Article

Impacts of Yield and Seasonal Prices on the Operation of Lancang Cascaded Reservoirs

Mengfei Xie [1], Suzhen Feng [2], Jinwen Wang [2], Maolin Zhang [1] and Cheng Chen [2,*]

1   Kunming Power Exchange Center Co., Ltd., Kunming 650011, China; xiemengfeihust@foxmail.com (M.X.); zhangmaolin_9901@163.com (M.Z.)

2   School of Civil & Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China; szfeng@hust.edu.cn (S.F.); jinwen.wang@hust.edu.cn (J.W.)

*   Correspondence: chencheng410@hust.edu.cn

Abstract: This work formulates a stochastic dynamic programming (SDP) model that incorporates seasonal electricity prices and can handle a constraint on power yield, which is assumed to be satisfied at any time it is possible, thus allowing for an analysis of their impacts on the operational performances of cascaded reservoirs. The model is applied to the Lancang Cascade, specifically its two largest reservoirs, Xiaowan and Nuozhadu. The results show that increasing the power yield of the cascade will reduce energy production unfavorably but will impact water spillage favorably, with a power yield of 2000 MW and with a 91% reliability suggested as being a satisfactory operational target. The case study also suggests that using seasonal electricity prices makes the power generation very unstable during weeks 12–20, which is a period of time that is critical to transferring from dry to flooding seasons.

Keywords: stochastic dynamic programming (SDP); power yield; seasonal price; reliability; cascaded reservoirs

1. Introduction

Due to the stochastic characteristics of inflow into reservoir and electricity prices, long-term hydropower operation is generally interpreted as a stochastic sequential decision-making process. Additionally, the nonlinear factors that involve hydropower output are interconnected, including the water head, reservoir storage, release, generating capacity, and water rate, which raises challenges for long-term hydropower operation [1]. Stochastic dynamic programming (SDP) is the most conventional methodology that can be used to address these issues. SDP has long been studied and successfully implemented in reservoir operation [2–4], which is attributable to its capability for dealing with the nonlinearity of functions involved in the model as well as in terms of its structure being inherently compatible with the stage-by-stage decision-making procedure often employed in real-time reservoir operation. For example, the water balance expresses the relationship of storages between two successive time-steps [5], and the Markov chain is commonly used to represent the stochastic relationship of inflows also between two successive time-steps [6]. Both the water balance and the Markov chain can serve well as the transition equations in dynamic programming (DP).

SDP can derive a close-loop operating rule, which defines the optimal decision at any possible state and is usually represented by a combination of discrete values of the state variables that traditionally include the storage at the beginning of the current or decision-making stage and the local inflow during the current stage of each reservoir [7–9]. The discretization in even a decent resolution of the state variables makes it problematic for SDP to be applicable to a reservoir system on a large scale such as in instances with more than three reservoirs. Some scholars [10,11] call it a "dimensional curse" since enumerating to obtain the optimal decisions for all the possible combinations of the discrete values of the

state variables takes a very long computational time, which is intolerable in engineering practice. For a hydropower system with no more than three cascaded reservoirs, however, SDP is always one of the most favorable options.

In China, where the advancement of power marketization and the market-oriented electricity price plays a vital role in promoting the optimal allocation of power resources [12], electricity price is the most pivotal factor in regulating the activity of the power producer. In deregulated electricity markets, price uncertainty must also be included in the scheduling models [13]. Due to the influence of uncertain quantities including electricity price and inflow, reliability in the long-term hydropower is a meaningful measurement to evaluate the hydropower system [14]. For instance, Saadat and Asghari [15] employed the reliability concept to maximize the reservoir releases to satisfy downstream demands. Reliability is an important indicator for power supply in the electrical system [16].

This work will apply SDP to the Xiaowan and Nuozhadu, the two biggest cascaded hydropower reservoirs in the Lancang River. The problem size is small and can be easily handled with SDP, which, however, will still encounter great challenges in dealing with the probabilistic constraints, such as those relating to the reliability, vulnerability, and resilience of the system, due to the difficulty of formulating these constraints into ones that are adaptable to the structure of the DP [17]. The Lancang hydropower cascade that is to be studied is required to yield energy at a high reliability, which is essentially a probabilistic constraint that is originally unable to be explicitly included in the SDP formulation but is likely to be converted into a normal constrain by assuming that the power yield must be satisfied whenever possible.

This work aims to investigate how the power yield will affect the operation of lancing cascaded reservoirs in terms of reliability, power production and water spillages, the proper power yield the cascade should target, as well as how the implementation of seasonal electricity prices will have impacts on the hydropower generation, in addition to simultaneously deriving the operational policies of the cascaded hydropower reservoirs that meet different objectives.

2. Problem Formulation

The reservoir operation, when formulated into SDP, may use one or a few state variables, usually selected from the storage S_t at the beginning of time-step t, the local inflow Q_{t-1} in previous time-step $t-1$, the local inflow Q_t in current time-step t, and the forecasted inflow H_t in time-step t [18]. The recursive objective function in the SDP, for one reservoir and when using the H_t as the inflow state variable, can be expressed as follows:

$$f_t(S_t, H_t) = \mathop{E}_{Q_t|H_t} \left\{ \max_{R_t} \left[B(S_t, Q_t, R_t) + \mathop{E}_{H_{t+1}|H_t,Q_t} [f_{t+1}(S_{t+1}, H_{t+1})] \right] \right\} \quad (1)$$

where $f_t(S_t, H_t)$ is the benefit-to-go function, representing the maximum expected benefit till the end of the planning horizon given the initial storage S_t at the beginning of time-step t and the hydrological state H_t in time-step t; $\mathop{E}_{Q_t|H_t}\{\cdot\}$ is the expectation operator at conditional probability of $P(Q_t \mid H_t)$; and $B(S_t, Q_t, R_t)$ is the stage benefit, determined by the initial storage S_t, the current inflow Q_t, and the release R_t in stage/time-step t.

This work applies SDP to a hydropower system involving two cascaded reservoirs, where the state variables include the storages S_{it} of reservoir i at the beginning of time-step t and the total inflow into the cascade in coming time-step $t-1$. The recursive objective function in SDP, when applied to two cascaded reservoirs, is expressed as follows:

$$f_t(S_{1t}, S_{2t}, Q_t) = \max \left\{ \mathop{E}_{Q_{t+1}|Q_t} [B_{1t}(S_{1t}, I_{1t}, R_{1t}) + B_{2t}(S_{2t}, I_{2t}, R_{2t}) + f_{t+1}(S_{1,t+1}, S_{2,t+1}, Q_{t+1})] \right\} \quad (2)$$

where the local inflow I_{it} to an individual reservoir is assumed as being perfectly correlated to the total inflow Q_t to the whole cascade:

$$I_{it} = \lambda_i \cdot Q_t \tag{3}$$

in which λ_i is the proportional coefficient of local inflow into reservoir i to the total inflow into the whole cascade. The constraints include the following:

(1) The water balance

$$S_{i,t+1} = S_{it} + (I_{it} + \sum_{k\in\Omega(i)} R_{kt} - R_{it})\cdot\Delta t \tag{4}$$

where the $\Omega(i)$ is the set of reservoirs immediately upstream of reservoir i and the Δt is the time length in time-step t.

(2) The lower $S_{it}^{\min}$ and upper bounds $S_{it}^{\max}$ on the storage

$$S_{it}^{\min} \le S_{it} \le S_{it}^{\max} \tag{5}$$

(3) The release being nonnegative

$$R_{it} \ge 0 \tag{6}$$

(4) The power yield Y at a certain reliability ε

$$\Pr\left(\sum_i P_{it} \ge Y\right) \ge \varepsilon \tag{7}$$

where the hydropower output P_{it} of i in time-step t is determined with

$$P_{it} = \eta_i \cdot \left[Z_i^{up}\left(\frac{S_{it} + S_{i,t+1}}{2}\right) - Z_i^{dn}(R_{it}) \right] \cdot \min(R_{it}, U_i^{\max}) \tag{8}$$

with the final storage $S_{i,t+1}$ being determined via the water balance (4), the η_i being the coefficient of generation efficiency, and the $U_i^{\max}$ being the capacity of turbine discharge of hydropower reservoir i.

The stage benefit $B_{it}(S_{it}, I_{it}, R_{it})$, dependent of different operational objectives, could be the energy production, the revenue gained at seasonal prices, and power yield in current stage, etc. For a certain stage benefit function, the recursive equation in the SDP will evolve to the optimal reservoir operational policy, which can then be used in long-term real-time operation, month by month, for instance. This work will compare the following three objectives with each other, denoted as follows:

(1) SDP-1: to maximize the energy production without power yield;

$$B_{it}(S_{it}, Q_{it}, R_{it}) = P_{it}\cdot\Delta t \tag{9}$$

(2) SDP-2: to maximize the revenue at seasonal prices without power yield, where ζ_t is the seasonal price of electricity in time-step t;

$$B_{it}(S_{it}, Q_{it}, R_{it}) = \zeta_t\cdot P_{it}\cdot\Delta t \tag{10}$$

(3) SDP-3: to maximize the energy production with power yield (7).

3. Solution Procedures

The reservoir operation problem will be solved with discrete stochastic dynamic programming (DSDP), which requires state variables to be discretized and both the objective and constraints to be decoupled into every stage, with only the state variables coupled via state variables between two adjacent stages.

3.1. The Typical Inflows and Their Transition Probabilities

The coming inflow into the cascade, as one of the state variables, is a random variable that can be represented by some typical values, making the monthly inflows a stochastic

chain, with the Markov Chain as being specific when considering only the first-order correlation between the inflows in two successive time-steps. Generally, based upon the mean and standard deviation of the random inflow, its distribution space can be divided into five intervals: below dry, dry, average, wet, and above wet, commonly practiced due to the simplicity in engineering application [19]. To derive the inflow transition probabilities, four methods—counting, ordinary least-squares regression, robust linear model regression, and multi-variate conditional distributions—are evaluated to determine how they influence water system performance [20]. The most accessible counting method is selected to acquire inflow transition probabilities in this work. The number of typical inflows should be neither too small nor too large to balance the computational burden with the fullness of their representation. The inflow in this work is represented with 7 typical values, determined by following these steps:

(1) Suppose the number of typical inflows in time-step t is K, and there are Y years of historical inflows Q_t^y observed, which ensure n ($=Y/K$) historical inflows be represented by one typical inflow, determined as the average over these n historical inflows.

(2) Arrange the historical inflows Q_t^y for $t = 1,2,\dots,T$ and $y = 1,2,\dots,Y$ in order from the smallest to the largest: $Q_t^{y(1)} \leq Q_t^{y(2)} \leq \cdots \leq Q_t^{y(Y)}$; the typical inflow for any interval k ($k = 1,2,\dots,K$) in time-step t is determined as the average over n historical inflows, expressed as

$$Q_t(k) = \frac{1}{n} \sum_{j=n(k-1)+1}^{nk} Q_t^{y(j)} \tag{11}$$

(3) Apparently, each historical inflow in any time-step t can be represented by one of the k typical inflows in this time-step, and the transition probability from the k-th in time-step t to the l-th in time-step $t + 1$ can be estimated as follows:

$$\Pr\left\{ Q_{t+1}^l \middle| Q_t^k \right\} = \frac{n_t(k,l)}{n} \tag{12}$$

where among n historical inflows represented by the k-th typical inflow in time-step t, there are $n_t(k,l)$ of their successive inflows that are represented by the l-th typical inflow in time-step $t + 1$.

3.2. The Representative Storages

The storage of a reservoir, which theoretically could be any value within its physically feasible region, is actually a continuous state variable that will also be discretized into a number of storage volume intervals representing the storage state at certain time. The number of the storage intervals should be good enough for the computational resolution. According to the work by Karamouz [21], increasing the number of storage intervals to more than 20 will help little in improving the operational results of a reservoir that has an active storage capacity 1.7 times more than the amount of its average annual runoff.

3.3. The Power Yield at Certain Reliability

As required by SDP, the constraints must be decoupled into every stage, with only the state variables coupled via state variables between two adjacent stages. The constraint (7) of power yield at certain reliability, however, cannot be easily decoupled into individual stages as the reliability of meeting the power yield is determined over the whole planning horizon and cannot be exactly enforced stage by stage. However, if assuming the power yield must be met whenever it is possible, then we have the following deterministic constraint:

$$\sum_i P_{it} \geq Y \tag{13}$$

to be satisfied in every stage, thereby making the constraint decoupled into individual stages. Under this assumption, the power yield actually determines the reliability, which

can be estimated by simulating the final optimal operation policy derived by solving the SDP.

3.4. Recursive Evolution

With the transition probabilities determined with (12), the recursive Equation (2) can be reformulated as follows:

$$f_t(S_{1t}^i, S_{2t}^j, Q_t^k) = \max_{R_{1t}, R_{2t}} \left\{ \sum_{l=1}^{K} \Pr(Q_{t+1}^l | Q_t^k) \cdot \left[B_{1t}(S_{1t}, I_{1t}, R_{1t}) + B_{2t}(S_{2t}, I_{2t}, R_{2t}) + f_{t+1}(S_{1,t+1}, S_{2,t+1}, Q_{t+1}^l) \right] \right\} \quad (14)$$

which is a backward recursive equation and, given a stationary stochastic streamflow process, will evolve to a steady operational policy of the cascaded reservoirs. To be specific, let the benefit-to-go function at the end of the planning horizon (T) be zero so that the recursive equation finishes one iteration after evolving from T to 1 backward over time. Then, subtract the benefit-to-go function by a constant to avoid numerical overflow, and repeat this procedure until the convergence has been achieved.

4. Engineering Applications

4.1. Engineering Background

Xiaowan and Nuozhadu are two large, cascaded hydropower reservoirs located in the Lancang River basin; both are situated in the middle and lower reaches of the Lancang River, and the reginal location is depicted in Figure 1. The Xiaowan reservoir, with an active storage capacity close to 10 billion cubic meters, has a strong over-year regulation capacity, the same as its downstream Nuozhadu reservoir that, with a total of 5850 MW capacity installed, is one of the key strategic projects for the West-to-East and the Yunnan-to-outside power transmission thanks to its strong power generation capacity. This work studies the joined optimal operation of the cascaded Xiaowan and Nuozhadu reservoirs, which are two main regulatory hydropower stations under the Yunnan Provincial Power Grids. The dimension problem will not be an obstacle for the case study of two reservoirs.

Figure 1. Reginal location of Xiaowan and Nuozhadu reservoir.

4.2. Data Preparation and Setting

There are 57 years of "Xunly" historical inflows during 1953–2009 available for the cascade. In China, a month is divided into three Xuns: the early, mid, and late, with each Xun being a period of about 10 days.

The number of inflow states are set to be 7 in each Xun; thus, the size of the transition probability matrix will be $7 \times 7 = 49$. It is worth noting that since the reservoir operational rule is annually periodic, the transition probability from the last Xun in a year is to the first Xun in its next year. The optimal reservoir operation rule derived with the SDP will

be simulated under the historical scenario of the inflows observed during 1953–2009. The basic parameters of both hydropower reservoirs are summarized in Table 1.

Table 1. The basic parameters of the reservoirs.

Parameters	Xiaowan	Nuozhadu
Dead storage capacity (bcm)	4.662	10.414
Normal storage capacity (bcm)	14.557	21.749
Minimum head (m)	164	152
Maximum head (m)	251	215
Generating discharge capacity (m^3)	2261	3429

This work studies the joint operational strategies of the cascaded Xiaowan and Nuozhadu hydropower reservoirs under three different objective conditions. The SDP-1 that maximizes the expected energy production is different from the SDP-2 that maximizes the expected revenue calculated based on seasonal electricity prices, which are given in Table 2.

Table 2. Monthly electricity prices.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Price (CNY/KWh)	0.39	0.39	0.39	0.39	0.26	0.19	0.19	0.19	0.19	0.19	0.26	0.39

The SDP-2 is equivalent to the SDP-1 when keeping the electricity unchanged over the planning horizon. As for the SDP-3, the operational strategies with different power yields can be derived and then simulated over many years to statistically estimate the reliabilities in meeting the power yields. Figures 2 and 3 reveal the water level, inflow, and release process of Xiaowan and Nuozhadu for SDP-1, SDP-2, and SDP-3 models during 2000–2009. For SDP-2, due to the dynamic electricity price, water level fluctuations are more obvious. Similar trends can be presented for SDP-3 models with different power yield. Statistical analysis will be described as follows:
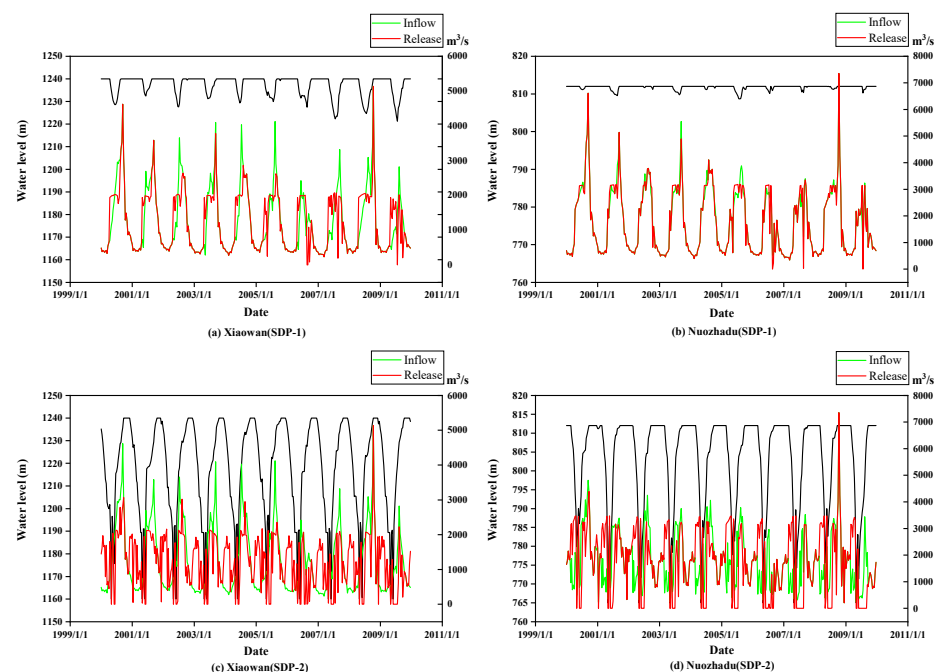

Figure 2. Water level, inflow, and release process of cascade reservoir for SDP-1 and SDP-2 during 2000–2009.
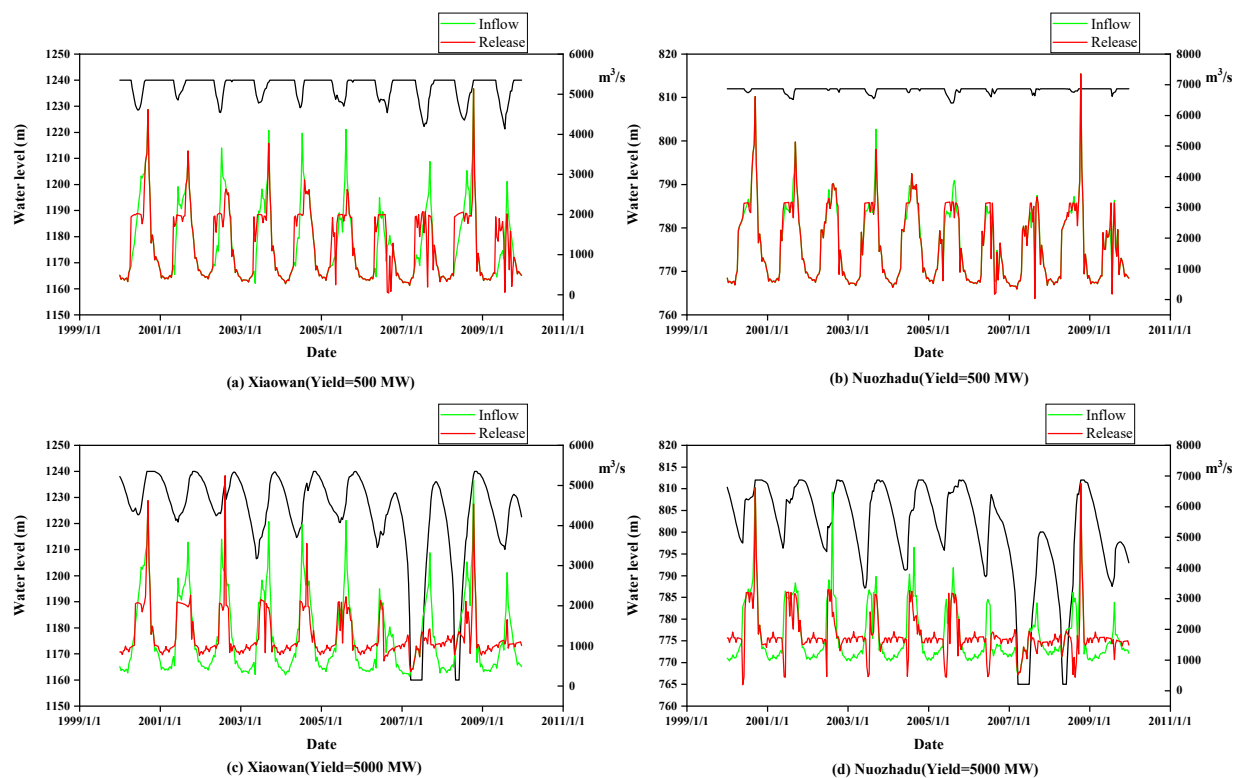
Figure 3. Water level, inflow, and release process of cascade reservoir for SDP-3 with power yield = 500 MW and 5000 MW during 2000–2009.

4.3. Comparison between the SDP-1 and SDP-2

Figure 4 compares the SDP-1 and SDP-2 of the annual energy production and revenue for both cascaded hydropower stations. When compared to the statistical average determined by simulating the operation strategies over many years, the SDP-1 gives an annual average energy production (AAEP) of 50,274,110 MWh, improved by 2.29% over the SDP-2 that produces an AAEP of 49,121,330 MWh. The SDP-2, however, gains an annual average revenue (AAR) of CNY 15,094 M, improved by 5.34% over the SDP-1 that will have an AAR of CNY 14,328 M when estimated at CNY 0.285/KWh, which is the average of the monthly electricity prices given in Table 2.
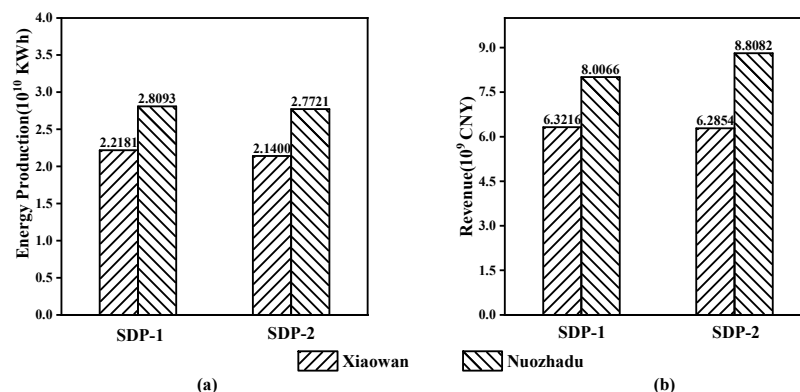

Figure 4. Comparison between the SDP-1 and SDP-2 on expected energy (**a**) and revenue (**b**).

Figure 5 shows the energy production processes from the SDP-1 and 2 for both the Xiaowan and Nuozhadu hydropower reservoirs. The SDP-1 that maximizes the expected energy production has a generation process that is similar to the inflow process in the trend, with less generation in dry seasons and more in wet seasons. However, the SDP-2

that maximizes the expected revenue will try to generate less in wet seasons when the electricity prices are low but will generate more in the dry seasons to take advantage of the higher electricity prices, which is particularly obvious from the Nuozhadu reservoir. The results suggest that using seasonal electricity prices makes the power generation very unstable during 12–20 Xun, which is a critical time period for transferring from dry to flooding seasons.
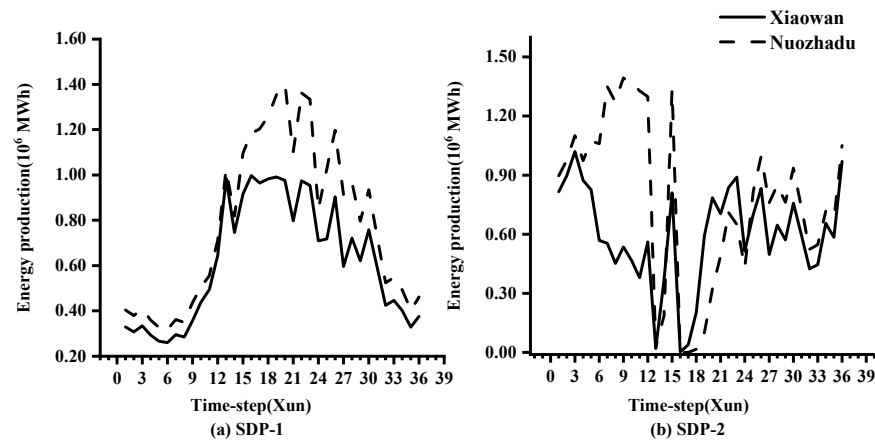

Figure 5. Comparison between the SDP-1 and SDP-2 on average energy production in each Xun (1/3 month).

4.4. The Results by the SDP-3

By setting the power yield to 500 MW, 1000 MW, 2000 MW, 3000 MW, 4000 MW, and 5000 MW, the SDP-3 problems are solved to derive the corresponding optimal operational strategies, which are then simulated over a time period of 1953–2009, when the historical inflows are available to estimate the reliabilities associated with the power yields. Table 3 summarizes the relationship between the power yield and its reliability, and Figure 6 illustrates the average energy production in each time-step or Xun (1/3 month), in this case. As Table 3 shows, when the power yield increase gradually, the reliability in meeting the power yield decrease gradually, and Figure 6 demonstrates that the energy production process becomes more even for both the Xiaowan and Nuozhadu hydropower reservoirs, with the higher reliability leading to more generation than the lower reliability in dry seasons. The reliability decreases sharply when increasing the power yield from 2000 MW to 3000 MW, suggesting the power yield of 2000 MW at 91% reliability should be a satisfactory operational target.

Table 3. The reliability of the power yield for the cascade.

Power Yield	500 MW	1000 MW	2000 MW	3000 MW	4000 MW	5000 MW
Reliability	99.4%	98.3%	90.7%	77.9%	69.8%	47.5%

The average annual energy productions and spillages under different power yields are summarized in Table 4, which shows that both the energy production and spillage of the Xiaowan and Nuozhadu decrease when the power yield of the hydropower cascade grows higher; in particular, the spillage from the Nuozhadu decreases sharply from 1073 to 595 million cubic meters when the power yield increases from 4000 to 5000 MW. Apparently, increasing the power yield of the cascade will reduce the energy production unfavorably but will affect the water spillage favorably.
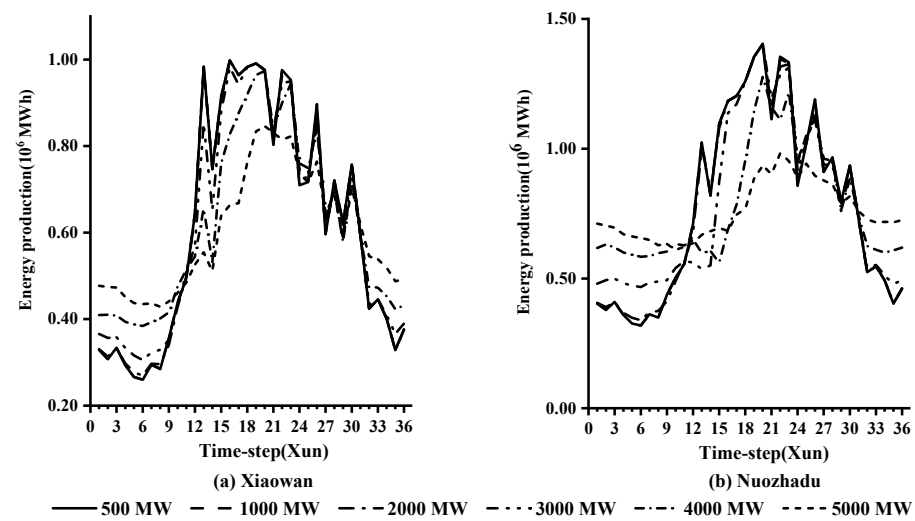
Figure 6. The average energy production of Xiaowan (**a**) and Nuozhadu (**b**) in each Xun (1/3 month) under different power yields for the cascade.

Table 4. Annual energy and spillage under different power yields by the SDP-3.

		500 MW	1000 MW	2000 MW	3000 MW	4000 MW	5000 MW
Energy (MWh)	Xiaowan	22,180,172	22,179,661	22,179,003	22,176,101	22,160,479	21,512,030
	Nuozhadu	28,093,066	28,093,517	28,092,104	28,071,902	27,933,721	27,214,184
Spillage (million m^3)	Xiaowan	1417.9	1416.8	1410.9	1367.1	1191.1	909.0
	Nuozhadu	1282.5	1280.8	1278.3	1231.6	1072.5	594.5

The average spillage processes derived by the SDP-3 under different power yields are illustrated in Figure 7, which demonstrates that spillage mainly occurs from early July to late August. With the power yield increased from 0 to 5000 MW, the spillages of Nuozhadu have a trend that is broadly similar to each other in terms of gradually increasing from early July, reaching the first peak in mid or late August, then decreasing until early September, reaching the second peak, then decreasing slowly. Xiaowan, however, has a different trend in spillage that reaches its peak in mid or late September when the power yield is 4000 MW and in mid or late June when it is 5000 MW.
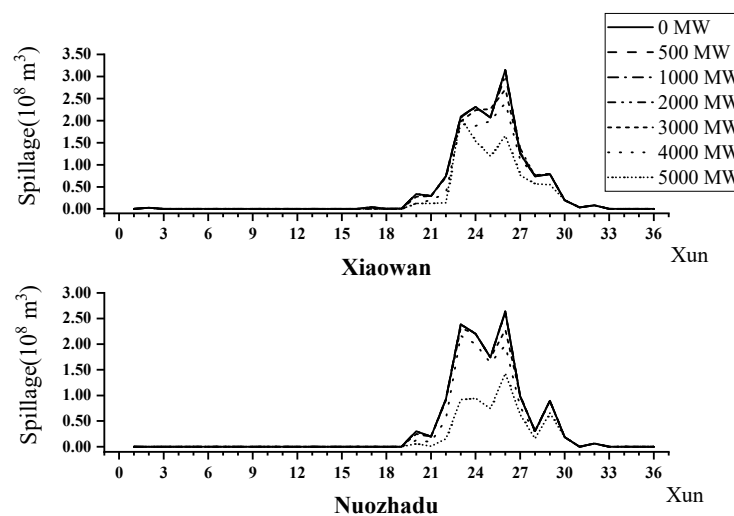

Figure 7. The average spillage in each Xun under different power yields.

5. Conclusions

This work formulates a stochastic dynamic programming (SDP) model that incorporates seasonal electricity prices and can handle the constraint on power yield at certain reliability that are assumed to be satisfied whenever it is possible. Despite the increasing number of reservoirs that will hinder the application of proposed SDP model to multi-reservoir operation, a cascade with two reservoirs in the case study makes it convenient to employ the proposed SDP model. When applied to the Lancang River consisting of its two major cascaded hydropower reservoirs, SDP derives the optimal operational strategies under three conditions: to maximize the energy production and revenue without power yield, and to maximize the energy production with power yield, both of which are compared with each other. The results show that increasing the power yield of the cascade will reduce the energy production unfavorably but will affect the water spillage favorably, with a power yield of 2000 MW with a 91% reliability being suggested as a satisfactory operational target. The case study also suggests that using seasonal electricity prices makes the power generation very unstable during weeks 12–20, which is a critical period of time for transferring from dry to wet seasons.

Author Contributions: M.X.—Conceptualization, formal analysis, writing—review and editing; S.F.—Writing—review and editing, visualization; J.W.—Methodology, software; M.Z.—Supervision, project administration; C.C.—Methodology, software, writing—original draft preparation, writing—review and editing. All authors have read and agreed to the published version of the manuscript.

Data Availability Statement: Not applicable.

References

1. Kang, C.X.; Chen, C.; Wang, J.W. An Efficient Linearization Method for Long-Term Operation of Cascaded Hydropower Reservoirs. *Water Resour. Manag.* **2018**, *32*, 3391–3404. [CrossRef]
2. Stedinger, J.R.; Faber, B.A.; Lamontagne, J.R. Developments in Stochastic Dynamic Programming for Reservoir Operation Optimization. In Proceedings of the World Environmental and Water Resources Congress 2013: Showcasing the Future, Cincinnati, OH, USA, 19–23 May 2013; pp. 1266–1278.
3. Ben Alaya, A.; Souissi, A.; Tarhouni, J.; Ncib, K. Optimization of Nebhana Reservoir Water Allocation by Stochastic Dynamic Programming. *Water Resour. Manag.* **2003**, *17*, 259–272. [CrossRef]
4. Stedinger, J.R.; Sule, B.F.; Loucks, D.P. Stochastic dynamic programming models for reservoir operation optimization. *Water Resour. Res.* **1984**, *20*, 1499–1505. [CrossRef]
5. Turgeon, A. Stochastic optimization of multireservoir operation: The optimal reservoir trajectory approach. *Water Resour. Res.* **2007**, *43*. [CrossRef]
6. Scarcelli, R.O.C.; Zambelli, M.S.; Soares, S.; Carneiro, A.A.F.M. Ensemble of Markovian stochastic dynamic programming models in different time scales for long term hydropower scheduling. *Electr. Power Syst. Res.* **2017**, *150*, 129–136. [CrossRef]
7. Tejada-Guibert, J.A.; Johnson, S.A.; Stedinger, J.R. The Value of Hydrologic Information in Stochastic Dynamic Programming Models of a Multireservoir System. *Water Resour. Res.* **1995**, *31*, 2571–2579. [CrossRef]
8. Mousavi, S.J.; Mahdizadeh, K.; Afshar, A. A stochastic dynamic programming model with fuzzy storage states for reservoir operations. *Adv. Water Resour.* **2004**, *27*, 1105–1110. [CrossRef]
9. Feng, Z.K.; Niu, W.J.; Cheng, C.T.; Liao, S.L. Hydropower system operation optimization by discrete differential dynamic programming based on orthogonal experiment design. *Energy* **2017**, *126*, 720–732. [CrossRef]
10. Zhao, T.T.G.; Cai, X.M.; Lei, X.H.; Wang, H. Improved Dynamic Programming for Reservoir Operation Optimization with a Concave Objective Function. *J. Water Resour. Plan. Manag. Asce* **2012**, *138*, 590–596. [CrossRef]
11. Cheng, C.T.; Wang, S.; Chau, K.W.; Wu, X.Y. Parallel discrete differential dynamic programming for multireservoir operation. *Environ. Model. Softw.* **2014**, *57*, 152–164. [CrossRef]
12. Liu, S.; Yang, Q.; Cai, H.; Yan, M.; Zhang, M.; Wu, D.; Xie, M. Market reform of Yunnan electricity in southwestern China: Practice, challenges and implications. *Renew. Sustain. Energy Rev.* **2019**, *113*, 109265. [CrossRef]
13. Hongling, L.; Chuanwen, J.; Yan, Z. A review on risk-constrained hydropower scheduling in deregulated power market. *Renew. Sustain. Energy Rev.* **2008**, *12*, 1465–1475. [CrossRef]
14. Ware, A. Reliability-constrained hydropower valuation. *Energy Policy* **2018**, *118*, 633–641. [CrossRef]
15. Saadat, M.; Asghari, K. Reliability Improved Stochastic Dynamic Programming for Reservoir Operation Optimization. *Water Resour. Manag.* **2017**, *31*, 1795–1807. [CrossRef]

16. Li, X.; Liu, P.; Ming, B.; Huang, K.; Xu, W.; Wen, Y. Joint Optimization of Forward Contract and Operating Rules for Cascade Hydropower Reservoirs. *J. Water Resour. Plan. Manag.* **2022**, *148*, 04021099. [CrossRef]
17. Chen, C.; Kang, C.; Wang, J. Stochastic Linear Programming for Reservoir Operation with Constraints on Reliability and Vulnerability. *Water* **2018**, *10*, 175. [CrossRef]
18. Mujumdar, P.P.; Nirmala, B. A bayesian stochastic optimization model for a multi-reservoir hydropower system. *Water Resour. Manag.* **2007**, *21*, 1465–1485. [CrossRef]
19. Kim, Y.-O.; Palmer Richard, N. Value of Seasonal Flow Forecasts in Bayesian Stochastic Programming. *J. Water Resour. Plan. Manag.* **1997**, *123*, 327–335. [CrossRef]
20. Haguma, D.; Leconte, R.; Côté, P. Evaluating Transition Probabilities for a Stochastic Dynamic Programming Model Used in Water System Optimization. *J. Water Resour. Plan. Manag.* **2018**, *144*, 04017090. [CrossRef]
21. Karamouz, M.; Houck, M.H. Annual and monthly reservoir operating rules generated by deterministic optimization. *Water Resour. Res.* **1982**, *18*, 1337–1344. [CrossRef]