energies

Article

Energy Design Synthesis: Algorithmic Generation of Building Shape Configurations

Tamás Storcz [1,*], Zsolt Ercsey [1], Kristóf Roland Horváth [2], Zoltán Kovács [3], Balázs Dávid [4,5] and István Kistelegdi [6]

1 Department of Systems and Software Technologies, Faculty of Engineering and Information Technology, University of Pécs, Boszorkány Street 2, 7624 Pécs, Hungary
2 Marcel Breuer Doctoral School, Faculty of Engineering and Information Technology, University of Pécs, Boszorkány Street 2, 7624 Pécs, Hungary
3 Optin Ltd., 6720 Szeged, Hungary
4 InnoRenew CoE, Livade 6a, 6310 Izola, Slovenia
5 Department of Information Sciences and Technologies, Faculty of Mathematics, Natural Sciences and Information Technologies, University of Primorska, Glagoljaška 8, 6000 Koper, Slovenia
6 Department of Simulation Driven Design, Ybl Miklós Faculty of Architecture and Civil Engineering, Institute of Architecture, Óbuda University, Thököly Street 74, 1146 Budapest, Hungary
* Correspondence: storcz.tamas@mik.pte.hu

Abstract: The building industry is responsible for a significant degree of energy consumption in the world, causing negative climate changes and energy supply uncertainties due to low energy efficiency as well as the high resource demand of construction. Consequently, energy design optimization has become an important research field. Passive design strategies are one of the most definitive factors concerning energy-related building development. The given architectural problem calls for a method that can create all potentially feasible building geometries, thus guaranteeing the optimal solution which is addressed in the current paper. To reach this requirement, the necessity of a modular space arrangement system and architectural selection rules were determined, focusing on the relationship between the rules and the generation of geometries with mathematical rigor. Next, the architecture-based congruency analysis performed, further reduced the number of simulation cases. With the simulations, it is illustrated how the building shape versions affect the heating energy demands: the performance of the configurations themselves. Results clearly illustrate the importance of the synthesis step of the architectural design.

Keywords: building energy design; heating energy assessment; geometry generation; modular space arrangement

check for updates

Citation: Storcz, T.; Ercsey, Z.; Horváth, K.R.; Kovács, Z.; Dávid, B.; Kistelegdi, I. Energy Design Synthesis: Algorithmic Generation of Building Shape Configurations. *Energies* **2023**, *16*, 2254. https://doi.org/10.3390/en16052254

Academic Editors: Paulo Santos and Chi-Ming Lai

Received: 18 January 2023
Revised: 17 February 2023
Accepted: 24 February 2023
Published: 26 February 2023

1. Introduction

Buildings produce more than 40% of the world's total energy consumption and CO_2 emissions due to construction and operations [1,2]. One of the main reasons is represented by the planning methodology of buildings. The prevailing modern, conventional building design includes a typically tight project schedule, making only a very limited number of concepts possible to consider. Through a linear process, the architect and client agree on one concept to be worked out. The services systems and further design disciplines subsequently integrate their contents to cover the evolved needs of the plan and planner cooperation is minimized due to time limits. Solar gains are not considered sufficiently, in general, and high solar loads (overheating) cause high cooling costs due to large glass surfaces in the envelope together with local thermal discomfort phenomena. Passive and active design strategies rely solely on experience and theoretical, general knowledge. With the comprehensive use of passive design concepts (e.g., space organization, building body shaping, envelope design, etc.) the highest degree (up to 80%) of energy conservation is achievable [3]. However, architects who are experts in designing spaces (generating the highest amount of energy needed), at the same time, do not handle the building's physics aspects, calculations, etc. in detail. Under such conditions, optimization is obviously not conceivable.

The next development step is performed by inventing an integrated design methodology that, still today, belongs to the exceptional examples of professional design teams worldwide [4–6]. During this iterative planning process, the ideas and knowledge of each participant are taken into consideration from the beginning and the design concept is created through teamwork. The energy and building services system concept does not follow the architect's plan; rather it is carried out parallel to the architectural design. The Energia Design (ED) method, developed at the beginning of the 2010s, has become one of the iterative tools that is used in practice when considering both energy and comfort issues in building design [7]. This method includes systemized heuristic building simulations in each design stage at different levels, and as a result, unique buildings were constructed with significant improvement in energy and comfort. However, neither this methodology nor other integrated design methods consider all possible solutions, and thus the optimal concept may be overlooked.

To overcome this drawback—and additionally motivated by the growing negative climate and environmental statistics—new efforts are continuously made on the development of building design methodology in order to gain optimized solution(s) in the fields of energy, and comfort, as well as in environmental impact performance. Since the complex building design objectives are characteristically ill-defined and often contradictory [5], rigorous mathematical optimization is required.

In architecture so far, numerous papers have been dedicated to problems of optimization methods related to some specific targets. In a comprehensive review, more than 100 studies about diverse building energy, comfort, and environmental design optimization (BECEDO) issues were evaluated [8]. The analysis states that most BECEDO studies' focus is set on the HVAC system and envelope parameter, and occupancy behavior optimization [9–12]. Even though the building geometry's positive effect on comfort and energy performance is more and more proven [13–15], over 60% of energy demand [16], as well as 80% of environmental impact reduction [17] is achievable with the combined optimization of the shape and further variables. Nevertheless, building geometry is still an underestimated and more or less neglected design variable, since numerical variables of the active systems and envelope materials are considerably easier to handle in the algorithms than space organization and building form describing mathematical models. There exist two main directions in BECEDO research, which integrate building geometry variables: the modification of the shape [18–20] that represents approx. 90% of the available literature, as well as shape generation [21–23] to create case versions of buildings. Though the dimensional properties of the geometry describing variables (e.g., depth, height, roof as well as diverse ratios) are frequently used in BECEDO, however, they are not able to provide information about form assembly of the geometry i.e., the position of the spaces, walls, edges, etc. relative to each other.

Another problem of BECEDO lies in the stochastic nature of the most frequently applied evolutionary (genetic) algorithms (GA), whereas due to the random move through the search space, the optimum solution is not guaranteed [24–27].

A promising development is proposed by the extension of the mentioned ED method, i.e., the Energia Design Synthesis (EDS) method has been introduced by Kistelegdi [28]. This method is intended to ensure de facto optimal buildings performing the highest energy and comfort efficiency, together with the lowest LCA impact. The EDS starts with the generation of all potentially feasible building shape configurations and extends these with various architectural aspects, i.e., building structure, fenestration ratio, and orientation. In this way, in the first stage of the method, all buildings are considered with the fundamental, most important passive measures, since these can enable architects to ensure a high degree (up to 80%) of energy savings [29,30]. When the buildings with passive measures are available, these are equipped with artificial illumination, occupants, and equipment, and in further steps with HVAC systems to serve as input for complex comfort and energy performance simulations. The simulation results are then evaluated, and a ranked order of the building cases is determined including the optimal design. The application of the synthesis step within the field of architecture is a novelty. The idea to start the algorithmic solution of a specific problem is well-known in the chemical engineering field, for example, Ref. [31] presented the p-graph methodology to successfully represent chemical engineering problems to serve as the basis of mathematical programming models. Now, this algorithmic method has been successfully adapted in other fields from handling sustainability issues [32], manufacturing problems by [33], energy storage and distribution by [34], and bus transport optimization [35] to scheduling problems [36].

The present work focuses on the first step of the EDS methodology, namely on the synthesis step, when during a BECEDO process all possible feasible building geometry configurations are generated, while all other cases are left out of consideration. This phase of the EDS method is performed algorithmically and with mathematical rigor. To support the algorithmic building shape generation, some definitions and a theorem are also presented together with technical details. The applicability of the result for further steps inevitably includes optimality to be considered throughout the whole EDS method.

The following questions are therefore proposed to be addressed:

- Does energy efficiency depend on building geometry?
- Is it possible to algorithmically generate all building geometries?
- What kind of architectural rules are required to define geometry feasibility?
- Can the number of feasible geometries be reduced based on architectural congruency?
- What are the formal characteristics of the gained geometry configurations meeting the required architectural rules?
- What are the energy performance characteristics of the selected geometry configurations meeting the required architectural rules?
- How intensively does the building shape affect the most relevant annual energy demand, the heating requirement?

The paper is constructed in the following manner (see Figure 1). Section 2 describes modular space organization and based on its result, the search space of the considered problem is determined. Section 3 talks about the inevitable architectural design rules, i.e., principles, that result in habitable and sustainable family houses. Section 4 explains how the search space is explored, i.e., all building geometries are set. Sections 5–7 detail how those building geometries that satisfy the architectural rules can be selected and how architecturally congruent geometries are eliminated, furthermore, which building geometries are configurations. Section 8 provides results and an analysis of energy simulations which clearly demonstrates the impact of the geometry dependencies.

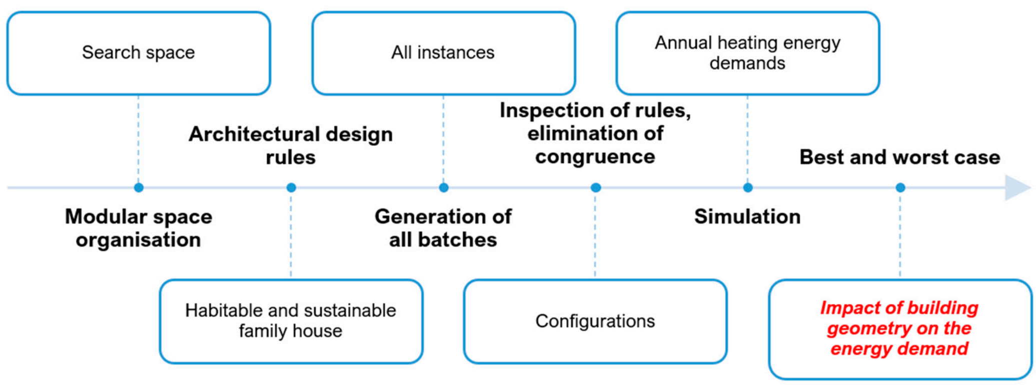
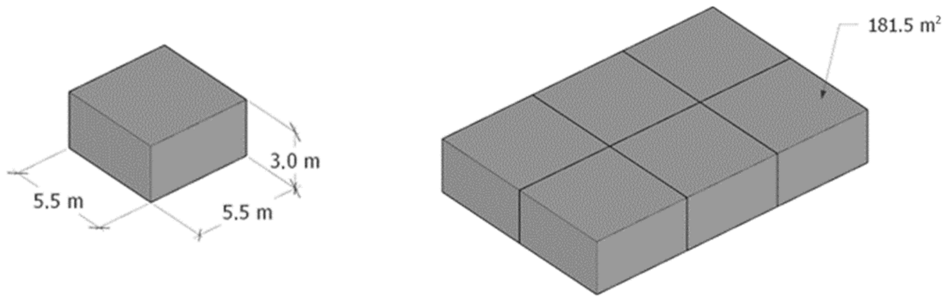


Figure 1. Process of configuration generation.

2. Modular Space Organization

The current problem from the architectural point of view is to develop all family house building configurations that must be considered when a house design is to be investigated, performing at as high energy and comfort performance as possible within the boundaries of the task. During this exemplary modeling, let us consider units or building blocks with the size of 5.5 m × 5.5 m × 3.0 m as general building elements; whereas the total of six blocks serves as the overall cubature of the housing. The size of the basic space unit provides generic room dimensions with a one-story height and the combination of the units results in larger spaces as living/dining/kitchen functionality or the division of the unit enables gaining smaller spaces such as bath, toilet, storage, etc. In other words, six blocks are to be placed next to each other according to architectural rules specified thereafter, thus, all family house building geometries of 181.5 m^2 floor area are sought, which meet the predefined architectural prerequisites (Figure 2).



(**a**) (**b**)

Figure 2. Basic space unit (**a**) and the combination of 6 units (**b**) to assemble a complete family house.

To easily describe the various building geometries and distinguish them from one another, let us consider an orthonormal coordinate system with X, Y, and Z axes and (5.5 m, 5.5 m, 3.0 m) unit size according to the architectural requirements. Moreover, let us consider a box of appropriate size within the positive orthant of the coordinate system, i.e., width × depth × height. In this specific family house generation case, where there are six building blocks under consideration, for the current problem let us consider 6 × 6 × 6 (width = depth = height = 6) as the size of this box. In other words, one vertex of this box is the origin of the coordinate (0, 0, 0) system and the opposite vertex is (6, 6, 6). Now, let us divide this box into 216 unit-sized cells. These will serve as cells to be used by the generation method. Any cell is to be identified by the triplet (x, y, z), which triplet now does not represent the coordinates of a point within the space but identifies the cells themselves. The triplet can be referred to as the coordinates of the vertex closest to the origin. In this particular case, there are 216 distinguished triplets, i.e., the set of triplets or the set of cells to be used during the algorithmic generation is the following: {(0, 0, 0), (1, 0, 0), . . . , (5, 5, 5)}. Now, let us place the blocks within these cells. It is worth mentioning that since the blocks are placed within cells of a coordinate system, it can be said that the blocks have floor, roof, and walls, which support the ease of understanding and the direct usability of the architectural rules. Should we place more than one block within the available cells of the box, then a batch is formed. For the specific architectural family house building geometry problem, let us consider batches of six blocks only since the total of six blocks serves as the overall cubature of the family house. As an example, the following description: {(0, 0, 0), (0, 1, 0), (1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 0, 0)} corresponds to the building geometry depicted in Figure 3. Since the description can be given in a lexicographical order, the various structures can be easily ordered.

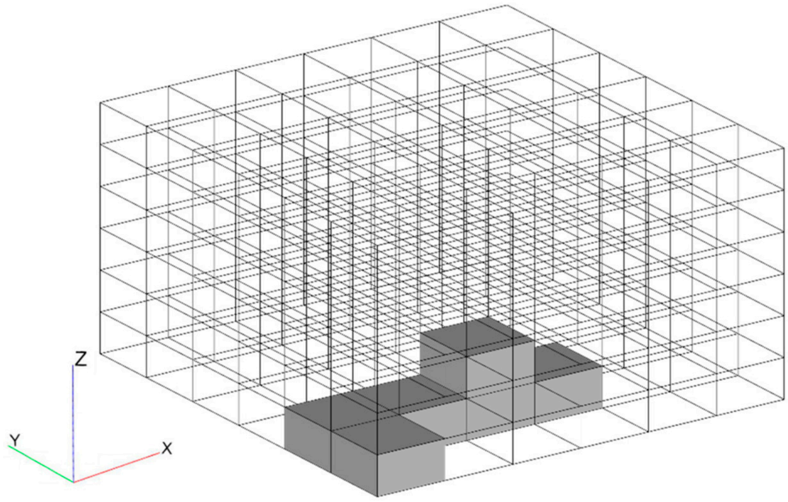


Figure 3. The batch.

3. Architectural Design Rules

In practice, no arbitrary batch can represent a building. A batch must satisfy certain characteristics to represent a family house appropriately. There are design principles that architects follow during the planning process of a family house; for instance functional dependencies between diverse spaces and use, shape design based on formal ideas, limitations due to legislation and regulation, visual view connections within the building and between inner and outer spaces, economical questions, site, and topography adaptation. However, these architectural design rules are diverse and often too complex to meet the abstraction level of the current investigation. Therefore, it is of utmost importance to adjust architectural rules to the requirements of the present scenario. Focus was set on avoiding sculpture-like shapes or futuristic, impractical batches. Shapes that are too high or too long are not supported either since these building shapes are impractical for family house use. In addition, building design regulations do not approve such solutions. Thus, the following set of rules was determined to express the necessary and sufficient architectural properties (elementary architectural requirements) to which a family house structure should conform:

(1) The blocks of the batch should be connected to each other with a minimum of one face, i.e., the batch should be compact.
(2) On the ground level, a minimum of two and a maximum of four blocks can be placed in a straight line. In this way, too short, too high as well as too long batches are avoided.
(3) There may be only one block within the batch, which is cantilevered. This means in architectural practice only one cantilevered space unit is allowed.
(4) The batch should not be more than three blocks high in order to exclude too high geometries, untypical for family housings.
(5) Above the ground floor, there should not be two blocks on top of each other with free walls, meaning at least one of the walls of the blocks should be connected to another block (no free-standing 'tower-like' building parts allowed).
(6) The length of the batches, which are taller than two blocks, should be a maximum of three blocks long in directions X and Y.

Please note, that for further architectural studies, only those batches are considered to be potentially feasible family house building configurations, which meet the above rules, while no other structures have to be considered. In other words, the search space is limited by the above-mentioned set of rules to fully control the optimization process by considering all possible and feasible solutions and thus ensuring that the optimal concept is also taken into account.

First, all batches within the search space should be assembled. When a batch is fully generated, it should be checked as to whether the batch satisfies the above-mentioned architectural rules. Should any of the rules be violated, the batch must be excluded from further consideration. Those building geometries that are not excluded, i.e., the remaining shapes satisfying the rules, are to be called building configurations hereinafter.

4. Generation Method

The given architectural problem specifies the generation of all potentially feasible building configurations, i.e., batches ofsix blocks satisfying the architectural rules. Please note that rule four limits the aforementioned original box size of $6 \times 6 \times 6$ cells to a maximum of three cells when considering the z-axis; while rules two and three limit the box size tofive cells when considering the axes x and y; resulting in the overall size of the box $5 \times 5 \times 3$, including 75 cells. This box will serve as the search space where batches of six blocks are sought that satisfy all of the above rules. It is worth mentioning that within this box there are $\binom{75}{6} = 201{,}359{,}550$ different ways to place the six blocks.

Now, as the first phase of the generation method, all possible placement combinations of the six blocks within this box are generated. Technically, a zero-based sequence number (SN) is assigned to each cell by formula or Equation (1):

$$SN = x + y \cdot width + z \cdot width \cdot depth \tag{1}$$

For example, the first cell (SN = 0) of the box within the positive orthant of the coordinate system is identified by the triplet (0, 0, 0), the 25th cell (SN = 24) is identified by the triplet (4, 4, 0) and the last one, i.e., the 75th cell (SN = 74) is identified by the triplet (4, 4, 2). This sequence number eases the generation process since the cells of the box can now be easily ordered, and it can be said that cells have predecessor and successor cells. The classical backtracking algorithm that incrementally builds partial candidates [37] finds all solutions to some computational problems. The candidates are represented in a tree structure, where the parent differs from the child by a single extension step. This backtracking serves as the guiding rule for the generation procedure in the present case in the following way (Figure 4): Let the initial state be, where the first block is in the first cell, the second block is in the second cell, and so on, and the sixth block is placed in the sixth cell. In other words, the six blocks are given as follows: {(0, 0, 0), (1, 0, 0), (2, 0, 0), (3, 0, 0), (4, 0, 0), (0, 1, 0)}. The final state of the procedure is given as follows: {(4, 3, 2), (0, 4, 2), (1, 4, 2), (2, 4, 2), (3, 4, 2), (4, 4, 2)}. Between the initial and the final states, the following is performed: the block in the cell with the highest sequence number is moved forward by one into the next cell. Should this not be possible, then the block with the predecessor's highest sequence number is moved forward by one, and all successors are moved behind. If the first block is in a cell from where it cannot be moved forward anymore, then the algorithm stops.

```
procedure Steps()
begin
    Size = width * depth * height
    for (i1=0; i1 < Size-5; i1++)
        for (i2 = i1+1; i2 < Size-4; i2++)
            for (i3=i2+1; i3 < Size-3; i3++)
                for (i4=i3+1; i4 < Size-2; i4++)
                    for (i5=i4+1; i5 < Size-1; i5++)
                        for (i6=i5+1; i6 < Size; i6++)
                            architectural compliance check
end
```

Figure 4. Generation procedure of the 6 blocks.

Each block is indexed with the zero-based sequence number (SN). Within the cycles, the blocks are placed into the cells of an index between 0 and 74 as mentioned earlier. The procedure given in Figure 3 represents a space-filling method based on SN. When all six blocks are placed, the batch is formed. The x, y, and z coordinates of the blocks must be calculated based on Equations (2)–(4).

In Equations (2)–(4) the // symbol denotes the integer division and % denotes the modulo division.

$$z = SN//(width \cdot depth) \tag{2}$$

$$y = (SN\%(width \cdot depth))//width \tag{3}$$

$$y = (SN\%(width \cdot depth))\%width \tag{4}$$

5. Meeting the Requirements

The above-described procedure generates all batches of six blocks within the search space. It is important that the procedure considers the whole search space and therefore it is sufficient to consider the generated batches only, and no other batches of six blocks should be taken into account. During the generation, when a new batch is available, it must be checked as to whether the architectural rules are satisfied or if any of the rules are violated. Should none of the rules be violated, then a potentially optimal configuration is found. To handle all rules the following process is performed.

Rule 1: corresponds to the elementary architectural requirement that the blocks, as general building elements of the house, should be connected, i.e., the configurations should be compact batches, see Figure 5. Obviously, a case where these blocks are separately scattered cannot be considered a family house. However, the connectivity requires further clarification, since the spatial connection of the blocks can be interpreted in various ways. For instance, blocks could be connected with their peaks or with their edges also, but these connections are also excluded from the current consideration since some space units become spatially separated from each other from an architectural point of view.

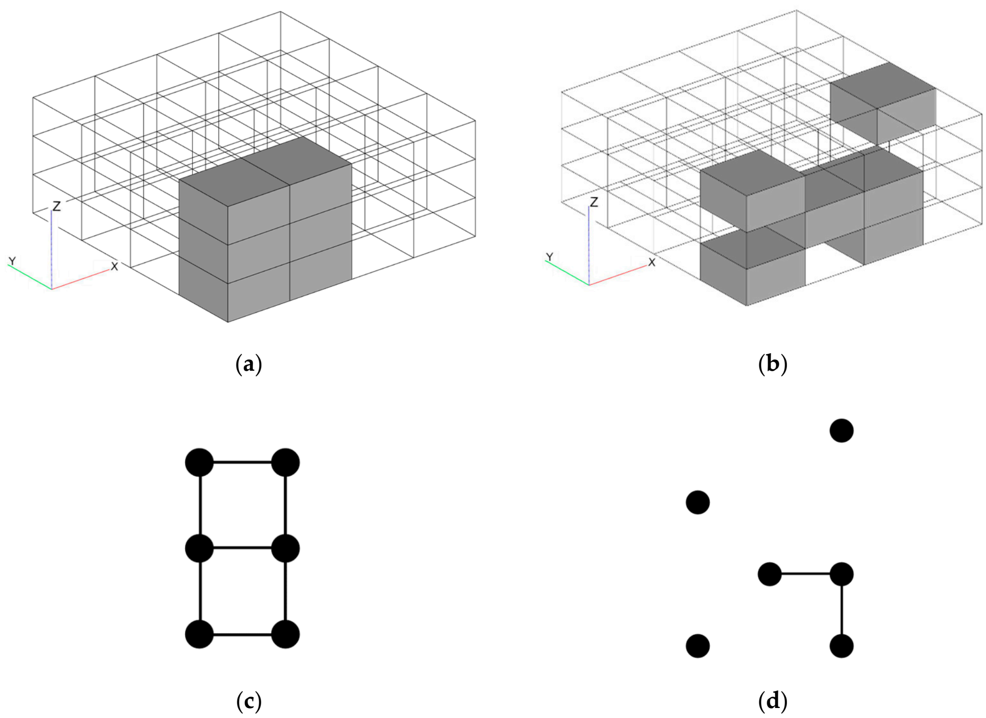


Figure 5. Compact batch (**a**) and a batch that is not compact (**b**). Underlying incidence graph (**c**) of the compact batch (**a**). Underlying incidence graph (**d**) of a batch that is not compact (**b**).

Hereinafter, a batch will be considered as connected, should the underlying incidence graph be connected. The underlying incidence graph can be formed as vertices of the graph are assigned to the blocks, namely, the node is assigned to the center of gravity of the block. Two vertices are linked together with an edge of the graph if, and only if, their corresponding blocks are connected to each other with their faces, i.e., the two blocks under consideration are connected through their floor, or roof, or one of their walls is common.

Rule 2: corresponds to the elementary architectural requirement that the blocks should not form a corridor-like long and narrow structure. Therefore, in the X or Y direction, in which the batch is longer, items on the ground and connected by walls, must form a 2–4 long line. This can be verified by counting connected elements in layers of the ground level of the incidence graph. Layers are created from the subgraph (ground level of the incidence graph) by slicing it parallel to X and Y axes. When counting the connected elements, the highest number must be in the range of 2–4.

Rule 3: corresponds to the elementary architectural prerequisite that accepts a cantilevered balcony under certain boundaries as a part of a family house. For example, a cantilevered shape with an open area below for cars, etc. may be accepted but this part of the building cannot become dominant, i.e., its size is limited to the size of only one block (Figure 6). This can be investigated by checking all blocks that are not on the ground level, i.e., whose z coordinate is not 0. Should there be no block below on the ground level, then the balcony index is increased by 1. All batches violate the rule where this index becomes greater than 1 at the end of the process. Moreover, should there be a block above a balcony-like block, the corresponding batch should be excluded from further considerations because due to the architectural request, unfeasible, impractical 'stacking' of the stories is also an unwanted solution.

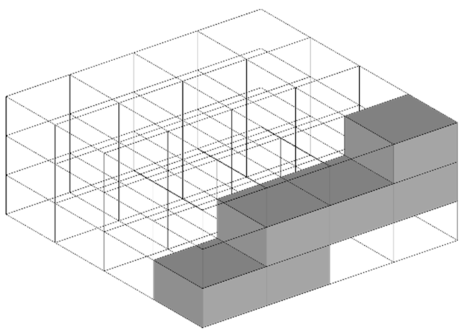


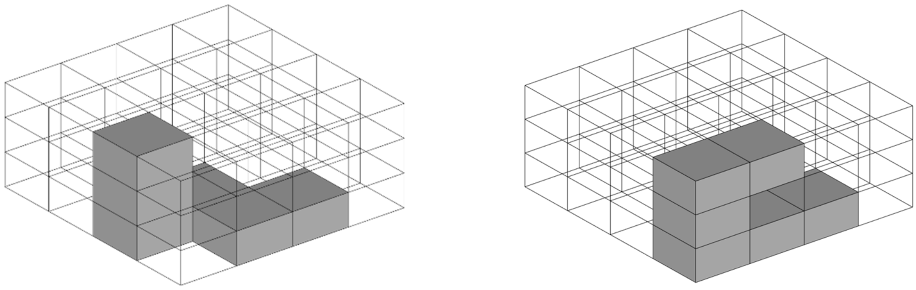
Figure 6. A batch of 6 blocks violating rule 3 as the cantilevered 'balcony' size exceeds the limit.

Rule 4: prescribes that only up to three blocks can be placed on top of each other, a higher building is not possible as a family house. In this way, unnecessary high-building shapes are avoided. This is obviously controlled by the search space, i.e., the height of the considered box is three.

Rule 5: does not allow the creation of a separate 2 story high (=stacked) stand-alone module, where each side of the module is free, and only the floor and roof sides of the blocks are connected to each other; in other words, no 'tower-like' shapes are allowed. That rule is satisfied when examining the underlying incidence graph without the ground level, there is no vertical subgraph with two nodes found. An example of violating rule 5, and an accepted configuration, are depicted in Figure 7.

Rule 6: buildings with a height of three blocks should not be longer than three blocks on the ground in any direction (X, Y). The total length of the batch along an axis could be longer. Figure 8 also violates rule 6.

It is important to emphasize that the rules include many other, elementary architectural requirements. For example, let us consider the requirement that a minimum of two blocks of the batch should stand on the ground floor. Should a batch satisfy the above rules, this requirement is also satisfied. It follows from the rules, namely, there can only be one cantilevered balcony and the batch cannot be higher than three blocks, therefore a minimum of two blocks should stand on the ground floor.



(**a**) (**b**)

Figure 7. Batch violating rule 5, 'towers' are not allowed (**a**); an accepted configuration (**b**).

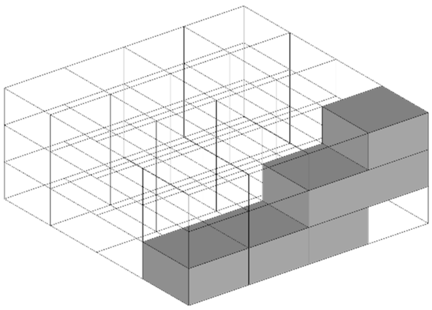
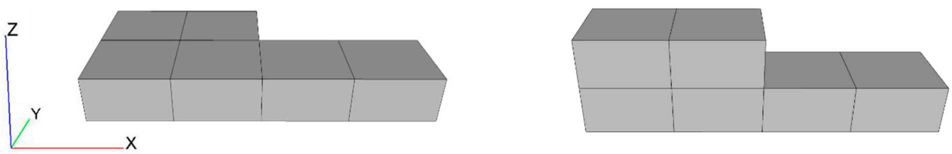


Figure 8. Batches of 6 blocks violating Rule 6. Total length is too long.

6. Identification of the Batches

It is easy to show that there is a large number of potentially feasible building configurations (batches ofsix blocks satisfying the rules), which have to be considered identical from the architectural point of view. Therefore, the problem of architectural identity or congruency must be solved. In the most common case, two batches can be considered to be identical, when one can become the other using diverse transformations such as shifts, rotations, and reflections. However, this is not appropriate for this specific architectural problem, since it is easy to transform a one-story batch into a two-story batch by only rotation around axis X, see Figure 9.



(**a**) (**b**)

Figure 9. Rotation around axis X is applied between a one-story batch (**a**) and a two-story batch (**b**).

Hereinafter, pursuant to an own developed transformation rule, two batches, or family house building configurations are considered to be identical or congruent, if they can become one another using rotation around a line parallel to axis Z, or reflection through a plane parallel to XZ and YZ planes, or shift parallel to the axes X or Y. All batches are accepted as identical, should they become one another by the application of any transformations or a series of the transformations below (Figures 10–12). The batch is considered to be in an initial position when any of its blocks touches the plane XZ, any of its blocks touches the plane YZ, and all of its blocks are within the positive orthant of the coordinate system.

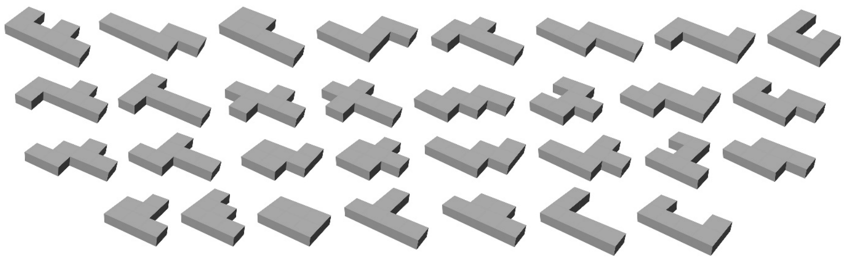


Figure 10. The generated 1-level, ground floor batches (31 building shapes).

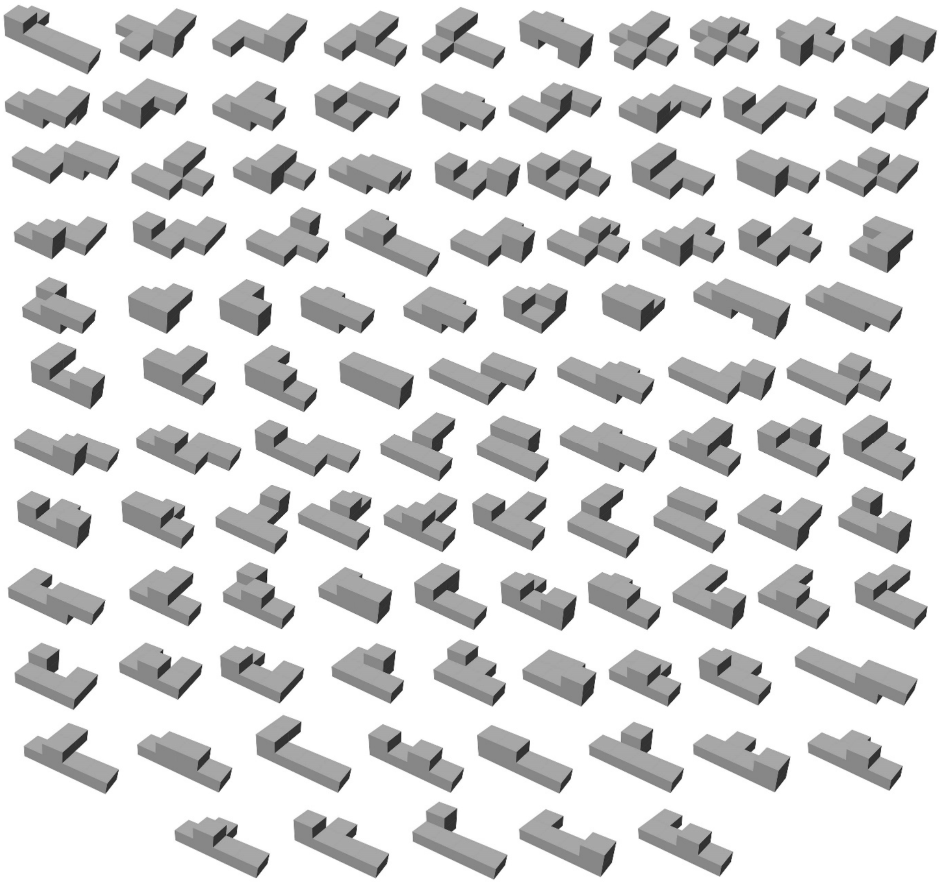


Figure 11. The generated 2-level batches (105 building shapes).

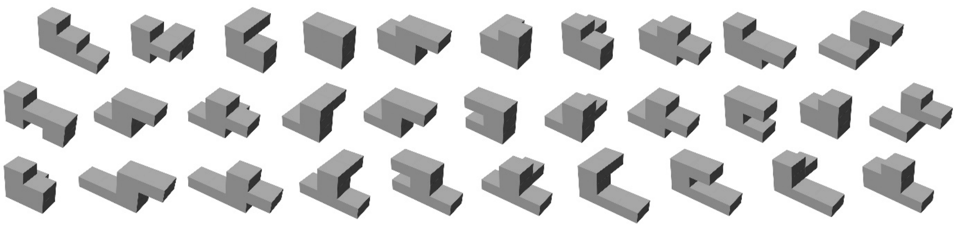


Figure 12. The generated 3-level batches (31 building shapes).

Transformation types:

Let there be a line e parallel to axis Z intersecting the point (2.5, 2.5, 0). Let us denote by R_1 the transformation that rotates the batch around the line e by $-90°$, R_2 the transformation that rotates the batch around the line *e* by −180°, and similarly, R_3 the transformation that rotates the batch around the line *e* by −270°. Please note that this transformation is very similar to the major orientation of the building, namely when the main façade of the building is turned in different directions. Nevertheless, it is worth mentioning that the orientation as well as the wall-to-window ratio (WWR) and other major architectural considerations are part of subsequent steps of the EDS methodology.

Let there be a plane *S (swap-plane)*, parallel to plane *XZ* intersecting the point (2.5, 0, 0). Let us denote by S_x the transformation reflection to the plane *S*. Similarly, let us denote by S_Y the transformation reflection to the plane that is parallel to the plane *YZ* intersecting the point (2.5, 0, 0). Therefore, the former transformation represents the swap between the front and the back of the building geometry, while the latter transformation represents the swap between the two sides of the batch.

Let us denote by T_x the transformation shift into the direction of the origin and parallel to axis *X*, and similarly T_Y the transformation shift into the direction of the origin and parallel to axis *Y*.

To support the algorithmic determination of whether two batches are identical or not, as well as to help further considerations, a new theorem is introduced.

Theorem 1. *Each and every identical batch can be constructed from an initial batch with a 1–2 steps long series of transformations of type R, and/or S, followed by at most one TX and at most one TY transformation. If the first part is 2 long, it has the form of* $R \circ S$.

Proof of Theorem 1. Let us perform all *R* and *S* transformations first and finally perform the required shift by T_x and T_y. First, the explicit form of the transformations is given.

$$R_{360} = R_0 : \ (x, y, z) \rightarrow (x, y, z) = I \tag{5}$$

$$R_{90} = R_1 : \ (x, y, z) \rightarrow (width - y, x, z) \tag{6}$$

$$R_{180} = R_2 : \ (x, y, z) \rightarrow (width - x, depth - y, z) \tag{7}$$

$$R_{270} = R_3 : \ (x, y, z) \rightarrow (y, depth - x, z) \tag{8}$$

$$S_X : \ (x, y, z) \rightarrow (x, depth - y, z) \tag{9}$$

$$S_Y : \ (x, y, z) \rightarrow (width - x, y, z) \tag{10}$$

The properties of used linear transformation are:

$$X \circ I = I \circ X = X \ where \ X \in \{R, \ S, \ T\} \tag{11}$$

$$(E \circ F) \circ G = E \circ (F \circ G) \ where \ E, \ F, \ G \in \{R, \ S, \ T, \ I\} \tag{12}$$

$$R_i = R_{i+k*4} \ where \ k \in N^+ \tag{13}$$

Based on Equations (5)–(10) and (11)–(13), the following identities can be defined:

$$R_i \circ R_j = R_k \ where \ k = (i + j) \ \% \ 4 \tag{14}$$

$$S_X \circ S_X = I \tag{15}$$

$$S_Y \circ S_Y = I \tag{16}$$

$$S_X \circ S_Y = S_Y \circ S_X = R_2 \tag{17}$$

$$S_Y = S_Y \circ I = S_Y \circ S_X \circ S_X = R_2 \circ S_X \tag{18}$$

$$S_X \circ R_i \circ S_X = R_{(4-i)\ \%\ 4} \tag{19}$$

$$S_X \circ R_1 \circ S_X = R_3 \tag{20}$$

$$S_X \circ R_2 \circ S_X = R_2 \tag{21}$$

$$S_X \circ R_3 \circ S_X = R_1 \tag{22}$$

$$S_X \circ R_i = R_{(4-i)\ \%\ 4} \circ S_X \tag{23}$$

$$S_X \circ R_1 = R_3 \circ S_X \tag{24}$$

$$S_X \circ R_2 = R_2 \circ S_X \tag{25}$$

$$S_X \circ R_3 = R_1 \circ S_X \tag{26}$$

Let us suppose that there exists an initial arbitrary series of transformations of type R and S. With the repeated application of the identities (14)–(17), a series of transformations can be constructed in which transformations of type R and type S are located alternately, i.e., there are neither two type R, nor two type S transformations beside each other. With the application of identity (18), S_Y transformations are eliminated from the sequence by being replaced by $R_2 \circ S_X$. After the application of identity (14), a series of transformations is made where transformations of type R and type S_X are located alternately. After the application of identities (19)–(22), the resulting series of transformations will be shortened to the length of one or two steps, or the resultant series will have the form of $R_i \circ S_X \circ R_j$. Should it have the latter type, one of identity (23)–(26) then Identity 1 has to be applied. Thus, the final series is an equivalent of the original series of transformations, has one step or two steps and if it is two steps long, it has the form $R \circ S$. After the application of the reduced sequence, the necessary shift must be applied by T_X and T_Y if required. Therefore, the theorem has been proved.

After the batch generation process, the identity of generated batches must be investigated. This investigation is performed pairwise. The new batch is compared to all previously generated unique batches collected in a list. First, the orientation and then the positioning of batches must be adjusted, then congruency can easily be checked by comparing batch elements. On orientation adjustment, instead of finding the transformation sequence which could transform one batch to match others' orientation, based on the above-presented theorem, we create one and two-step long transformation sequences from R and S transformations. That will result in 12 transformations, explained in (27) (including Identity to also check the generated batch), for which transformation matrices could be pre-generated to make transformation faster.

$$|Identity| + |R| + |S| + |R| \cdot |S| = 1 + 3 + 2 + 3 \cdot 2 = 12 \tag{27}$$

After the transformation of the new batch, shifting initial and all transformed cases to the X and Y axes handles positioning differences as described in Equations (28) and (29).

$$T_x = min\left(C_{i_x}\right)\ ;\ \text{minimum of X coordinates of cells of batch} \tag{28}$$

$$T_Y = min\left(C_{i_Y}\right)\ ;\ \text{minimum of Y coordinates of cells of batch} \tag{29}$$

If none of the 12 new cases are found in the list of previously generated batches, the shifted initial batch can be saved as a new unique batch by adding it to the list. □

7. Configurations

With the presented method, instead of generating the total number of 201,359,550 batches, only a total number of 167 batch configurations were necessary to be generated (Figures 10–12). The building geometries (=batches) satisfying the rules are presented in the following figures in form of an overview. 18.6% (31 pieces) of the geometries are one-level (ground floor) building bodies, 62.8% (105 pieces) consist of two stories and the remaining 18.6%

(31 pieces) include three levels. The ground-level configurations represent a marginal part of the complete geometry package, based on the multiple, transformation-based congruency instances. The three-level shapes contribute with a smaller number of examples, mainly due to the 'tower'-rule (Rule 6) that limits the possible, acceptable block-unit constellations above the ground level. A major part of the shapes is two-level configuration, where a higher number of batch arrangements of blocks were possible. This is mainly caused by the possibility to set blocks with unconnected walls (only floor-roof connection between the stories), next, the 'balcony'-rule (Rule 4) enables further variations as well. According to statistics [38], the greatest number of family housing are two-level buildings with a sloped roof (with or without attic rooms) or a flat roof. This shows an interesting correlation since in this particular study the largest number of generated geometry configurations possess two-levels as well. However, to see whether the optimal solution belongs to this configuration group or not is subject to further simulation-based investigation.

8. Simulations and Results

The total number of 167 batch configurations (Figures 10–12) gained were modeled in an indoor climate and energy simulation framework (IDA ICE 4.8 SP2) to assess the energy performance of each configuration. At this stage of the investigations, the shape versions do not possess glazed façade openings and orientation settings, therefore no solar radiation-based cooling and lighting demand permutations evolve among the versions. Although this simulation setup is not realistic, it was chosen to emphasize the effect of configuration (building shape) on energy demand. Consequently, only the annual heating energy demand is of particular interest, because under the chosen climate conditions, heating energy consumption plays the greatest role in the yearly energy balance of the residential building. This character was underlined in a preliminary study [39], whereas the heating energy demand was weighted at 80% of the total energy requirement (heating + cooling + lighting). The structures of the geometries were modeled from conventional construction materials, and they all meet the minimum requirements of the Hungarian building energy regulations (7/2006 V.24. TNM) as follows

- External wall structure (from inside to outside), U_{wall} = 0.24 W/m^{2}K:
  - render 1.5 cm
  - brick wall 30.0 cm
  - external thermal insulation 8.75 cm
  - render 1.5 cm
- Slab on ground structure (from inside to outside), U_{floor} = 0.283 W/m^{2}K:
  - ceramic tiles 0.8 cm
  - concrete screed 8.0 cm
  - thermal insulation 10 cm
  - reinforced concrete 8.0 cm
  - consolidated layer of gravel (hardcore) 15 cm
- Flat roof structure (from top to bottom), U_{roof} = 0.17 W/m^{2}K:
  - consolidated layer of gravel (hardcore) 15 cm
  - thermal insulation 20 cm
  - reinforced concrete 20.0 cm
- Cantilever slab (floor) structure, U_{roof} = 0.17 W/m^{2}K:
  - ceramic tiles 0.8 cm
  - concrete screed 8.0 cm
  - thermal insulation 5.66 cm
  - reinforced concrete 15.0 cm
  - external thermal insulation 15 cm

Figure 13 displays the ordered list of simulated annual heating energy demands (grey and black colored graphs) of the configurations; in other words, it is visible how the building shape versions affect the results. The maximum value is 18.461 kWh/year while the minimum is 13.163 kWh/year. Thus, the deviation range is 5297 kWh/year. The plus-minus deviation of the heating demand values related to the average heating value of all configurations amounts to 16%. In addition, a 40% difference between the minimum (optimal) energy demand and the worst-case heating value can be observed, demonstrating how high the increment of heating demand develops by the suboptimal solution(s). Both results indicate a significant measure of how building shape permutation affects annual building heating energy performance.

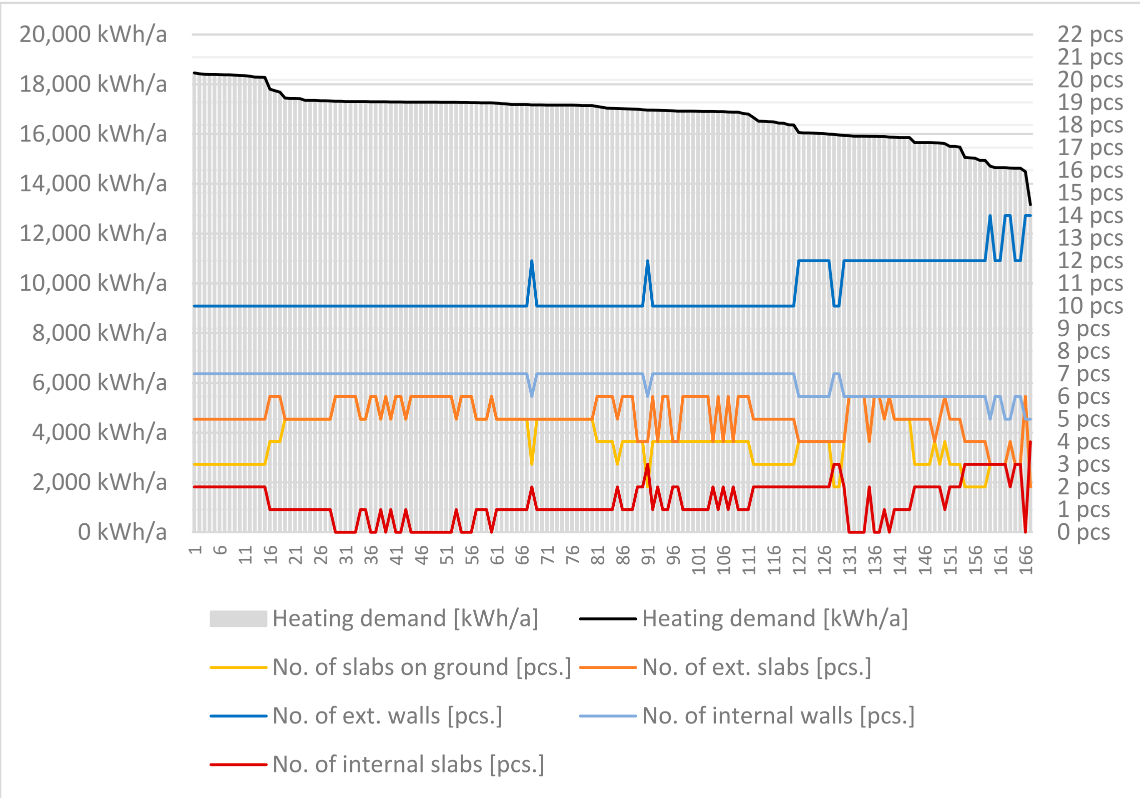


Figure 13. Heating energy demand [kWh/m^{2}a] of the 167 batches with indicated amount of external heat loss surface and internal connection surface in each configuration.

Figure 13 includes the heat loss surfaces of the building envelope structures, separated into vertical (external walls) and horizontal (external slabs and slabs on the ground) components. The number of external wall components (blue colored graphs) possesses no significant effect on the energy results, moreover, it has a counterproductive effect: the higher the number of external walls, the lower the heating demand develops. However, the reason for that is clearly seen in the graph of external slabs (yellow- and orange-colored graphs). Compared to an external wall, one slab measures a 40% larger transmission heat loss surface, therefore with decreasing number of external slab surfaces (more stories and hence a higher number of external walls in the shape configurations), the heating demand improves (becomes lower) as well. In other words, by increasing the number of internal slabs, the number of internal walls and transmission losses decrease as well. This interdependency offers the potential to elaborate an energy performance-related geometry generation rule, whereby for the number of horizontal internal connections of the space units (number of internal walls), as well as for the vertical connections of the space units (number of internal slabs) defined threshold values can be linked. Under a certain minimum and/or maximum threshold value, the given shape configuration is considered an energy-inefficient case during the geometry generation procedure. However, further investigation is required to gain precise statements about the modular connectivity rules, especially when in further simulation-based investigations the geometries are combined with further passive design elements (wall-to-window ratio, structures, materials, shading, orientation), as well as active planning factors such as HVAC and energy supply systems and operation control mechanisms. Interesting aspects may arise, for example, when the Weather Research and Forecast meteorological model (WRF) is used to predict future climate scenarios [40], or when state-of-the-art visualization techniques are applied to the simulation results [41].

9. Conclusions

In the framework of the given architectural problem, by using an appropriate algorithmic method, it is possible to specify the process of the guaranteed generation of all potentially feasible building configurations. The current study proposed a mathematically verified method that includes a modular space unit system (blocks) to arrange them in building shapes (batches) according to predefined architectural rules. The space organization codes served the goal to create only such shape configurations, which are feasible from the architectural point of view (e.g., appropriate compactness, etc.). All possible connection solutions of the space units can be ensured by a classical back-tracking algorithm to meet the prerequisites. To avoid congruency of the configurations and thus to further reduce the number of simulation cases, an algorithmic congruency detection is applied to check the shapes via a transformation analysis. Through changing the models by rotation, reflection, and shifting, the transformation of one batch into the other can be successfully proven. To further reduce computation time, the series of transformations can be shortened to the length of one or two steps by using an own developed theorem.

By using the above method, a significant reduction of the possible geometry variants can be carried out, in this particular case it is more than 99%. It can be stated that the guaranteed number (167) of geometry configurations was successfully generated, satisfying the given architectural rule system. The generation of all configurations, which meet the predefined rules, would logically result in the case of a changed predefined rule system another number of batch configurations.

Since scientific studies focus mainly on the numerically easy-to-define design variables such as heating, ventilation, and air conditioning (HVAC) system parameters, energy management system values as well as system operation data, and only a few efforts tried to integrate geometry-related architectural design variables, the present paper focuses on the effect of building shape on the annual heating energy demand. Thus, simulations were performed in a limited manner, i.e., without taking into account, for example, solar radiation by not using fenestration and/or main façade orientation. By simulating the most dominant (heating) energy performance of all gained batches, the energy-related significance of the building geometry modifications was decisively underlined.

In a future step, the result of this research enables the integration of the proposed generative step in a comprehensive sustainable building performance optimization system: by simulating the geometries equipped with glazed façade partitions (windows), diverse structures, materials, orientation, as well as varieties of HVAC and electrical systems or seasonal operation strategies (natural vs. mechanical ventilation, passive and active lighting, passive and active heating-cooling, etc.) all relevant passive, shape-related building design strategies, as well as active, system related solutions can be evaluated. This gives a helping hand to the architects to fully control their pursuit towards designing energy, and comfort-optimal residential buildings.

Author Contributions: Conceptualization, I.K. and Z.E.; Mathematical modeling, Z.E., T.S. and Z.K.; Software, Z.K.; Validation, B.D.; Architecture and simulation, K.R.H. and I.K.; Visualization, T.S.; Supervision, Z.E. and I.K. Founding, Z.E., I.K. and B.D. All authors have read and agreed to the published version of the manuscript.

Funding: The publication was granted by the Faculty of Engineering and Information Technology, University of Pécs, Hungary, within the framework of the “Call of Grant for Publication”.

Data Availability Statement: The building configuration data can be generated using the procedure described in this article. The stated thesis can be verified by the presented proofs and does not require a concrete implementation. The simulated energy data mentioned in the introduction are only used data, not relevant to the actual content, and therefore do not need to be shared.

Acknowledgments: This work was partially supported by the [2019-2.1.11-TÉT Bilateral Scientific and Technological Cooperation]. B.D. gratefully acknowledges the European Commission for funding the InnoRenew CoE project (Grant Agreement #739574) under the Horizon2020 Widespread-Teaming program, and the Republic of Slovenia (Investment funding of the Republic of Slovenia and the European Union of the European Regional Development Fund). He is also grateful for the support of the Slovenian National Research Agency (ARRS) through grant BI-HU/21-22-004.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

BECEDO	Building Energy: Comfort and Environmental Design Optimization
ED	Energy Design architectural design method
EDS	Energia Design Synthesis architectural design method
GA	Genetic Algorithm
HVAC	Heating, Ventilation and Air Conditioning
LCA	Building Life-Cycle Assessment
SN	Sequence Number
WRF	Weather Research and Forecast
WWR	Wall-Window Ratio

References

1. Cao, X.; Dai, X.; Liu, J. Building energy-consumption status worldwide and the state-of-the-art technologies for zero-energy buildings during the past decade. *Energy Build.* **2016**, *128*, 198–213. [CrossRef]
2. IEA; UNEP. *2019 Global Status Report for Buildings and Construction*; UNEP: Gigiri Nairobi, Kenya, 2019; Volume 224.
3. Hausladen, G.; Saldanha, M.; Liedl, P.; Sager, C. *Climate Design*; Birkhäuser: Basel, Switzerland, 2005.
4. Lemke, C.R. *Architectur Form & Solarenergie*; Cuvillier: Gottingen, Germany, 2009.
5. Nguyen, A.-T.; Reiter, S.; Rigo, P. A review on simulation-based optimization methods applied to building performance analysis. *Appl. Energy* **2014**, *113*, 1043–1058. [CrossRef]
6. Szalay, Z. Modelling building stock geometry for energy, emission and mass calculations. *Build. Res. Inf.* **2008**, *36*, 557–567. [CrossRef]
7. Baranyai, B.; Póth, B.; Kistelegdi, I. Planning and research of smart buildings and constructions with the 'Energydesign Roadmap' method. *Pollack Period.* **2013**, *8*, 15–26. [CrossRef]
8. Kistelegdi, I.; Horváth, K.R.; Storcz, T.; Ercsey, Z. Building Geometry as a Variable in Energy, Comfort, and Environmental Design Optimization—A Review from the Perspective of Architects. *Buildings* **2022**, *12*, 69. [CrossRef]
9. Mert, Y.; Saygın, N. Energy efficient building block design: An exergy perspective. *Energy* **2016**, *102*, 465–472. [CrossRef]
10. Granadeiro, V.; Duarte, J.P.; Correia, J.R.; Leal, V.M. Building envelope shape design in early stages of the design process: Integrating architectural design systems and energy simulation. *Autom. Constr.* **2013**, *32*, 196–209. [CrossRef]
11. Rodrigues, E.; Fernandes, M.; Soares, N.; Gomes, A.; Gaspar, A.R.; Costa, J.J. The potential impact of low thermal transmittance construction on the European design guidelines of residential buildings. *Energy Build.* **2018**, *178*, 379–390. [CrossRef]
12. Lopes, M.A.R.; Antunes, C.H.; Reis, A.; Martins, N. Estimating energy savings from behaviours using building performance simulations. *Build. Res. Inf.* **2016**, *45*, 303–319. [CrossRef]
13. Depecker, P.; Menezo, C.; Virgone, J.; Lepers, S. Design of buildings shape and energetic consumption. *Build. Environ.* **2001**, *36*, 627–635. [CrossRef]
14. AlAnzi, A.; Seo, D.; Krarti, M. Impact of building shape on thermal performance of office buildings in Kuwait. *Energy Convers. Manag.* **2009**, *50*, 822–828. [CrossRef]
15. Yu, Z.; Gou, Z.; Qian, F.; Fu, J.; Tao, Y. Towards an optimized zero energy solar house: A critical analysis of passive and active design strategies used in Solar Decathlon Europe in Madrid. *J. Clean. Prod.* **2019**, *236*, 117646. [CrossRef]
16. Ciardiello, A.; Rosso, F.; Dell'Olmo, J.; Ciancio, V.; Ferrero, M.; Salata, F. Multi-objective approach to the optimization of shape and envelope in building energy design. *Appl. Energy* **2020**, *280*, 115984. [CrossRef]
17. Kiss, B.; Szalay, Z. Modular approach to multi-objective environmental optimization of buildings. *Autom. Constr.* **2020**, *111*, 103044. [CrossRef]
18. Waibel, C.; Evins, R.; Carmeliet, J. Co-simulation and optimization of building geometry and multi-energy systems: Interdependencies in energy supply, energy demand and solar potentials. *Appl. Energy* **2019**, *242*, 1661–1682. [CrossRef]

19. Caldas, L.G.; Norford, L.K. Genetic Algorithms for Optimization of Building Envelopes and the Design and Control of HVAC Systems. *J. Sol. Energy Eng.* **2003**, *125*, 343–351. [CrossRef]
20. Saratsis, E.; Dogan, T.; Reinhart, C.F. Simulation-based daylighting analysis procedure for developing urban zoning rules. *Build. Res. Inf.* **2017**, *45*, 478–491. [CrossRef]
21. Michalek, J.; Choudhary, R.; Papalambros, P. Architectural layout design optimization. *Eng. Optim.* **2002**, *34*, 461–484. [CrossRef]
22. Javanroodi, K.; Nik, V.M.; Mahdavinejad, M. A novel design-based optimization framework for enhancing the energy efficiency of high-rise office buildings in urban areas. *Sustain. Cities Soc.* **2019**, *49*, 101597. [CrossRef]
23. Gan, V.J.; Wong, H.; Tse, K.; Cheng, J.C.; Lo, I.M.; Chan, C. Simulation-based evolutionary optimization for energy-efficient layout plan design of high-rise residential buildings. *J. Clean. Prod.* **2019**, *231*, 1375–1388. [CrossRef]
24. Yi, Y.K.; Malkawi, A.M. Optimizing building form for energy performance based on hierarchical geometry relation. *Autom. Constr.* **2009**, *18*, 825–833. [CrossRef]
25. Wang, W.; Rivard, H.; Zmeureanu, R. Floor shape optimization for green building design. *Adv. Eng. Informatics* **2006**, *20*, 363–378. [CrossRef]
26. Tuhus-Dubrow, D.; Krarti, M. Genetic-algorithm based approach to optimize building envelope design for residential buildings. *Build. Environ.* **2010**, *45*, 1574–1581. [CrossRef]
27. Bichiou, Y.; Krarti, M. Optimization of envelope and HVAC systems selection for residential buildings. *Energy Build.* **2011**, *43*, 3373–3382. [CrossRef]
28. Kistelegdi, K.H.; Ercsey, Z. Energia design synthesis. In Proceedings of the VOCAL 2018. 8th VOCAL Optimization Conference: Advanced Algorithms, Esztergom, Hungary, 10–12 December 2018.
29. Sadoughi, A.; Kibert, C.; Sadeghi, F.M.; Jafari, S. Thermal performance analysis of a traditional passive cooling system in Dezful, Iran. *Tunn. Undergr. Space Technol.* **2019**, *83*, 291–302. [CrossRef]
30. Ochoa, C.; Capeluto, I.G. Strategic decision-making for intelligent buildings: Comparative impact of passive design strategies and active features in a hot climate. *Build. Environ.* **2008**, *43*, 1829–1839. [CrossRef]
31. Friedler, F.; Tarján, K.; Huang, Y.; Fan, L. Graph-theoretic approach to process synthesis: Axioms and theorems. *Chem. Eng. Sci.* **1992**, *47*, 1973–1988. [CrossRef]
32. Cabezas, H.; Heckl, I.; Bertok, B.; Friedler, F. Use the P-graph framework to design supply chains for sustainability. *Chem. Eng. Prog.* **2015**, *111*, 41–47.
33. Ercsey, Z. Process network solution of a clothing manufacturer's problem. *Pollack Period.* **2017**, *12*, 59–67. [CrossRef]
34. Bertok, B.; Bartos, A. Renewable energy storage and distribution scheduling for microgrids by exploiting recent developments in process network synthesis. *J. Clean. Prod.* **2020**, *244*, 118520. [CrossRef]
35. Nagy, Z.; Ercsey, J.T.; Kovács, Z. Bus transport process network synthesis. *Acta Polytech. Hung.* **2019**, *16*, 25–43. [CrossRef]
36. Holczinger, T.; Ősz, O.; Hegyháti, M. Scheduling approach for on-site jobs of service providers. *Flex. Serv. Manuf. J.* **2019**, *32*, 913–948. [CrossRef]
37. Russel, S.; Norvig, P. *Artificial Intelligence A Modern Approach*, ISBN-13: 978-0-13-604259-4, 3rd ed.; Pearson Education, Inc.: London, UK, 1995.
38. American Housing Survey. Available online: https://www.census.gov/programs-surveys/ahs/data.html (accessed on 12 April 2022).
39. Kistelegdi, K.R.; Kovács, Z.; Ercsey, Z. Energy Design Synthesis For Buildings: An Example. In Proceedings of the 22nd Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction, Crete, Greece, 20–23 October 2019.
40. Giama, E.; Chantzis, G.; Kontos, S.; Keppas, S.; Poupkou, A.; Liora, N.; Melas, D. Building Energy Simulations Based on Weather Forecast Meteorological Model: The Case of an Institutional Building in Greece. *Energies* **2022**, *16*, 191. [CrossRef]
41. Vera-Piazzini, O.; Scarpa, M.; Peron, F. Building Energy Simulation and Monitoring: A Review of Graphical Data Representation. *Energies* **2022**, *16*, 390. [CrossRef]