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Partial Y-Bus Factorization Algorithm for Power System Dynamic Equivalents

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Abstract: This paper presents a partial Y-bus factorization algorithm to reduce the size of a power system model for transient stability analysis. In the proposed approach, steady-state operating conditions for dynamic equivalents are maintained using the traditional Ward admittance method. Fictitious generators are attached at boundary buses to preserve transient behavior following a disturbance. The equivalent dynamic effects from eliminated generators can be maintained by choosing appropriate dynamic parameters of fictitious generators, including machine inertia, transient reactance, and the damping coefficient. Parameters are determined using the idea that the contributions from external generators mostly depend on the network configuration and impedance characterized by the Y-bus matrix. The fictitious generators' dynamic parameters are determined by conducting partial Y-bus factorization on dynamic parameter matrices. The proposed method's performance is validated by conducting case studies with the IEEE 118-bus system and a 10,000 synthetic western U.S. power grid model and comparing simulation outcomes between the full system and reduced equivalent models. Simulation comparisons show that the equivalent model maintains high accuracy. The proposed method is promising alternative solution for power system dynamic equivalents.

Keywords: dynamic equivalents; ward equivalents; partial Y-bus factorization; transient stability simulation; computational efficiency

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1. Introduction

Electric grids worldwide are rapidly changing, partially because of the integration of newer types of generation and load, with one result being an increased need to study and simulate electric grid dynamics [1]. To ensure a secure energy delivery system, power system control centers must accurately and quickly analyze the electric power system's behavior. However, because interconnections within modern power grids have increased system complexity, and power system equations are inherently nonlinear, such analyses are computationally expensive, especially when contingency analysis is performed. Furthermore, the recent massive use of renewable energy sources with power electronic interfaces has increased modeling complexity. These computational limits impose severe constraints on power system security assessments. For decades, developing powerful computational tools for rapid and precise power system analysis has remained an open challenge [2,3].

Many studies to speed up the simulation in power system areas have focused on developing efficient network-reduction algorithms. These approaches partition electric power systems into internal and external systems. The internal system denotes the area of interest for the study. The external system, connected to the internal system, is replaced with smaller equivalents. Thus, the power system model size is reduced, while the internal system remains unchanged. Based on the system model equations and the analysis purpose, network-reduction techniques are divided into static and dynamic equivalents.

Static equivalent methods reduce a system model for power-flow studies. The Ward equivalent has been the most widely used [4]. This approach eliminates the external system by performing Gaussian elimination on a complex nodal admittance matrix (Y-bus matrix) representing the system network. The Ward equivalent approach has two versions of how to model external bus power injections [5]. The Ward injection method converts the injected power at each bus to the injected current. The Ward admittance method converts all injected powers to shunt admittances, resulting in zero external system injections. More details about the Ward equivalent and extended versions can be found in [6,7]. Recently, power transfer distribution factor (PTDF)-based equivalents have been proposed that correctly reflect the PTDF characteristics and are effective for power system planning studies [8–10].

The dynamic equivalent reduces the computational requirements for transient stability analysis. Transient stability analysis determines whether power systems will reach a new operating point and is used to examine how system properties undergo transient deviations from equilibrium following a disturbance. Three principal categories of dynamic equivalent models are modal-, coherency-, and measurement-based methods [11]. The modal method eliminates insignificant modes in the external system based on a linearized model analysis [12]. The coherency method utilizes the concepts of coherency and aggregation to create reduced models preserving the power system model structure [13]. The measurement method determines the parameters of simplified equivalent models using external system responses [14,15].

This paper presents a promising alternative approach to creating a dynamic equivalent for the computational reduction of transient stability analysis. A steady-state operating point is maintained using the traditional Ward admittance method. Fictitious generators are attached at the boundary buses dividing the external system from the internal system. To achieve high-level simulation accuracy using a reduced model, external generators' effects on the internal system through the boundary buses should be carefully maintained, and fictitious generators with appropriate dynamic parameters should mimic the external effects. External generators' dynamic influences on the internal system depend on the network configuration characterized with the Y-bus matrix. In the proposed method, fictitious generators' appropriate dynamic parameters, including machine inertia, transient reactance, and damping coefficients, are determined based on the network information using a partial Y-bus factorization method on the matrices of dynamic parameters. Partial Y-bus factorization deals with only a portion of the Y-bus matrix; therefore, heavy additional computations are not needed to create a dynamic equivalent. The reduced dynamic equivalent retains the power system model structure.

This paper is organized as follows: Section 2 presents the proposed approach. In Section 3, simulation comparisons between full and reduced system models are performed using the IEEE 118-bus system. The final conclusions are presented in Section 4.

2. Methodology

The proposed methodology comprises steady-state network reduction and determining the dynamic parameters of the fictitious equivalent generators at the boundary buses. Figure 1 shows the procedure used to derive the reduced model. For the equivalent, power systems are divided into three mutually exclusive subsystems, depending on the area of interest, called the internal system, the external system, and boundary buses. The internal system is connected to neighboring systems, called the external system. Buses in the external system connected to a bus in the internal system are called boundary buses.

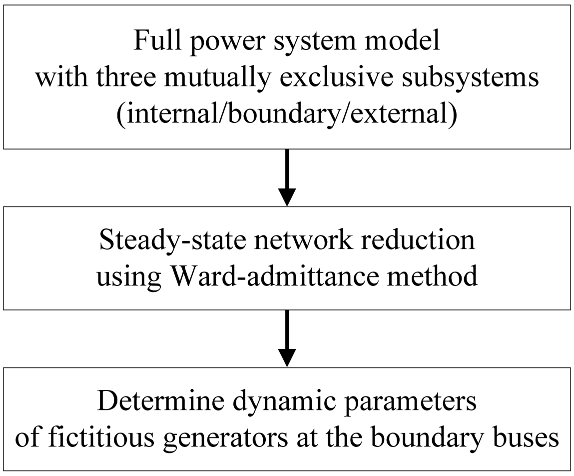

Figure 1. Procedure of the proposed approach.

2.1. Steady-State Network Reduction

In the proposed method, the power system network is reduced using the traditional Ward admittance method, which preserves steady-state operating points [6]. The method converts all external powers, including generations and loads, to shunt admittances using voltage and current information. The external system becomes a passive network without any current injections. Equation (1) shows the Y-bus matrix for the entire system. The coupling (Y_{be}) between the external system and boundary buses is eliminated using Gaussian elimination, and Equation (3) shows the equivalent network, including the internal system and boundary buses. The power system network's size is reduced while the internal system is unchanged, decreasing the required computations for power system analysis. During the reduction process, the Y-bus matrix for boundary buses is changed accordingly. Numerous equivalent lines joining boundary buses are created and associated with $Y_{be}Y_{ee}^{-1}Y_{be}$ in Equation (3).

$$\begin{bmatrix} Y_{ee} & Y_{eb} & 0 \\ Y_{be} & Y_{bb} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} 0 \\ I_b \\ I_i \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} I & Y_{ee}^{-1}Y_{eb} & 0 \\ 0 & Y_{bb} - Y_{be}Y_{ee}^{-1}Y_{be} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} = \begin{bmatrix} 0 \\ I_b \\ I_i \end{bmatrix} \quad (2)$$

$$\left(Y_{bb} - Y_{be}Y_{ee}{}^{-1}Y_{be}\right)V_b + Y_{bi}V_i = I_b Y_{ib}V_b + Y_{ii}V_i = I_i \quad (3)$$

where Y is the partial Y-bus matrix corresponding to each area; V is the voltage vector; I is the current vector; and subscripts i, b, and e represent the internal system, boundary buses, and the external system, respectively.

Figure 2 shows the network diagram corresponding to the reduction process. In the reduced system (Figure 2B), the external system is removed, and additional transmission lines are created among boundary buses.

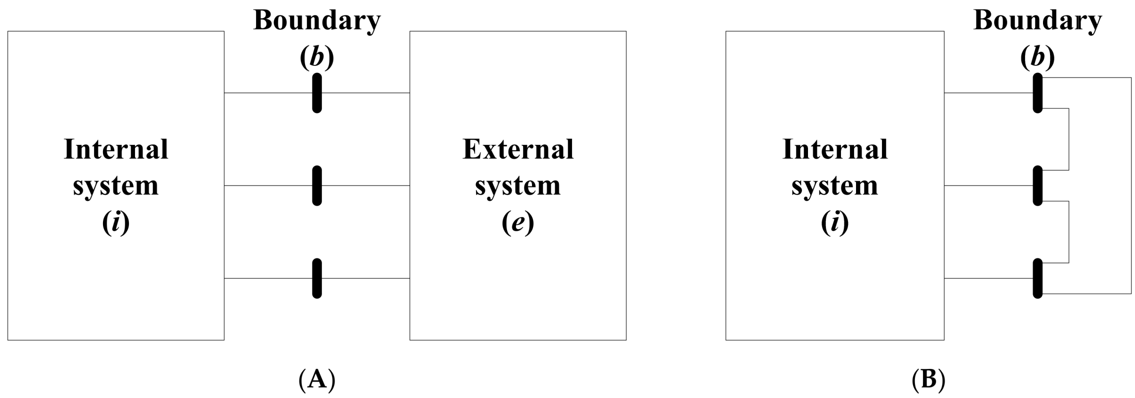

Figure 2. Ward admittance method for steady-state network reduction. (**A**) Original system corresponding to (1); (**B**) reduced system corresponding to Equation (3).

2.2. Dynamic Parameters of Fictitious Generators

After steady-state network reduction, the effects of the external generators eliminated in the reduced system must be preserved. Careful considerations are needed to maintain high-level transient simulation accuracy. In the proposed method, the equivalent effects from external generators are represented by fictitious generators at the boundary buses. The external generators' dynamic effects on the internal system depend on the network configuration, characterized by the Y-bus matrix. The fictitious generators' dynamic parameters are thus determined based on the nodal equations shown in Equation (1). Using Equation (4), the proposed method obtains a vector of the generator parameters and treats them like the current vector in Equation (1). During the equivalencing procedure, the fictitious generators' dynamic parameters are determined using Gaussian elimination. For example, the calculation with machine inertia (H) is shown in Equations (4) and (5). All other dynamic parameters, including the transient reactance (X_{dp}) and damping coefficient (D), can be obtained using the same approach. It is assumed that the fictitious generator has a classic machine model. From Equation (5), the equivalent dynamic parameters at the boundary buses deal with only partial components of the Y-bus matrix, which are Y_{be} and ${Y_{ee}}^{-1}$.

$$\begin{bmatrix} Y_{ee} & Y_{eb} & 0 \\ Y_{be} & Y_{bb} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} \propto \begin{bmatrix} H_e \\ H_b \\ H_i \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} I & Y_{ee}^{-1}Y_{eb} & 0 \\ 0 & Y_{bb} - Y_{be}Y_{ee}^{-1}Y_{be} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_i \end{bmatrix} \propto \begin{bmatrix} Y_{ee}^{-1}H_e \\ H_b - Y_{be}Y_{ee}^{-1}H_e \\ H_i \end{bmatrix} \tag{5}$$

where H is a vector of the machine inertia constant; and subscripts i, b, and e represent the internal system, boundary buses, and the external system, respectively.

3. Case Study

The proposed method was implemented in the transient stability package, PowerWorld [16]. Its performance was validated using the IEEE 118-bus system that comprised 118 buses, 186 branches, 19 generators, and 99 loads [17]. Figure 3 shows the test system. The system dynamics comprised the classic machine model in Equations (6) and (7) [18]. The constant impedance model represented the loads. Table 1 shows the system generators' dynamic model parameters.

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \tag{6}$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i(\omega_i - \omega_s) \tag{7}$$

where δ is the rotor angle position, ω is the rotor angle velocity, H is the inertia constant, T_M is the mechanical torque, T_E is the electrical torque, D is the damping coefficient, and subscript i represents machine i.

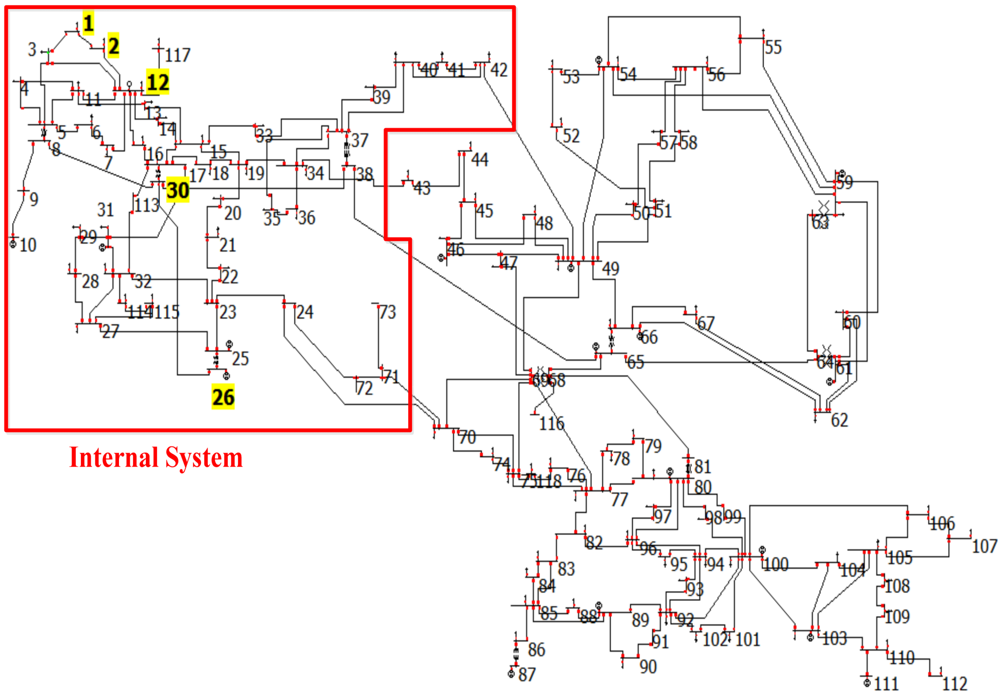

Figure 3. One-line diagram of the IEEE 118-bus system.

Table 1. Machine dynamic parameters for the IEEE 118-bus system (machine base: 100 MVA).

Bus No.	H	X_{dp}	D	Bus No.	H	X_{dp}	D
10	5.66	0.059	3	65	7.41	0.067	3
12	9.97	0.22	3	66	7.41	0.067	3
25	8.24	0.139	3	69	5.26	0.053	3
26	6.01	0.096	3	80	5.26	0.053	3
31	12.37	0.247	3	87	12.37	0.247	3
46	12.37	0.247	3	89	4.64	0.047	3
49	8.24	0.139	3	100	8.26	0.095	3
54	9.97	0.22	3	103	9.97	0.22	3
59	7.93	0.153	3	111	9.97	0.22	3
61	7.93	0.153	3				

Table 2 provides the system division we used to create the equivalent model. The internal system had 49 buses. The external system had 4 boundary buses and 65 buses. Figure 3 shows the division, where the red box denotes the internal system.

Table 2. Details of the system division.

	Buses
Internal system	1–42, 71–73, 113–115, 117 (49 buses)
Boundary buses	43, 49, 65, 70

3.1. Equivalent System

The reduced system was derived using the proposed method. We eliminated 65 buses and 11 generators in the external system, and we newly created 4 fictitious generators at the boundary buses. In the reduced system, 53 buses and 11 generators are placed. Table 3 shows the equivalent dynamic parameters of the fictitious generators. Those parameters were obtained using Equation (5) in Section 2.2.

Table 3. Dynamic parameters of the newly added fictitious generators (machine base: 100 MVA).

Generator No.	H	X_{dp}	D
43	1.4625	1.6138	0.371
49	31.9228	0.0374	10.221
65	68.7234	0.0114	25.2638
70	22.8318	0.0313	8.4201

For validating the performance, three-phase bus-to-ground faults were applied by changing a fault location, and dynamic responses from the full system and equivalent models were compared. The differences were measured using root mean square error (RMSE) over the simulation period, which is calculated using Equation (8).

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_{i\_full} - x_{i\_equivalent}\right)^2} \tag{8}$$

where N is the number of simulation time steps and x is the time-series data that we compared.

3.2. Bus-to-Ground Fault at Bus 1

First, a bus-to-ground fault was simulated at bus 1. A three-phase bus-to-ground fault was applied at 1 s and was cleared at 1.05 s. Figures 4 and 5 show the voltage magnitude of bus 2 and the real and reactive power outputs of the generator at bus 12, respectively.

Figures 4 and 5 show simulation comparisons of the time-domain responses. Initial operating points were maintained well. An overall good agreement in bus voltage magnitude and generator responses between the full and equivalent models was achieved. Figure 6 shows the RMSE of bus voltage magnitude from 0 to 10 s in Equation (8) for all buses in the internal system and the boundary buses. Zero represents the RMSE for external buses.

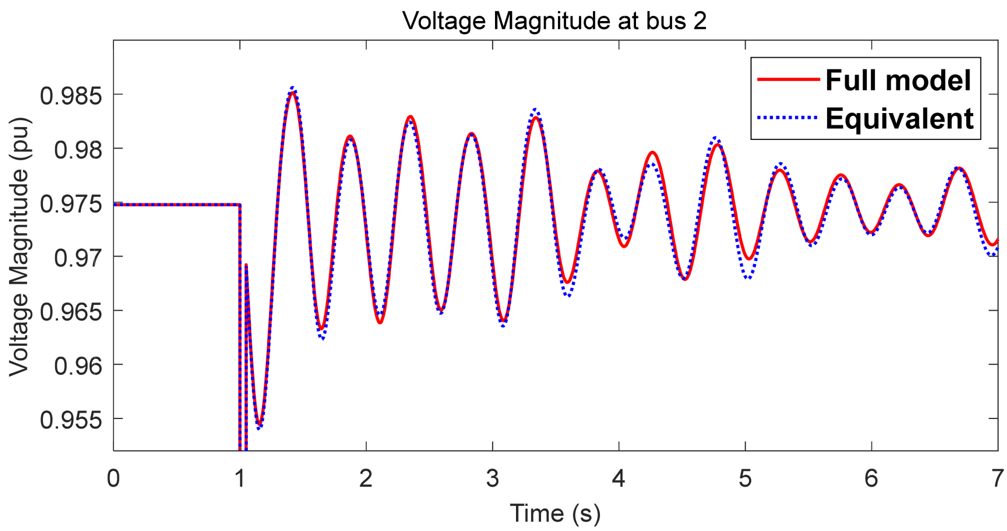

Figure 4. Bus 2 voltage magnitude comparison with bus-to-ground fault at bus 1.

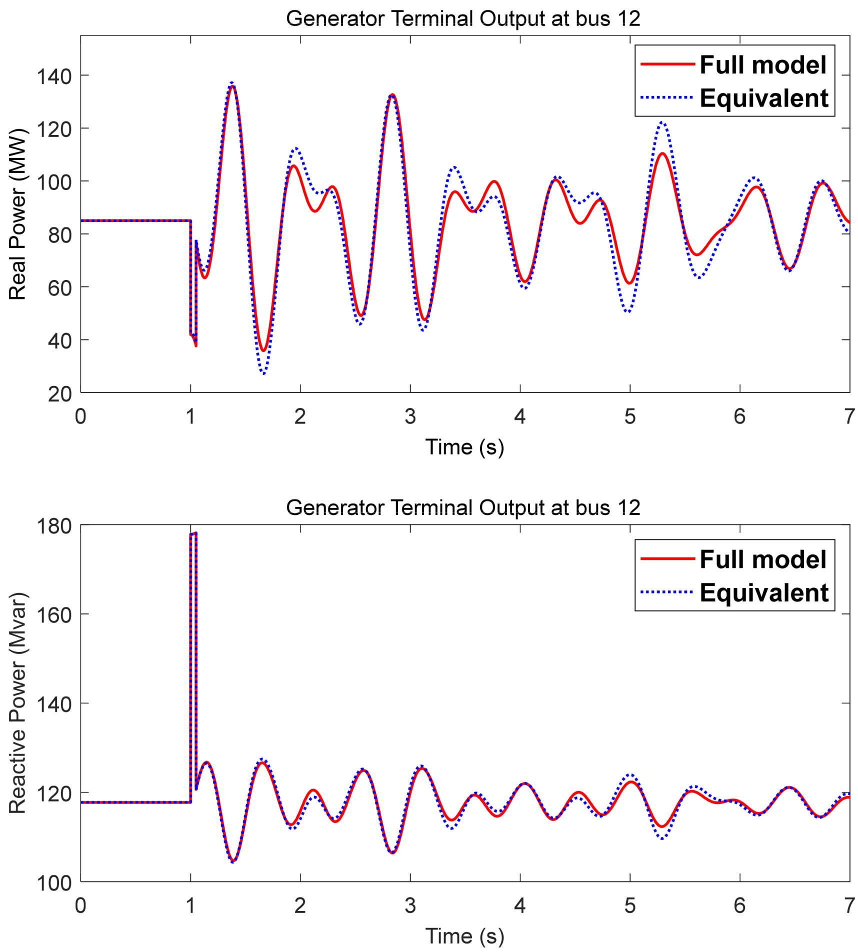

Figure 5. Real and reactive power comparisons of Gen #12 with a bus-to-ground fault at bus 1.

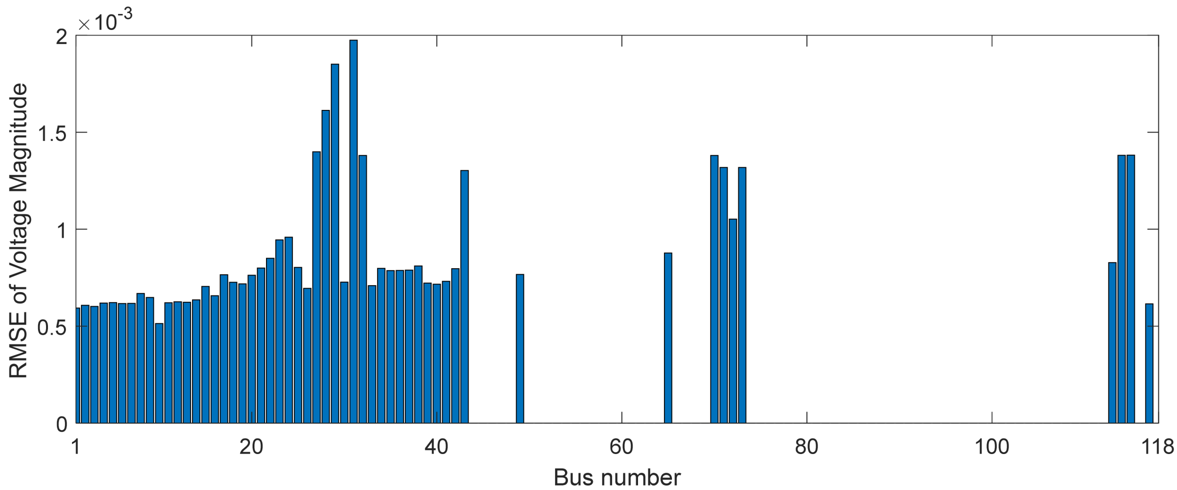

Figure 6. RMSE of bus voltage magnitude with bus-to-ground fault at bus 1.

3.3. Bus-to-Ground Fault at Bus 30

A second comparison was made considering a bus-to-ground fault at bus 30. The three-phase bus-to-ground fault was simulated at 1 s and cleared at 1.05 s. Figure 7 shows the voltage magnitude at bus 30, and Figure 8 shows the real and reactive power of the generator at bus 26. Their differences are reasonably small. The RMSE was calculated for the area of interest (Figure 9). These simulation outcomes confirmed that the reduced model using the proposed method matched the full system model well.

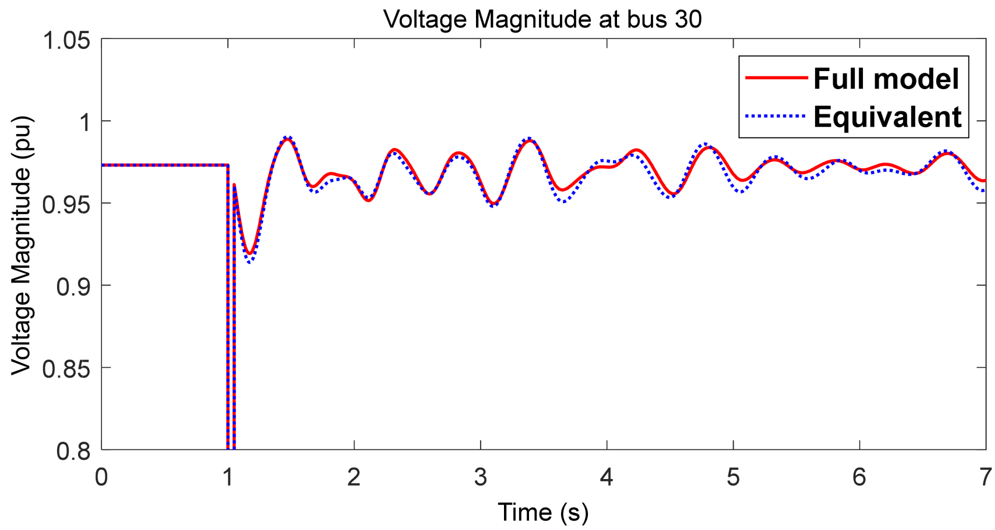

Figure 7. Bus 30 voltage magnitude comparison with bus-to-ground fault at bus 30.

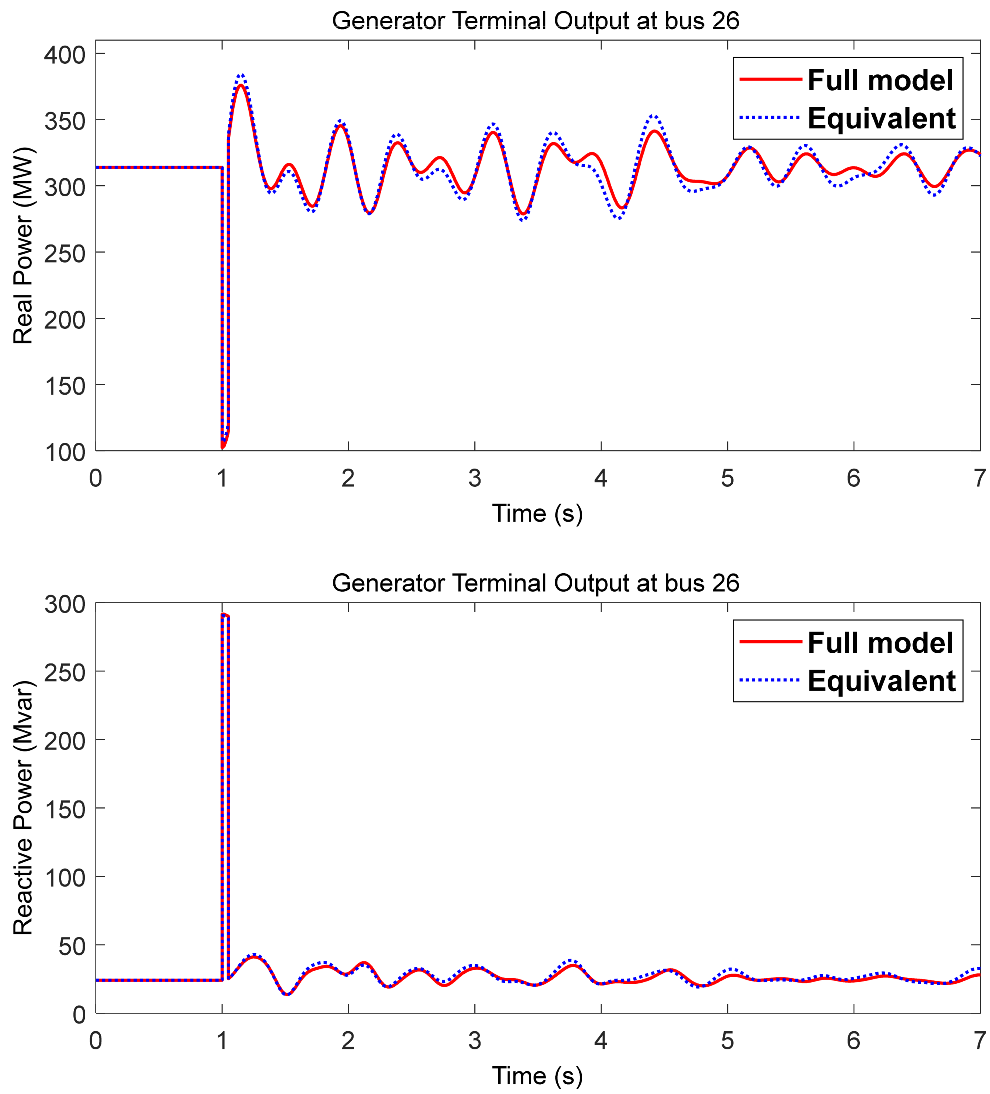

Figure 8. Real and reactive power comparisons of Gen #26 with bus-to-ground fault at bus 30.

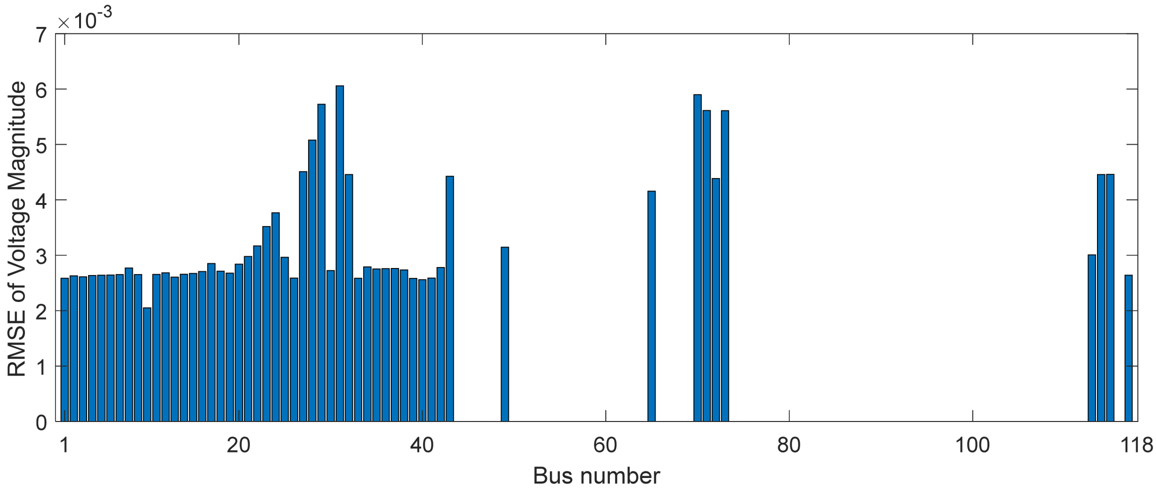

Figure 9. RMSE of bus voltage magnitude with bus-to-ground fault at bus 30.

3.4. Comparison of Computation Time

Table 4 shows the computational benefits of using the proposed equivalent approach with the IEEE 118-bus system and a 10,000 synthetic western U.S. power grid model. A simulation of the bus-to-ground fault in the internal system was performed. The computation time is the average execution time of multiple 10 s simulations. The equivalent model provided a faster solution than the full model. For the system configuration where the internal to external bus ratio was 1–1.3 with the IEEE 118-bus system, the equivalent model showed a 25% computation time reduction. When the external system is much bigger than the internal system, more computational benefits can be expected. For comparison, a practical larger power system case of a 10,000-bus synthetic grid in the western U.S. [17,19] was considered. With a higher ratio of internal to external buses of 1 to 8.2, the computation time was significantly reduced by about 94%.

Table 4. Computational time comparison.

Model Used		Computation Time (s)	Ratio of Computation Time
118 buses	Full model	0.436	1
	Equivalent (53 buses)	0.328	0.75
10,000 buses	Full model	32.002	1
	Equivalent (1082 buses)	1.978	0.06

4. Conclusions

This paper presented a new dynamic equivalent approach, enhancing the computational efficiency and maintaining high-level simulation accuracy for transient stability analysis. The approach reduces the power system network using the traditional Ward admittance method, maintaining the steady-state operating points. Fictitious generators are attached to retain the critical dynamics from the external system. The generators' dynamic parameters are obtained using partial Y-bus factorization on the vector of dynamic parameters, such as machine inertia, transient reactance, and damping coefficients. The equivalent dynamic parameters are determined from the Y-bus matrix representing a network configuration. Case studies using the IEEE 118-bus system and a 10,000-bus case from the western U.S. confirmed that the reduced equivalent model from the proposed method achieves faster accurate simulation outcomes. When the ratio of internal to external buses was ~1–8.2 in the 10,000 bus system, the reduced model achieved ~16.2 times faster simulation than the conventional full model method. More computational benefits for speed can be achieved with a larger power system and a higher dimension of the external system than the internal system. The proposed method is a promising alternative solution for power system dynamic equivalents. In future work, this method can be extended to include newer types of power-electronics-based generation and nonlinear loads.

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