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Abstract: The study aimed to present the features of a 2-DOF vibration reduction system (VRS) equipped with a linear electrodynamic motor in passive, semi-active and active mode. At first, the VRS model was formulated. Then, simulation tests of the VRS were conducted to distinguish its most advantageous features in each mode and to analyze dynamic performance and energy consumption. Next, the VRS was experimentally tested to evaluate its effectiveness in each mode for the assumed excitations and to compare tests results against simulation data. In active mode, a sliding mode algorithm was employed for motor control, while in semi-active mode, the equivalent damping coefficient analysis was used.

Keywords: linear electrodynamic motor; vibration; passive mode; semi-active mode; active mode; slide mode control; dynamic performance; energy consumption



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1. Introduction

The division of vibration reduction methods into passive, active and semi-active is based on the possibility of using an additional energy source. Passive methods require that the facility's operating conditions are defined and appropriate structural changes are made. The effectiveness of these methods is limited by the level of interference not considered by the designer. There is rich literature on this topic, such as from several decades ago, e.g., [1-3], and more contemporary literature, e.g., [4,5]. Active methods ensure the effectiveness of activities in the field of excitations limited by the adopted control law and the possibility of supplying energy from an additional source. These methods and their application also have rich literature. Among the older works, we should mention, e.g., [6], among the more recent ones [7–9]. On the other hand, semi-active methods use the control law to change system parameters. In contrast to active methods, the additional energy is only used to change the parameters of the actuator employed in the system. The intensive development of semi-active methods over the last six decades has also led to numerous papers, e.g., [10-12]. The common point of each method is to consider the dynamics of the facility and the availability of energy sources. For example, in [13,14] a semi-active vibration reduction system with self-powered and self-sensing capability is reported, and in [15] the energy flows in this type of system are studied. When two or more reduction methods are used in one system, it is referred to as a hybrid system or hybrid application, as exemplified by [16,17] for example. Structural dynamics issues are also widely described in the literature. The theoretical basis for this is contained in [18]. Among the numerous practical applications, special attention should be paid to the automotive sector [19–21] and the civil engineering sector [22].

Various types of control algorithms are used in vibration reduction systems where active and semi-active methods are applied. In addition to classical PID (proportional integral derivative regulator), skyhook, groundhook, LQR (linear–quadratic regulator), on/off switching algorithms and their modifications, e.g., cascade, optimal, slide mode or

adaptive regulators, fuzzy logic or neural network regulators are also used. For example, in [17], the skyhook algorithm was applied for control in the magnetorheological elastomerbased vibration reduction system, in [23,24] the fuzzy logic algorithm for suspension control, and in [25] the negative stiffness algorithm for seat vibration reduction system. Conversely, the use of slide mode algorithms is described in [26,27].

Many researchers focus on an analysis of vibration reduction systems operating in only one mode and do not consider the benefits resulting from the possibility of their combination. In addition, they omit the problem of energy source availability for system power. Therefore, the purpose of the present work is to develop a vibration reduction system (VRS) operating in hybrid mode, taking into account the limitation resulting from the finite capacity of the energy source. To achieve this, we carried out numerical simulations of this system in the passive, semi-active and active modes from the point of view of dynamic performance and energy consumption. Based on the results, the VRS operating in hybrid mode was proposed, and its effectiveness was confirmed experimentally.

The study is structured as follows. Section 2 contains a brief description of the VRS in the test rig and presents governing equations of the system. Section 3 highlights details of the VRS modelling in the passive, semi-active and active modes and presents the sliding mode algorithm employed for motor control. Section 4 discusses the simulation results of the VRS in each mode. Section 5 presents and summarizes data obtained from the carried out experiments. Conclusions are drawn in Section 6. In Table 1, we present the designations used in the study.

Symbol	Description	Symbol	Description
t	time	σ	sliding variable
t_{max}	simulation time	С	angle of the sliding surface on a state trajectory
Z	displacement of shaker plate	V	Lyapunov function
x_1	displacement of plate 1	Fmax	maximum value of F_d
<i>x</i> ₂	displacement of plate 2	β	control algorithm parameter chosen to offset the impact of F_{max}
m_1	mass of plate 1	A_z	amplitude of displacement z
<i>m</i> ₂	mass of plate 2	A_{x1}	amplitude of displacement x_1
b_1	damping coefficient	A_{x2}	amplitude of displacement x_2
b_2	damping coefficient	f	frequency of the input signal
k_1	spring coefficient	f_1	starting frequency
k_2	spring coefficient	f_2	ending frequency
F_m	motor control force	f_{x1}	vibration frequency determined for x_1
κ_1	force sensitivity coefficient	Kp_{x1}	overshoot of displacement x_1
κ_2	back emf coefficient	Kp_{x2}	overshoot of displacement x_2
e_m	electromotive force	T_{x1z}	displacement transmissibility
R_m	motor resistance	T_{x2z}	displacement transmissibility
L_m	motor inductance	T_{px1z}	value of T_{x1z} in case PM at the frequency $f = 2$ Hz
i_m	current in motor circuit	T_{px2z}	value of T_{x2z} in case PM at the frequency $f = 2 \text{ Hz}$
u_m	motor circuit voltage	D_p	dynamic performance coefficient
R_l	additional resistance	$t_n - t_{n-1}$	time duration of <i>n</i> -th displacement cycle
Cm	equivalent damping coefficient	E_{dy}	dissipated energy in the time duration of one cycle of displacement
F_c	control force	E_{de}	delivered energy in the time duration of one cycle of displacement
F_d	disturbing force	E_c	energy consumption coefficient

Table 1. List of symbols used in the formulas.

2. Vibration Reduction System

The 2-DOF VRS investigated in the study is shown in Figure 1 (see dotted line). The first degree of freedom consists of two springs and a movable plate 1. The second degree of freedom is composed of a movable plate 2, two springs and a linear motor acting as an actuator. Plate 1 and plate 2 are mounted on linear guides. The springs making up the first degree of freedom are attached to the shaker plate. The test rig comprises a VRS housed in an aluminum frame, a shaker, a control system and a potentiometer.



Figure 1. Vibration reduction system in the test rig—general view.

The VRS uses an LA25 series electrodynamic linear motor of Sensata Technologies [28]. The motor is composed of two elements (able to move relative to each other), coils system made of resistance wire wound on a diamagnetic core and a ferromagnetic core with an array of permanent neodymium magnets. The VRS extortion is implemented as a shaker plate displacement generated by the LA30 series actuator of Sensata Technologies [29]. S1 and S2 displacement sensors of SMK series [30] are mounted on the shaker plate and plate 1. SMS12 series sensor [31] is mounted on plate 2 (S3). Figure 2 shows a diagram of the VRS. The designations used in the figure mean: m_1 and m_2 —mass of plate 1 and plate 2; z, x_1 , x_2 —displacement of shaker plate, plate 1 and plate 2; k_1 and k_2 —spring coefficients; b_1 and b_2 —damping coefficients in kinematic pairs; and F_m —motor control force. The governing equations of system motion can be written in the form:

$$\begin{cases} m_1 \ddot{x_1} + b_1 (\dot{x_1} - \dot{z}) + b_2 (\dot{x_1} - \dot{x_2}) + 2k_1 (x_1 - z) + 2k_2 (x_1 - x_2) = -F_m \\ m_2 \ddot{x_2} + b_2 (\dot{x_2} - \dot{x_1}) + 2k_2 (x_2 - x_1) = F_m \end{cases}$$
(1)



Figure 2. Diagram of the vibration reduction system.

In view of the fact that the force F_m produced by the motor depends on the force sensitivity κ_1 coefficient and the current i_m in the circuit, and the induced electromotive force e_m is the product of the back emf κ_2 coefficient and the relative velocity $(x_2 - x_1)$. The constitutive equations of the motor can be written as:

$$\begin{cases} F_m = \kappa_1 i_m \\ e_m = \kappa_2 (\dot{x}_2 - \dot{x}_1) \end{cases}$$
(2)

The designations R_m and L_m introduced in Figure 2 indicate the resistance and inductance of the motor coil. Table 2 provides the experimentally determined values of the test rig's mechanical and electrical parameters (see Figure 1).

Mechanical Parameters						
<i>m</i> ₁ 6 kg	b_1 10 N · s/m	<i>k</i> ₁ 2400 N/m	<i>m</i> ₂ 13.8 kg	b ₂ 13 N · s/m	<i>k</i> ₂ 2700 N/m	
Electrical Parameters						
R_m 2.4 Ω	<i>L_m</i> 2.5 mH	κ ₁ 21.35 N/A		$\frac{\kappa_2}{21.36 \mathrm{V}\cdot\mathrm{s/m}}$		

Table 2. Mechanical and electrical parameters of the vibration reduction system.

3. Modelling

The structure of the motor allows the VRS to operate in passive, semi-active or active mode (Figure 3). In passive mode, the model of the motor is achieved by including a fixed resistance R_l in the circuit. In semi-active mode, a controllable load R_l should be included in the motor circuit, and in active mode a controllable voltage or current amplification circuit.



Figure 3. Schematic diagram of the linear motor circuit: (**a**) passive mode, (**b**) semi-active mode, and (**c**) active mode.

3.1. Passive and Semi-Active Mode

Passive and semi-active VRS mode require the inclusion of additional resistance R_l in the motor circuit. According to Faraday's law of electromagnetic induction, an electromotive force e_m is induced in the motor circuit proportional to the relative velocity $(\dot{x}_2 - \dot{x}_1)$. The governing equation of the motor circuit is as follows:

$$\kappa_2(\dot{x_1} - \dot{x_2}) = L_m i_m + (R_m + R_l)i_m \tag{3}$$

Taking into account that at low frequencies of z, the inductance L_m takes on small values, the force generated by the motor can be written as:

$$F_m \approx c_m (\dot{x_1} - \dot{x_2}) \tag{4}$$

where c_m is an equivalent damping coefficient:

$$c_m = \frac{\kappa_1 \kappa_2}{R_m + R_l} \tag{5}$$

Equation (4) shows that the motor influence on the VRS is viscous so that the generated force depends on the damping coefficient c_m . The value of c_m can be modified by changing the value of the resistance R_l according to the graph shown in Figure 4.



Figure 4. Damping coefficient c_m vs. resistance R_l .

3.2. Active Mode

In active mode, a source of voltage u_m or current i_m must be included in the linear motor circuit. Then, the equation of the motor circuit takes the form:

$$u_m = L_m i_m + R_m i_m + e_m \tag{6}$$

The purpose of the active mode is to minimize the amplitude of vibration in the system. To achieve this, a sliding mode control algorithm was chosen. The equation of plate 1 motion can be written as:

$$\ddot{x_1} = \frac{1}{m_1} F_d(x_1, \dot{x_1}, x_2, \dot{x_2}, z, \dot{z}, t) + \frac{1}{m_1} F_c(i_m, t)$$
(7)

where F_c is the control force that should counteract the force F_d disturbing the accepted equilibrium point ($x_1 = 0$ and $\dot{x_1} = 0$). Considering Equation (2), the force F_c and F_d can be expressed as:

$$F_c(i_m, t) = -\kappa_1 i_m \tag{8}$$

$$F_d(x_1, \dot{x_1}, x_2, \dot{x_2}, z, \dot{z}, t) = b_1(\dot{z} - \dot{x_1}) + b_2(\dot{x_2} - \dot{x_1}) + 2k_1(z - x_1) + 2k_2(x_2 - x_1)$$
(9)

Let us assume the sliding variable σ in the form:

$$\sigma = \dot{x_1} + cx_1 \tag{10}$$

where c is the angle of the sliding surface on a state trajectory. To formulate the control law, it is necessary to take the Lyapunov function V, which is a measure of the distance of the state trajectory from the sliding surface [32]:

$$V = \frac{1}{2}\sigma^2 \tag{11}$$

To ensure asymptotic stability at the equilibrium point $\sigma = 0$, the function *V* must satisfy the following conditions [33]:

$$\lim_{|\sigma| \to \infty} V = \infty \tag{12}$$

$$V < 0 \text{ for } \sigma \neq 0$$
 (13)

Taking into account Equations (7), (8), (10) and (11), the derivative of the function V can be expressed as follows:

$$\dot{V} = \sigma \dot{\sigma} = \sigma \left(\ddot{x_1} + c \dot{x_1} \right) = \sigma \left(\frac{1}{m_1} F_d \left(x_1, \dot{x_1}, x_2, \dot{x_2}, z, \dot{z}, t \right) - \frac{\kappa_1}{m_1} i_m + c \dot{x_1} \right)$$
(14)

If the control signal is assumed to be:

$$i_m = \frac{m_1}{\kappa_1} c \dot{x_1} + \frac{1}{\kappa_1} \beta \cdot \operatorname{sign}(\sigma)$$
(15)

Then, satisfying condition (13) requires constraining the maximum value of V according to the inequality:

$$\sigma\left(\frac{1}{m_1}F_d - \frac{1}{m_1}\beta \cdot \operatorname{sign}(\sigma)\right) < |\sigma|\left(\frac{1}{m}F_{max} - \frac{1}{m_1}\beta \cdot \operatorname{sign}(\sigma)\right)$$
(16)

where F_{max} is the maximum value of F_d , and β is a parameter chosen to offset the impact of F_{max} . Note that inequality (16) will be satisfied when $\beta \ge F_{max}$.

The use of a control signal as in Equation (15) requires the introduction of an additional current control system to compensate current changes due to the induced electromotive force e_m . Assuming the energy source is a voltage generating system u_m and ignoring the small influence of the inductance L_m , two components can be distinguished in the force F_m resulting from the interaction of the external voltage source and changes in relative velocity $(\dot{x}_2 - \dot{x}_1)$:

$$F_m \approx \frac{\kappa_1}{R_m} u_m + \frac{\kappa_1 \kappa_2}{R_m} \left(\dot{x_1} - \dot{x_2} \right) \tag{17}$$

The component $\frac{\kappa_1 \kappa_2}{R_m} (\dot{x_1} - \dot{x_2})$ can be treated as an additional component of the disturbing force F_d . In connection with the control force F_c can be written as:

$$F_c(u_m, t) = -\frac{\kappa_1}{R_m} u_m \tag{18}$$

The adopted modification of the force f_c allows the assumption of the control signal u_m in the form:

$$u_m = \frac{m_1 R_m}{\kappa_1} c \dot{x_1} + \frac{R_m}{\kappa_1} \beta \cdot \text{sign}(\sigma)$$
(19)

4. Simulation Tests

Numerical simulations of the VRS in each mode (see Figure 3) were carried out using Equation (2). In passive mode (case PM), Equation (3) was additionally used, assuming $R_l = 200 \Omega$. In the semi-active mode, Equation (3) was used, assuming that the control system can modify the value of the resistance R_l in the range from 0 to 25 Ω . Exemplary simulations were performed with $R_l = 0 \Omega$ (case SM1), $R_l = 2 \Omega$ (case SM2), $R_l = 5 \Omega$ (case SM3), and $R_l = 10 \Omega$ (case SM4). In the active mode (case AM), Equation (6) and the control law written by Equation (19) were used, assuming from work [34] that $\beta = 50 N$ and $c = 10 \text{ s}^{-1}$. Simulations were performed with a step signal:

$$z(t) = \begin{cases} 0 \text{ m,} & \text{for } t < 0\\ 0.1 \text{ m,} & \text{for } t \ge 0 \end{cases}$$
(20)

Additionally, a sinusoidally varying displacement with amplitude $A_z = 0.01$ m and slowly increasing frequency from $f_1 = 0.1$ to $f_2 = 10$ Hz in $t_{max} = 300$ s:

$$z(t) = A_z \sin\left[2\pi t \left(f_1 + t \frac{f_2 - f_1}{2t_{max}}\right)\right]$$
(21)

To evaluate the effectiveness of VRS under step signal [35], the following overshoot of displacement x_1 and x_2 were introduced:

$$Kp_{x1} = \frac{\max(x_1) - 0.1}{0.1} \cdot 100\%$$

$$Kp_{x2} = \frac{\max(x_2) - 0.1}{0.1} \cdot 100\%$$
(22)

For sinusoidally varying excitation, displacement transmissibility was defined as:

$$T_{x1z}(f) = \frac{A_{x1}}{A_z}$$
$$T_{x2z}(f) = \frac{A_{x2}}{A_z}$$
(23)

where the coefficients A_{x1} and A_{x2} denote the displacement amplitudes x_1 and x_2 . Finally, it was assumed that the dynamic performance coefficient D_p would determine effectiveness of the VRS:

$$D_p = 100 - \frac{1}{4}Kp_{x1} - \frac{1}{4}Kp_{x2} - \frac{T_{x1z}(2)}{T_{px1z}} \cdot 25\% - \frac{T_{x2z}(2)}{T_{px2z}} \cdot 25\%$$
(24)

where D_p is expressed as a percentage, $T_{x1z}(2)$ and $+T_{x2z}(2)$ refer to the frequency f = 2 Hz, and T_{px1z} and T_{px2z} specify the value of T_{x1z} and T_{x2z} in case PM at the frequency f = 2 Hz. The dynamic performance coefficient D_p allows the evaluation of the performance of the VRS for frequencies below or close to the first resonant frequency of the system.

Assuming the existence of an external energy source in VRS in the case AM, the development of the energy consumption coefficient requires separate consideration of the cases PM, SM1, SM2, SM3, and SM4, in which there is no external power source. In PM, SM1-SM4 cases, the mechanical energy provided by the shaker is converted to electrical energy and dissipated at the resistances R_m and R_l . The amount of energy E_{dy} that can be generated and stored in the time duration of one cycle of displacement z is equal to the energy dissipated on the resistance R_l :

$$E_{dy} = -\int_{tn-1}^{tn} R_l \cdot i_m^2 \cdot \mathrm{d}t \tag{25}$$

where $t_n - t_{n-1}$ is the time duration, and *n* is the cycle number.

If an external energy source is connected, the amount of delivered energy E_{de} can be defined as:

$$E_{de} = \int_{tn-1}^{tn} \frac{u_m^2}{R_m} \cdot dt$$
(26)

Considering Equations (24) and (26), the energy consumption coefficient is given by:

$$E_c = \begin{cases} E_{dy}(2.15), \text{ case PM, SM1-SM4} \\ E_{de}(2.15), \text{ case AM} \end{cases}$$
(27)

where $E_{dy}(2.15)$ and $E_{de}(2.15)$ refer to the frequency f = 2.15 Hz. The frequency value was selected based on the $E_{dy}(f)$ local maximum value.

Simulation results under such assumptions are shown in Figures 5–7. The plots in Figure 5b,c shows the VRS response to the step signal (Figure 5a) in the form of a time

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waveform x_1 and x_2 displacements. It can be noted that the smallest overshoot value occurs in AM case and the largest in PM case. Moreover, when analysing cases SM1–SM4, it can be seen that increasing the value of R_l causes an increase in the value of overshoot.



Figure 5. Time waveform of displacement. (a) z, (b) x_1 , and (c) x_2 .

The simulation results of the VRS under sinusoidally varying excitations are shown in Figure 6a,b. The plots show displacement transmissibility T_{x1z} and T_{x2z} . For greater readability of the graphs, cases SM3 and PM have been omitted. Considering transmissibilities T_{x1z} and T_{x2z} at low frequencies f, the most favourable case for VRS operation is the AM case. In other cases, effectiveness depends on frequency. In the range from 0.1 to 2.3 Hz and from 5.7 to 10 Hz, a small value of R_l improves the efficiency of the VRS. However, between 2.3 and 5.7 Hz, a small value of R_l reduces the effectiveness of the VRS.



Figure 6. Displacement transmissibility. (a) T_{x1z} and (b) T_{x2z} vs. frequency f.



Figure 7. Dissipated energy E_{dy} and delivered energy E_{de} vs. frequency f.

Figure 7 shows the relationship of the dissipated energy E_{dy} and the energy supplied E_{de} from an external power source under sinusoidally varying excitations. It can be seen that in the case AM, there is high energy consumption. In case PM and SM2–SM4, the amount of energy E_{dy} depends on the frequency f and the resistance R_l . Analysing the influence of the resistance R_l on the energy E_{dy} , it can be noticed that at each of the frequency

values adopted for the simulation, there is a particular value R_l , for which E_{dy} takes on a maximum value. For example, Figure 8 presents the dependence of the dissipated energy E_{dy} on the resistance R_l at frequency f = 2.1 Hz.



Figure 8. Dissipated energy E_{dy} vs. resistance R_l at f = 2.1 Hz.

In Table 3 we show the factors of VRS in each considered mode. In addition to the previously introduced parameters Kp_{x1} , Kp_{x2} , T_{x1z} , and T_{x2z} the dynamic performance coefficient D_p and energy consumption coefficient E_c are also given. The D_p coefficient confirms the VRS effectiveness in case AM. Considering the other cases, the value of the coefficient D_p depends on the R_l resistance and decreases with its increase. Considering the E_c coefficient, it should be noted that the amount of recovered energy can be optimized by matching the resistance R_l to the frequency f of the sinusoidal excitation. In case AM, the amount of energy supplied from an external source is at least two times greater than the amount of energy recovered in each of the analyzed cases. The obtained results make it possible to define the principles of VRS operating in hybrid mode. Within the adopted range of frequency f, five ranges can be distinguished that define the operation of the VRS in a particular mode:

- 1. Up to 1.9 Hz, SM at $R_l = 11 \Omega$, increasing the amount of recovered energy;
- 2. From 1.9 to 2.7 Hz, AM in the case of available energy from an external source or SM at $R_1 = 0 \Omega$ when no external source is available, reducing the values of T_{x1z} and T_{x2z} ;
- 3. From 2.7 to 5.7 Hz, SM at $R_l = 11 \Omega$, T_{x1z} value reduction and increase the amount of recovered energy;
- 4. From 5.7 to 9.0 Hz, SM at $R_l = 0 \Omega$, value reduction of T_{x1z} ;
- 5. Ranging from 9.0 Hz, SM at $R_l = 2 \Omega$, value reduction of T_{x1z} , increasing the amount of recovered energy.

Table 3. Factors of the vibration reduction system in the passive, semi-active and active mode.

Mode	$\substack{Kp_{x1} \\ [\%]}$	Кр _{х2} [%]	<i>T</i> _{x1} (2 Hz) [-]	$T_{x2}(2 \text{ Hz})$ [-]	$egin{array}{c} D_p \ [\%] \end{array}$	<i>Ec</i> [J]
PM	79.3	103.7	19.69	33.55	4.25	-0.67
SM1	56.8	76.3	2.02	2.94	61.97	0
SM2	59.2	77.2	2.97	4.74	58.60	-0.74
SM3	66.5	86.5	4.36	7.23	50.83	-1.49
SM4	71.6	93.1	6.35	10.7	42.79	-2.14
AM	43.5	45.1	0.75	0.70	76.38	8.11

5. Experimental Tests

A block diagram of the control system designed for the test rig is shown in Figure 9. The control system is composed of an industrial cRIO 9063 controller [36], a potentiometer, a PWM module, and a PC. The cRIO controller includes an FPGA module, RT processor and chassis designed for measurement cards. Communication between the RT processor and the FPGA module is via DMA bus. On the other hand, the FPGA module communicates with the measurement cards and the PWM module by I/O bus. The chosen architecture enables the implementation of control algorithms hardware-based on the FPGA module and software-based in the RT-Linux operating system.



Figure 9. Diagram of the control system implemented in the test rig.

Measurement cards were configured to allow the measurement of displacement z, x_1 , x_2 voltage u_m , and current i_m . The sampling rate of the acquired data archived on the solid-state drive was 4 kHz. The defined process data and settings were verified in a control panel application running on an independent PC. The controller software and the control panel application were developed in the LabView environment.

The linear motor can operate in active mode by drawing energy from an external source or in semi-active mode by dissipating the generated energy in a resistive load circuit. In active mode, the power conditioner is a commercial NI 9505 PWM [37] module. In semi-active mode, the resistive nature of the load was achieved using a potentiometer R_l (operating in range from 1 to 25 Ω). When working in active mode, the physical limitations of the linear motor must be taken into account. The instantaneous value of the current i_m is limited by the amount of heat energy that must be dissipated by the cooling system. In practice, assuming motor operation at room temperature and passive air cooling, the motor can be loaded momentarily with 9 A or continuously with 4 A. Therefore, the continuous force should not exceed 80 N, and the voltage u_m should be between -9 and 9 V.

The control algorithm implementation in VRS is divided into three steps: data acquisition, frequency determination, and frequency range selector (see Figure 10).



Figure 10. The implementation of the control algorithm.

In step 1, implemented in the FPGA module, the acquired data were subjected to conditioning and filtering the displacement x_1 and velocity $\dot{x_1}$. The displacement x_1 was sent to the RT system using a DMA buffer. Step 2 was implemented in the RT system in a separate thread sampled at a frequency of 100 Hz. In each processing cycle, the RT system updated a buffer containing the 8192 most recently received displacement x_1 was sent to the FPGA module (using a shared memory f_{x1} . In step 3, the frequency f_{x1} was sent to the FPGA module (using a shared memory buffer) to calculate the values of β , *c* and R_l . Assuming VRS in semi-active and hybrid mode, the frequency range selector determined the coefficients β , *c* and R_l following the frequency ranges provided in Table 4. The VRS was investigated in a test rig (see Figure 1) by a sinusoidally varying *z* with amplitude $A_z = 1.5$ mm and slowly increasing frequency *f* from 0.1 to 10 Hz. Figure 11a,b shows the test results for both modes. The plot in grey refers to semi-active mode and in red to hybrid mode. In addition, the results in semi-active mode with a fixed value of $R_l = 1 \Omega$, marked in green, and $R_l = 25 \Omega$, marked in blue, are compared.

Semi-Active Mode							
Freque	Frequency Range Hz		$c~{ m s}^{-1}$	$R_l \Omega$			
Ι	$f_{x1} \le 2.1$	0	0	1			
II	$2.1 < f_{x1} \le 4.1$	0	0	25			
III	$4.1 < f_{x1}$	0	0	1			
Hybrid Mode							
Freque	ncy Range Hz	βΝ	$c \ \mathrm{s}^{-1}$	$R_l \Omega$			
T							
1	$f_{x1} \le 1.9$	0	0	11			
I II	$f_{x1} \le 1.9 \ 1.9 < f_{x1} \le 2.7$	0 20	0 10	11 25			
I II III	$f_{x1} \le 1.9$ $1.9 < f_{x1} \le 2.7$ $2.7 < f_{x1} \le 5.7$	0 20 0	0 10 0	11 25 11			
I II III IV	$f_{x1} \le 1.9$ $1.9 < f_{x1} \le 2.7$ $2.7 < f_{x1} \le 5.7$ $5.7 < f_{x1} \le 9.0$	0 20 0 0	0 10 0 0	11 25 11 1			

Table 4. Parameters of the control algorithm; semi-active and hybrid mode.

The simulation data in semi-active mode show that the analyzed frequency f_{x1} maybe divided into ranges I, II and III (see Table 4). In the range I and III, it is preferable to assume the smallest value of R_l . In range II, the value of the R_l should be as large as possible. The test results confirm that VRS is able to operate without an external energy source.

In hybrid mode, the frequency range may be divided into ranges I, II, III, IV, and V (see Table 4). The values of coefficients β , *c*, and *R*_l assumed in the range I, III, and V directly correspond to the parameters as in Section 4. In range IV, due to hardware limitations, the value of *R*_l was assumed to be 1 Ω . In range II, the coefficients $\beta = 20$ N, $c = 10 \text{ s}^{-1}$, and $R_l = 25 \Omega$ were introduced. The high value of the resistance *R*_l minimizes the energy loss in the potentiometer. The value of parameter *c* was assumed according to

numerical simulations. The value of the parameter β was chosen to present that VRS is able to operate with powering from the limited energy source. According to this, the initial increase in the displacement transmissibility T_{x1z} value in the range II results from the small value of parameter β . Assuming a high value of the parameter β lead us to unnecessary energy losses in the linear motor and also requires additional filters to prevent against chattering effect.



Figure 11. Displacement transmissibility T_{x1z} vs. frequency f; (a) semi-active mode, (b) hybrid mode.

6. Conclusions

In the study, dynamic performance and power consumption in 2-DOF VRS with a linear motor were investigated. Simulation studies of the VRS in passive, semi-active and active mode were carried out. In passive and semi-active mode, the effect of additional resistance R_l on the equivalent damping coefficient c_m was considered. In active mode, the control signal u_m which determines the amount of energy supplied from an external source was determined. Based on the simulation data, VRS in hybrid mode operation was proposed.

The analysis of the simulation results led us to the following conclusions:

The effectiveness of VRS in passive and semi-active mode depends on the equivalent damping coefficient c_m . Therefore, at low frequencies f it is advantageous to adopt a low resistance R_l (see Figure 4).

In passive and semi-active mode, taking a small R_l results in a reduction in recoverable energy E_{dy} . The amount of this energy can be optimised by matching the resistance R_l to the frequency f (see Figure 8).

The effectiveness of VRS in active mode is highest at low frequencies f up to 2.8 Hz.

In active mode, the amount of energy consumed significantly exceeds the amount of energy that can be recovered in other modes.

Analysis of D_p , E_c coefficients and $T_{x2z}(f)$ indicates the validity of using the semiactive mode to change the resistance R_l for frequency f up to 2.3 Hz.

In the assumed frequency range f, five ranges were separated. The principle of hybrid mode operation was proposed by checking the effectiveness of the VRS in each of these ranges.

The slide mode control algorithm implemented in frequency range II of hybrid mode can be improved by using anti-chattering filters or setting the β value as a function of frequency.

Further work will aim to develop a new VRS powered by the energy harvested from vibrations. For this purpose, an energy conditioning sub-system will be developed in conjunction with a buffer energy storage and low-energy control system.

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