



Optimal Power Flow in Distribution Network: A Review on Problem Formulation and Optimization Methods[†]

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Abstract: Distributed generators (DGs) have a high penetration rate in distribution networks (DNs). Understanding their impact on a DN is essential for achieving optimal power flow (OPF). Various DG models, such as stochastic and forecasting models, have been established and are used for OPF. While conventional OPF aims to minimize operational costs or power loss, the "Dual-Carbon" target has led to the inclusion of carbon emission reduction objectives. Additionally, state-of-the-art optimization techniques such as machine learning (ML) are being employed for OPF. However, most current research focuses on optimization methods rather than the problem formulation of the OPF. The purpose of this paper is to provide a comprehensive understanding of the OPF problem and to propose potential solutions. By delving into the problem formulation and different optimization techniques, selecting appropriate solutions for real-world OPF problems becomes easier. Furthermore, this paper provides a comprehensive overview of prospective advancements and conducts a comparative analysis of the diverse methodologies employed in the field of optimal power flow (OPF). While mathematical methods provide accurate solutions, their complexity may pose challenges. On the other hand, heuristic algorithms exhibit robustness but may not ensure global optimality. Additionally, machine learning techniques exhibit proficiency in processing extensive datasets, yet they necessitate substantial data and may have limited interpretability. Finally, this paper concludes by presenting prospects for future research directions in OPF, including expanding upon the uncertain nature of DGs, the integration of power markets, and distributed optimization. The main objective of this review is to provide a comprehensive understanding of the impact of DGs in DN on OPF. The article aims to explore the problem formulation of OPF and to propose potential solutions. By gaining in-depth insight into the problem formulation and different optimization techniques, optimal and sustainable power flow in a distribution network can be achieved, leading to a more efficient, reliable, and cost-effective power system. This offers tremendous benefits to both researchers and practitioners seeking to optimize power system operations.

Keywords: optimal power flow; distributed generators; distribution network; mathematical optimization; artificial intelligence



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1. Introduction

The proportion of DGs connected to DNs has increased significantly due to the implementation of the "Dual-Carbon" strategy. DGs, which typically consist of photovoltaic cells (PVs) and wind turbines (WTs), offer clean and renewable energy. However, the output from these DGs is often intermittent, resulting in challenges for grid stability. To address these challenges, a battery energy storage system (BESS) is commonly used to smooth out the output curve. The integration of DGs has transformed the traditional DN model from a passive energy receiver into an active network that participates in energy exchange. The energy exchange of DGs is typically achieved through power routers (PRs). OPF is a common technique used to reduce network loss and the cost of DN [1,2]. The objective of OPF is to minimize system losses and to allocate power rationally among different nodes while ensuring system safety and stability. In addition to these objectives, carbon emissions [3] are also considered as optimization targets.

To achieve OPF with DGs, understanding their impact on DNs is of utmost importance. Ref. [4] investigated the relationship between nodal voltage and the placement of DGs. Two types of DG models, namely stochastic and forecasting, were established to account for their uncertainty. Additionally, the combination of OPF with the electricity market is a hot topic, with recent studies, as seen in references [5–7], establishing a demand response (DR) model for calculating locational marginal prices (LMP) and scheduling DG output.

The power flow model, which is the basis of the OPF problem, is also reviewed. The model can be categorized into conventional nonlinear and linear approximation models. The conventional model can reflect the power flow distribution directly, but its nonlinear nature makes it less desirable for calculations. Linear models, on the other hand, can speed up the calculation process but may lose some accuracy. In addition, new component models have been added to the equality and inequality of the model to reflect the new components in DNs.

After defining the basic problem of OPF, various methods for solving the OPF problem are reviewed. However, changes and uncertainties in energy demand, energy flow, and load changes in active distribution networks (ADNs) can lead to voltage offset, frequency dislocation, and power losses at various nodes in the network. As a result, the scheduling and optimization of ADNs can be complex. To address these challenges, it is necessary to optimize the energy distribution and load control of ADNs through an optimal power flow analysis.

In this paper, state-of-the-art methods for solving the OPF problem are reviewed. The optimization methods reviewed in this paper are divided into four categories: mathematical, heuristic, ML, and mixed methods. Each method has its own strengths and limitations, and its suitability depends on the specific characteristics of the ADN under analysis.

Mathematical optimization methods for solving the OPF problem include conventional OPF, Alternating Direction Method of Multipliers (ADMM), Mixed Integer Linear Programming (MILP), Semi-Definite Programming (SDP), Second-Order Conic Programming (SOCP), Quadratic Programming (QP), and others. While mathematical optimization methods can provide rapid convergence and are easy to implement, they may require significant computation resources. The primary concept behind mathematical optimization is to transform the original non-convex OPF model into convex models, thus improving the accuracy and convergence time of the model.

Heuristic optimization methods for solving the OPF problem include Sunflower Optimization (SFO), Particle Swarm Optimization (PSO), Genetic Algorithm (GA), and others. Heuristic optimization methods can handle non-convex and nonlinear problems, but they may not guarantee convergence or optimality and may depend on random factors. These methods reveal the design principles of optimization algorithms by understanding the behavior, function, experience, rules, and mechanisms of action in biological, physical, chemical, social, artistic, and other systems or domains. Under the guidance of specific problem characteristics, the corresponding feature models are refined to design intelligent iterative search-type optimization algorithms.

To the best knowledge of the authors, there is no previous review that extensively covers problem formulation and optimization methods for the OPF problem. Refs. [8–14] have reviewed optimization methods to solve OPF problem, Ref. [12] has reviewed heuristic and conventional algorithms in the DN while introducing some basic principles of OPF; Ref. [8] focused on the mathematical model of OPF, but has not considered other methods or modeling of new models in DN. In their articles, Refs. [13,14] individually reviewed the ADMM and heuristic methods for OPF. However, it is important to note that these reviews do not encompass the entirety of available optimization methods for OPF. Refs. [9,10] mainly focused on the problem formulation and the benefits of different methods but only presented a table for the drawbacks without providing a detailed explanation of how the algorithms work. On the other hand, Ref. [11] explained and analyzed some of the methods through case studies but included only a few methods and only in a certain network. In summary, this paper makes a significant contribution by addressing the lack of extensive coverage in previous reviews regarding the problem formulation and optimization methods for the OPF problem. While several existing reviews have reviewed optimization methods for OPF, they either focus on specific aspects or do not provide a comprehensive analysis. Some reviews cover heuristic and conventional algorithms, others focus on mathematical models, and a few address individual methods such as ADMM. However, none of these previous reviews encompass the entirety of available optimization methods for OPF or provide a detailed explanation of algorithm workings. Moreover, the limited case studies conducted in previous research further highlight the need for a more comprehensive and holistic assessment. Therefore, this review paper fills an important gap by providing a comprehensive analysis of problem formulation and optimization methods, establishing its novelty and significant value in the field.

The main contributions of this paper are detailed next.

- Through a comprehensive analysis of the model development and optimization methods for the optimal power flow problem in distribution power systems, this study provides a profound understanding and valuable references for solving this problem. Therefore, it establishes a solid theoretical framework for promoting the application and advancement of optimal power flow solutions in distribution power systems, thus making notable contributions to this field.
- A detailed discussion of the power flow and component models necessary for implementing the various applications mentioned above is provided.
- Furthermore, future opportunities and challenges for the application of OPF in DN are identified.

The organization of this paper is as follows: the OPF problem is formed in Section 2, then optimization approaches are reviewed in Section 3, and the advantage or disadvantages of these approaches and future works are discussed in Section 4. The reviewed OPF methods are shown in Figure 1.



Figure 1. Reviewed OPF formulation and methods.

2. Problem Formulation

In general, distribution networks comprise loads, power lines, transformers, and DGs. However, the widespread deployment of DGs such as PV and WT, in addition to the use of various new smart meter technologies, has played a crucial role in transforming distribution networks. As a result, the nature of the distribution network has significantly changed.

2.1. Impact of DG in DN

When DGs are connected to DNs, they typically have an impact on nodal voltage and power loss in the network. A comparison of DNs before and after any DGs are connected to the DN is shown in Figures 2 and 3.



Figure 2. DN with DGs.



Figure 3. DN without DGs.

In [4], the influence mechanism of DGs on reactive power and voltage characteristics of DNs was explained. Additionally, Ref. [15] showed that the placement of loads in different positions can have varying impacts on voltage stability. Loads located too close to the substation can cause the voltage to be too high and can overload the load, while loads located too close to the end of the line can cause the voltage to be too low and can create an unstable power supply.

To determine the optimal placement of DGs, Ref. [16] evaluated the impact of solar PV systems on the reliability of power distribution systems using various methods. They used energy not supplied (ENS) as the primary indicator to evaluate the impact. The results revealed that distributed solar PV input significantly affects the ENS value, and the ENS value was reduced by 50% when the capacity factor of the solar PV system was equal to

one. The researchers also assessed the impact of installing solar PV systems under different conditions and further evaluated the improvement in grid reliability by adding energy storage facilities.

In [17], the impact of integrating squirrel cage induction generator (SCIG) WTs at different positions on the power system was investigated. The study concluded that integrating SCIG WTs into the power system can achieve better voltage and can reduce power losses. However, voltage stability issues may arise when SCIG WTs are integrated into the power system at certain positions.

2.2. Generator Modeling

Due to the uncertainty aspect of PVs and WTs, the uncertainty of DGs can be addressed in the following ways: stochastic and forecasting models. A brief summary of the two models is expressed in Figure 4.



Figure 4. Brief summary of DG models.

2.2.1. Stochastic Model

The output of DGs is a stochastic event and, hence, can be expressed by a set of random variables. This uncertainty arises from various factors such as weather conditions, load variations, and system disturbances. In their research, Ref. [18] addressed the uncertainty of power outputs of DGs based on probability. They proposed a probabilistic approach to model the output fluctuations of DGs within a certain range. They noted that the output of DGs fluctuates within a range, and thus, they proposed a probabilistic approach to model the uncertainty.

Similarly, in their study, Ref. [19] expressed uncertainty via a range of random variables, which were observed over a specified time interval. The goal of these approaches is to effectively capture the uncertainty associated with the output of DGs, thereby supporting the optimization of power system operations.

Stochastic models offer several advantages in the evaluation of the reliability and stability aspects of photovoltaic systems. By analyzing probability distributions, these models facilitate the identification and resolution of potential issues. Moreover, they consider uncertainties associated with DGs, effectively incorporating risk factors into system design and planning. This integration enhances the reliability of decision-making processes. Furthermore, stochastic models are particularly well suited for long-term planning purposes. Leveraging historical data, these models can forecast and evaluate future power generation over extended time periods, providing valuable guidance for system design and investment decisions.

However, stochastic models also have certain limitations. They heavily rely on the quality and reliability of available data, making them highly dependent on data quality. Insufficient or inaccurate data can adversely impact the accuracy and efficacy of probability models, potentially leading to flawed results and incorrect applications. Furthermore, stochastic models prove inadequate in handling short-term fluctuations and exceptional cases. Due to their inherent characteristics, these models encounter challenges in effectively addressing short-term power generation variations and individual anomalous situations. Consequently, these limitations can introduce forecasting biases and should be carefully considered when utilizing stochastic models in practice.

2.2.2. Forecasting Model

Utilizing the relation between weather conditions and output, it is possible to establish a mathematical function to estimate the DG output. Ref. [5] estimated the output of DG by using the weather conditions. The active power output of a PV module was determined based on the input solar irradiance and ambient temperature. The equation was used in the paper to generate an active power curve that corresponds to the solar irradiance and ambient temperature data entered [20]. The commonly used PV power prediction formula calculates the predicted PV power (PV) based on weather conditions such as solar radiation intensity (G) and PV panel temperature (T). The formula is as follows:

$$PV(t) = A \times G(t) \times (1 - \beta \times (T(t) - T_{ref}))$$
(1)

where PV(t) represents the predicted PV power, A is the rated power or area of the PV panel, G(t) is the solar radiation intensity, T is the PV panel temperature, T_{ref} is the reference temperature, and β is the temperature coefficient.

The commonly used wind turbine power prediction formula calculates the predicted wind turbine power *P* based on weather conditions such as wind speed (*V*), air density (ρ), and the rated power of the turbine (P_{rated}). The formula is as follows:

$$P = 0.5 \times \rho \times A \times C_p \times V^3 \tag{2}$$

where *P* represents the predicted wind turbine power, ρ represents air density, *A* represents the swept area of the turbine blades, C_p is the power coefficient, and *V* represents the wind speed.

Forecasting models offer several advantages in the context of DGs. These models leverage advanced algorithms and real-time data to provide accurate short-term predictions, enabling informed decision-making in electricity market operations and real-time dispatch. By considering multiple influencing factors such as weather conditions, seasons, and system parameters, forecasting models enhance the accuracy and reliability of predictions. Additionally, these models support intelligent operation and optimization, providing decision support for power plant operations to optimize generation scheduling and to improve energy utilization efficiency. However, forecasting models also have certain limitations that should be taken into account. They require high-quality and timely input data to ensure accuracy and reliability. Particularly, accurate weather data and real-time operational data are crucial for achieving optimal performance. Moreover, the establishment and maintenance of high-quality forecasting models involve substantial costs. These costs include the acquisition of extensive data, the implementation of complex algorithms, and the allocation of appropriate computational resources. Therefore, cost-related challenges can arise in the development and maintenance phases. Finally, forecasting models are unable to account for future uncertainties, such as unforeseen weather events. While they provide accurate short-term predictions, unexpected uncertainties in the future cannot be accurately predicted using these models.

2.3. Demand Response

DR is a mechanism by which power users receive direct notifications or price increase signals from the power supplier to induce load reduction when the wholesale market price of electricity rises or the system reliability is threatened. This allows users to modify their inherent electricity consumption habits and to reduce or shift their electricity load over a certain period in response to the power supply, ensuring the stability of the power grid and suppressing short-term behavior of electricity price increases.

In [5–7], the commonly used model for the OPF problem was the electricity elasticity model, which changes the price of electricity for consumers. The article [5] provided an implementation of electricity elasticity in a DR model. The DR model assumes that multiple consumers are connected to each node and that their loads are classified as either flexible or non-flexible. Furthermore, this paper considered the difference between expected load and actual load to ensure the accurate implementation of DR.

Another approach presented in [6] used electricity prices to build a DR model that modifies the load curve to obtain an equivalent daily load curve. The model considers the elasticity coefficient of the electricity price, which measures the sensitivity of electricity demand to price changes. The elasticity coefficient matrix was used to model user demand response behavior, ensuring that electricity demand in each period is related not only to the current price but also to the electricity price of other periods. This approach is particularly effective when Time-of-Use (TOU) rates are adopted.

Finally, the paper [7] proposed a combination of DR and OPF problems, the former being combined with LMP through a Lagrangian function in the latter. This approach optimizes load management by ensuring that LMP reflects the real cost of operating the power system while optimizing the use of flexible loads.

2.4. Power Flow Model

Power flow calculations, which involve solving complex equations and handling large-scale networks, often require significant computational time and resources. Ref. [21] introduced a general power flow calculation method for DN with DGs and voltage regulators. The power line can be represented by the π model. Transformers can be represented by admittance matrices, while the load can be represented by a vector. The voltage output of a step-voltage regulator can be described using a function of tap, and the impedance and voltage drop of a compensator can be computed using the ratio of the voltage transformer and current transformer and the impedance of the line. A brief summary of the power flow models is expressed in Figure 5.



Figure 5. Brief summary of power flow models

In [11], the Backward/Forward Sweep (BFS) method, a well-established and highly regarded power flow calculation algorithm, was shown to be a popular choice for accurately evaluating power flows in distribution networks. This method was used to calculate the power flow in the network. In [22], the Bus Injection Model (BIM) was discussed, which is used for analysis and optimization using nodal variables such as voltages, current, and power injections. This model does not directly deal with power flows on individual branches.

Based on the conventional power flow model, several approximation models have been established. The simplest approximation is to ignore some nonlinear factors in power flow calculations. In [1], a linearized power flow model was introduced, which assumes the network to be lossless. In [23], a linearized model was proposed that assumes the voltage drop to be small and uses per-unit (p.u.) values to replace real values. The model also introduces loss factors for distribution systems, which linearizes and divides the distribution grid into a few sub-areas to improve accuracy and efficiency. In [24], the Zero-Inflated Poisson (ZIP) model was combined with the current injection method to linearize the four-wire system. In [25], two linearizations based on polar coordinates were mentioned. The first approximation considers the phase angle deviation caused by impedance to be small, making sin θ 0 and cos θ 1 [26]. The second approximation is based on the same assumption; [27] proposed a novel power flow model, the decoupled linearized power flow model, which is state-independent and highly accurate in voltage magnitude.

Another approach to obtain linear equations is to use other methods for linearization. In [28], an approximate model for the power flow problem in three-phase unbalanced ADNs was proposed based on the Wirtinger model. In [29], a linearization method for three-phase unbalanced power systems was proposed. The method linearizes the magnitude and phase of voltage on the complex plane by representing small perturbations in voltage amplitude and phase as complex numbers and by performing a Taylor series expansion on the complex numbers to approximate the voltage changes. In [25], Taylor expansion was used for DC networks. Certain variables, such as voltage magnitude or voltage squares, are treated as independent variables, and the power flow equations are expressed using these variables to obtain linear formulas.

Similarly, Ref. [7] used first-order Taylor expansion for power loss linearization. Ref. [30] used a linearized Euclidean norm for branch current magnitudes and nodal voltages. The voltage angle range was limited for better accuracy, and an approximation of DG power injection was acquired via first-order expansion of the estimated value. In [23,31], a linearized power flow model was proposed. The model was linearized based on the loss factor-based linearization method, which is commonly used in most electricity markets in the U.S. The losses are naturally quadratic and cannot be linearized using a cold start in a non-iterative manner. Therefore, the loss factor-based linearization method was used to facilitate the fully linear formulation of the network model. The losses were linearized based on a base case system operating condition.

The conventional power flow calculation model offers high accuracy and reliability, making it suitable for various scales and types of power systems. However, it is limited by its high computational complexity and stringent data requirements, making it unsuitable for real-time calculations. Conversely, the approximate power flow calculation model provides fast computation speeds and is applicable for real-time calculations.

Considering different scenarios, some devices are used for optimization, these devices need to be modeled and added to the power flow model:

- Various models have been proposed from the grid perspective to enhance and optimize power system operations. These models include the superconducting cable model, voltage regulator model, flexible loop converter, and energy router model. Each model serves a specific purpose, such as improving transmission efficiency, voltage regulation, and interconnection between microgrids. In [32], the zero-resistance characteristic of superconducting material was considered, and the superconducting magnet parameters were introduced to reflect the superconducting properties. The modeling of the superconducting cable was constructed using a nonlinear inductance, current source, and leakage resistance to model the cable as an element within the circuit analysis. Voltage regulators play a crucial role in maintaining stable power transmission by adjusting the voltage levels of transformers. These devices ensure that voltage remains within the desired range, mitigating potential issues associated with voltage fluctuations. The power flow calculation method proposed in [21] considers the influence of distributed generation and voltage regulators. The method adds the power equation of distributed generation to the power flow equation of the system and uses the three-phase power injection method in the calculation, which overcomes the convergence problem that traditional power flow calculation methods face. In [33], the modeling of step voltage regulators (svrs) was achieved by assuming that the svr is an ideal component and by modeling it as a three-phase element. The series impedance of the single-phase autotransformer of the svr wes assumed to be zero. The Virtual Power Plant (VPP) model was proposed in [34], which allows for the integration of distributed energy resources as a virtual unit and can participate in the energy market. To interconnect microgrids, power routers must be used. In [35], a stable model for PRs was proposed based on the steady-state power flow calculation model. The line structure in the hybrid AC/DC distribution system based on PR is divided into eight types according to the bus type at both ends of the line and the line type. The power transaction between microgrids relies on energy routers, as proposed by [36]. The energy router is modeled using the port-bus incidence matrix by expressing the energy router using nodal currents and by applying constraints such as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) before embedding it into the existing power system power flow model.
- The flexible closed-loop converter, constructed using power electronic devices, can achieve DC transmission and can solve the impact of closing AC load loop on the power grid, providing ideas for the closed-loop control of distribution networks. According to [37], there are four operational modes for the flexible closed-loop converter:

closed-loop operation mode, power flow transfer mode, circulating current limiting mode, and power flow control mode. The power flow transfer and power flow control modes involve solving the power flow equations. In the power flow transfer mode, when a line of a substation fails or stops operating due to maintenance, the closed-loop controller transfers the remaining load to another line. In the power flow control mode, the closed-loop controller outputs adjustable voltage amplitudes and phases according to the power demand of the secondary power supply line, such as load balancing and minimum line loss, and inserts it into the line for the control of four-quadrant power flow.

- From the regional perspective, a Community Load Aggregator (CLA) is established. In some studies, a whole residential area is considered to be a node. Ref. [38] established the CLA model by integrating and modeling the residential load, electric vehicle load, and communication load in a community. The model is divided into static and dynamic loads, where static loads refer to the residential and communication loads that have relatively stable characteristics, while dynamic loads refer to the electric vehicle load, which has significant time-varying characteristics and requires optimized scheduling strategies.
- For a single node, the Soft Opening Point (SOP) model and the State of Charge (SOC) model are established. Ref. [39] modeled SOP and estimated the loss of SOP with the least square estimator. To describe the status of energy storage of a single nodal DG, Ref. [40] used SOC. The formula was used to determine the SOC of a battery based on the battery's charging and discharging power, charging and discharging efficiency, rated capacity, and calculation time interval.

2.5. Objective Function

There are two main types of objective functions with constraints: one considers network loss, while the other also considers the cost of generators and node voltages. A brief summary of the objective functions is expressed in Figure 6.

In [29], the objective function was set as the minimization of power loss after deploying DGs in a distribution network. In [41], the objective function was adjusted to include power loss as a Lagrangian function for ADMM. In [2], power transfer distribution factors (PTDFs) were introduced to calculate the estimated contributions of changes in power output from each generator to each flow into the corresponding transmission line/interface based on the topology of the power system's transmission network. This approach is used to establish mathematical relationships between characteristics such as line resistance and node stress, representing the interconnectivity and power transfer capacity between nodes and lines. In [34], the objective function was set as the Source–Load–Storage Matching Index, which aims to allocate distributed energy resources (DERs) to support the grid's load demand while minimizing the economic cost. In [23,42], not only power loss but also the LMPs were considered for pricing. The LMPs reflect the power production and demand characteristics of the network, and the pricing information can provide economic incentives to encourage optimal utilization of resources.

In addition to traditional objectives such as power loss and cost, carbon emission is also being considered in power system optimization due to the "Dual-Carbon" policy. For example, in [43], an electrothermal model for thermal loads in DNs was established, and a carbon dioxide emission cost was put into consideration when optimizing, in addition to the general loss and cost functions. Similarly, in [44], the effect of temperature on transmission line load was considered, and the goal was to reduce power flow losses accordingly. In [3], the carbon dioxide generated when renewable generators cannot supply the loads was considered as a goal when optimizing. In [45], security was also introduced as one of the objectives of power system optimization.



Figure 6. Brief summary of objective functions [2,41,43–45].

3. Optimization Methods

This paper discusses three optimization methods for the OPF problem: mathematical optimization, heuristic optimization, and ML-based optimization. While mathematical optimization is a widely used approach for solving OPF problems, heuristic optimization methods are known to provide more flexibility and dependability. In recent years, ML-based optimization has emerged as a promising technique for handling complex and large-scale OPF problems. Thus, the selection of the most appropriate optimization method depends on the specific problem requirements and the available resources for implementing the solution. In this section, optimization methods are introduced and compared.

3.1. Mathematical Approach

OPF models are often solved using mathematical methods, which are scalable and easy to implement. While the simplest and most direct way to solve the OPF model is to do so directly without an optimization model, this approach can be impractical due to the model's non-convex and nonlinear nature requiring significant computational resources. The conventional OPF problem can be expressed in (3)

min
$$F(x)$$

s.t. $P_i - P_{Di} + \sum (G_{ij} \times V_i \times V_j - B_{ij} \times V_i \times V_j \times \cos(\theta_i - \theta_j)) = 0$
 $Q_i - Q_{Di} + \sum (G_{ij} \times V_i \times V_j \times \sin(\theta_i - \theta_j) + B_{ij} \times V_i \times V_j \times \cos(\theta_i - \theta_j)) = 0$
 $P_i(V, \delta) = P_i^G - P_i^L \forall i \in \mathbf{N}$
 $Q_i(V, \delta) = Q_i^G - Q_i^L \forall i \in \mathbf{N}$
 $P_i^{G,\min} \le P_i^G \le P_i^{G,\max} \forall i \in \mathbf{G}$
 $Q_i^{G,\min} \le Q_i^G \le Q_i^{G,\max} \forall i \in \mathbf{G}$
 $V_i^{\min} \le V_i \le V_i^{\max} \forall i \in \mathbf{N}$
 $\delta_i^{\min} \le \delta_i \le \delta_i^{\max} \forall i \in \mathbf{N}$

F is the objective function to be optimized, for example, minimizing the total generation cost or minimizing the transmission loss. P_i and Q_i are the active and reactive power injections at node *i*, respectively. $P_D i$ and $Q_D i$ are the active and reactive load demands at node *i*, respectively. G_{ij} and B_{ij} are the admittance matrix elements between node *i* and node *j*. V_I and v_j are the voltage between node *i* and node *j*.

Combining conventional OPF problems with power flows, the direct solution can be sped up. In [46], the Forward/Backward Sweep (FBS) load flow algorithm and the Exhaustive Search Algorithm were used to compute load flow parameters and to inject optimal reactive power in the distribution system. In [23], the nonlinear OPF problem was transformed into a convex optimization problem. In [47], based on the linear power flow model, the power flow model was iteratively calculated, starting with no loss. The criterion for convergence was the maximum voltage deviation and error.

Other techniques to reduce power loss were also adopted. In [48], network reconfiguration was achieved during the optimization process by strategically opening or closing certain lines in the power system's topology to alleviate congestion and to reduce overall system costs following N-1 contingencies. In [49], two new linear approximation methods were proposed, i.e., the Enhanced DC (EDC) OPF and Coupled OPF (COPF) approximations. The EDC OPF method considers the impact of resistance by establishing an equivalent inductance between resistance and inductance. Specifically, for a given line with resistance and reactance, an equivalent inductance is introduced to simplify the calculation. This improves the accuracy of the model and makes the EDC OPF method more applicable to distribution network optimization problems. The COPF method can be regarded as an improved version of the EDC OPF by optimizing the entire network OPF problem to minimize the potential energy loss of the base station while considering the characteristics limitation of each component, such as load and converter, in the system. To achieve fault current-constrained optimal power flow on unbalanced distribution networks, the algorithm presented in [50] encodes the full short-circuit current flow as a constraint. This constraint limits fault current by adjusting the DG set points to redistribute power while ensuring that load is supplied. The method is applicable to generalized unit commitment and determines the optimal configuration for generation deployment while limiting fault current at fault locations.

As mentioned in Section 2, the uncertainty of DGs can be expressed with the stochastic model. The OPF problem can utilize this model for optimization. In [51], a probabilistic model based on the traditional static power flow optimization mathematical model was constructed, considering the uncertainty of photovoltaic power generation and electric vehicle loads. The model uses a stochastic optimization algorithm to solve the problem and describes the probability distribution of the total network loss using probability density curves and cumulative distribution curves.

To achieve multiple objectives, the conventional OPF model can be adjusted. In [52], a weighted sum method was used to combine different optimization objectives, resulting in a comprehensive single-objective problem. The Newton–Raphson (N-R) method was employed to solve the problem. Ref. [53]'s objective function was to increase the maximum utilization ratio of DR in the voltage and reactive power optimization process. Capacitor banks (CBs), reactor banks (RBs), and static var generators (SVGs) as well as DR are used as control variables to optimize the voltage and reactive power of the power grid, aiming to achieve the best regulation effect. Ref. [54], based on the three-phase optimal power flow model at medium voltage level, adjusted the output of distributed power sources in the medium voltage distribution network based on targets such as network losses, voltage offsets, and distributed power consumption.

The conventional OPF model could achieve the optimal value of a certain performance indicator of the system while satisfying all operating constraints; however, in some cases, this optimization problem itself may not have a solution. To further improve the convergence time of the OPF problem, researchers have taken measures to convert OPF models into mathematical models like QP, SDP, ADMM, etc.

3.1.1. Quadratic Programming

QP is the process of solving certain mathematical optimization problems involving quadratic functions. Specifically, one seeks to optimize (minimize or maximize) a multivariate quadratic function subject to linear constraints on the variables. To convert a conventional OPF problem into a QP model, we can express it using Formula (4):

min
$$\frac{1}{2}x^{\top}Px + q^{\top}x$$

s.t. $Ax = b$ (4)

where *x* is a vector containing the variables to be solved, typically representing power generation, node voltages, or other relevant quantities. *P* is a positive semi-definite matrix that describes the quadratic coefficients in the objective function. *q* is a vector representing the linear coefficients in the objective function. *A* is the constraint matrix that defines the physical constraints related to the power system. *b* is a column vector representing the right-hand side of the equality constraints. Ref. [55] transformed the general AC OPF model into a QP model by making the variables that were originally *V* (voltage), *I* (current), *P* (power), and $a/d \times Q$ (var) into V^2 , I^2 , P, and Q for QP. Ref. [56] applied a linear PF model and then a combination of QP and Benders decomposition for acquiring the solution. Ref. [57] proposed a convex iteration technique to simultaneously achieve optimal and feasible solutions. The algorithm obtains linear inequality constraints by analyzing the quadratic equality constraints mathematically, ensuring problem feasibility.

For multi-objective optimization, some improved QP methods have been introduced. Ref. [58] proposed a method for simultaneously optimizing the location and capacity of DGs in radial power distribution networks using a Mixed Integer Linear Programming (MIQP) approach. The optimization formula of the MIQP was constructed by utilizing the modified DistFlow method and by linearizing products of real and binary variables.

Multi-objective optimization would add constraints to the model. The model construction involves two stages, where the first stage is used to determine the upper and lower bounds of the McCormick envelopes proposed in [59] and the environmental variables, and the second stage involves eliminating the non-convexity introduced via the McCormick relaxation.

QP offers several advantages, including the ability to accurately solve OPF problems with quadratic objective functions and linear constraints, along with a faster computation speed compared with other methods, particularly for moderate-sized OPF problems. Additionally, QP methods allow for the transformation of the OPF problem into a linear model, resulting in a more efficient and concise modeling and solving process. However, there are also some disadvantages to consider. QP methods may encounter numerical instability, requiring additional efforts for stability checks and handling. They may also have higher memory requirements for large-scale OPF problems, as storing related matrices and vectors consumes computational resources. Furthermore, QP methods face limitations in solving non-convex OPF problems, finding only local optimal solutions instead of global ones.

3.1.2. Alternating Direction Method of Multipliers

ADMM is a powerful algorithm used to solve convex optimization problems by decomposing them into smaller and more manageable subproblems. The solutions to these subproblems are then coordinated to find a global solution to the original problem. In the context of the OPF problem in power systems, ADMM has been implemented using various techniques to enhance convergence speed and accuracy. These techniques include the use of different penalty methods, employing preconditioning techniques, adjusting the step size parameter in the algorithm, and incorporating real-time measurements and control strategies into the ADMM framework. To transform the conventional OPF problem into ADMM form, auxiliary variables and an augmented Lagrangian function were introduced, as shown in Formula (5):

$$L(x, z, u) = F(x) + u^{\top} (Ax - z) + (\rho/2) ||Ax - z||^2.$$
(5)

Here, auxiliary variable $z = [P, Q, V, \theta]$, the multiplier variable $u = [u_P, u_Q, u_V, u_\theta]$, A is the linear transformation matrix, ρ is a non-negative multiplier factor, and ||.|| denotes the Euclidean norm.

Repeated ADMM algorithm steps are as shown in Formula (6):

$$x_{k+1} = \arg\min_{x} L(x, z_k, u_k)$$

$$z_{k+1} = \arg\min_{z} L(x_{k+1}, z, u_k)$$

$$u_{k+1} = u_k + \rho(Ax_{k+1} - z_{k+1}).$$
(6)

Through the iterative process of ADMM, it can transform the conventional OPF problem into a series of subproblems and gradually approach the optimal solution by alternating between optimizing variables, auxiliary variables, and multipliers. In each subproblem, various mathematical optimization techniques can be used, such as gradient methods, interior point methods, and others.

One technique involves using a variable separation method to relax the original problem, as discussed in [60]. Then, the relaxation variables and multipliers are updated iteratively through the ADMM process, using the augmented Lagrangian method to handle the relaxed constraints. These improvements have been shown to enhance the effectiveness and efficiency of the algorithm.

In another approach, as discussed in [61], conventional OPF was converted to SDP-OPF by adopting semi-definite relaxation. The model was then given to an accelerated ADMM algorithm, which breaks the OPF problem into a few subproblems, each of which adopts a self-adapted penalty parameter. This technique has shown promising results in solving complex optimization problems in power systems.

A few improvements to ADMM were made as follows:

- Ref. [62] discussed Harmonic Optimal Power Flow (HOPF) problems that involve harmonic coupling factors. To efficiently solve these problems, they employed SDP optimization techniques. SDP is an effective approach for handling non-convex and nonlinear constraints, particularly for complex power system models that incorporate significant harmonic components. The authors used SDP to optimize the power flow and to ensure voltage stability even under harmonic conditions. Additionally, by incorporating harmonic coupling factors into the HOPF problem, they were able to achieve higher accuracy in their power flow solutions under harmonic scenarios.
- Ref. [41] improved the convergence speed of ADMM by proposing an adaptive scheme to improve the convergence of the ADMM on the component-based dual decomposition of the AC OPF. The proposed method incorporates a local curvature approximation scheme with underlying parameter update steps inspired by the local residual balancing scheme to automatically tune the penalty parameters locally without central oversight.
- Ref. [63] iteratively applied ADMM and the Sequential SOCP algorithm (SSA) with ADMM for optimal gas power flow and SSA for OPF and optimal gas flow.
- Ref. [64] made improvements by proposing a new penalty factor selection method. This method can achieve better optimization results by designing a smaller penalty factor while keeping the voltage deviation and the relative objective function value small and dynamically adjusting the step size parameter at each iteration cycle, which can improve the convergence speed and stability of the algorithm.

ADMM offers several advantages, including convergence properties, distributed solving capabilities for accelerated computation, and effective handling of constraints. However, it has certain drawbacks. For instance, when seeking high-precision solutions, its convergence performance is not optimal. Additionally, the choice of step size is complex, where a value that is too small can result in insufficient constraint satisfaction, whereas a value that is too large can weaken the optimization strength of the objective equation. It is worth noting that the applicability of ADMM depends on the conditions of the given scenario. Specifically, the sub-problems to be addressed after fixing a set of constraint variables should be simple and linear equality constraints should be in place.

3.1.3. Mixed Integer Linear Programming

MILP is a mathematical method for determining the optimal outcome of an objective function, subject to a set of linear constraints, where some of the variables are required to be integers. To convert a conventional OPF model into an MILP model, we can express it using (7):

- ()

min
$$F(x)$$

s.t. $G(x) = 0$
 $h(x) \ge 0$
 $x \in Z$
(7)

where *x* is a vector containing variables, including both continuous and integer variables; F(x) represents the objective function to be minimized; and G(x) and h(x) are the equality and inequality constraint functions, respectively.

Ref. [30] made the OPF problem into an MILP model for optimization. In [65], the SVG MILP model was built by setting upper and lower constraints for variables such as SVG capacity and level, active and reactive power of each branch, SVG output power, voltage at each node, and SVG installation location.

The MILP model in [66] was formed through mathematical equations to minimize the peak load of the electric power distribution network and annual energy losses. At the technical level, the model considers the lifespan of battery energy storage systems to minimize energy losses and peak load of the power distribution network, while at the economic level, the model takes into account the costs associated with application, maintenance, and degradation of BESS, as well as costs related to energy losses and peak demand–supply.

To implement parallel optimization, Ref. [67] proposed an algorithm that uses adaptive sampling and divided the parameter space into small partitions. The algorithm then applies MILP to solve the problem in each partition. During optimization, the algorithm adaptively adjusts the sampling strategy and the number of parallel processors based on the error tolerance and the number of partitions.

MILP offers advantages such as accuracy in finding optimal solutions, flexibility in handling various constraints and objective functions, and efficiency for medium-sized problems. However, MILP can become computationally complex for large-scale OPF problems with integer variables, and the vast number of possible combinations can increase the difficulty of finding solutions. Careful consideration is also needed for numerical stability during the solution process. Therefore, a large amount of research can only use heuristic strategies to find local optimal solutions.

3.1.4. Semi-Definite Programming

SDP deals with the optimization of a linear objective function over the intersection of the cone of positive semi-definite matrices with an affine object, such as a spectrahedron. To convert a conventional OPF model into an SDP model, we can express it using Formula (8):

min
$$c^{\top}x$$
 (8)

s.t.
$$Ax = b$$
 (9)

$$Gx \le h$$
 (10)

where *x* represents the variables to be optimized. In a conventional OPF problem, it may include continuous variables such as generator outputs, node voltages, etc. *c* is the coefficient vector of the objective function. *A*, *b*, *G*, and *h* are coefficient matrices and vectors representing the constraints. x_i denotes semi-definite matrix variables. Covariances or covariance matrices involved in the conventional OPF problem can be modeled using semi-definite constraints.

In [22], optimization was performed through SDP. The nodal voltage constraint in the Bus Injection Model is converted into a 2 by 2 matrix to fit SDP. In [68], a power flow equation was first established based on the branch flow model. Then, the S-lemma was used to prove that the original ACOPF problem is a convex optimization problem. Next, the problem was reformulated by introducing equivalent variables, and an SDP problem was established based on the original problem.

SDP could also combine with other methods.

- Ref. [69] established an SDP-OPF model which focuses on branch current and branch power flows rather than nodal injections. To perform convex relaxation of power flow equations, new variables were introduced to define branch currents and voltages. Ref. [33] reduced the complexity of OPF model with a sparse decomposition technique. Then, to further reduce the complexity of the optimization problem, it was transformed into a first-order semi-definite-constrained nonlinear minimization problem which was solved using a branch flow Semi-Definite Programming relaxation method.
- Ref. [70] replaced the nonlinear terms with new variables. The original AC-OPF problem was transformed into a "graph + relaxation"-based SDP problem. Ref. [71], based on [70], further proposed an intermediate variable that turns the general OPF problem into a mixed-integer SDP model. Ref. [72] combined SDP-OPF with the Branch Injection Model OPF (BIM-OPF) and then added constraints for nodal voltage, P, Q, and voltage angle to ensure the convex relationship of the model.

SDP offers several advantages, such as its ability to handle quadratic and semi-definite constraints, allowing for greater flexibility in dealing with nonlinear and non-convex constraints and objective functions in the OPF problem. SDP provides precise solutions by finding the global optimal solution for the OPF problem and is compatible with other optimization methods, enabling integration to improve solution efficiency. However, SDP can be more complex to solve, particularly for large-scale OPF problems, and requires higher memory usage due to the storage of large matrices. Numerical stability challenges may also arise during the solution process, requiring careful numerical treatment and stability checks.

3.1.5. Second-Order Cones

SOCP is a type of convex optimization problem where a linear function is minimized over the intersection of an affine set and the product of second-order (quadratic) cones. To convert a conventional OPF model into an SOCP model, we can express it using (11):

min
$$c^{\top}x$$

s.t. $Ax = b$
 $Gx \le h$
 $F_ix + g_i \ge 0$
 $l_i^T x + m_i \ge ||(F_ix + g_i)||$
(11)

where *x* represents the variables to be optimized and in a conventional OPF problem, it may include continuous variables such as generator outputs, node voltages, etc.; *c* is the coefficient vector of the objective function; *A*, *b*, *G*, and *h* are coefficient matrices and vectors representing the constraints; F_i and g_i are coefficient matrices and vectors representing the equality constraints; l_i and m_i are vectors representing the inequality constraints; and ||.|| denotes a norm operator, usually the Euclidean norm.

Ref. [73] used the standard SOCP algorithm to transform the distributed power generation modeling problem into a SOCP model that can be solved. In [74], the SOCP model was established by transforming the capacity constraint of flexible open point into rotated cone constraints and by converting the power flow constraints of the system into standard second-order cone constraints.

Some articles have focused on improving the SOCP model:

- Ref. [75] improved the SOCP algorithm by introducing relaxation variables to simplify the power flow equations and by deriving a SOCP optimization problem, also considering the stochastic model, resulting in a chance-constrained optimal power flow model based on the SOCP algorithm.
- In the field of optimization, combining different models can often lead to further improvements. For instance, Ref. [76] introduced the SDP framework for modeling and the SOCP method for solving. The SDP framework leverages generalized matrix inequalities to handle problems with quadratic cost functions and arbitrary inequality constraints, while the use of the SOCP method for solving ensures high efficiency and universality.

Similarly, the approach in [18] is a second-order method based on Robust Optimization (RO) and solved using the Column-and-Constraint Generation (C&CG) algorithm. This approach transforms the model into a problem of normal state optimization and corrective dispatch confirmation, enabling the variables and constraints involved in corrective dispatch to be introduced into normal state optimization. This yields a tight and efficient representation, resulting in accurate solutions. By utilizing different optimization frameworks in conjunction with one another, the resulting solution can be more robust, efficient, and flexible in handling complex problems.

- In [77], the authors proposed a mixed-integer SOCP model that comprises two significant parts. The first part involves the minimization of generation cost with Conditional Value at Risk (CVaR) as the constraint function. CVaR is a risk measure commonly used in finance and risk management that captures the expected loss beyond a certain confidence level. In this approach, CVaR is utilized as a constraint function to ensure the system's stability while minimizing the generation cost. The second part of the model is the complete optimization model, which consists of multiple constraint conditions. The constraints ensure that the system operates within specific parameters, such as generator output limits, transmission line capacity, and demand balance. By using a mixed-integer SOCP model and by incorporating the CVaR constraint function, the optimization solution can simultaneously address the issues of cost and risk in power systems.
- In [78], the problem addressed is how to maximize economic, environmental, and reliability objectives under PV uncertainty. To solve this problem, the authors decomposed the original problem into multiple sub-problems. For each storage node, they used a SOCP model to solve the investment and operation problem. Then, they combined all the sub-problems using the Benders decomposition method. The Benders decomposition method is a powerful technique for solving large-scale optimization problems, particularly in cases where the problem structure is separable. This approach can achieve efficient and accurate solutions by solving each sub-problem independently and then by integrating the results of each sub-problem.

SOCP offers several advantages, such as its flexibility in handling non-convex constraints and nonlinear objective functions, making it suitable for problems involving convex functions like square roots and absolute values. Additionally, SOCP methods exhibit improved numerical stability, providing reliable solutions even in situations with singular or infeasible matrices. They also offer compact and efficient problem representations by utilizing second-order cone constraints, resulting in optimized memory usage and faster solution times. However, it is important to acknowledge that solving SOCP problems can be more complex, especially for large-scale scenarios, and global optimality cannot always be guaranteed, potentially leading to local optimal solutions.

3.1.6. Multi-Level OPF

OPF tends to be multi-objective, and using a single mathematical method may not achieve global optimization. Thus, a multi-layer structure is often considered to be a possible solution. By considering one or a few targets in each layer and facilitating interactions between every layer, the best compromise could be achieved. Ref. [3] proposed a two-layer optimization with the top-level adopting SOCP for maximum usage of renewable power, and the bottom-level aiming for max usage of electric vehicle (EV) aggregators that purchase and sell electricity to consumers through ADMM. In [79], a bi-level optimization problem was used to manage the day-ahead scheduling of multiple microgrids in an unbalanced distribution system. The first level minimizes costs, and the second level adjusts the transformer and capacitor banks to minimize power loss. Ref. [80] proposed a bi-level optimal dispatching model considering dynamic reconfigurations. The model achieves coordinated optimization by defining identical optimization objective functions for both upper and lower levels, which are the minimization of operating cost and maximization of the fast voltage stability index. Ref. [81] also proposed a bi-level framework, in the upper layer, that determines the curtailment rates of distributed generators and injection power of tied microgrids. In the lower layer, an energy management model was proposed to schedule the power generation of tied microgrids and to determine the adjustable range of their injection power, which was set as a complementary constraint in the upper layer model.

Apart from considering different objectives, the combination of different mathematical methods is also a solution. As an example, Ref. [82] proposed a model-free coordinated control algorithm based on a two-dimensional adaptive control method to enable the provision of transmission-level services. In this control algorithm, each DER device utilizes an orthogonal detection signal and modulation signal's sine and cosine components to sample real power.

Another example is the strategy model proposed in [83], which is a Nash bargaining solution to the power flow coordination problem among smart microgrids based on game theory. The lower level involves the coordination of control agents within each microgrid towards their local objective, while the upper level considers the interaction between power exchange coordination agents in the distributed system and each microgrid to satisfy power balance constraints among multiple microgrids.

3.1.7. Other Mathematical OPF Models

Some other mathematical methods are also used in the problem:

- Other mathematical models are also used in power system optimization. For example, in [84], Model Predictive Control (MPC) was used to predict future energy demand and renewable energy generation, and these predictions were used to schedule the active distribution network, minimizing overall costs while increasing consumption of renewable energy. In [85], two indices were introduced: Power Stability Index (PSI) and Power Loss Index (PLI). PLI uses the reactive power values of all nodes to determine the optimal placement location for a Distribution Static Synchronous Compensator (D-STATCOM), while PSI determines the optimal placement location for a D-STATCOM by considering the stable voltage values of nodes. In [86], a Gaussian mixture model was established to describe the complementarity of power flows amongst AC/DC lines, which is a probabilistic distribution model obtained by linearly combining several Gaussian distributions. The Expectation-Maximum (EM) algorithm was used to estimate the parameters of the Gaussian mixture model. In [87], the C&CG algorithm was adopted. This algorithm was used to transform two-stage robust optimization problems into a master problem and subproblems and was iteratively solved using the CPLEX solver. In [88], Generalized Generation Distribution Factor (GGDF) was adopted to replace the voltage and angle variables in the general DC OPF model.
- Another approach to power system optimization is to use graph-theory-based methods. In [89], a minimum-spanning-tree-based approach was used, taking into account the measurement cost of each measurement in the optimization. In [90], the traditional minimum spanning tree algorithm was modified by incorporating microgrids as vertices and by connecting them with their corresponding load nodes. In [91], an Equivalent Network Approximation (ENApp) algorithm was proposed, which can be

solved through reduced network equivalents and the augmented Lagrangian multiplier method. Building on [91], in [92], the impact and properties of communication network topology on the cooperative operation between microgrids were discussed. It was concluded that the network topology has a significant effect on the performance of the OPF algorithm and an efficient communication network topology can improve the algorithm's performance and convergence speed, enabling microgrids to achieve optimal operation quickly.

Some articles have established a sensitivity index for determining the optimal placement of DGs. For example, in [93], a DG placement optimization method was proposed based on a sensitive index algorithm. This method evaluates the sensitivity of each node in the power distribution grid to voltage and determines the optimal placement of DGs according to this sensitivity index to improve voltage and to minimize power losses. In [17], the Sensitivity Sequence Energy Algorithm (SSEA) was relied upon to adjust loads. The algorithm estimates the sensitivity of the flow based on a sensitivity matrix and calculates an implicit representation of reactive power based on energy scheduled for each management period. The energy scheduling was adjusted by translating some of the demand response strategies into load adjustment strategies in each management period. Additionally, corresponding power scheduling decisions can be made for each management period.

As the scale of the DN increases, the amount of data generated using DGs also increases, which can make the calculations for a centralized distribution system operator (DSO) too time-consuming. To address this issue, it is common to adopt a distributed optimization approach, whereby optimization calculations are performed locally at each DG site. This reduces the computational burden on the centralized DSO and creates a more resilient and fault-tolerant system, as each DG can operate independently if communication with the DSO is lost.

- Ref. [94] proposed a smart power distribution system for both residential and industrial applications. The system utilizes decentralized methods to determine optimized real and reactive power set points for inverters that provide auxiliary services such as voltage support and harmonic filtering. The entire distribution system is divided into multiple zones, and controllers that make use of local measurements to calculate the optimized power points are deployed in each zone. These controllers also communicate with each other to share information and to create a cohesive system-level strategy. The system is built to be scalable and flexible, easily adjusting to various applications and system sizes. By employing decentralized optimization techniques, the suggested system efficiently distributes the workload, ensures power quality, and maintains the stability and reliability of the distribution system.
- The paper in [95] presented a decentralized algorithm for reducing allocation costs and optimizing the deployment of DERs. The algorithm consists of two steps: first, a distributed optimization model calculates the allocation amount of each DER; second, local information exchange among allocation agents enables quick convergence on the optimal allocation scheme. The goal is to improve system efficiency while reducing allocation costs. Through decentralization, the algorithm can handle large-scale systems with multiple DERs and can maintain scalability and flexibility. The algorithm's fast convergence speed makes it suitable for real-time applications.
- The paper in [96] proposed a modular structure for addressing the OPF problem in distribution networks. The structure consists of two parts: the first part solves the OPF problem using a gradient-descent algorithm, while the second part sends signals to devices in the distribution network, such as BESS, to control their power output.
- In [97], the authors proposed a method for integrating and coordinating multiple energy sources to optimize the energy flow system. The proposed method uses a unified energy pathway approach to model and analyze all the energy sources, treating them as a single integrated system comprising all energy sources and transmission lines in a unified model. By considering all energy sources and transmission lines

together, this unified approach enables efficient optimization of the energy flow system. The authors devised an optimization algorithm that coordinates the operation of diverse energy sources while considering the constraints and uncertainties associated with each source.

• The paper in [98] introduced a hierarchical distributed algorithm designed for solving large-scale mixed-integer convex problems. The algorithm presented in [98] employed an enhanced Generalized Benders Decomposition (GBD) technique, aiming to ensure computational efficiency and to effectively handle intricate problems. The algorithm's ability to optimize integer variables is useful for optimization problems in the energy and power systems domain.

Other articles have provided an implementation of the above OPF method:

- The paper in [99] presented a hardware implementation of an optimal power flow (OPF) algorithm for distribution networks using decentralized measurements. The algorithm was based on the MATPOWER OPF and optimizes power flow in the network using a decentralized approach. In [100], the cost optimization problem of energy storage systems was solved using Gurobi. The paper in [101] addressed the analysis and management needs of large-scale integrated transmission and distribution networks through the implementation of an integrated power flow algorithm. The algorithm in [101] was optimized through the utilization of an adaptive Lavenberg–Marquardt method and an incomplete LU decomposition. An improved DC OPF model with linear constraints was constructed. An adaptive interior point method was proposed to solve the problem based on branch loss. This approach provides an efficient solution for the operation and planning of integrated transmission and distribution networks, taking into account technical and economic aspects.
- In [102], a comprehensive optimization model was proposed to model active distribution networks, and the primal-dual interior point method was used to solve the optimization problem. The approach offers a promising solution for optimal operation and planning of distribution systems, considering both technical and economic aspects. The paper in [103] proposed the use of topology adjustment as a means to optimize the power generation of offshore wind farms. The proposed approach focuses on optimizing the power output of wind turbines and reducing transmission losses by making adjustments to the network topology. An efficient solution to the optimization problem is achieved through the utilization of the interior point method. The proposed approach shows promise in improving the performance and efficiency of offshore wind power generation systems.

The comparison between mathematical methods is shown in Table 1.

Method	Advantages	Limitations
Conventional [51–54]	Simple and easy to understand/implement; Efficient computation	Can only handle linear constraints and variables; Cannot handle certain nonlinear constraints; Results may be local optima
MILP [30,66,67]	Handles discrete variables and logical constraints; High flexibility	Computational complexity for large-scale problems; Difficulties in solving non-convex problems
ADMM [41,62-64]	Handles structured constraints; Suitable for large-scale problems; Convergence guaranteed	Limited by sparsity; Requires proper problem decomposition; Parameter tuning required
QP [55–58]	Widely used; Efficient solution algorithms	Unable to handle certain nonlinear constraints; Computational complexity for large-scale problems
SOCP [18,73-78]	Handles some nonlinear and convex constraints; Efficient solution	Requires higher modeling requirements for the problem; Not suitable for general non-convex problems
SDP [22,33,68–72]	Capable of representing a wide range of constraints and variables; Precise convex optimization solution	Higher computational complexity; Suitable for medium-sized problems

Table 1. Comparison of mathematical methods.

3.2. Heuristic Approach

This section discusses various heuristic methods for OPF in distribution networks, such as Sunflower Algorithm, Bees Algorithm, Particle Swarm Algorithm, sin–cos algorithm, and Whale Algorithm.

3.2.1. Sunflower Algorithm

The SFO is a new meta-heuristic algorithm inspired by the movement of sunflowers toward the sun. The algorithm mimics sunflowers' movement to absorb solar radiation and comprises two phases: pollination and movement.

In their work [104], the authors proposed an enhanced version of the SFO algorithm. This improved version incorporates an adaptive mechanism that adjusts the pollination rate and death rate of individuals within the population. In [105], a further enhancement was suggested, incorporating several improvements to expedite convergence and to decrease computational complexity. These enhancements involve variable normalization, inclusion of extra constraints, employment of smaller time constants, and utilization of variable speed factor coefficients.

SFO is a versatile algorithm that can be applied to a wide range of optimization problems, including continuous optimization, discrete optimization, and multi-objective optimization, among others. The algorithm is straightforward to understand, and its parameters can be easily adjusted for optimal performance. In some problems, SFO has a fast convergence speed and can find the global optimal solution. However, its drawbacks include a slow convergence speed in some problems that may require a long time to find the global optimal solution. Achieving optimal performance with the algorithm relies heavily on appropriately adjusting its parameter settings, as they significantly impact its overall performance. While SFO is a versatile algorithm with easy-to-adjust parameters, it is worth noting that its convergence and stability may not match those of other optimization algorithms like GA and PSO.

3.2.2. Particle Swarm Algorithm

PSO is a meta-heuristic algorithm designed to optimize a problem by iteratively enhancing a candidate solution using a predefined quality measure. The algorithm utilizes a population of particles, each representing a candidate solution, and guides their movement within the search space using mathematical formulae that modify their position and velocity. This iterative process aids in solving the problem at hand.

In PSO, each particle's movement is influenced by its local best-known position but is also guided toward the best-known positions in the search space, which are updated as other particles find better positions. The algorithm uses this global and local optimization approach to converge to the optimal solution.

Some articles focus on improving PSO:

• To improve the PSO algorithm, several researchers have proposed different variations of the algorithm. For instance, Ref. [106] proposed an adaptive weight updating mechanism and an early stopping strategy to improve the algorithm's performance and to reduce computation time. Likewise, another study [107] proposed an Enhanced PSO (EPSO) approach, which incorporates various heuristic operators, inertia weight adjustment, and chaotic mapping mechanisms. These enhancements were introduced to improve the overall performance of the traditional PSO algorithm. The study aimed to optimize the algorithm by introducing additional features and mechanisms. A study by [108] presented an improved PSO algorithm that enhances the analysis capability of each optimization direction by dividing the single-particle vector into sub-vectors. This partitioning approach was introduced to improve the algorithm utilizes a regionally coordinated control structure for intelligent distribution networks. It incorporates the energy generated via controllable distributed generators and energy storage systems and takes into account the charging and discharging

process of the energy storage system. This holistic approach aims to optimize the coordination and utilization of energy resources within the distribution network. In another research paper [109], a framework was proposed to enhance the performance of Performance Index Optimization (PIO). This framework introduced the utilization of Power Management Unit (PMU) data, leading to the development of the PMU-PIO algorithm. The incorporation of PMU data aimed to improve the accuracy and effectiveness of the optimization process in the PIO algorithm. The PSO-PIO algorithm combines the strengths of both approaches to achieve more efficient and accurate optimization results.

• Several articles focus on using multi-objective optimization methods with the PSO algorithm to solve complex optimization problems. For instance, Ref. [110] improved the traditional Network-Structured Multi-Objective Particle Swarm Optimization (NS-MOPSO) algorithm by introducing crowding distance as a feasibility evaluation criterion. The paper also proposed a new weight coefficient updating strategy and a solution set pruning strategy to better manage the solution set size and to maintain its global optimality. Furthermore, the algorithm uses Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to calculate the optimal solution for multi-objective problems. Similarly, Ref. [111] presented an improved version of the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm for solving problems in electrical distribution systems. The algorithm coordinates transformer tap settings, capacitor bank injection, and DG active power output to minimize power flow losses and voltage deviations.

In another study, Ref. [112] introduced the Pareto Entropy-based Multi-Objective Particle Swarm Optimization (PE-MOPSO) algorithm to solve optimal control problems in distribution networks. The PE-MOPSO algorithm is based on an elite strategy in the multi-objective PSO algorithm, which addresses the problem of non-uniform distribution of non-dominated solutions in the traditional MOPSO algorithm. Additionally, the algorithm introduces a region-updating strategy to balance the trade-off between global and local searches. The research paper in [113] introduced an enhanced version of the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm known as Competitive Multi-Objective Particle Swarm Optimizer (CMOPSO). The Improved CMOPSO algorithm improves upon the traditional Particle Swarm Algorithm by incorporating an archive strategy that utilizes an adaptive grid. During the iteration process, the paper proposes updates to the position and velocity of each particle using different equations. These enhancements aim to improve the overall performance and optimization capabilities of the CMOPSO algorithm.

Finally, in [114], a multi-objective optimization algorithm based on PSO is presented. This algorithm aims to minimize two objective functions concurrently. By utilizing different weighting factors and random coefficients, the algorithm calculates the velocity and displacement of each particle in order to search for the global optimal point during iterations. Through this approach, the algorithm aims to achieve superior solutions by efficiently exploring the solution space and by balancing between the two conflicting objectives.

- Ref. [115] improves upon the traditional PSO algorithm by applying it to the distribution network reconfiguration problem and by optimizing multiple objective functions such as power loss and the voltage level of the system.
- The research paper [116] introduces a novel hybrid algorithm called Hybrid Firefly and Particle Swarm Optimization (HFPSO). This algorithm combines the Firefly Algorithm (FA) with PSO to leverage their respective strengths. By incorporating the velocity concept from PSO and the brightness concept from FA, a mixed velocity is obtained and used to update the position of fireflies. This combination enables the algorithm to benefit from the fast convergence speed of PSO, while also mitigating the risk of getting trapped in local optima, which is a common issue in FA. The HFPSO algorithm

provides a promising approach for optimization problems by effectively combining the advantages of both FA and PSO.

The PSO algorithm can handle multi-dimensional optimization problems, including continuous optimization, discrete optimization, multi-objective optimization, and more. The algorithm is simple and easy to understand, and its parameters are easy to adjust. The PSO algorithm's parallelization in a distributed environment can also improve its efficiency. By distributing the particles and their computations across multiple processors, the algorithm's search capability and speed can be significantly enhanced.

However, the PSO algorithm suffers from poor local search ability and low search accuracy. Due to PSO's stochastic nature, the algorithm cannot guarantee finding the global optimal solution, and it may converge prematurely or get stuck in sub-optimal solutions. Therefore, researchers have proposed various modifications to address these issues and to improve the PSO algorithm's performance and effectiveness.

3.2.3. Genetic Algorithm

The Genetic Algorithm (GA) is a metaheuristic that belongs to the broader class of evolutionary algorithms. It is inspired by the process of natural selection and mimics the principles of survival of the fittest. GAs are widely employed in addressing optimization and search problems. They make use of biologically inspired operators, namely mutation, crossover, and selection, to generate high-quality solutions. These operators mimic the processes of genetic variation, reproduction, and natural selection in biological organisms.

In particular, Refs. [117,118] have used GA to determine the optimal active and reactive power outputs of DG converters to minimize power losses. The GA-based approach enables these papers to search for optimal solutions efficiently and effectively by using genetic operators and processes similar to natural selection and evolution. The optimal operation of DGs is critical to improving the efficiency and stability of power systems, so the use of GA in these studies exemplifies the wide-ranging applications of GA as a flexible and versatile optimization technique.

Several studies have sought to enhance the performance of GAs:

- Several papers have proposed improvements to GA algorithms to optimize power systems and control mechanisms. For instance, Ref. [119] combined GA with an N-R method to obtain the necessary reactive power factor that minimizes power losses across different PV-BESS connection schemes in a distribution network. Similarly, Ref. [120] used a combination of GA and a FBS power flow method to optimally place BESS in a unidirectional power distribution network. The authors used mathematical modeling and heuristic algorithms to quickly evaluate different scenarios and to determine the optimal placement of BESS. In another study, Ref. [121] proposed a GA algorithm that uses a search algorithm based on spanning trees during the selection operation to filter out invalid configuration combinations effectively. Another improvement was proposed in [122], where an improved GA based on dynamic weighting was presented. The paper considered both the weight of active power loss and grid voltage to achieve better coordination and more efficient solutions. In [123], a method to preserve the optimal solution from disruption caused by genetic crossover and mutation was proposed. This was achieved by incorporating an elitism selection mechanism. This mechanism ensures that the best solution discovered so far is preserved in the population, even if it is not improved in the current generation. By including elitism, the algorithm can prevent the loss of favorable solutions and can maintain the progress made towards finding the optimal solution. This enhances the effectiveness and robustness of the Genetic Algorithm in solving optimization problems.
- Various variants of GA algorithms have been applied to optimize OPF models, aiming to enhance performance and optimize capabilities. For example, in [124], the Nondominated Sorting Genetic Algorithm II (NSGA-II) is utilized as a foundation, with several improvements proposed to enhance its performance. NSGA-II is a multiobjective Genetic Algorithm known for maintaining the diversity of Pareto-optimal

solution sets. The paper introduced strategies such as an elite strategy, crowding distance comparison, and crowding distance control to further augment the algorithm. In another study [125], the Immune Genetic Algorithm (IGA) was employed to determine the optimal allocation and sizing of shunt capacitor banks in distribution networks while considering technical constraints. In this approach, chromosomes represent potential solutions, and a fitness function is employed to assess the quality of each chromosome. Genetic operators are employed to generate fresh populations, promoting the search for the global optimal solution. The fitness function is then used to evaluate the best solution obtained through IGA.

Furthermore, in [126], a Multi-Objective Genetic Algorithm (MOGA) was employed to simultaneously optimize multiple objective functions. MOGA, which is a multi-objective variant of the original GA, offers improved optimization performance and the ability to effectively handle problems involving multiple objectives.

GAs have several advantages, such as the ability to perform global search in the solution space efficiently, without falling into the rapid descent trap of locally optimal solutions. Its inherent parallelism also enables distributed computing, which can speed up the search.

However, GA also faces challenges and disadvantages. Notably, the local search ability of GA is often poor, and it is prone to premature convergence in practical applications. Additionally, selecting a method that preserves excellent individuals while maintaining group diversity is a difficult problem in GA. These limitations call for continued efforts to develop new and improved variants of GA, such as hybrid methods with other optimization algorithms, to overcome these challenges and to optimize complex problems more effectively.

3.2.4. Other Heuristic Approaches

Apart from the above frequently used algorithm, other heuristic methods were also used for OPF:

Apart from the previously mentioned algorithms, there are several other heuristic optimization algorithms that have been utilized for OPF problems. For instance, Ref. [127] used the Increasing Population size Covariance Matrix Adaptation Evolution Strategy (IPOP-CMA-ES) algorithm, which is a type of optimization algorithm that introduces a new start trigger and can increase population size for Covariance Matrix Adaptation Evolution Strategy (CMA-ES). The IPOP-CMA-ES algorithm has the ability to cover a large search space in a shorter time, which results in faster convergence to global optimal solutions. In another study, Ref. [128] proposed improvements to the Water Cycle Algorithm (WCA). These improvements include including nondominated sorting, a new solution type called "sea", and a phase-based strategy for raindrop generation. Moreover, Ref. [129] used the Flower Pollination Algorithm (FPA) to optimize the optimal capacity and location of PV distributed generation to minimize power loss on the distribution grid. In another study, Ref. [130] used the Grey Wolf Optimization (GWO) algorithm to determine the optimal placement and size of PV stations in rural unbalanced distribution networks to minimize losses and to reduce the imbalance. The algorithm uses a set of randomly generated grey wolf populations to estimate the prey's position using alpha, beta, and gamma wolves, and to optimize the objective function and position with the corresponding wolf population to obtain the optimal PV placement. Additionally, Ref. [131] introduced a modified heuristic function for updating the positions of teachers and students in Teaching Learning-Based Optimization (TLBO). The new function called the Weighted Parameter and Resonance Frequency Heuristic Function, defines a weighted parameter based on a concept similar to the resonance frequency. Furthermore, Ref. [132] used a heuristic multi-objective coordinated search algorithm based on the Multi-Objective Harmony Search (MOHS) algorithm to solve the problem of optimizing and dispatching DGs and obtained a set of Pareto optimal solutions. Finally, Ref. [133] proposed improvements

to the Cuckoo Search algorithm by introducing a parameter that balances global and local random walks and by using the Levy distribution to describe the step size of the random walk. The Mantegna equation and gamma function were also used to generate random numbers, resulting in an improved trade-off between convergence time and global optimum.

• Some of the articles proposed improvements based on other articles. For instance, in [134], an improved version of the Whale Optimization Algorithm (WOA) was proposed based on the original WOA proposed in [135]. This improved version introduced a nonlinear update factor to increase the diversity of the population and to improve speed and accuracy. Similarly, in [136], an improved Multi-objective Bees Algorithm (BA) was proposed based on the BA proposed in [137,138]. This improved version incorporated both crowding distance and fuzzy mechanism to manage the size of solutions and to select non-dominating solutions in the algorithm. Moreover, in [139], an improved version of the Sine Cosine Algorithm (SCA) was proposed based on the original SCA proposed in [140]. The improvement involved the use of different operators to update position variables in the optimization problem, such as mutation operation to enhance exploration performance and to improve exploitation performance by targeting the position.

A comparison between heuristic methods is shown in Table 2.

Table 2. Comparison between common heuristic methods.

Method	Advantages	Limitations
SFO [104,105]	Strong global optimization capability.	High computational complexity, especially for large-scale problems. Requires parameter tuning and appropriate convergence criteria. Lack of maturity compared to GA and PSO.
GA [117–125]	Widely applicable to various optimization problems. Can handle both discrete and continuous variables. Suitable for problems with multiple constraints and uncertainties.	Relatively slower convergence speed, especially for complex problems. May converge to local optima. High complexity for large-scale problems.
PSO [106–116]	Powerful global search capability. Suitable for optimization problems with constraints. Can handle high-dimensional problems.	Performance may degrade for complex problems. Requires parameter tuning and appropriate convergence criteria. May become trapped in optima when multiple exist.

3.3. Machine Learning Approach

In addition to using mathematical models, machine learning is also widely used in solving the OPF problem. This is particularly useful for predicting the output of PV and WT generators using historical data. For instance, in [141], a statistical machine learning model, involving Non-negative Matrix Factorization (NMF) and Stochastic Robust Search Method (SRSM), was utilized to optimize stochastic optimal planning by reducing the dimensionality and correlation of input data. Similarly, in [142], decision trees, support vector machines, and k-nearest neighbors were used to build a forecasting model for OPF.

Furthermore, in [39], a polynomial-based estimation algorithm was proposed to estimate the loss of SOP for a High Voltage Direct Current (HVDC) network. This algorithm was trained with past OPF data and weather data for better accuracy, even under the circumstance of communication interruption. In [143], Multiple Linear Regression and Random Forest Regression algorithms were used to predict the output of solar power plants.

One of the common machine learning models used in the OPF is the Neural Network (NN). The NN consists of layers of interconnected nodes that process information and learn by adjusting their weights and biases according to the feedback from training examples. For instance, in [144], an NN was trained using data from smart meters, which means that the electrical model of the distribution network was not required. Additionally, this approach has led to the proposal of several improved NN models.

A few improvements to NN were made as follows:

- Ref. [145] NN in this paper is designed as a deep recurrent NN that has more hidden layers than a simple recurrent NN. The additional layers enable the network to learn more complex historical and temporal information and to provide a more compact representation of the input–output relationship.
- Ref. [146] applies an Artificial Neural Network (ANN) to control the direct and quadrature axis currents of D-STATCOM improving the damping performance of power systems. The nonlinear characteristics of the ANN controller make it more adaptable to different operating conditions. In addition, by optimizing the parameters of the NN such as gain constants, the control performance and stability of D-STATCOM can be further improved.
- Ref. [147] overcomes the long convergence time of a conventional NN by updating the weight of the NN model with Lavenberg–Marquardt and Bayesian Regularization.

Other ML methods can also be implemented into the OPF problem:

• Supervised learning is another approach to dealing with the OPF problems. For instance, in [1], supervised learning was used to map the solutions of linear OPF to nonlinear control variables using a two-node approximation of radial networks with a radial basis function network. Similarly, in [148], a Message Passing Graph Convolution OPF (MPGCN-OPF) model based on graph convolution and message passing interface was proposed. This approach utilizes the property of graph convolution feature mapping and collects, aggregates, and updates the feature matrix from neighboring nodes through information passing. The performance of this approach was found to be better than Deep Neural Network (DNN) models in terms of the loss function and performance evaluation parameters, and it has been validated through accurate predictions.

Moreover, in [149], a data-driven approach was used to combine Support Vector Regression (SVR) and Ridge regression (RR) to minimize the linearization error of the power flow model. The accuracy of the model was improved by considering the covariance of the input data and using Principal Component Analysis (PCA) to reduce the impact of data collinearity. This data-driven approach was found to be effective for solving OPF problems.

• Multiple methods of deep reinforcement learning have been applied in the field of power system optimization. For example, Double Deep Q-learning has been used in [150,151], where two NNs are used for optimization simultaneously.

Another method called Soft Actor-Critic (SAC), has been used in [152], where it was combined with Multi-Agent Reinforcement Learning techniques such as DDPG to solve the OPF problem. In [153], the optimal power flow problem over multiple time periods was described as a Markov process to control PV and WT output, thus avoiding local optima. Additionally, in [154], Deep Deterministic Policy Gradient (DDPG) was employed to solve dynamic OPF problems by describing the problem over multiple time periods as a Markov process and by utilizing active and reactive outputs of PV generation and an energy storage system as control actions. The algorithm then used deep learning to train the agent for decision-making.

Similarly, in [155], the DDPG algorithm was used to solve the optimal control problem in the ADN, which is a Markov Decision Process. DDPG is a policy gradient method consisting of two neural networks—an actor and a critic—both based on DNNs. The critic network learns a Q-function that estimates the expected future rewards of state– action pairs. DDPG can handle continuous action spaces without discretization and learn deterministic policies that are more efficient and less noisy.

• A power flow model can also be added to the ML solution. Ref. [156] combines an exact nonlinear AC power flow model and an approximate linear power flow model to reflect the system's response under uncertainty and proposed a data-drive AC-OPF model. ML has the ability to process large amounts of data quickly and can improve model performance through continuous learning. ML algorithms can also handle complex nonlinear relationships and detect patterns automatically, making it a powerful tool for solving problems in various fields, including power systems.

However, the dependence on large amounts of data can result in high computational costs and can make it difficult to train models effectively. ML algorithms can also suffer from overfitting, which can lead to poor performance on new data. To mitigate overfitting, techniques such as regularization and early stopping can be used. Finally, the tuning of hyperparameters can be complex, requiring significant expertise and time, which can make model selection and adjustment challenging.

3.4. Mixed Approach

The combination of different methods in power system optimization is an effective way to harness the strengths of each method and to achieve greater effectiveness and performance. Mathematical methods offer a rigorous theoretical foundation and framework for solving problems, while machine learning algorithms can extract useful information from large amounts of data and can make accurate predictions.

For example, Ref. [157] proposed a strategy for optimizing SOC by using GWO under normal circumstances, but when communication interruptions occur, a neural-networkbased algorithm is used to find the most likely status from historical data according to the available data. In [158], a chance-constrained OPF approach was used for central control, while Support Vector Machines (SVM) were used for determining the operation of local DGs. Similarly, Ref. [44] proposed a method called Artificial Neural Network-Primal-Dual Interior Point Method-Temperature (ANN-PDIPM-T), which combines ANNs and the primal-dual interior point method algorithm, a type of interior point method, for reactive OPF calculation based on Temperature-Dependent Power Flow.

To achieve multi-dimensional optimization, Ref. [159] proposed a multi-layer optimization approach where the top-level optimizes through a mathematical approach for potential solutions, while the middle and low-level optimizes through GA and PSO to determine the output, location, and number of devices such as capacitors and BESSs.

Other studies, such as [160], proposed a bi-level structure with Linear Programming for feeder optimization at the top-level and heuristic methods like PSO and GA for coordinating between regions at the bottom level. An improvement to the PSO algorithm was used in [161], which introduced an adaptive inertia coefficient ω and a compression coefficient ϕ to reduce the risk of the algorithm falling into local optimal solutions [162]. Additionally, Ref. [40] solved the optimization problem of multi-terminal DC distribution networks by using multi-objective optimization and fuzzy decision-making methods to select the final solution from a non-dominated set of solutions obtained by using a hybrid PSO.

In summary, combining different optimization methods can help us better understand and solve complex problems in power systems, leading to greater efficiency and performance.

4. Discussion

Figure 7 summarizes the steps involved in the technical process of optimal power flow.

To compare the three methods (mathematical, heuristic, and machine learning) mentioned earlier, Table 3 summarizes the experimental data from the literature, all of which utilize the IEEE-33 bus case study. From the table, we can observe that, in the IEEE-33 bus case study, the mathematical method exhibits shorter computational time but relatively fewer optimization gains. The heuristic algorithm requires slightly more computation time than the mathematical method, but it achieves better optimization results. On the other hand, the machine learning method, despite the need for learning from historical data and parameter tuning, outperforms both methods in terms of optimization effectiveness and computational time.



Figure 7. Steps involved in the technical process of optimal power flow.

Optimizing using mathematical methods often yields results in a shorter time frame but may be limited to finding local optima. Heuristic algorithms, on the other hand, employ intelligent search strategies and can discover better solutions, albeit at a slightly longer computational cost. Machine learning methods, leveraging historical data for learning and parameter adjustment, can generate more accurate results while being more efficient in terms of computation time compared with the other two methods.

In summary, based on these experimental data, it can be inferred that heuristic algorithms and machine learning methods hold greater optimization potential in the IEEE-33 bus case study, offering improved solutions. However, the selection of a specific method still depends on the application context and problem requirements.

Туре	Method	#	Loss Reduced (%)	Compared to	Simulation Platform	Computation Time (s)
	QP	[56]	7.9	Conventional OPF	MATLAB	0.0750
Mathematic	ADMM	[56]	-	-	MATLAB	2.1030
	MILP	[34]	2.42		MATLAB	-
	Conventional OPF	[54]	30.0	·	CPLEX	-
	PSO	[114]	25.4		-	-
	IPOP-CMA-ES	[127]	62.9	- No optimization	MATLAB	24.9000
Heuristic	GA	[127]	61.6		MATLAB	9.1000
	PSO	[127]	62.8		MATLAB	12.9000
	SCA	[139]	25.8		MATLAB	-
	DDQN	[150]	18.4		-	-
ML	DDPG	[154]	64.4		-	-
	NN	[157]	45.3		-	-
	ANN	[147]	-	-	-	0.0095

Table 3. Case study of three types of methods.

- represents item not mentioned in reference.

Mathematical optimization methods are commonly used to solve optimal power flow problems. They are fast, able to handle large-scale problems, and can obtain global or local optimal solutions with some accuracy guarantees. However, due to OPF's nonconvex nature, mathematical optimization methods may lose accuracy. Additionally, certain requirements on the form of the problem must be met, such as differentiability and convexity, or the solution may be difficult or fail. Mathematical optimization methods are also sensitive to the choice of initial values and parameters, which can lead to different results or convergence.

Heuristic methods can handle non-convex and nonlinear problems that may be difficult for mathematical optimization methods. They can explore a large and diverse search space, making them more adaptive to different problem settings and constraints. However, they may not guarantee convergence or optimality and may depend on random factors. It may require a lot of trial and error, guesswork, or historical data analysis to find a satisfactory solution. Additionally, heuristic methods may have high computational costs and memory requirements, especially for large-scale problems.

Machine learning methods can learn from data and can adapt to changes in the system, such as load demand, generation capacity, and network topology. However, they may suffer from overfitting or local minima, which means that they may perform well on the training data but poorly on the testing or new data. Machine learning methods may also require a lot of data and computational resources to train and test the models. Furthermore, they may have low interpretability or transparency, which means that they may not explain how they reach the solution or what factors influence the solution.

In summary, the paper highlights the advantages and limitations of three different approaches for solving optimal power flow problems: mathematical optimization methods, heuristic methods, and machine learning methods.

Mathematical optimization methods offer speed and the ability to handle large-scale problems while providing accurate global or local optimal solutions. However, they may lose accuracy due to the non-convex nature of the OPF problem, and specific problem requirements must be met for successful solution convergence.

Heuristic methods, on the other hand, excel in handling non-convex and nonlinear problems and can explore diverse search spaces. However, they may not guarantee convergence or optimality and can be computationally expensive, particularly for largescale problems.

Machine learning methods have the advantage of learning from data and adaptability to system changes. However, they may suffer from overfitting, require substantial computational resources and data, and lack interpretability.

In conclusion, combining multiple methods can leverage the strengths of each approach and compensate for their limitations, leading to improved results in solving optimal power flow problems. For a better understanding, Table 4 illustrates the advantages and limitations of the three optimization methods.

Table 4. Advantages and limitations of three optimization methods.

Comparison Factor	Mathematical	Heuristic	Machine Learning
Interpretability	High	Medium	Low
Accuracy	High	Medium-High	Medium-High
Scalability	Medium-High	Medium-High	High
Computational Complexity	High	Medium-High	Low-Medium
Data Requirement	Low	Low-Medium	High
Algorithm Tuning Requirement	High	Medium	Low-Medium
Robustness	High	Low-Medium	Medium-High
Applicability	Complex Problems	Medium-Complex Problems	Complex Problems

Given these considerations, our next step is to further refine the DG model by integrating both power market optimization and distributed optimization techniques. By combining mathematical optimization methods with market optimization, we can optimize the scheduling and dispatch of DG units, considering factors such as electricity prices and demand fluctuations. This approach takes advantage of the speed and accuracy offered by mathematical optimization methods. Additionally, by incorporating distributed optimization techniques, we can improve efficiency and scalability, addressing the limitations of both mathematical optimization and heuristic methods. Furthermore, machine learning methods can be applied to leverage historical and real-time data for improved predictions and operational optimization, enhancing the overall performance of the DG model. Thus, combining these approaches will enable us to pursue a comprehensive solution that optimizes the DG model, taking into account power market dynamics and the distributed nature of the problem. The future developments of OPF can be summarized as follows:

4.1. Further Exploiting Uncertainty of DG

The uncertainty of the DGs still plays a significant role in optimization, as accurate predictions can result in better optimization results. To address this issue, ML can be used to analyze historical data and to improve prediction accuracy. By training models on large amounts of historical data, machine learning can help capture complex patterns and correlations in the data, which can improve the accuracy of predictions for power generation, demand, and other critical factors. These more accurate predictions can then be used to inform optimization decisions and to improve overall system performance. Machine learning can play an important role in enabling smarter decision-making and effective system management, especially in complex and dynamic power systems.

4.2. Distributed Optimization

DG, which is often referred to as prosumer, has created challenges for DSOs since it consumes and produces electricity, making power flow dataset management difficult. DN are becoming larger and more complex. The optimization of power flow in such large-scale grids poses challenges in terms of computational speed, complexity, and convergence. Addressing these challenges requires the development of efficient algorithms and optimization techniques to improve computational efficiency and accuracy. Therefore, distributed optimization is becoming a new trend that optimizes power flow management locally and regionally. However, communication interruptions may occur among different regions, and optimization should consider them.

To address this issue, a combination of heuristic and mathematical optimization methods can be used since they are easier to implement. Heuristic optimization methods are flexible, adaptive, and suitable for handling non-convex problems. They can explore a large solution space to escape from local minima. Mathematical optimization methods provide a rigorous theoretical foundation and can obtain global or local optimal solutions with accuracy guarantees. Combining these two methods can take advantage of their strengths and compensate for their limitations.

By utilizing distributed optimization based on a combination of heuristic and mathematical optimization methods and considering communication interruption, future power systems with high DG penetration can be managed more efficiently and effectively. This would allow DSOs to improve power flow management and ensure the reliability and stability of the power grid.

4.3. Combination of OPF with Electricity Market

The DR model formulated in Section 2 provides a solid foundation for the integration of OPF and electricity market, enabling the development of innovative algorithms and methodologies for the optimal coordination of energy resources in the power grid. One of the primary objectives of OPF is to minimize costs, which can play a significant role in calculating electricity prices. In the electricity market, OPF technology can help fully consider the adjustment of active and reactive power, take into account safety constraints such as line overload, and provide complete information on optimized power flow. By doing so, it becomes possible to generate an optimized dispatch plan for the power system while meeting safety requirements.

With the OPF technology, electricity pricing can be optimized, allowing for more efficient pricing in the electricity market. In addition, safety constraints, such as line overload, can be taken into account, ensuring the safe and reliable operation of the power system. Furthermore, the complete information provided by the OPF technology can aid in

generating an optimized dispatch plan, which can enable a more efficient use of resources. In summary, the electricity market has become a critical component of the power industry, playing a crucial role in the operation and management of power systems. Focusing on its development as a future direction is supported by the need for effective demand response management, the flexibility and scalability offered by the electricity market model, and the alignment with policies promoting market-oriented operations and renewable energy integration. Additionally, the incorporation of optimal power flow provides the foundation for nodal electricity pricing, which further enhances the efficiency and effectiveness of the electricity market. By optimizing the electricity market and integrating it with distributed generation, we can achieve sustainable development, efficient resource allocation, and better adaptability to changing market conditions.

5. Conclusions

This paper provides an extensive overview of DG models and power flow calculation models, along with a comprehensive analysis of optimization methods for solving the OPF problem. These optimization methods are categorized into three main approaches: mathematical optimization, heuristic optimization, and ML. The study reveals that mathematical and heuristic optimization methods have been widely adopted and proven to be effective in solving OPF problems. However, the ML method shows promise in addressing the uncertainty inherent in DNs, presenting a potential avenue for resolving OPF problems.

While mathematical optimization methods provide a solid theoretical foundation for finding optimal solutions, their applicability may be limited in certain situations, particularly when dealing with high-dimensional problems. On the other hand, heuristic optimization methods demonstrate greater flexibility and suitability for handling nonconvex problems. However, they do not always guarantee optimality.

The application of machine learning methods offers a data-driven approach to effectively tackle the OPF problem by addressing uncertainty. Nonetheless, it is important to note that ML methods may face challenges related to overfitting and the requirement of significant computational resources and data. Additionally, the interpretability of ML solutions may vary.

In conclusion, this paper contributes valuable insight into the optimization methods employed for addressing the OPF problem. It emphasizes the advantages and limitations of each approach and provides a potential direction for future research to further enhance the efficiency and effectiveness of the OPF problem. This includes exploring approaches that combine OPF with the electricity market and distributed optimization, as well as addressing the uncertainty of distribution systems.

The findings presented in this study serve as a foundation for researchers and practitioners to make informed decisions in selecting appropriate optimization methods for solving OPF problems. The identified potential for applying machine learning methods opens up avenues for further advancements in this field.

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Abbreviations

The following abbreviations are used in this manuscript:

ADMM	Alternating Direction Method of Multipliers
ADN	Active Distribution Network
ANN	Artificial Neural Network
ANN-PDIPM-T	Artificial Neural Network-Primal-Dual Interior Point Method-Temperature
BA	Bees Algorithm
BJM-OPF	Branch Injection Model OPF
OPF	Optimal Power Flow
DG	Distributed Generator
DN	Distribution Network
D-STATCOM	Distribution Static Synchronous Compensator
BESS	Battery Energy Storage System
BIM	Bus Injection Model
CVaR	Conditional Value at Risk
C&CG	Column-and-Constraint Generation
СВ	Capacitor Banks
CLA	Community Load Aggregator
COPF	Coupled OPF
CMA-ES	Covariance Matrix Adaptation Evolution Strategy
CMOPSO	Competitive Multi-Objective Particle Swarm Optimizer
DDPG	Deep Deterministic Policy Gradient
DNN	Deep Neural Network
DRL	Deep Reinforcement Learning
DSO	Distribution System Operator
EA	Evolutionary Algorithm
EDC	Enhanced DC
EM	Expectation-Maximum
ENApp	Equivalent Network Approximation
ENS	Energy Not Supplied
EPSO	Enhanced PSO
EV	Electric Vehicle
FA	Firefly Algorithm
FBS	Forward / Backward Sweep
FPA	Flower Pollination Algorithm
GA	Genetic Algorithm
GBD	Generalized Benders Decomposition
GGDF	Generalized Generation Distribution Factors
GWO	Grev Wolf Optimization
HFPSO	Hybrid Firefly and Particle Swarm Optimization
HOPF	Harmonic Optimal Power Flow
IGA	Immune Genetic Algorithm
IPOP-CMA-ES	Increasing Population size Covariance Matrix Adaptation Evolution Strategy
IPSO	Intelligent Single Particle Optimization
LF	Loss Factor
LMP	Local Marginal Price
MILP	Mixed Integer Linear Programming
MIOP	Mixed Integer Quadratic Programming
ML	Machine Learning
MOGA	Multi-Objective GA
MOHS	Multi-Objective Harmonic Search
MOPSO	Multi-Objective Particle Swarm Optimization
MPC	Model Predictive Control
MPGCN-OPF	Message Passing Graph Convolution OPF
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IN-K	Newton–Kaphson
NMF	Non-negative Matrix Factorization
NN	Neural Network
NSGA-II	Non-Dominated Sorting Genetic Algorithm II
NS-MOPSO	Network-Structured Multi-Objective Particle Swarm Optimization
OPF	Optimal Power Flow
PCA	Principal Component Analysis
PE-MOPSO	Pareto Entropy-Based Multi-Objective Particle Swarm Optimization
PIO	Performance Index Optimization
PLI	Power Loss Index
PMU	Power Management Unit
PR	Power Router
PSI	Power Stability Index
PSO	Particle Swarm Optimization
PTDF	Power Transfer Distribution Factors
PV	Photovoltaic Cell
QCP	Quadratic Constraint Programming
QP	Quadratic Programming
RB	Reactor Banks
RO	Robust Optimization
RR	Ridge Regression
SAC	Soft Actor–Critic
SCA	Sine Cosine Algorithm
SCIG	Squirrel Cage Induction Generator
SDP	Semi-Definite Programming
SFO	Sunflower Optimization
SOC	State Of Charge
SOCP	Second-Order Conic Programming
SOP	Soft Opening Point
SRSM	Spectral Regularized Similarity Matching
SSA	Sequential SOCP algorithm
SSEA	Sensitivity Sequence Energy Algorithm
SVG	Static Var Generator
SVM	Support Vector Machines
svr	step-voltage regulators
SVR	Support Vector Regression
TLBO	Teaching Learning-Based Optimization
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
TOU	Time of Use
WCA	Water Cycle Algorithm
WOA	Whale Optimization Algorithm
WT	Wind Turbine

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