e_{ijt}, p^g_{ijt}, q^e_{ijt}, and q^g_{ijt} are pre-determined parameters. All primary variables of the retailer and prosumer problems are calculated by the gradient boosting method from the corresponding sub-problems, and the dual variable of the problem is updated iteratively:

$$p_{ie}{}^{m+1} = arg\min_{p^e} L(p^e_i, q_{jite}{}^m, \gamma^m_{ijte}) \tag{35}$$

$$q_{je}{}^{m+1} = arg\min_{q} L(p_{ijte}{}^{m+1}, q^e_i, \gamma^m_{ijte}) \tag{36}$$

$$\gamma^{m+1}_{ijte} = \gamma^m_{ijte} - \rho(p_{ijte}{}^{m+1} - q_{jite}{}^{m+1}) \tag{37}$$

$$p_{ig}{}^{m+1} = arg\min_{p^g} L(p^g_i, q_{jitg}{}^m, \gamma^m_{ijtg}) \tag{38}$$

$$q_{jg}{}^{m+1} = arg\min_{q^g} L(p_{ijtg}{}^{m+1}, q^g_i, \gamma^m_{ijtg}) \tag{39}$$

$$\gamma^{m+1}_{ijtg} = \gamma^m_{ijtg} - \rho(p_{ijtg}{}^{m+1} - q_{jitg}{}^{m+1}) \tag{40}$$

A variable update step Φ^m can be used instead of a constant update step (ρ) as presented in Relations (41), (42) and (44) [34] :

$$\kappa^{m+1} = \frac{1 + \sqrt{1 + 4(\kappa^m)^2}}{2} \tag{41}$$

$$\Phi^{m+1} = \frac{\kappa^m - 1}{\kappa^{m+1}} \tag{42}$$

In this relation, $\kappa^0 = 1$ is assumed. The stopping condition for the proposed algorithm is presented as follows:

$$\left|\gamma^{m+1}_{ijte} - \gamma^m_{ijte}\right| \le \epsilon \tag{43}$$

$$\left|\gamma^{m+1}_{ijtg} - \gamma^m_{ijtg}\right| \le \epsilon \tag{44}$$

ϵ is a positive infinitesimal value and is close to zero.

4. Simulation

This section evaluates the feasibility of the proposed P2P gas and electrical energy market using a distribution system test platform with two retailers and three prosumers. The wholesale market price is extracted from the average New York wholesale market price [35]. Figure 3 illustrates the predicted electrical and thermal demand of each prosumer. The private parameters of retailers and prosumers are taken from [36]. The stopping criterion is assumed as 0.0001. The present paper considers the following two case studies:

1. Case study 1: P2P electrical energy and gas trading between retailers and prosumers without considering uncertainties of wholesale electricity market price and prosumers' electrical demand (deterministic programming).
2. Case study 2: P2P electrical energy and gas trading between retailers and prosumers considering uncertainties of wholesale electricity market price and prosumers' electrical demand (robust possibilistic programming)

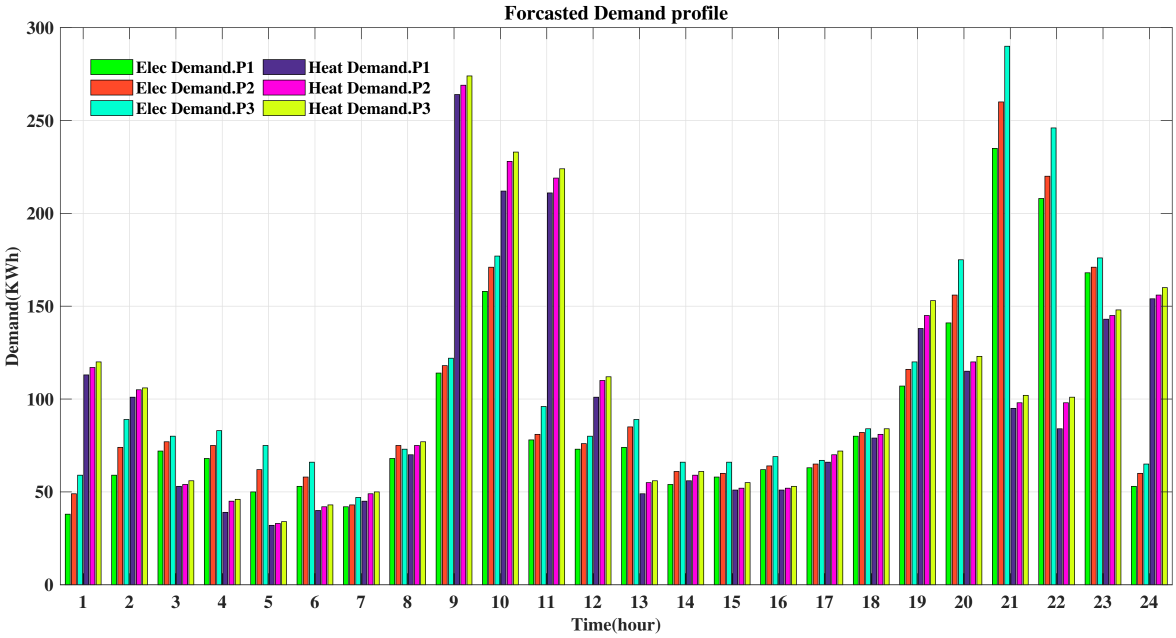

Figure 3. The predicted electrical and thermal demand of each prosumer.

4.1. Case Study 1

This case study evaluates the results of electrical energy and gas trading between retailers and prosumers without considering the uncertainties of wholesale electricity market price and prosumers' power demand.

The retailers are equipped with storage and self-generation units to profit more from electricity price fluctuations. The electrical energy transactions of retailers with the upstream network and prosumers, as well as the self-generation and storage units, are illustrated in Figure 4. As shown in this figure, the retailers charge their batteries at low electricity price hours, including 1:00, 2:00–6:00, and 12:00–18:00. Contrarily, they discharge their batteries during periods of high electricity price, including 5:00–7:00, 10:00, and 18:00–23:00, to supply power to the prosumers. At 7:00–8:00. and 17:00–19:00., due to high electricity prices, the retailers sell power to the grid. Moreover, the self-generation units operate at higher capacities during periods of peak electricity price, such as 17:00–22:00. This figure reveals that the storage and self-generation units have led to optimal energy management from the grid perspective while increasing the retailers' profit. At most hours, the power purchased from the network is almost in a certain range. At peak hours, i.e., 17:00–19:00, no power is purchased from the network, which reduces the pressure on the grid.

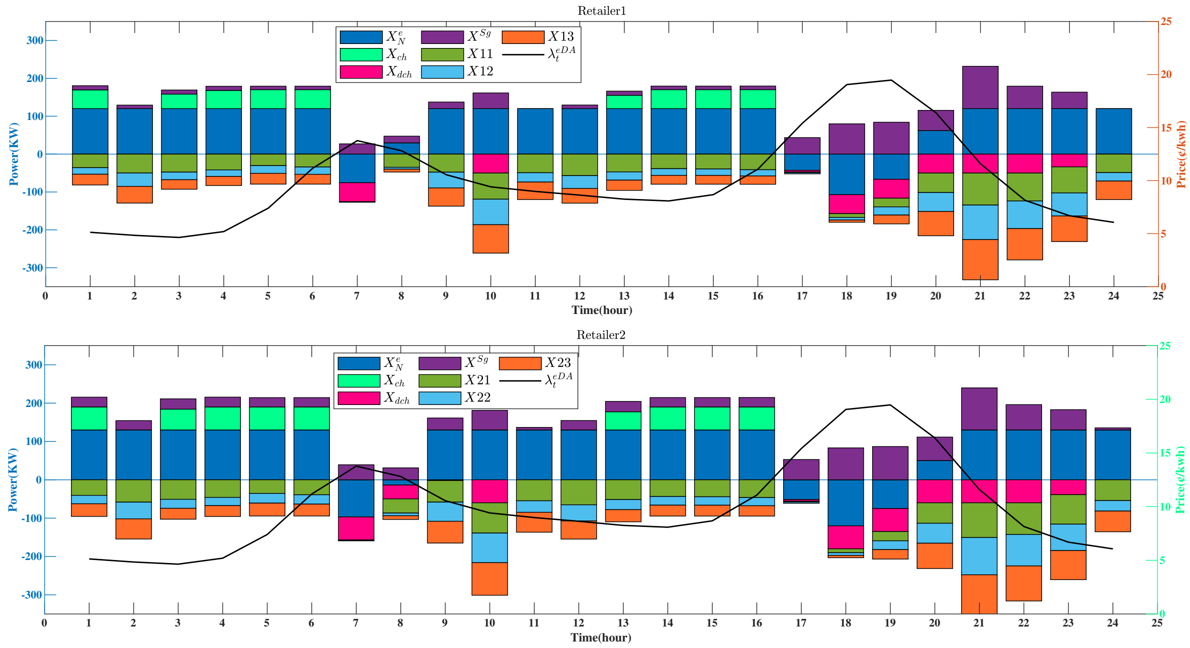

Figure 4. The plot of retailers' power production from self-generation units, their transactions with the upstream network and prosumers, and the charge and discharge of electrical storage, along with the price variation curve in the deterministic model.

Figure 5 displays the stored energy level in the electrical energy storage system. As already mentioned, the energy storage system increases the system's flexibility during peak hours and lowers the costs due to reduced power generation of the costly units.

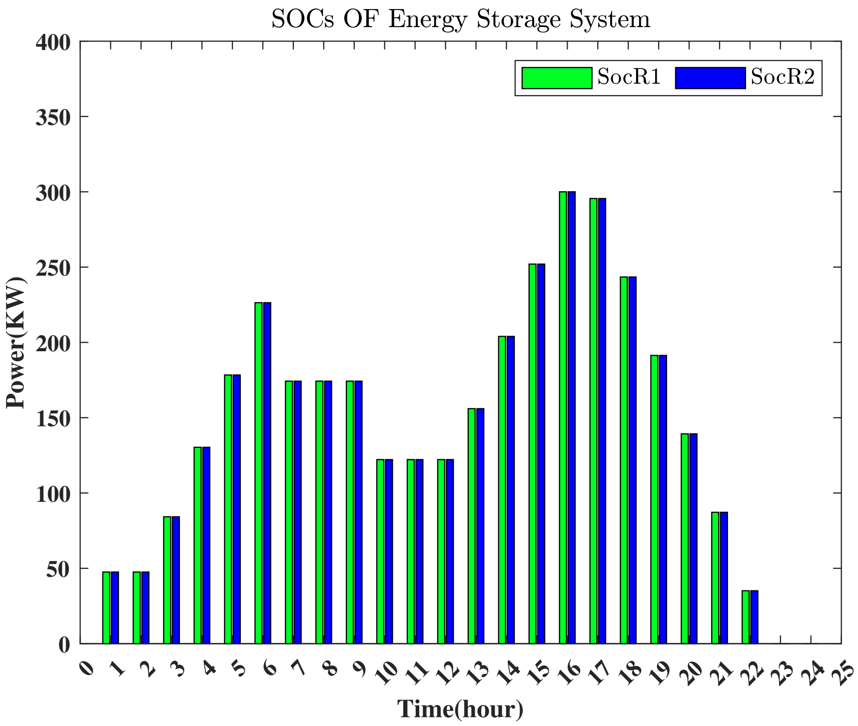

Figure 5. The stored energy level in the electrical energy storage system.

4.2. Case Study 2

This case study presents the P2P energy trading between retailers and prosumers, considering the uncertainties in the wholesale electricity market price and prosumers' demand.

A robust possibilistic programming method is applied to capture these uncertainties. In this case study, the prosumers' costs are expected to increase, whereas the retailers' profits are expected to drop. This reduction depends on ψ and δ coefficients. In the following, a sensitivity analysis is carried out to assess how variations of these coefficients affect the total welfare of market players. In this case study, δ and ψ are taken as 8 and 0.35, respectively.

For a simple analysis of the obtained results, the electrical energy transaction in robust mode is shown at 14:00 between retailer 1 and prosumer 1, at 9:00 between retailer 1 and prosumer 2, and 4:00 between retailer 1 and prosumer 3. Furthermore, gas trading in the robust case is depicted at 20:00 between retailer 1 and prosumer 1, at 17:00 between retailer 2 and prosumer 2, and 8:00 between retailer 1 and prosumer 3. Similar to the deterministic case, the electrical energy and gas transactions between retailer 1 and prosumers are presented at some hours. According to Figure 6, the convergence of the proposed decentralized approach is evident in both deterministic and robust cases.

Figure 7 illustrates the electrical energy transactions of retailers with the upstream network and prosumers, along with power production of self-generation and storage units. As expected, the transactions are similar to the deterministic model. The storage systems are charged at hours of low electricity prices and discharged at peak hours. During periods of high electricity prices, the retailers supply the required power to prosumers from their self-generation and storage units, as well as the upstream network. In contrast, low power prices at off-peak hours encourage them to purchase power from the network for storing and selling to the prosumers.

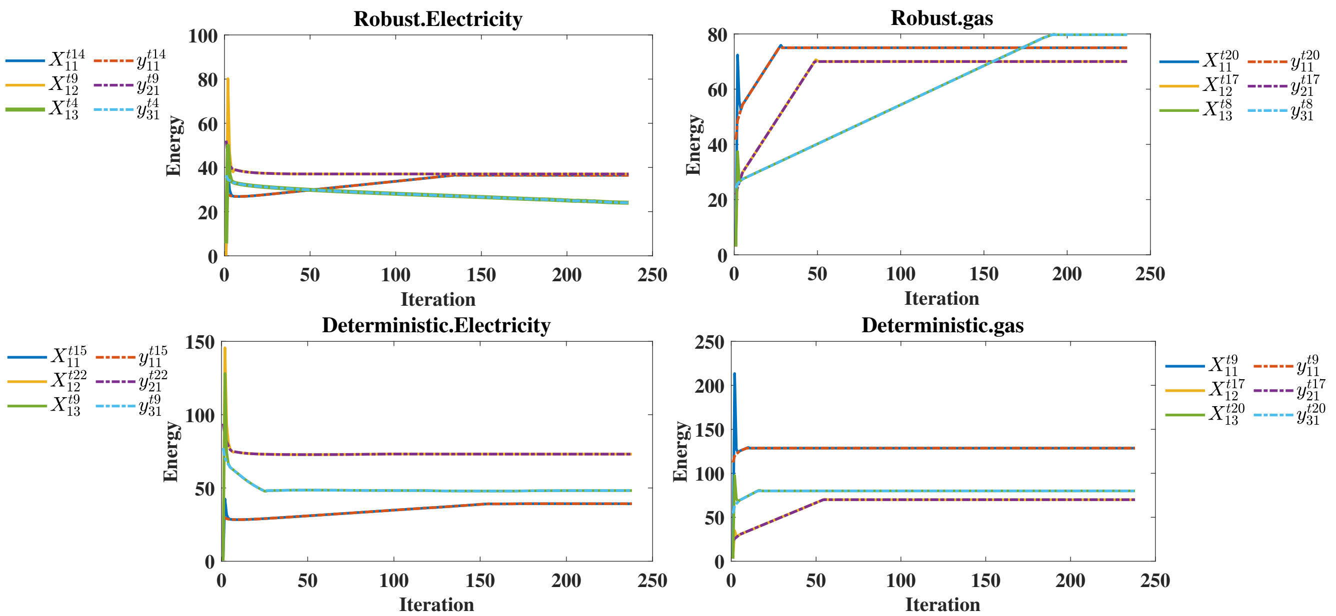

Figure 6. The prediction of electrical and thermal loads of each prosumer.

As stated before, the prosumers are equipped with CHP units and heat pumps to manage their energy consumption and supply their thermal load optimally. Figure 8 demonstrates the electrical power supply of each prosumer and the price bid of each retailer. At hours of peak electricity price, including 17:00–20:00, the prosumers purchase less power from retailers compared to other hours. At 7:00–11:00 and 17:00–23:00, they purchase gas from retailers to supply a portion of the load using CHP units. Moreover, at 1:00–6:00 and 9:00–12:00, when the electricity price is low, they use heat pumps to meet a portion of their heat demand.

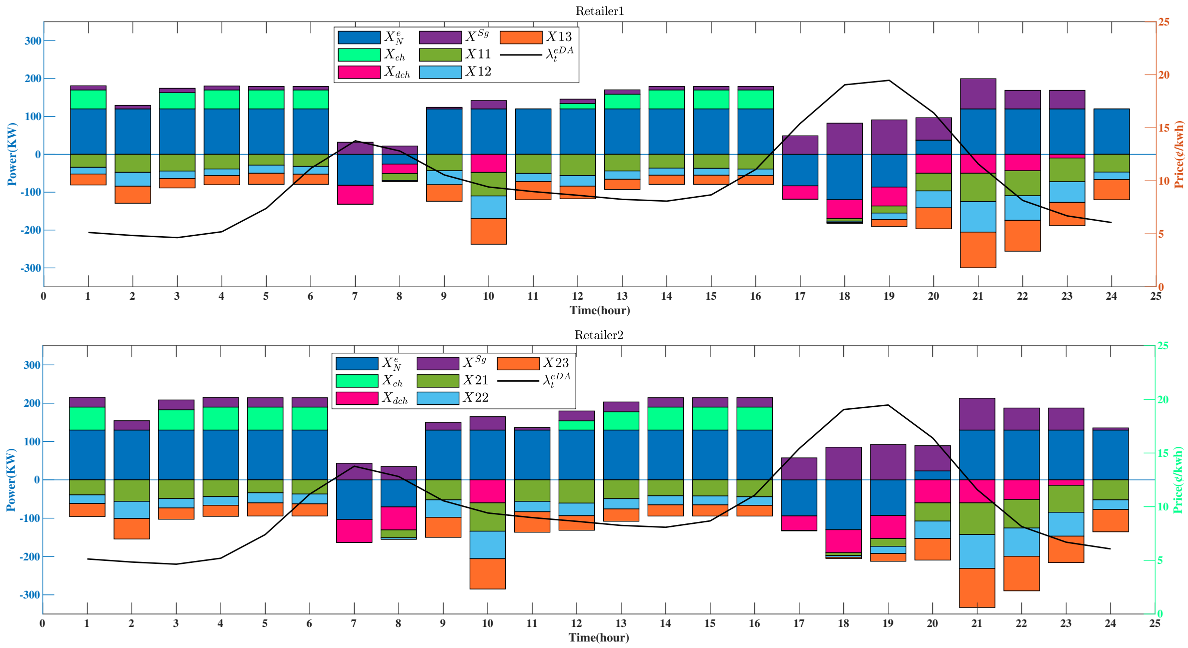

Figure 7. The plot of retailers' power production from self-generation units, their transactions with the upstream network and prosumers, and charge and discharge of electrical storage, along with the price variation curve in the robust model.

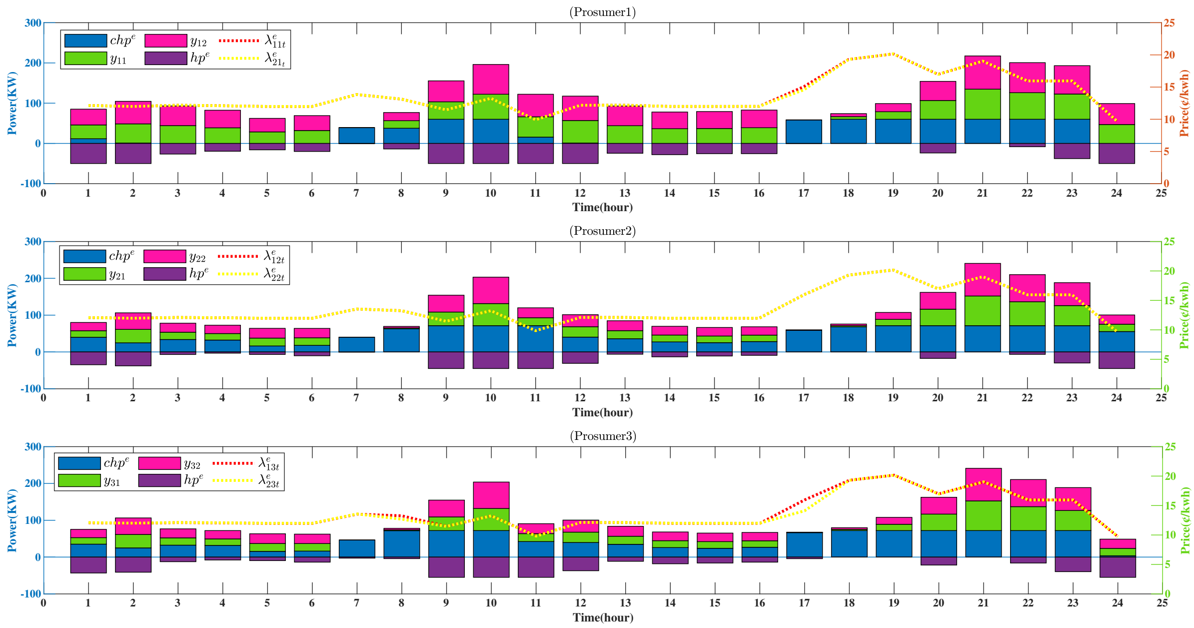

Figure 8. The electrical power balance of prosumers along with electricity price variations.

Figure 9 exhibits the thermal load supply of each prosumer and the gas price bid of each retailer. At hours with high electricity prices, the prosumers use CHP and boiler units to supply heat power. For instance, prosumer 1 uses a combination of boilers, heat pumps, and CHP units for power supply at 7:00–11:00 and 17:00–23:00 and utilizes the heat pump at off-peak hours, including 1:00–6:00, when the electricity price is low.

Table 1 presents the profit and cost of retailers and prosumers for both centralized and decentralized cases. The results suggest only a 171.5 difference between the two approaches and verify the effectiveness of the decentralized approach. Moreover, given that the maximum deviation is considered below the expected optimal value in the robust case, the retailer's profit has decreased by 12.1 percent, and the prosumers' cost has increased by 27.4 percent due to the feasibility penalty term.

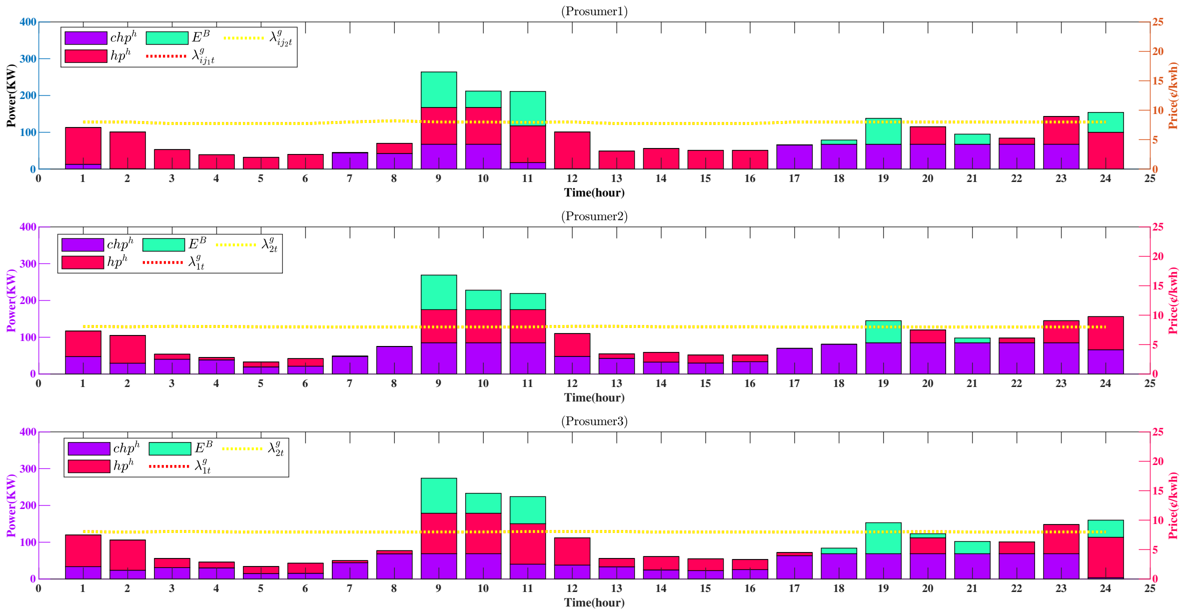

Figure 9. The thermal power balance of prosumers along with the gas price.

Table 1. A comparison of the proposed decentralized and centralized methods for deterministic and robust cases.

		Profit (¢)		Cost (¢)			Total Cost (¢)	Total Profit (¢)
		Retailer		Prosumer				
		R1	R2	U1	U2	U3		
Robust	Centralized	13,474.1	15,122.2	−58,763.3	−61,540.8	−66,341.9	186,646	28,596.3
	Decentralized	12,726.7	15,240.9	−58,821.4	−61,623.2	−66,372.9	186,817.5	27,967.6
Deterministic	Centralized	14,655.6	17,362.8	−46,509.9	−48,192.5	−51,900.9	146,603.3	32,018.4
	Decentralized	14,584.8	17,249.7	−46,472.9	−48,209.9	−51,913.3	146,596.1	3183.5

5. Conclusions

This paper implemented a flexible, dynamic, competitive, fully decentralized P2P electricity and gas market for retailers and prosumers of an energy hub, considering the uncertainties of the wholesale electricity market price and prosumers' electrical demand. The retailers supplied the electricity and gas needs of prosumers by purchasing from the wholesale market. Additionally, they were equipped with electrical storage and self-generation units and could leverage the wholesale electricity market price difference to sell power to the upstream network. The prosumers could engage in electrical energy and gas transactions with retailers while protecting their privacy. The energy hub prosumers had CHP units, heat pumps, and boilers and exploited the price difference between electricity and gas to minimize the costs. The presence of various players, storage systems, and self-generation units, as well as using a multi-energy system, contributed to the dynamicity and flexibility of the proposed model for profit maximization and cost minimization.

The uncertainties were addressed using robust possibilistic programming, which searches for a trade-off between the performance average, optimality robustness, and feasibility robustness. As shown in case study 2, considering the uncertainty resulted in a 12.1 percent reduction in retailers' profit and a 27.1 percent increase in prosumers' costs. Applying the ADMM approach for market clearing enabled the different market players to reach the optimal solution with minimal information exchange and without disclosure of their private information. A comparison of the obtained solutions to the centralized approach suggested a 0.09 percent difference, indicating a global solution similar to the centralized approach. The following suggestions are proposed for future studies:

1. Analyze the human factor impacts and human energy consumption on distribution networks.
2. Design a business model for P2P energy trading, which should provide proper incentives to participate in a market, especially a suitable pricing mechanism.
3. Considering the electricity and gas network constraints.

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Nomenclature

Indexes	**Defintion**	**Unit**
t	Time index	h
i	Retailer index	-
j	Prosumer index	-
k	Repetition index	-
Parameters		-
Φ_i, π_i and ψ_i	Cost function parameters for retailer i	$/kWh^2, $/kWh , $
$p_{ch}^{ESS,\,max}$, $p_{dch}^{ESS,max}$	Maximum of charging/discharge power of electrical energy storage system	kW
η_h^{chp}	Efficiency of gas consumed by the CHP unit	-
η^b	Efficiency of gas consumed by the boiler unit	-
a^h	Efficiency of electrical consumed by the heat pump unit	-
Υ^{gDA}	The gas prices of the wholesale market	¢/m^3
η^{ESS}	Efficiency of charging/discharging of storage system	-
ρ	the penalty parameter	-
Φ	the confidence level	-
$Loss^{ESS}$	Losses rate of storage system	%

Variables

C_j	The gas and electricity cost of prosumer j	¢
R_i	Revenue of retailer$i$	¢
p^e_{ijt}	The quantities of electricity sold to each prosumer	kW
p^g_{ijt}	The quantities of gas sold to each prosumer	m^3
p^{eDA}_{it}	Energy purchased from the wholesale market by retailer i at time t	kW
p^{gDA}_{it}	Gas purchased from the wholesale market by retailer i at time t	m^3
Y^e_{ijt}	The electricity trading price between retailers and prosumers	¢/kW
$\check{Y}^e_{ijt}$	The constant fluctuation of wholesale market price	-
Y^g_{ijt}	The gas trading price between retailers and prosumers	¢/m^3
p^{sg}_{it}	The power of self-generation units	kW
$p^{BSS,ch}_{it}$, $p^{BSS,dch}_{it}$	Charging/discharging power of electrical energy storage system	kW
$C(p^{sg}_{it})$	The cost function of a self-generator belonging to retailer i	¢
b^{ch}_{jt}, b^{dch}_{jt}	Binary variable for determining the state of charging/discharging of system storage	-
q^{gb}_{jt}	Gas consumed by boiler units	m^3
q^{gchp}_{jt}	Gas consumed by CHP units	m^3
q^{ehp}_{jt}	Electricity consumed by the heat pump units	kW

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