



Article Energy Performance Curves Prediction of Centrifugal Pumps Based on Constrained PSO-SVR Model

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Abstract: It is of great significance to predict the energy performance of centrifugal pumps for the improvement of the pump design. However, the complex internal flow always affects the performance prediction of centrifugal pumps, particularly under low-flow operating conditions. Relying on the data-fitting method, a multi-condition performance prediction method for centrifugal pumps is proposed, where the performance relationship is incorporated into the particle swarm optimization algorithm, and the prediction model is optimized by automatically meeting the performance constraints. Compared with the experimental results, the performance under multiple operating conditions is well predicted by introducing performance constraints with the mean absolute relative error (MARE) for the head, power and efficiency of 0.85%, 1.53%,1.15%, respectively. By comparing the extreme gradient boosting and support vector regression models, the support vector regression is more suitable for the prediction of performance curves. Finally, by introducing performance constraints, the proposed model demonstrates a dramatic decrease in the head, power and efficiency of MARE by 98.64%, 82.06%, and 85.33%, respectively, when compared with the BP neural network.

Keywords: centrifugal pump; performance relationship; support vector regression; particle swarm; performance prediction

1. Introduction

With the significant development of the economy and society, the utilization of energy resources by humans is gradually increasing. Centrifugal pumps are key to energy transmission and utilization systems. The fluid driven by the mechanical energy generated by the prime mover can be transported to a designated target. At present, many industries rely on energy transportation because of the huge demand for centrifugal pumps [1], such as municipal sewage, power systems, agricultural irrigation, and chemicals. Therefore, an inevitable problem is proposed regarding the effective design of centrifugal pumps. In traditional design and manufacturing methods, a large amount of time is consumed owing to the complicated operation process, which leads to high costs [2]. Performance prediction is one of the most effective ways to improve the optimization design of centrifugal pumps, which helps researchers quickly understand the performance of the designed pump, thereby accelerating the development of pump products and saving costs.

In recent years, numerical simulation methods based on computational fluid dynamics (CFD) have always been the main method used by researchers for performance prediction [3–14]. The 3D simulation of the impeller and its stationary flow in the centrifugal pump casing were analyzed using CFD [3,14], and the head and efficiency of the pump under different flow rates were obtained. Yang [4] used computational fluid dynamics



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). (CFD) to predict the performance of a single-stage centrifugal pump in forward and reverse modes and verified it with experimental data. To improve the performance of a centrifugal pump, a new impeller structure was explored using a numerical simulation method [5]. By calculating the head and efficiency of the centrifugal pump and comparing them with the experimental data, it was found that the overall performance of the centrifugal pump was improved based on the proposed impeller design. However, complex numerical simulation of turbulent flow has always been an unsolved difficulty in computational fluid dynamics (CFD) [11]. High-quality grids and appropriate boundary conditions require a strong work experience for designers, and the long simulation time of CFD is also not conducive to the design of centrifugal pumps.

Theoretical models have also been used for the performance prediction of pumps [15-31], which is a prediction method based on empirical formulas and various assumptions. Among these, theoretical loss models are widely used for performance prediction [22–25]. This method, which classifies the losses in the suction chamber, impeller, and pressurized water chamber of the pumps, uses different calculation methods to calculate the losses of each part and then obtains the performance curves according to the basic equation of the pump [25]. A theoretical model based on an Oseen vortex was proposed to optimize the performance of multistage multiphase pumps. Through verification of the three-stage multiphase pump, the optimized pump head and efficiency can be increased by an average of 0.29% and 0.19%, respectively. To explore the changing condition of the pump performance curves under different speeds, three different methods were summarized by Pedersen [30] to predict the pump performance curves based on proportional law. The results show that the highest prediction accuracy occurs when the speed difference method is used. Although the influence of various factors, such as secondary flow and return flow, is fully considered by the loss model, it is an assumption made under certain conditions, and satisfactory results can only be obtained within a certain range. When conditions change, the model is no longer applicable and is not universal.

Over the last 10–20 years, tremendous progress has been made in the computer industry, and the computing power of computers has become very powerful. In this process, a large amount of data is accumulated, and there is an urgent need for a method that can reasonably utilize the data for analysis. In recent years, machine learning has attracted widespread attention as a method for processing large amounts of data. In the pump industry, machine learning is widely used for pump fault diagnosis [32,33] and performance prediction [34–44]. Deep learning methods based on neural networks are widely used for pump performance prediction [35–38]. The BP neural network is one of the most widely used deep learning methods. Considering the huge computing resources and running time required for numerical simulation, a hybrid neural network based on a theoretical loss model and a BP neural network was proposed in [37]. By introducing the theoretical loss model, the mean-squared error of the head and the efficiency are both significantly reduced. A performance prediction method for a centrifugal pump based on the Levenberg-Marquardt training algorithm and a double-hidden-layer BP neural network [38] is proposed to solve the shortcomings of the traditional single-hiddenlayer BP neural network. Compared with the traditional single hidden layer structure, the convergence time of the improved neural network is significantly reduced, and the situation of low learning efficiency and falling into a local optimum is effectively solved. Because the isentropic efficiency plays a significant role in the system performance of the eddy current pump, a prediction model of the isentropic efficiency of the eddy current pump was constructed by Ping [39], which was combined with experimental data and deep learning methods. In [41], to improve the performance of a centrifugal pump, a global optimization algorithm combining an artificial neural network with an artificial bee colony was developed to redesign the geometry of the impeller, and the CFD method was used to analyze and verify all the regions inside the centrifugal pump. However, a slow learning rate and complex network topology have always been problems with neural networks [37]. Even though the desired neural network structure can be searched using a

genetic algorithm, the selection of parameters of the genetic algorithm also has a significant impact on the convergence accuracy [43]. In addition, it is cumbersome to readjust the structure and weights of the neural network when new data dimensions need to be added to the training data to improve the prediction accuracy of pump performance [33].

Unlike the way in which the neural network continuously approximates the true value, support vector regression (SVR) has a good prediction effect on small-scale and multidimensional data based on its solid theoretical foundation [43]. However, the relationship between centrifugal pump performance can be fragmented when only machine-learning methods are used to make predictions. Furthermore, different model structures can yield different prediction results. Therefore, an energy performance prediction method for centrifugal pumps based on performance constraints was proposed. By combining the relationship between centrifugal pump performance, particle swarm optimization (PSO), and support vector regression, a performance prediction model that satisfies the performance constraint is found. The remainder of this paper is organized as follows. A detailed theoretical description of the proposed method is provided in Sections 2 and 3, respectively. Section 4 discusses the building process of the performance-prediction model and determines the final model structure. Section 5 presents the effects of the model structure. In addition, a comparison of XGBoost with performance constraints and BP neural networks without performance constraints is provided. The conclusions are presented in Section 6.

2. Geometric Features

Centrifugal pumps are regarded as important devices in drainage systems. The wide applications of centrifugal pumps benefit from their easy installation, low maintenance cost, etc. Three main components are passed by the fluid when the centrifugal pump operates: inlet tube, impeller, and volute, as shown in Figure 1. The impeller is the core component of the centrifugal pump that converts the mechanical energy of the rotor into the kinetic energy of the fluid. Therefore, a commonly used method is to optimize the parameters of the impeller to obtain a satisfactory pump performance [45]. The main geometrical parameters are described in Figure 2, including the inlet diameter of the impeller D_j , the inlet diameter of the blade D_1 , the hub diameter of the impeller d_h , the inlet angle of the blade β_2 , the wrap angle of the blade φ and the number of blades z.



Figure 1. The inlet pipe, the impeller and the volute of the centrifugal pump.



Figure 2. Geometric parameters of centrifugal pump impeller.

Another important component is the volute, which plays a significant role in improving the performance of a centrifugal pump, as shown in Figure 3. The main structure of the volute is the diameter of the base circle D_3 , placement angle of the cut tongue φ_0 , and width of the inlet b_3 . The structural diagram of the eighth section of the volute is shown in the upper-left corner. It is accepted that kinetic energy can be converted into pressure energy using a reasonable volute structure.



Figure 3. Geometric parameters of the volute.

Usually, energy performance is described by head *H*, efficiency η , and power *P*. In addition, the flow rate *Q* is usually regarded as a significant operating condition. Therefore, nine geometric parameters, namely D_j , d_h , D_2 , b_2 , β_2 , φ , D_3 , b_3 , z, and the flow rate *Q* are considered as the input variables, which represent the major parameters of the centrifugal pump. Mathematically, it can be expressed as $\mathbf{S} = {\mathbf{X}, \mathbf{Y}}$, where $\mathbf{X} = {\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m}^T \in \mathbb{R}^{m \times 10}$, $\mathbf{Y} = {\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m}^T \in \mathbb{R}^{m \times 3}$, and *m* are the dimensions of the variable. Furthermore, the *i* th sample of the centrifugal pump can be written as Equation (1).

$$\mathbf{S}_{i} = \left\{ \begin{array}{l} \mathbf{x}_{i} = \left[D_{j}(i), d_{h}(i), D_{2}(i), b_{2}(i), \beta_{2}(i), \varphi(i), z(i), D_{3}(i), b_{3}(i), Q(i) \right]^{T} \\ \mathbf{y}_{i} = H(i), P(i), \eta(i) \end{array} \right\}$$
(1)

3. Theoretical Analysis

3.1. Performance Relationship of Pumps

The enhancement of the energy per unit weight of fluid from the pump inlet to the outlet is called head H (the height of the fluid pumped by the pump, m).

$$H = E_d - E_s \tag{2}$$

where E_d and E_s are the energy per unit weight of fluid at the pump outlet and inlet, respectively.

There is always a close relationship between energy and power. The effective energy obtained in the fluid pump per unit time is called output power P_e . According to the definition of head H, the effective power P_e can be expressed as:

$$P_e = \rho g Q H \tag{3}$$

where ρ , *g*, and *Q* are the density (kg/m³), acceleration of gravity (m/s²), and flow rate (m³/s) of the fluid delivered by the pump, respectively.

Usually, the input power of the pump is provided by default, which is transmitted by the prime mover to the pump shaft, also called shaft power *P*. During the operation of the pump, it is inevitable that a certain energy loss can be generated, namely loss power ΔP , thus, the effective power of the pump is equal to the difference between the shaft and loss power:

$$P_e = P - \Delta P \tag{4}$$

To clarify the state of the pump during running time, the ratio between the effective power and shaft power is defined as the efficiency η of the pump. Thus, the relationship between the head *H*, power *P*, and efficiency can be derived as follows:

$$\eta = \frac{P - \Delta P}{P} = \frac{\rho g Q H}{P} \tag{5}$$

3.2. Support Vector Regression

It is widely acknowledged that support vector regression is suitable for processing small-scale samples. By mapping the data into a high-dimensional space, Support vector regression expected to find the "support vector factor" that minimizes the distance between sample points and the hyperplane [46,47]. The specific representation of the model can be written as:

$$f(\mathbf{x}) = \boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x}) + b \tag{6}$$

where ω , *b* are the parameters to be obtained by the model, *x* is the input data, which are represented in the form of a matrix, and ϕ represents the mapping of the samples to the high-dimensional space.

A relaxation factor is introduced to adjust the fault tolerance of the model and prevent overfitting. Therefore, the optimization function of support vector regression can be shown in Equation (7):

where *C* is the penalty coefficient, which is used to adjust the generalization performance of the model, ξ_i and $\hat{\xi}_i$ are the introduced relaxation factors; and ℓ_{ε} is the ε - insensitive loss function.

Finally, the solution of support vector regression can be obtained by transforming the problem into a dual problem:

$$f(\mathbf{x}) = \sum_{i=1}^{m} (\hat{\alpha}_i - \alpha_i) \kappa(\mathbf{x}, \mathbf{x}_i) + b$$
(8)

where α_i and $\hat{\alpha}_i$ are Lagrange multipliers, and $\kappa(\mathbf{x}, \mathbf{x}_i)$ is the kernel function of the model.

3.3. Particle Swarm Optimization

A particle swarm optimization (PSO) algorithm is applied to combine the performance relationship with the SVR model. One of the most important characteristics of a particle swarm optimization algorithm is information sharing between the particle and swarm. During the optimization process, each particle has two attributes: speed v and position x, where speed v determines the pace of particle motion, and position x determines the direction of particle motion. The particle is allowed to search for the optimization approach is shown in (9).

$$v_i = \omega v_i + c_1 \alpha (p_{op_i} - x_i) + c_2 \alpha (g_{op_i} - x_i)$$

$$x_i = x_i + v_i$$
(9)

where g_{op} is the global optimal solution, the speed v and position x of each particle are always adjusted continuously according to its own optimal solution p_{op} and the global optimal solution g_{op} , i represents the ith index of the particle; ω is the inertia factor, which is used to adjust the global optimization ability of the particle, c_1 and c_2 are the particle and swarm learning factors, which have the ability to weigh the local and global optimization of particles, respectively.

4. Model Construction

To obtain sufficient samples, 30 performance curves of single-stage and single-suction centrifugal pumps in different models were selected, with specific speeds n_s ranging from 23.1 (m/s²)^{3/4} to 195.6 (m/s²)^{3/4}; the specific speeds formula is shown in Equation (10). From each curve, the performance indices of 14–20 operation points were collected. In total, the geometric and performance parameters were recorded under 535 different working conditions. The samples were then divided into two groups: 428 for training and 107 for testing, in proportion to 4:1.

$$n_s = \frac{3.65n\sqrt{Q}}{H^{3/4}} \tag{10}$$

4.1. Data Porcessing

There are many geometric parameters for centrifugal pumps with different data ranges for each parameter. In the process of training, the parameters of small data ranges tend to be covered by those of large ones, so the small data range parameters cannot be well fit into prediction models. Normalization is an effective method for addressing this problem. By reducing the data range from 0 to 1, the geometric parameters could be fully trained in the prediction model. The specific normalization formula is shown in Equation (11), and a portion of the normalized data is shown in Table 1.

$$x^* = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{11}$$

where x_{max} and x_{min} are the maximum and minimum values of the samples, respectively.

| Number Series | Q | D_j | d_h | D_2 | b ₂ | β_2 | z | φ | D_3 | b_3 |
|------------------|--------|--------|--------|--------|-----------------------|-----------|-----|--------|--------|--------|
| 1 | 0.0196 | 0.0606 | 0 | 0.4156 | 0 | 0.36 | 0 | 0.8276 | 0.4060 | 0.0588 |
| 2 | 0.0283 | 0.2272 | 0.6944 | 0.5121 | 0.0385 | 0.52 | 0 | 0.9342 | 0.5639 | 0.1324 |
| 3 | 0.1907 | 0.3485 | 0 | 0.1596 | 0.3077 | 0.48 | 0.5 | 0.2884 | 0.1429 | 0.3235 |
| 4 | 0.9212 | 1 | 1 | 0.5121 | 1 | 0.4 | 0.5 | 0 | 0.5639 | 1 |
| ÷ | ÷ | : | ÷ | : | ÷ | ÷ | ÷ | ÷ | : | ÷ |
| 533 | 0.0176 | 0.0606 | 0 | 0.4156 | 0 | 0.36 | 0 | 0.8276 | 0.4060 | 0.0588 |
| 534 | 0.0434 | 0.1591 | 0 | 0.0074 | 0.1154 | 0.6 | 1 | 0.3135 | 0 | 0.1471 |
| 535 | 0.2738 | 0.6136 | 0 | 0.3191 | 0.4327 | 0.1 | 1 | 0.1693 | 0.3383 | 0.4412 |

Table 1. Partially normalized dimensionless geometric parameters.

4.2. Implementation Steps of Prediction Model

Based on the particle swarm optimization algorithm, a performance prediction model of a centrifugal pump that considers performance constraints is proposed. Because there were deviations around the experimental value for the predicted performance, and undeniable error occurred in the calculation efficiency. Taking the constraint relationship indicated by Equation (5) as the constraint condition of the model, the predicted value of each sample will be close to or satisfy the equation. The specific form is shown in (12).

$$\frac{1}{m} \sum_{i=1}^{m} (\eta_i - \eta_i')^2 \tag{12}$$

where *m* is the number of samples, η_i is the test efficiency of the *i*th sample, and η'_i is the calculation efficiency, which is calculated from the predicted head H'_i and predicted power P'_i according to Equation (5).

A few-shot kernel machine-learning method (SVR) is adopted in this study. The predicted performance curves of the SVR model are smoother than that of other machine learning methods because it finds a high-dimensional hyperplane, which makes it widely used in few-shot prediction models [48]. The prediction accuracy of the model depends mainly on the type of kernel function, coefficient γ of the kernel function, penalty coefficient *C*, and tolerance parameter ε . Among these, the kernel function is of utmost importance. However, there is no efficient method for selecting a kernel function. Currently, some kernel functions are widely used: linear, polynomial, sigmoid, and radial basis functions (RBF). RBF is one of the most widely applied functions for good spatial mapping.

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = \mathrm{e}^{-\gamma \|\mathbf{x}_i - \mathbf{x}_j\|^2} \tag{13}$$

where *i*, *j* are the index of the samples.

The prediction accuracy is significantly affected by the SVR model parameters. The penalty coefficient *C* and kernel function coefficient γ are the most important parameters of the SVR model. The penalty coefficient *C* is used to prevent overfitting so that the test samples can be better predicted. Kernel function coefficients γ are used to adjust the complexity of the model, which increases when the dimensionality of the sample is too high. In this study, the penalty coefficient *C* and kernel function coefficient γ were selected to minimize the performance constraint functions built by Equation (12). The specific steps are shown in Figure 4.



Figure 4. Construction Process of PSO-SVR model.

Step 1: Collect 535 samples under different working conditions to build the prediction model and divide 428 samples for training and 107 samples for testing. All samples were normalized using Equation (11).

Step 2: Establish the relationship between the optimization variables and performance constraint functions, as shown in Equation (14), determine the optimization range of the optimization variables, and select the appropriate kernel function according to the sample characteristics.

$$\varsigma = \frac{1}{m} \sum_{i=1}^{m} \left(\frac{\rho g Q f_{H_i}(\gamma, C)}{f_{P_i}(\gamma, C)} - \eta_i \right)^2$$
(14)

where ς is the constraint error, $f_{H_i}(\gamma, C)$ and $f_{P_i}(\gamma, C)$ are the predicted head and power, respectively.

Step 3: Randomly distribute 100 particles in the optimization range, adjust the local learning factor c_1 and the global learning factor c_2 , and assign a relatively large value for the inertia factor ω to improve the global optimization ability.

Step 4: Introduce Equation (14) into the objective function to determine the particle with the smallest constraint error. The maximum evolution generations were set to 15.

Step 5: Determine whether the particle velocity tends to converge. If it converges, switch to Step 6. Otherwise, go back to Step 2.

Step 6: Establish and train the SVR model based on the position of the globally optimal particle.

Step 7: Predict the head and power based on 107 test samples. Calculate the efficiency using Equation (5) and compare it with the experimental efficiency to determine whether the constraint accuracy is satisfied. If not, return to Step 2. Otherwise, switch to Step 8. Step 8: Output the SVR model to predict the energy performance of the centrifugal pump.

4.3. Establishment of PSO-SVR

Overall, 100 particles were randomly generated within the optimization range, and each particle contained two pieces of variable information: the penalty coefficient *C* and the kernel function coefficient γ . In the iterative process, apart from advancing around itself, the particle is affected by the global optimal particle. Figure 5 illustrates the changing situation of two

random particles during the iterative process. This reveals that the initial position of the particle is randomly assigned, which can be an ideal or relatively large value. As the number of iterations increased, the particle continuously updated its speed and direction.



Figure 5. Iterative process of random particles. (**a**): Iterative process of particle A; (**b**): Iterative process of particle B.

Particle A changes greatly in the process of updating, and its maximum mean square error reaches over 10,000 after the seventh iteration. Then the particle direction is changed, oscillating back and forth where the penalty coefficient *C* ranges from 9000 to 10,000 and the kernel function coefficient γ ranges from 1 to 2. The iterative process of particle B resembles that of particle A, which also constantly changes its speed and direction, finally oscillating around *C* = 9000–10,000 and γ = 1–2. The difference lies in the fact that Particle B is at an ideal position, and the overall change is not significant.

The iterative process of the optimal particle Figure 6a and the overall constraint error variation surface Figure 6b of the performance prediction model is shown in Figure 6. As can be seen from Figure 6a, the optimal particle starts to iterate from around C = 9000, $\gamma = 4$, and then the iteration moves toward the direction where *C* is decreasing. When the constraint error is perceived to be increasing, both *C* and γ simultaneously begin to change, in the direction of γ decreasing and *C* increasing. Finally, it reaches the global optimal value after 13 iterations, which is 2.165. The constraint error changing surface is shown in Figure 6b. It reveals that the constraint error has multiple peaks and troughs. When *C* and γ both approach 0, the peak increases rapidly, and the larger the peak, the less the predicted performance satisfies the relationship between the performance. The trough appears where $\gamma = 1-2$ or 2-3. Furthermore, the deeper the trough, the better the predicted performance is in line with the relationship among the performance. The minimum value occurs where $\gamma = 1-2$, and the minimum value is 1.48. The specific parameters of the model are shown in Table 2.

Name of Parameters Value number of particles 100 range of optimization $1-10,000 \setminus 1-5$ PSO times of iteration 15 learning factor 0.5\0.5 weight factor 0.8 RBF kernel function 10,000 С SVR 1.48 γ tolerance 0.001 ε 0.1

Table 2. Parameters of performance prediction model.



Figure 6. Iterative process of optimal particle. (a): Optimal particle change path; (b): Constraint error change surface.

5. Discussion and Results

Optimal Particle

5.1. Prediction Analysis of Tset Sample

A total of 107 test samples were used to predict the head and power of the singlestage single-suction centrifugal pump, and the predicted results are shown in Figure 7. The comparative results of the predicted and experimental performances are presented in Figure 7a,b, respectively. It can be observed that the prediction results of the head and power are close to the experimental results, indicating that the performance prediction accuracy of centrifugal pumps meets the requirements of engineering practice by considering the performance constraints. The absolute relative errors (ARE) of the head and power are shown in Figure 7d,e. This reveals that the prediction results of the model are in line, with expectations, with the absolute relative errors of the head and power being less than 12%. The experimental value of the head in the entire fluid field is generally large, such that the denominator of the relative error equation is large, thus leading to small calculation results for the head. The specific ARE equation is

$$\hat{\mathbf{y}} = |(y_i - y'_i) / y_i|, i = 1 \sim 107$$
(15)

where *i* is the index of the test sample, y_i is the test experiments of the *i*th sample, and y'_i is the predicted value of the *i*th sample.

A comparison of the predicted and experimental efficiencies is shown in Figure 7c. In the figure, there is only a slight error between the predicted and test efficiencies, which proves that the predicted head and power of the model are close enough to the test head and power so that the efficiency is calculated by the performance constraint Formula (5) is also similar. In Figure 7f, the ARE of the efficiency is less than 13%, and the mean absolute relative error (MARE) is only 1.82%, which further indicates that the centrifugal pump performance prediction model that is based on the performance constraints has sufficient prediction accuracy.



Figure 7. Comparison results of test sample. (**a**–**c**) are the comparison of test samples and prediction samples; (**d**–**f**) are the absolute relative error of head, power, and efficiency.

5.2. Prediction of Multi-Condition Samples

To verify the applicability of the prediction model, five single-stage single-suction centrifugal pumps with specific speeds of 23.4, 46.2, 63.2, 90.7, and 125.3 are selected as the research objects to predict their head and efficiency under 7–10 operating conditions. The geometric and performance parameters of the single-stage single-suction centrifugal pump are listed in Table 3.

| Fable 3. Geometric p | arameters of cer | itrifugal | pump. |
|----------------------|------------------|-----------|-------|
|----------------------|------------------|-----------|-------|

| n _s | Q/(m ³ /h) | D _j /mm | d _h /mm | D_2/mm | <i>b</i> ₂ /mm | $\beta_2/(^\circ)$ | z | <i>φ</i> /(°) | D ₃ /mm | b ₃ / mm |
|----------------|-----------------------|--------------------|--------------------|----------|---------------------------|--------------------|---|---------------|--------------------|-----------------------------------|
| 23.4 | 12.5 | 50 | 14 | 251 | 6.5 | 32.5 | 4 | 142 | 255 | 18 |
| 46.2 | 50 | 80 | 0 | 252 | 6.5 | 39 | 5 | 135 | 260 | 22 |
| ÷ | ÷ | | | ÷ | : | ÷ | ÷ | ÷ | ÷ | : |
| 125.3 | 100 | 90 | 0 | 173 | 20 | 29 | 5 | 112 | 178 | 34 |

Figure 8a–e correspond to five single-stage single-suction centrifugal pumps with specific speeds of 23.4, 46.2, 63.2, 90.7, and 125.3, respectively. There was a good performance in the prediction results of the five centrifugal pumps under multiple operating conditions. The best prediction effect appears for the centrifugal pump with a specific speed of 46.2, in which the MARE of the head, power and efficiency are 0.36%, 0.46%, and 0.55%, respectively. For the centrifugal pump with a specific speed of 90.7, the MARE of the head, power and efficiency were 0.56%, 1.52%, and 1.15%, respectively. The worst prediction effect occurs in the centrifugal pump with a specific speed of 125.3, and the ARE of the head and power is up to 5.76% and 6.43%, respectively. The remaining two sets of validation samples are sufficiently close to the experimental values.



Figure 8. Comparison of performance prediction results of five single-stage single-suction centrifugal pumps. (**a**–**e**) are the prediction results of centrifugal pumps with n_s = 23.4, 46.2, 63.2, 90.7 and 125.3, respectively.

By observing the prediction results of five single-stage single-suction centrifugal pumps, it can be found that: the overall prediction effect is poor when the specific speed is too low or too high. The initial value of the performance curve has a significant impact on the prediction results, and the larger the relative error of the performance under lowflow conditions, the worse the effect of the model under the entire flow field. In most cases, the maximum ARE of centrifugal pumps occurs outside the design conditions. The prediction results show that the multi-condition performance prediction model can be established effectively by particle swarm optimization and constraint errors, and the prediction accuracy for engineering practice is satisfied.

5.3. Comparsion with XGBoost Model

Extreme gradient boosting (XGBoost) is widely used in the field of regression prediction owing to its advantages of fast running speed, good prediction effect, and ability to handle large-scale data [49]. The performance prediction of centrifugal pumps is being studied using a variety of machine learning algorithms, but the performance prediction of centrifugal pumps based on XGBoost is rare. In this study, XGBoost was used to predict the performance of centrifugal pumps using performance constraints. The specific parameters for optimizing the XGBoost are listed in Table 4.

| | Name of Parameters | Value | | |
|---------|-----------------------|--|--|--|
| | number of particles | 100 | | |
| | range of optimization | $3 \sim 10 \\ 0.1 \sim 0.8 \\ 300 \sim 1000$ | | |
| PSO | times of iteration | 15 | | |
| | learning factor | $0.5 \setminus 0.5$ | | |
| | weight factor | 0.8 | | |
| | Max depth | 10 | | |
| | Learning rate | 0.1204 | | |
| XGBoost | Estimators | 720 | | |
| | objective | Squared error | | |
| | jobs | -1 | | |

Table 4. Performance prediction model of centrifugal pump based on XGBoost.

Two sets of single-stage single-suction centrifugal pumps with specific speeds of 46.2 and 90.7 were used to verify the applicability of the PSO-XGBoost prediction model and were compared with the PSO-SVR prediction model. The comparison results are shown in Figures 9 and 10, respectively.



Figure 9. Comparison of performance prediction models for single-stage single-suction centrifugal pumps with a specific speed of 46.2. (**a**–**c**) are the predictions of head, power and efficiency; (**d**–**f**) are the absolute relative error of head, power and efficiency.



Figure 10. Comparison of performance prediction models for single-stage single-suction centrifugal pumps with a specific speed of 90.7. (**a**–**c**) are the predictions of head, power and efficiency; (**d**–**f**) are the absolute relative error of head, power and efficiency.

In summary, the prediction accuracy of the XGBoost model is good for the MARE of the head, and the power and efficiency of the two centrifugal pumps are 2.11%, 2.61%, 4.03%, 2.50%, 4.08%, and 5.95%, respectively. The overall mean absolute error is less than 6%.

Compared with the SVR prediction model, it was found that the performance predicted by the XGBoost prediction model had a large ARE for individual operating conditions, as shown in the red boxes in Figure 9a,c and Figure 10a,c. The sudden shift in the prediction trend is poor, which can cause unknown difficulties in the prediction of centrifugal pump performance. In addition, the performance curves of XGBoost model prediction are not smooth compared with the SVR model, mainly attributed to the fact that SVR has better prediction continuity by finding hyperplanes in high-dimensional space, which is only affected by a small number of samples. Compared with the ARE of the two centrifugal pumps, the ARE of the XGBoost model is higher than that of the SVR model, with the MARE of head, power and efficiency rising by 486%, 467%, 633% and 346%, 168%, and 417%, respectively. Therefore, it proves that the SVR prediction model is more suitable for the study of performance curves prediction of centrifugal pumps.

5.4. Comparisons with BP Neural Network

A BP neural network model that does not consider performance constraints is used for comparison with the PSO-SVR model and the comparison results are shown in Figure 11. The absolute relative error between the predicted and experimental performance for the five centrifugal pumps shows that the prediction error of the BP neural network model is higher than that of the SVR model with performance constraints. Although the MARE of the head, power and efficiency are 2.24%, 8.53%, and 7.85%, respectively, which are all less than 9%, the maximum ARE the of three types of performance predicted by the BP neural network model are all greater than 15%. In contrast, the maximum ARE of the performance predicted by the constrained SVR model was less than 10%, indicating that the method of introducing performance constraints can effectively reduce the prediction error. It is worth mentioning that the ARE of the three performances based on the BP neural network model oscillate with a certain regularity, mainly because of the inaccurate prediction under small flow conditions. In other words, a small test value leads to a low denominator of Equation (14), resulting in a large calculation value. Table 5 presents a comparison of the SVR, XGBoost, and BP neural networks.



Figure 11. Comparison of BP neural network and SVR performance prediction model. (**a**–**c**) are the are the comparisons of head, power and efficiency.

Table 5. Comparison of PSO-SVR, PSO-XGBoost, and BP neural network models.

| | BPNN | PSO-XGBoost | PSO-SVR |
|----------------------|--------|-------------|---------|
| MSE | 26.225 | 14.37 | 0.638 |
| R ² score | 0.9914 | 0.9913 | 0.9993 |
| MARE | 6.21% | 3.76% | 1.18% |

6. Conclusions

Based on the geometric parameters of the impeller and volute of centrifugal pumps, a multi-condition performance prediction model of centrifugal pumps is proposed that incorporates the performance relationship into the particle swarm optimization algorithm. The performance (i.e., head, power and efficiency) of the centrifugal pumps can be predicted simultaneously and satisfied with the performance relationship. The performance prediction model proposed in this study can be used as a reference for the prediction method of centrifugal pump performance curves. A total of 428 samples were used to train the performance prediction model, 107 samples to test the generalization ability of the model, and 46 samples to verify the prediction effect of the model. The following conclusions were drawn:

(1) The structure of the energy performance prediction model under multi-condition operations can be effectively determined based on the particle swarm optimization and performance relationship. The penalty coefficient *C* and kernel function coefficient γ of the regression are 10,000 and 1.48, respectively.

(2) The multi-condition performance is well predicted by considering the performance constraints, the maximum ARE of the head, power and efficiency of the 46 verification samples are 5.76%, 6.42%, and 5.02%, respectively, and the MARE are 0.85%, 1.53%, and 1.15%, respectively. The overall MARE is less than 3%.

(3) The MARE of the head, power and efficiency corresponding to the PSO-SVR model decreased by 58.54%, 65.38%, and 76.05%, respectively, compared with those of the PSO-XGBoost model, indicating that the SVR model is more suitable than XGBoost for the performance prediction of centrifugal pumps. When compared with the BP neural network, the MARE of the head, power, and efficiency corresponding to the SVR prediction model with performance constraints decreased by 62.21%, 82.06%, and 85.33%, respectively, indicating that the introduction of performance constraints can effectively improve the overall prediction accuracy.

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Nomenclature

| ARE | Absolute relative error |
|---------|------------------------------|
| MARE | Mean Absolute Relative Error |
| CFD | Computational Fluid Dynamics |
| SVR | Support Vector Regression |
| PSO | Particle Swarm Optimization |
| XGBoost | Extreme Gradient Boosting |
| BP | Back Propagation |
| | |

| D_j | Inlet diameter of impeller |
|--------------------|--|
| D_1 | Inlet diameter of blade |
| d_h | Hub diameter of impeller |
| β_1 | Inlet angle of blade |
| D_2 | Outlet diameter of impeller |
| b_2 | Outlet width of impeller |
| β_2 | Outlet angle of blade |
| φ | Wrap angle of blade |
| z | Number of blades |
| D_3 | Diameter of base circle |
| φ_0 | Placement angle of cut tongue |
| b_3 | Width of inlet |
| H | Head |
| H! | Predicted head of the <i>i</i> th sample |
| P | Shaft power |
| P! | predicted shaft power of the <i>i</i> th sample |
| P_{i} | Output power |
| ΛP | Loss power |
| 1 11 | Efficiency |
| η 11. | Tost officional of the <i>i</i> th sample |
| η_1 | Calculation officiancy of the <i>i</i> th sample |
| η_i | Flow |
| £. | Energy per unit weight of fluid at the nump outlet |
| E_d | Energy per unit weight of fluid at the pump outer |
| | Density |
| ρ σ | Acceleration of gravity |
| ð C | Penalty coefficient |
| ک . | Relavation factor |
| 51 2. | Relaxation factor |
| ς1 β | Loss function |
| vе N· | Lagrange multipliers |
| $\hat{\alpha}_{i}$ | Lagrange multipliers |
| \mathbf{x}_{l} | Kernel function |
| $\kappa(x, x_1)$ | Sneed of particle |
| r | Position of particle |
| n | Current on timal value |
| o an | Global optimal solution |
| 80p W | Inertia factor |
| C1 | Particle learning factor |
| C2 | Swarm learning factor |
| Xmax | Maximum value of sample |
| Xmin | Minimum value of sample |
| m | Number of samples |
| γ | Coefficient of the kernel function |
| ε | Tolerance parameter |
| ς | Constraint error |
| U; | Test experiments of the <i>i</i> th sample |
| v'_i | Predicted value of the <i>i</i> th sample |
| . / 1 | |

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