

Article

Investigation of the Temperature Field Distribution in an EI Type Iron-Cored Coil Using 3D FEM Modeling at Different Load Conditions

Denitsa Darzhanova ¹ and Ilona Iatcheva ^{2,*}

¹ Department of Electrical Measurements, Technical University of Sofia, 1756 Sofia, Bulgaria; dpetrova@tu-sofia.bg

² Department of Fundamentals of Electrical Engineering, Technical University of Sofia, 1756 Sofia, Bulgaria

* Correspondence: iiach@tu-sofia.bg

Abstract: In this present paper, results for the temperature distribution in electromagnetic construction in the form of coil coupled with EI-type ferromagnetic core are presented. This construction is typical for small electric transformers (AC operation mode) or chokes—DC or AC operation mode. Investigation has been provided using 3D finite element method computer modeling at various load conditions to simulate power dissipation both in the coil and the core volumes. The results obtained were used to calculate overall thermal resistances toward ambient free air and the thermal resistance between the coil and the core. These results show the important role the thermal resistance between the coil and the core may play for both steady-state and transient device operation.

Keywords: transformer; temperature field; 3D FEM modeling; thermal resistances



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1. Introduction

Engineering practice often deals with electromagnetic devices in the form of a current-carrying coil, which is surrounded by a ferromagnetic core. These devices vary from the view-point of construction and dimension, starting from low frequency small electric transformers for radio–electronic purpose, medium power air or oil-cooled transformers for public service use [1], some special transformers for medium frequency [2] and ending with high frequency planar transformers for special electronic applications [3]. For all of them, the most important task of the designer is to ensure that the coil average temperature will not exceed certain limits [4], which depends on the selected wire, and more specifically, on the thermal properties of any additional electrical insulation used. This guarantees the reliability and endurance of the relevant device.

An example of that group of devices [5] is presented in Figure 1. It represents the special transformer subgroup family, namely the air-cooled EI laminated transformers. Their assigned power varies mainly in the range of 50 VA up to 1500 VA.

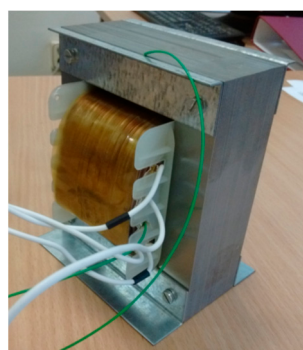


Figure 1. Studied transformer.

Usually, in steady-state conditions and at a predetermined constant load, calculations for the coil/core temperature of the transformers are based on the thermal equivalent scheme, presented in Figure 2. The concept of thermal resistances [6] has been applied to this scheme, which is also based on the analogy between the heat exchange processes and the electrical circuit theory. The scheme refers to the coil/core temperature rise in relation to the predetermined ambient air temperature T_0 . Here, P_1 is the heat power dissipated in the coil volume, while P_2 is the power dissipated in the core volume. The coil temperature rise is denoted by θ_1 , while θ_2 is the corresponding temperature rise of the core. Both of them are considered to be the average calculated values for the corresponding volumes V_1 and V_2 .

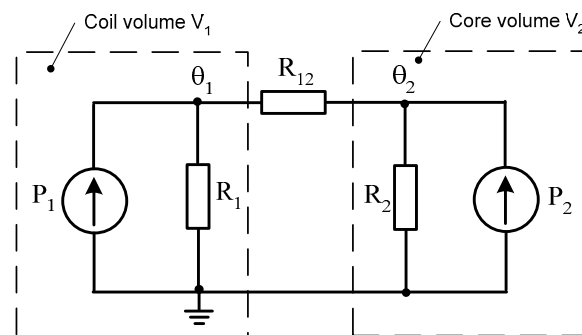


Figure 2. Equivalent scheme for the steady-state thermal analyses.

In particular, with regard to the temperature rise θ_1 , it has been observed that for the special transformers group, that the temperature differences within the framework of the coil volume V_1 do not exceed 2–3 °C. Thus, at levels $\theta_1 = 80$ °C, the calculation accuracy is approximately 3%, which from a practical engineering point of view is considered quite acceptable. From an electrical circuit analyses point of view both P_1 and P_2 are considered to be “current sources”. R_1 and R_2 are the thermal resistances responsible for the convection–radiation heat exchange from the coil and the core open surfaces, while R_{12} is the thermal resistance between the coil and the core. This thermal resistance will include the thermal resistances of the spool walls and those of the inevitable adjacent air layers between the coil and the core.

It is worth noting that different modifications of the equivalent scheme presented in Figure 2 have been reported, which reflect specificity of other design solutions and the author’s preferences. For example, in [7] the ambient medium is expressed through a “voltage (temperature) source”. For that case, the core is enveloped by the transformer windings, and due to the lack of direct contact between the core and ambient air, the thermal resistance R_2 has to be omitted. In [8], the authors offer an equivalent scheme for a dry-transformer, where the thermal resistance R_{12} is omitted. In addition, the two power sources P_1 and P_2 are combined to form only one ($P = P_1 + P_2$), which may lead to calculation inaccuracy when $R_{12} > 0$. For the very special design solution of the toroidal inductor [9], an extended equivalent scheme is proposed, with a large number of thermal resistances to reflect the variety of the model areas. Again, a single heat source is included to energize the scheme, which for that case is acceptable due to the unique coil-core construction. That same idea for a single heat source has also been utilized for the investigation of a high MHz frequency planar transformer [3]. For large oil-immersed power transformers, the problem with a single or divided heat source is presented and discussed in [10,11]. Additional considerations for the transformer oil properties as a cooling medium for those transformers are presented and discussed in [12]. In the thermal equivalent scheme proposed to investigate a dry-type 5 kVA transformer [13], a thermal contact resistance R_{12} between the core and windings exists. This way it separates the core and both windings as heat generating sources. It has to be mentioned here that together with the widely-used-in-engineering thermal equivalent schemes, modern computing tools have already been proposed and introduced [14]. The author’s conclusion in that study is that the

implementation of neural networks may help substantially to predict a power transformer's dynamic thermal behavior.

For the samples such as the one presented in Figure 1, obviously the ideal case from an engineering point of view is when $R_{12} = 0$. This will secure maximum cooling efficiency for the coil. Extended analysis provided in [5] has revealed the supplement of the thermal resistance R_{12} to the temperature rise θ_1 and θ_2 . In [5], calculations of R_{12} component values and its contribution are based on a one-dimensional [6] “plane wall” simplified for engineering use.

An extended version [15] of the scheme, shown in Figure 2, has been proposed to study the dynamic transient response of the transformer. As presented in Figure 3, thermal capacitances C_1/C_2 of the coil/core are included in the scheme. This analysis revealed the important role that the resistance R_{12} may play for the coil temperature time-flow curve, and to what extent this curve will differ from the well-known and widely-accepted-in-engineering practice single exponential curve. In [16], a similar scheme has been proposed to study the transient behaviour of small power pulse transformers, including some nonlinear thermal effects, by converting the current sources into controlled ones. A SPICE subcircuit was created on that ground to help investigations for that special group of transformers.

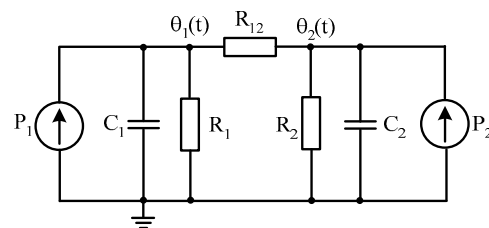


Figure 3. Thermal equivalent scheme for transient mode of operation.

The main objective in this present paper is, by more accurate calculation of the thermal resistances R_1 , R_2 and R_{12} using 3D FEM analysis, to reveal the actual distribution of the temperature field in the construction presented in Figure 1, thus validating previously formulated [5,15] conclusions. Next, attention is additionally focused on how and to what extent both θ_1 and θ_2 depend on the total heat power impact $P = (P_1 + P_2)$ at the given value of R_{12} . In the broad engineering practice, and some academic teaching courses, a concept [17] is formulated, that there exists a linear functional relationship for $\theta_1(P)$ and $\theta_2(P)$. A glance at the equivalent scheme in Figure 2 shows that it will occur only when $R_{12} = 0$, otherwise, deviations shall be expected. The rest of the paper is organized as follows. In Section 2, the developed 3D finite element model is described. In Section 3, experimental and numerical results are presented. In Section 4, discussion of the results is carried out.

2. Description of the 3D FEM Model

The 3D FEM model has been developed using COMSOL Multiphysics software [18] and its heat transfer module. Due to the construction symmetry, as illustrated in Figure 4, only 1/8 part of the construction in Figure 1 has been exempted to create the model. A simplified view is also presented in Figure 5, to illustrate some details of the coil placement inside the core and the real construction dimensions in mm. The air gaps δ_1 (between the spool and the central core leg) and δ_2 (between the winding surface and the external core leg) are also illustrated in this Figure. These two parameters, together with the spool thickness designated as Δs , play an important role for the thermal resistance R_{12} evaluation.

The number of finite elements in the model is about 700,000 and the number of degrees of freedom is about 1,200,000. The typical error of the solution of the system of linear equations is about 5×10^{-5} .

- SA_2 (b,e₁,e,f,f₁,c)—free air coil thermal contact (XY-direction);
- SA_3 (a,b,b₂,a₂)—contact between the coil and upper core part (Z-direction);
- SA_4 (b₂,c₂,d₂,a₂)—contact between the coil and the core central leg via spool wall (X-direction) and the air gap δ_1 ;
- SA_5 (a,b,c,d)—contact between the coil and external core leg via air gap δ_2 (X-direction);
- SA_6 (b₂,c₂,c₄,b₄)—contact between the coil and central core leg via spool frame wall (Y-direction).

Based on the above designations, the thermal contact resistances of interest have been calculated as a ratio between the average (by integration provided) temperature of the respective surface and the total heat flow through that same surface:

- R_1 —thermal contact via surfaces SA_1 and SA_2 . The thermal resistance R_1 is formed by two components R_{1_1} and R_{1_2} , connected in parallel:

$$R_{1_1} = (T_{av_SA1} - T_0)/q_{n_SA1}; \quad (1)$$

$$R_{1_2} = (T_{av_SA2} - T_0)/q_{n_SA2}; \quad (2)$$

$$R_1 = 1/(1/R_{1_1} + 1/R_{1_2}); \quad (3)$$

where T_{av} is the average temperature of the respective surface, q_n is the normal thermal flow through the surface and T_0 is the ambient temperature;

- R_{12} —thermal contact via surfaces SA_3 , SA_4 , SA_5 and SA_6 . The thermal resistance R_{12} is formed by four components $R_{12_1} \dots R_{12_4}$, connected in parallel:

$$R_{12_1} = (T_{av_SA3} - T_{av_SA3_core})/q_{n_SA3}; \quad (4)$$

$$R_{12_2} = (T_{av_SA4} - T_{av_SA4_core})/q_{n_SA4}; \quad (5)$$

$$R_{12_3} = (T_{av_SA5} - T_{av_SA5_core})/q_{n_SA5}; \quad (6)$$

$$R_{12_4} = (T_{av_SA6} - T_{av_SA6_core})/q_{n_SA6}; \quad (7)$$

$$R_{12} = 1/(1/R_{12_1} + 1/R_{12_2} + 1/R_{12_3} + 1/R_{12_4}); \quad (8)$$

where $T_{av_SA1,2,3,4}$ is the average temperature of the respective coil surface; $T_{av_SA1,2,3,4_core}$ is the average temperature of the respective core surface and q_n is the normal thermal flow through the surface. Different materials are present in the gaps between the respective coil and core surfaces: for R_{12_1} and R_{12_4} it is only spool material, for R_{12_2} it is spool material and air and for R_{12_3} —only air;

- R_2 —thermal contact between the external core surface and air. The thermal resistance R_2 is determined by the expression

$$R_2 = (T_{av_core_out} - T_0)/q_{n_core_out}, \quad (9)$$

where $T_{av_core_out}$ is the average temperature of the outer core surface and $q_{n_core_out}$ is the total normal thermal flow through this surface.

3. Experimental and Modeling Results

There is a large variety of AC and DC investigation cases for the sample presented in Figure 1 and its 3D FEM model presented in Figure 4. The DC case meets, e.g., a situation, when a choke of that construction is included in series with the DC welding arc

to prevent a sudden severe arc short circuit. Attention has been mainly focused on some specific cases to meet different combinations of heat powers P_1 dissipated in the coil and P_2 dissipated in the core, together with variation of the air gap δ_2 . It comes naturally that other (additional) required model parameters, including constructive ones and those reflecting thermal properties of the materials used, have to be fixed. These are specified below:

1. The convection coefficient from the open surfaces of the core and coil: fixed to $10 \text{ W/m}^2 \cdot ^\circ\text{C}$;
2. The equivalent thermal conductivity λ in the coil volume: fixed constant $\lambda = 4 \text{ W/m} \cdot ^\circ\text{C}$;
3. The thermal conductivity in the core volume (considered anisotropic): fixed constant $\lambda_{x,y,z} = (44.5; 2.5; 44.5) \text{ W/m} \cdot ^\circ\text{C}$;
4. Ambient temperature: $T_0 = 20 \text{ }^\circ\text{C}$;
5. The thermal conductivity of the spool material $\lambda = 0.06 \text{ W/m} \cdot ^\circ\text{C}$;
6. The thickness of the plastic spool EI 120 wall $\Delta s = 1.8 \text{ mm}$ [19];
7. The air gap between spool wall and central core leg (X direction) $\delta_1 = 0.2 \text{ mm}$.

Figure 7 shows the electrical scheme for the experimental measurements provided. The experimental sample, presented in Figure 1, is a small electric transformer with a rated power of 230 VA. A copper enameled wire with a diameter of 0.78 mm has been used for the winding, which consists of two equal sections S_1 and S_2 , each one having 420 turns. The winding together with the plastic spool forms the transformer coil as an individual constructive unit. The total resistance of the two sections connected in series (Figure 7a) at $25 \text{ }^\circ\text{C}$ was measured to be $6.31 \text{ } \Omega$. A thermocouple K type NiCr-Ni is located in the very center between the two sections for temperature rise measurement purposes, and the coil is not impregnated. The same second thermocouple was installed to control the ferromagnetic core surface temperature. The thermocouple sensitivity used in the range up to $100 \text{ }^\circ\text{C}$ was $40.9 \text{ } \mu\text{V}/^\circ\text{C}$, and the readings were digitally measured.

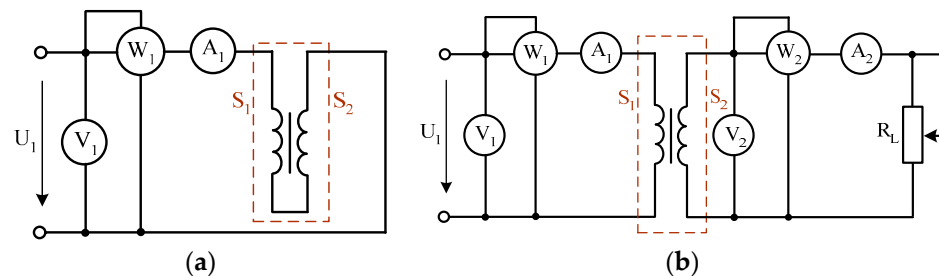


Figure 7. Electrical experimental study measurement schemes: (a) DC case and (b) AC case.

Measures have been taken to keep the heat power impact constant, reflecting the inevitable change of the section S_1 and S_2 conductor's resistance.

For verification purpose, experiments have been provided to cover both DC and AC heat impact cases on the sample. The model temperature rise readings are at the points where the experimental thermocouples were located.

- For the DC experiment (Figure 7a), the two sections S_1 and S_2 were connected in series. The power P_1 was measured by wattmeter W_1 and controlled by adjusting the voltage V_1 . Here at $P_1 = 16 \text{ W}$, $P_2 = 0 \text{ W}$, the steady-state experimental data, obtained after 2.5 h and indexed by (E), were $\theta_{1E} = 53.3 \text{ }^\circ\text{C}$ and $\theta_{2E} = 25.6 \text{ }^\circ\text{C}$. The corresponding model results, at $\delta_1 = 0.6 \text{ mm}$, and $\delta_2 = 3.5 \text{ mm}$, indexed by (M) are $\theta_{1M} = 52.35 \text{ }^\circ\text{C}$ and $\theta_{2M} = 26.06 \text{ }^\circ\text{C}$. Thus, adequate agreement was found.
- For the AC experiment (Figure 7b), a preliminary no-load test for the power loss in the core was provided in a range of the primary voltage V_1 up to 250 V. Table 1 shows the data obtained for the no-load transformer current I_0 and the power loss P_2 in the ferromagnetic core.

Table 1. Experimental data for AC case at no load.

U_1 [V]	51	71.4	90	110.5	129.8	149.9	170.2	190.6	210.8	230
I_0 [mA]	30	35	40	49	61	84	115	156	218	332
P_2 [W]	0.90	1.70	2.49	3.69	4.99	6.78	9.26	12.42	16.15	20.25

Under load at rated voltage $U_1 = 200$ V, the difference between the readings of the digital wattmeters W_1 and W_2 was recorded to get the total heat power impact $P = (P_1 + P_2)$ on the sample. The digital voltmeter V_2 reading was used to recalculate the power P_2 based on the relationship $P_2 = f(V_2)$, having in mind that sections S_1 and S_2 have an equal number of turns. The test was carried out for 4 h, keeping the total power at the level $P = 23 \pm 0.5$ W. Its components were close to $P_1 = 8.7$ W and $P_2 = 14.3$ W, which were used to find the computer model response. The steady-state temperature rise's recorded experimental values are $\theta_{1E} = 56.9$ °C and $\theta_{2E} = 45.6$ °C, while the model results are $\theta_{1M} = 55.0$ °C and $\theta_{2M} = 47.3$ °C, so again adequate agreement is considered to exist.

The model load conditions were varied to meet various typical DC and AC load cases, together with different air gap δ_2 variation.

In Figures 8 and 9, results for the temperature distribution for two cases, ($P_1 = 10$ W, $P_2 = 0$ W) and ($P_1 = 20$ W, $P_2 = 20$ W), are given for two values of the air gap δ_2 (1 and 4 mm).

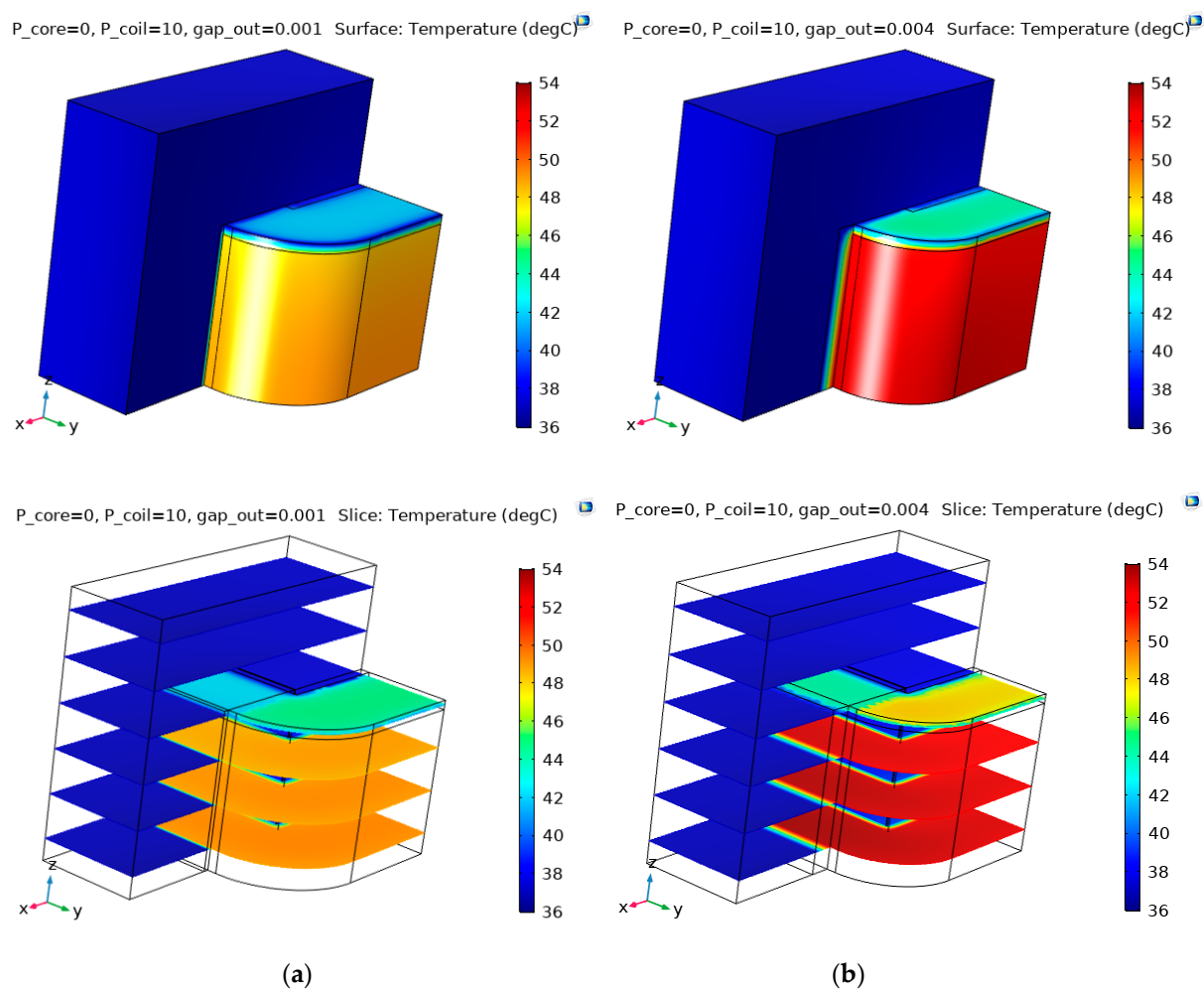


Figure 8. Temperature distribution for the case $P_1 = 10$ W, $P_2 = 0$ W: (a) air gap $\delta_2 = 1$ mm; (b) air gap $\delta_2 = 4$ mm.

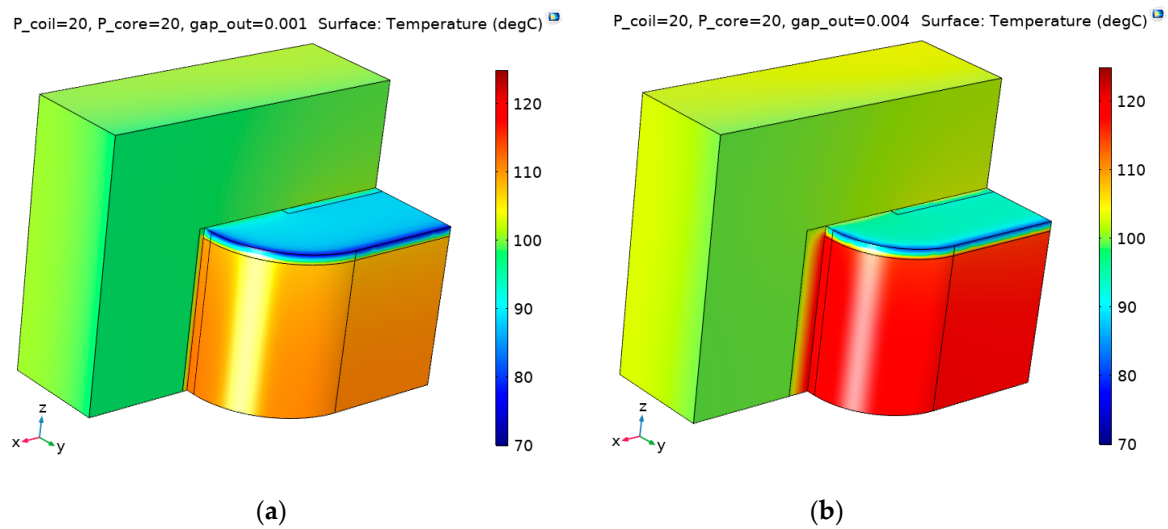


Figure 9. Temperature distribution for the case case $P_1 = 20$ W, $P_2 = 20$ W: (a) air gap $\delta_2 = 1$ mm; (b) air gap $\delta_2 = 4$ mm.

Several typical cases have been studied by varying the coil power P_1 , the core power P_2 and the air gap δ_2 . The combination of the studied parameters, together with the results for both average temperature rises θ_1 and θ_2 , are given in Table 2. In the last three columns, the thermal resistance R_1 , R_{12} and R_2 calculated values are presented.

Table 2. Average coil and core temperature rise and thermal resistances for typical cases.

Case	P_1 [W]	P_2 [W]	δ_2 [mm]	θ_1 [°C]	θ_2 [°C]	R_1 [°C/W]	R_{12} [°C/W]	R_2 [°C/W]
Cs 0	10	0	1	28.87	17.35	6.76	2.32	3.13
Cs 01			2	30.49	17.27	7.08	2.70	3.13
Cs 02			3	31.64	17.42	7.42	2.87	3.13
Cs 03			4	32.65	17.65	7.79	2.97	3.13
Cs 1	10	10	1	46.15	40.61	6.75	2.45	3.12
Cs 11			2	47.71	40.88	7.06	2.89	3.13
Cs 12			3	49.00	41.30	7.40	3.08	3.13
Cs 13			4	50.25	41.77	7.77	3.17	3.12
Cs 2	16	10	1	63.33	51.08	6.76	2.34	3.13
Cs 21			2	65.88	51.29	7.07	2.76	3.13
Cs 22			3	67.88	51.79	7.41	2.95	3.12
Cs 23			4	69.73	52.40	7.78	3.05	3.13
Cs 3	20	10	1	74.75	58.07	6.76	2.30	3.13
Cs 31			2	77.97	58.24	7.08	2.72	3.13
Cs 32			3	80.43	58.80	7.42	2.90	3.12
Cs 33			4	82.69	59.49	7.78	3.00	3.12
Cs 4	20	20	1	92.07	81.31	6.75	2.35	3.13
Cs 41			2	95.21	81.84	7.06	2.81	3.12
Cs 42			3	97.82	82.66	7.40	3.00	3.13
Cs 43			4	100.31	83.60	7.77	3.10	3.12

Temperature distribution for the cases Cs 12, Cs 22 and Cs 33 is shown in Figure 10.

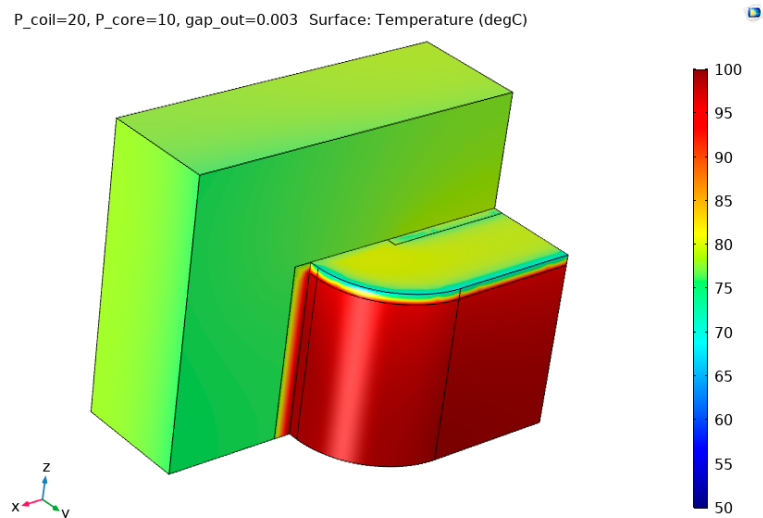
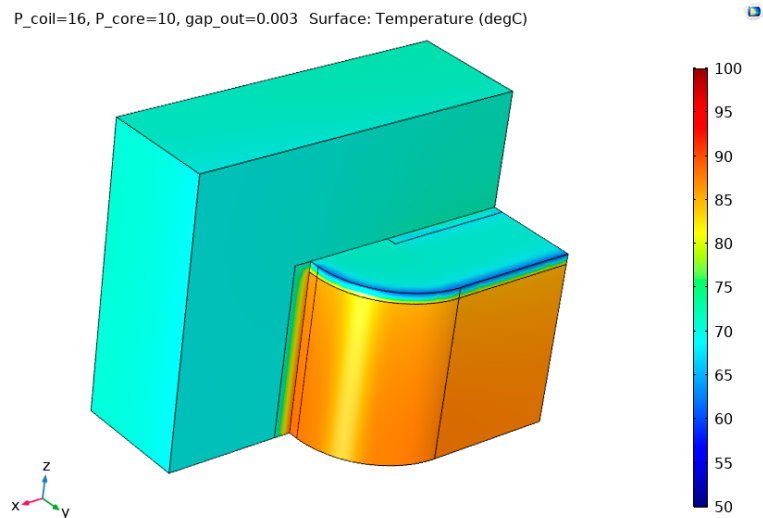
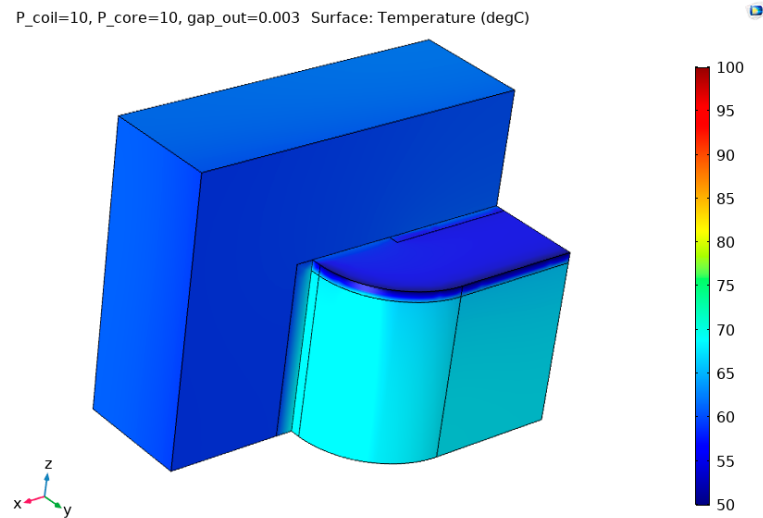


Figure 10. Temperature distribution for the cases from Table 1: (a) Cs 12; (b) Cs 22; (c) Cs 32.

For the Cs 2 case series, at a load typical for a transformer presented in Figure 1, the variation of the temperature rise along the x axis at $z = 0$ and $y = 0$ is displayed in Figure 11. The coordinate $x = 0$ corresponds to the geometry centre of the core central leg, while $x = 60$ mm meets the free air open surface of the core external leg. The four curves presented cover air gap values $\delta_2 = 1, 2, 3$ and 4 mm. The average temperature rises fixed at a gap value $\delta_2 = 3$ mm are $\theta_1 = 67.88$ °C and $\theta_2 = 51.79$ °C.

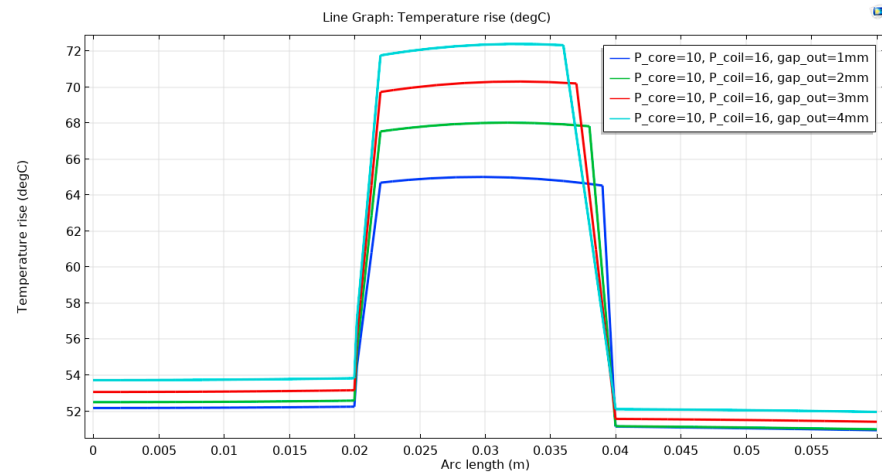


Figure 11. Variation of the temperature rise $\theta(x)$ at $y = 0$, $z = 0$, $x = 0$ –60 mm for the case series Cs 2.

Some additional data in Table 3 illustrate the temperature rise θ at key points shown in Figure 6.

Table 3. Temperature rise θ [degC] at some key points from Figure 6 for the case Cs 22.

Key Point (Figure 6)											
a	a ₁	a ₂	a ₃	a ₄	b	b ₁	b ₂	b ₃	b ₄	e	e ₁
68.1	51.7	67.5	52.7	52.7	66.1	50.0	66.5	65.9	65.0	64.6	64.8
Key point (Figure 6)											
c	c ₁	c ₂	c ₃	c ₄	d	d ₁	d ₂	d ₃	d ₄	f	f ₁
68.4	49.7	68.6	68.2	67.3	70.2	51.6	69.7	53.2	53.1	66.9	67.2

In Table 4, additional results are shown to check the model response at different combinations of AC load. For this extended Cs 22 case, both air gaps are kept constant ($\delta_1 = 0.2$ mm, $\delta_2 = 3$ mm). The total heat impact P on the construction increases, but its components P_1 and P_2 take a different distribution. This approach is illustrated by two specific sub-cases, namely:

Table 4. Average coil and core temperature rise θ_1 and θ_2 at various total AC load P .

Case Cs 22 A				Case Cs 22 B				Total Power P $= P_1 + P_2$ [W]
P ₁ [W]	P ₂ [W]	θ_1 [°C]	θ_2 [°C]	P ₁ [W]	P ₂ [W]	θ_1 [°C]	θ_2 [°C]	
5	3	21.1	15.9	8	0	25.3	13.9	8
10	6	42.0	31.7	10	6	42.0	31.7	16
14.3	8.7	60.3	45.7	10	13	54.2	48.5	23
20	12	83.9	63.6	10	22	69.6	69.4	32
30	18	125.6	95.5	10	38	98.0	108.0	48

Cs 22 A—the ratio between P_1 and P_2 maintained constant— $P_1/P_2 = 1.66$.

Cs 22 B—the coil power P_1 remains practically constant, and only the core power P_2 increases. This variant meets two load cases that may happen practically: DC choke case ($P_1 = 8 \text{ W}$, $P_2 = 0$), and AC case—designed under different quality of the selected transformer core steel ($P_2 = 6; 13; 22; 38 \text{ W}$).

4. Discussion

The construction in Figure 1 presents a complex case to study from a heat transfer point of view. It is a non-homogeneous 3-dimensional body, containing:

- Two main heat-generating parts: the winding itself (including enameled copper wire, dielectric insulation paper etc.) and the iron core around it (produced by use of ferromagnetic electro technical steel lamellae);
- Additional materials: the coil plastic spool;
- Technological airgaps between the coil and the core.

Convection is responsible for that complex body cooling from all open external surfaces. Conduction heat transfer exists between the coil, the spool and the core. The coil temperature evaluation is vital in order to observe standard recommendations of ISO 2578 [4] for the copper conductors' electrical insulation. This temperature is considered as the average value for the entire coil volume V_1 (Figure 2) due to small temperature differences observed in the volume V_1 (of about $2 \div 3 \text{ }^\circ\text{C}$)—a result of the coil's usually high thermal equivalent conductivity, as mentioned in the introduction and based on the authors' experiences.

The equivalent scheme for engineering calculation in Figure 2 may serve to study not only the steady-state behavior [5] of the electromagnetic devices such as the one in Figure 1. If extended by adding thermal capacitances of the coil and the core [15], the modified equivalent scheme presented in Figure 3 may also be used to study different dynamic mode operations. However, here, again, a question is raised for the calculation accuracy of the thermal resistance R_1 , R_2 and R_{12} values. Results for those resistances extracted from the 3D FEM analysis and presented in Table 2 can be considered as supervision of the simplified 1D thermal field approach reported in [15]. These results for R_1 , R_2 and R_{12} show:

1. As expected, they are not influenced by the heat dissipated in the coil and core volumes.
2. Thermal resistance R_1 only slightly depends on the air gap δ_2 . It grows by 15% when the gap δ_2 substantially increases from 1 to 4 mm.
3. Thermal resistance R_{12} increases, as expected, when the gap δ_2 changes from 1 to 4 mm. For the case Cs 1, this change reaches almost 30%.
4. Comparing the values of R_1 , R_2 and R_{12} in Table 2 with the corresponding values reported in [5] shows:
 - No difference observed for resistance R_2 .
 - The R_1 resistance evaluated in [5] is $R_1 = 9.1 \text{ }^\circ\text{C/W}$, i.e., is overestimated by some 15–20%.
 - The R_{12} resistance evaluated in [5] is $R_{12} = 2.0 \text{ }^\circ\text{C/W}$, i.e., is underestimated by 14% for the case Cs 0, and by 33% for the case Cs 03.
5. Comparing the obtained values of R_{12} with the values of the thermal resistances between the core and the two windings of a planar transformer obtained in [16] shows that the values are of the same order, even though the construction of the planar transformer is different from the one studied in this present paper.

The temperature distribution in the coil volume, as presented in Table 3, looks quite reasonable. For the case Cs 22, the powers $P_1 = 16 \text{ W}$ and $P_2 = 10 \text{ W}$ are uniformly distributed in the corresponding volumes of the coil and the core. The temperature hot spot is at the centre of the coil (X direction—points d and d_2). The difference of $0.5 \text{ }^\circ\text{C}$ between these two points can be explained by a slightly better cooling condition through the central core leg with respect to the one through the air gap $\delta_2 = 3 \text{ mm}$. A more substantial difference (more than $3 \text{ }^\circ\text{C}$) is observed between the points (d, d_2) and the free (open-to-air)

coil surface SA_2 . It can be explained by a more favored path for the heat flow from the coil volume enclosed between the core and the free surface SA_2 .

The diagram depicted in Figure 12 illustrates the results presented in Table 4. For sub-case A, including the point $\theta_1 = 0$ and $\theta_2 = 0$ at $P = 0$, the run of the curves $\theta_1(P)$ and $\theta_2(P)$ is practically linear. It is not so for sub-case B. The model data obtained were additionally processed mathematically, using the Least Square Method. Results show deflections from a virtual approximation straight line of the order up to 40%. The two curves $\theta_1(P)$ and $\theta_2(P)$ even cross each other at the point $P = 32$ W.

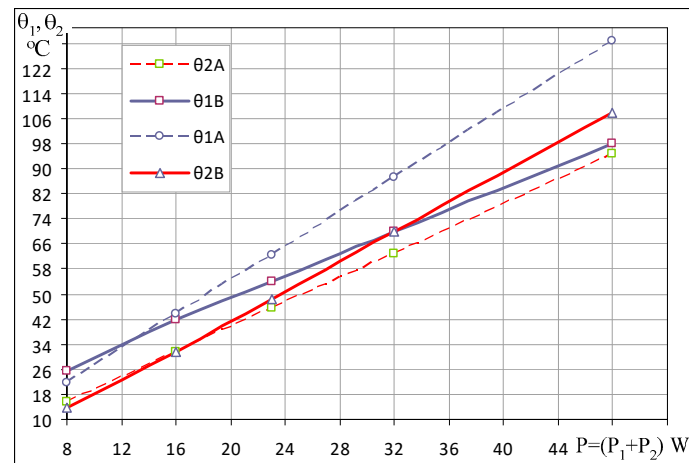


Figure 12. Temperature rise θ_1 and θ_2 vs total power P dissipated, Table 4, case Cs22 A and B.

5. Conclusions

- The data presented in Table 2 show that, expressed in %, the thermal resistance R_{12} may reach values of about 35–38% of the resistance R_1 value. Thus, results drawn in [5,15], but based on simplified engineering calculation of the resistances R_{12} and R_1 , are additionally and precisely supported by the use of modern and more accurate calculations. In this way, without any doubt, it is claimed that the role ratio R_{12}/R_1 plays for both the steady-state and dynamic behavior of the iron-cored coils under analyses above may become significant.
- Regarding the assumed linear flow concept suggested in [17] for the curves $\theta_1(P)$, FEM analysis provided proof that serious deviations may occur when the thermal resistance R_{12} takes figures about 20–30% of the thermal resistance R_1 value. Referring to Figure 2, theoretically, linearity may take place only when:
 - $R_{12} = 0$; this is an ideal heat exchange case between the coil and the core;
 - Heat powers dissipated in the coil P_1 and the core P_2 simultaneously increase, but the ratio P_1/P_2 remains practically the same (dotted lines presented in Figure 12).

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